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Theoretical Limits in Constraining Tidal Quality Factors of Binary Stars

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ABSTRACT

1. INTRODUCTION

We aim to place theoretical limits on the constraints we can get on $\mathcal{Q}$ and τ using tidal equillibrium models, and disentangle different sources of uncertainty:

- **Observational:** To what degree is the limitation on the uncertainty of $\mathcal{Q}$ and τ due to a limitation of observational (e.g. ability to observe a parameter, data precision, sample size), which might be improved by better data? That is, how does the accuracy and precision of our observations affect the derived uncertainty of $\mathcal{Q}$ and τ and which observational constraints matter most when it comes to inferring $\mathcal{Q}$ and $\tau_?$ What types of systems are most promising for constraining $\mathcal{Q}$ and $\tau_?$ Limitation of sample size or limitation of constraints? How do constraints from synchronization (rotation period + eccentricity + orbital period) compare to constraints from circularization (eccentricity + orbital period)? Smaller sample/more constraints vs. larger sample/less constraints

- **Model:** To what degree to inherent degeneracies/pathologies in the model formulation contribute to uncertainty? Is this a problem that is *well posed* for (Bayesian) inference–are the observables sensitive enough to your parameters of interest?

- **Hypothesis:** To what degree does our hypothesis fail to reproduce reality? (comparison between simulated data uncertainties and real data uncertainties; model comparison between CTL, CPL). Can theoretical limits to the uncertainties on $\mathcal{Q}$ and τ derived on *simulated data* give us context to intepret uncertainties on *real data*? Given enough free parameters, it can become possible for a model to reproduce any range of outcomes. How in this situation, can a model be invalidated?

2. METHODS

2.1. *Tidal Evolution Model*

2.2. *Stellar Evolution + Magnetic Braking Model*

2.3. *Global Sensitivity Analysis*

2.4. *Markov Chain Monte Carlo*

2.5. *Observational Constraints*

Model initial conditions and observational constraints:

	Model Input	Prior	Observational Constraint	Good Unc
M_1	primary mass [$M_\odot$]	$\mathcal{N}(m, s)$	kepler solution (lc eclipse + rvs)	0.001
M_2	secondary mass [$M_\odot$]	$\mathcal{N}(m, s)$	kepler solution (lc eclipse + rvs)	0.001
$P_{\mathrm{rot1},i}$	pri init rotation period [days]	$log\mathcal{N}(m, s)$	dist in young open clusters	
$P_{\mathrm{rot2},i}$	sec init rotation period [days]	$log\mathcal{N}(m, s)$	dist in young open clusters	
$P_{\mathrm{orb},i}$	init orbital period [days]	$\mathcal{U}(4.0, 10.0)$	uninformed	
e_i	init eccentricity	$\mathcal{U}(0, 0.5)$	uninformed	
age	system age [yr]	$\mathcal{N}(m, s)$	open cluster age	10%
$\varepsilon_{1,i}$	pri init obliquity [deg]	$\mathcal{U}(0, 30)$	uninformed	
$\varepsilon_{2,i}$	sec init obliquity [deg]	$\mathcal{U}(0, 30)$	uninformed	
$\mathcal{Q}_1$	pri tidal phase lag	$\mathcal{U}(4, 9)$	uninformed	
$\mathcal{Q}_2$	sec tidal phase lag	$\mathcal{U}(4, 9)$	uninformed	
τ_1	pri tidal time lag [log(s)]	$\mathcal{U}(-4, 2)$	uninformed	
τ_2	sec tidal time lag [log(s)]	$\mathcal{U}(-4, 2)$	uninformed	

Model final conditions and observational constraints:

	Model Output	Likelihood	Observational Constraint	Good Unc
$P_{\mathrm{rot1},f}$	pri final rotation period [days]	$\mathcal{N}(m, s)$	lc autocorrelation function	0.1
$P_{\mathrm{rot2},f}$	sec final rotation period [days]	$\mathcal{N}(m, s)$	spectroscopic $v \sin i$	0.1
$P_{\mathrm{orb},f}$	final orbital period [days]	$\mathcal{N}(m, s)$	lc lomb scargle	10^{-5}
e_f	final eccentricity	$\mathcal{N}(m, s)$	lc eclipse + rvs	0.001
$R_{1,f}$	pri final radius [$R_\odot$]	$\mathcal{N}(m, s)$	stellar models + photometry	0.01
$R_{2,f}$	sec final radius [$R_\odot$]	$\mathcal{N}(m, s)$	eclipse shape + pri radius	0.01
$L_{1,f}$	pri final lumniosity [$L_\odot$]	$\mathcal{N}(m, s)$	stellar models + photometry	0.1
$L_{2,f}$	sec final lumniosity [$L_\odot$]	$\mathcal{N}(m, s)$	stellar models + photometry	0.1
$T_{\mathrm{eff1},f}$	pri final temperature [K]	$\mathcal{N}(m, s)$	stellar models + spectra	
$T_{\mathrm{eff2},f}$	sec final temperature [K]	$\mathcal{N}(m, s)$	stellar models + spectra	

