Algorithm 1 MC Path Integral

procedure SAMPLEQUANTUMDIST
  $\hbar, m, k_b, \omega \leftarrow 1$
  $\theta \leftarrow$ temperature of heat bath
  $\beta \leftarrow \hbar/k_b\theta$
  $N_s \leftarrow$ # of steps
  $N_d \leftarrow$ # of lattice points
  $\epsilon \leftarrow T/N_d$

  // Initialize lattice
  $x \leftarrow \{x_0, ..., x_{N_d-1}\}$ // $x_k \sim \mathcal{N}(0,1)$
  $x' \leftarrow x$

  for $i = 0, ..., N_s$ **do**
    for $k = 0, ...N_d$ **do**
      $x'[k] \leftarrow x[k] + \mathcal{N}(0,1)$ // Proposition
      $E \leftarrow$ **energy**(x, ϵ)
      $E' \leftarrow$ **energy**(x', ϵ)
      $\pi \leftarrow e^{\beta(E'-E)}$
      if $(E' \leq E)$ **then**
        $x[k] \leftarrow x'[k]$
      else
        if (randUniform(0,1) $\leq \pi$) **then**
          $x[k] \leftarrow x'[k]$

$$E_{\text{path}} = \frac{m}{2\epsilon}\sum_{i=1}^{N}(x_i - x_{i-1})^2 + \frac{\epsilon}{8}m\omega\sum_{i=1}^{N}\left(x_{i-1} + x_i\right)^2$$

$$\langle E(T)\rangle = \frac{\sum\limits_{n=0}^{\infty} E_n e^{E_n/kT}}{\sum\limits_{n=0}^{\infty} e^{E_n/kT}} = \frac{\sum(n+\frac{1}{2})\hbar\omega\, e^{(n+\frac{1}{2})\hbar\omega/kT}}{\sum e^{(n+\frac{1}{2})\hbar\omega/kT}} = \frac{\hbar\omega}{2}\coth\left(\frac{\hbar\omega}{2kT}\right)$$

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega^2 x^2$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\langle E \rangle = \frac{\int \mathcal{D}[x(\tau)] \left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega^2 x^2\right) e^{-\frac{1}{\hbar}S[x]}}{\int \mathcal{D}[x(\tau)] e^{-\frac{1}{\hbar}S[x]}}$$

$$S[x] = \int_0^T \left[\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}m\omega^2 x^2\right] d\tau$$

$$E_o = \lim_{T\to\infty} \frac{\int \mathcal{D}[x(\tau)] \left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega^2 x^2\right) e^{-\frac{1}{\hbar}S[x]}}{\int \mathcal{D}[x(\tau)] e^{-\frac{1}{\hbar}S[x]}}$$

$$\psi_o(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$