3. RESULTS

- sensitivity analysis (full 9 parameters + age) - CTL, CPL, CTL + STELLAR, CPL + STELLAR

  free parameters: M_1, M_2, $\varepsilon_{1,i}$, $\varepsilon_{2,i}$, $P_{\mathrm{rot1},i}$, $P_{\mathrm{rot2},i}$, e_i, $P_{\mathrm{orb},i}$, $\mathcal{Q}$

  computation time: ∼30 min eqtide only, ∼1 day for eqtide + stellar

- sensitivity analysis (6 parameters + age) - CTL, CPL, CTL + STELLAR, CPL + STELLAR

  free parameters: M_1, M_2, $P_{\mathrm{rot1},i}$, $P_{\mathrm{rot2},i}$, e_i, $\mathcal{Q}$ (without obliquity, prior for porb based on final porb, ecc)

  computation time: ∼30 min eqtide only, ∼1 day for eqtide + stellar

- MCMC recovery test on synthetic data (4 parameters + age)

  free parameters: $P_{\mathrm{rot1},i}$, $P_{\mathrm{rot2},i}$, e_i, $\mathcal{Q}$ (fix masses, without obliquity, prior for porb based on final porb, ecc)

  computation time (per MCMC run): ∼ few hours for eqtide only, ∼ few weeks for eqtide + stellar

- 1D likelihood tests

  free parameters: $\mathcal{Q}$

- MCMC recovery test on real data (4 parameters + age)?

    if there's an open cluster system in a tide-sensitive age range

    comparison with the synthetic data tests may be able to tell us how much posterior uncertainty is due to which input parameters/observational uncertainties/degeneracies?

4. DISCUSSION

5. CONCLUSION

Figure 1. EQTIDE + STELLAR: CPL model

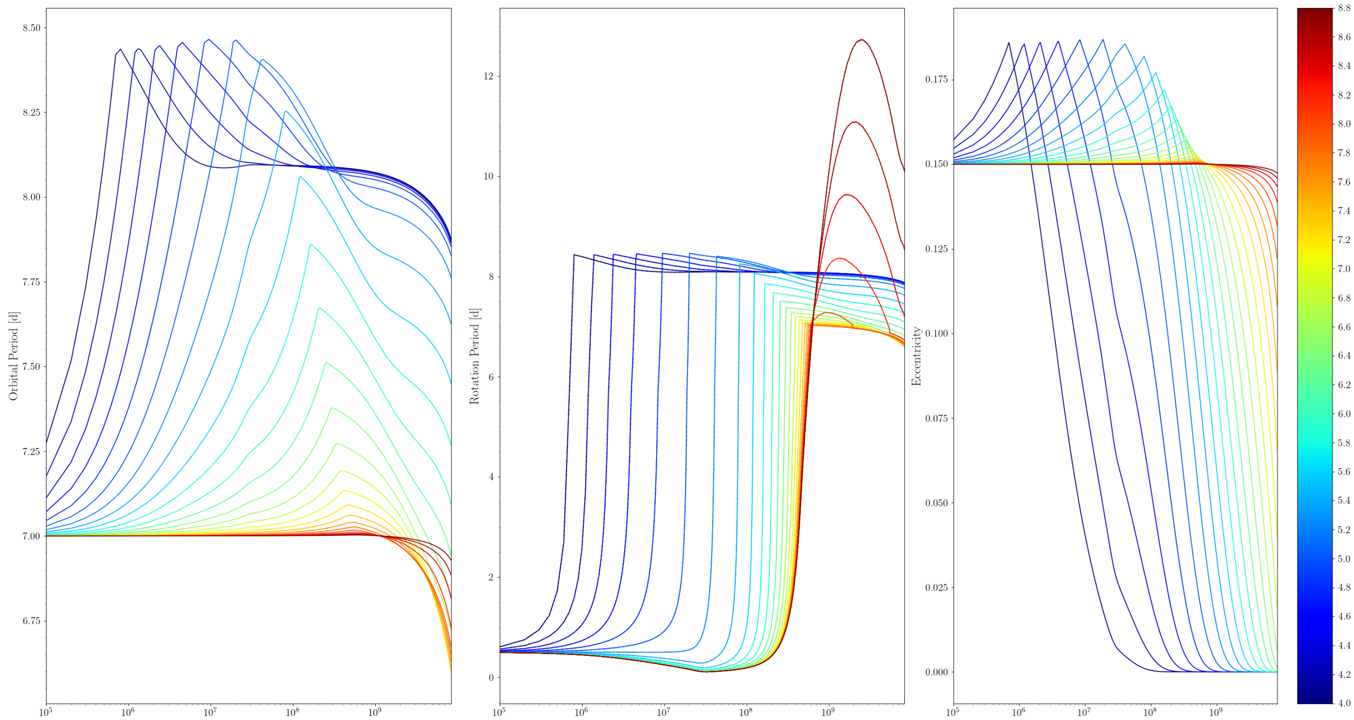
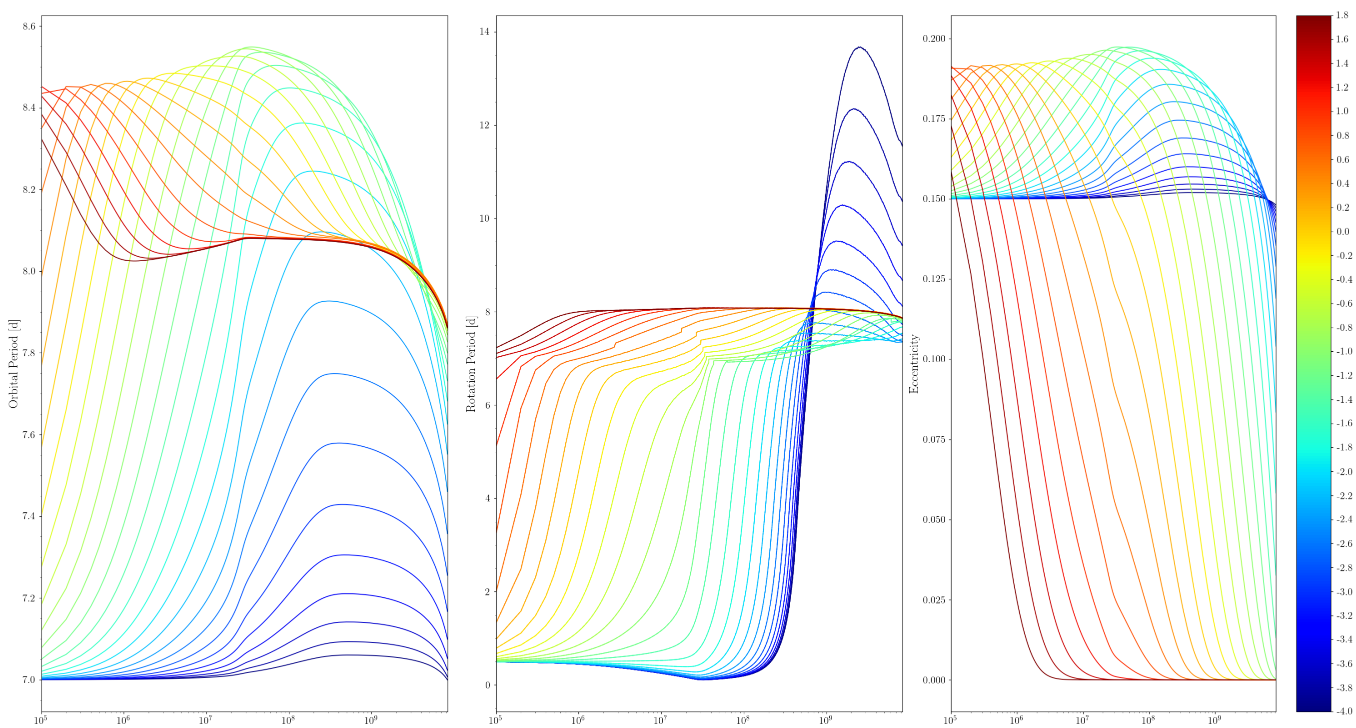
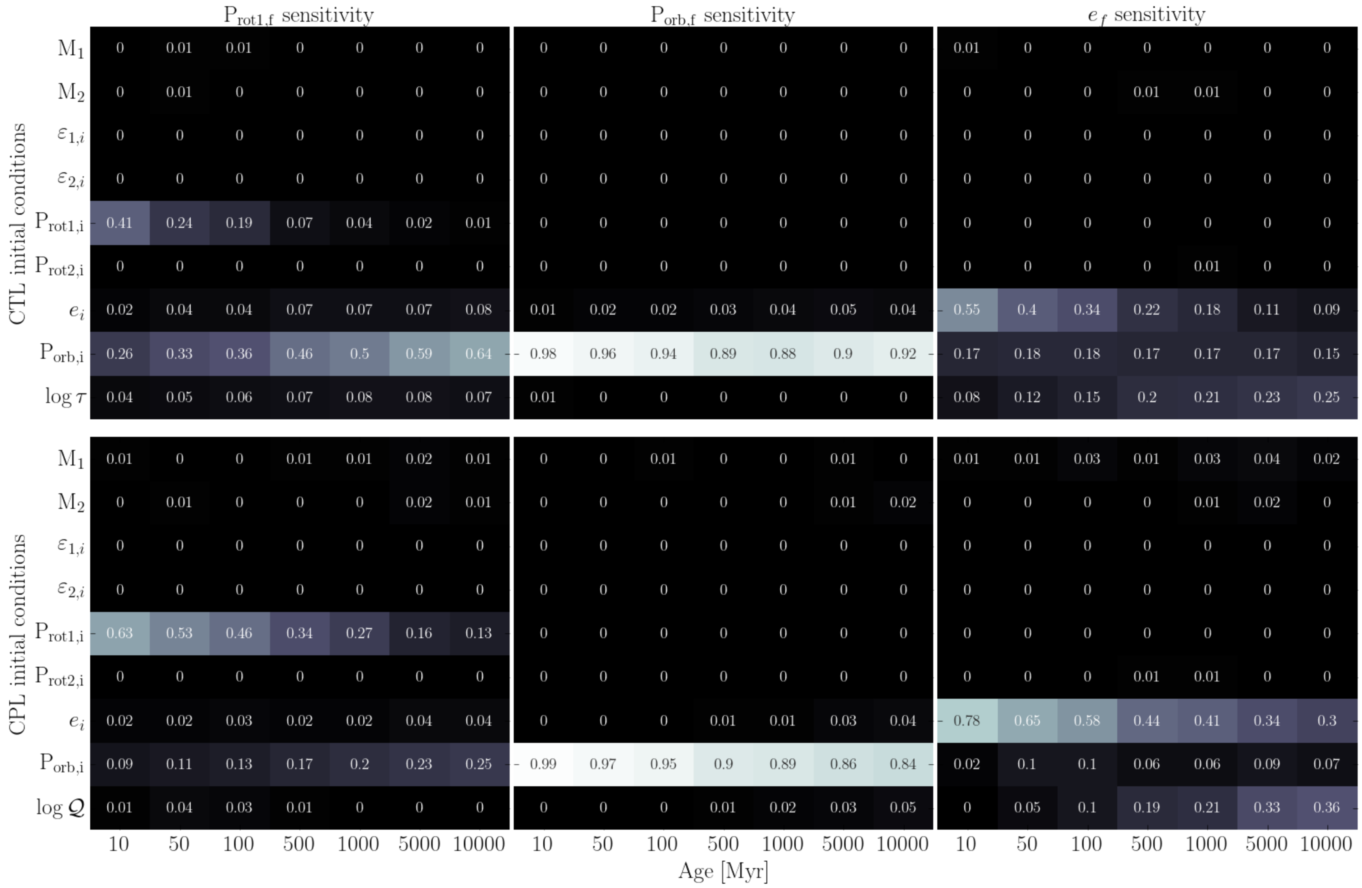


Figure 2. EQTIDE + STELLAR: CTL model

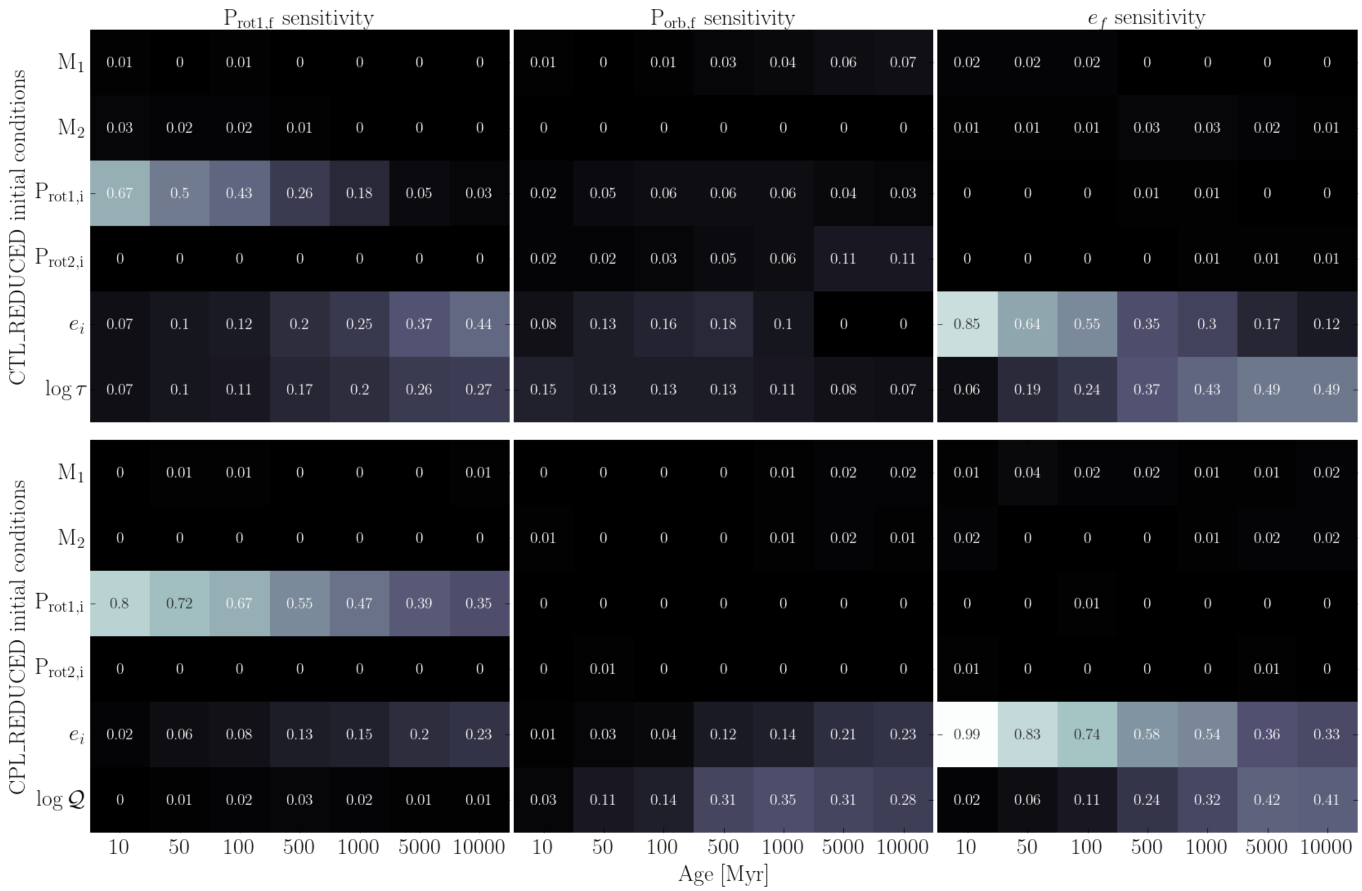


APPENDIX



$P_{rot1,f}$ sensitivity
$P_{orb,f}$ sensitivity
e_f sensitivity
CTL initial conditions
M_1 0 0.01 0.01 0 0 0 0 0 0 0 0 0 0 0 0.01 0 0 0 0 0 0
M_2 0 0.01 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.01 0.01 0 0
$\varepsilon_{1,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$\varepsilon_{2,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$P_{rot1,i}$ 0.41 0.24 0.19 0.07 0.04 0.02 0.01 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$P_{rot2,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.01 0 0
e_i 0.02 0.04 0.04 0.07 0.07 0.07 0.08 0.01 0.02 0.02 0.03 0.04 0.05 0.04 0.55 0.4 0.34 0.22 0.18 0.11 0.09
$P_{orb,i}$ 0.26 0.33 0.36 0.46 0.5 0.59 0.64 0.98 0.96 0.94 0.89 0.88 0.9 0.92 0.17 0.18 0.18 0.17 0.17 0.17 0.15
$\log\tau$ 0.04 0.05 0.06 0.07 0.08 0.08 0.07 0.01 0 0 0 0 0 0 0.08 0.12 0.15 0.2 0.21 0.23 0.25
CPL initial conditions
M_1 0.01 0 0 0.01 0.01 0.02 0.01 0 0 0.01 0 0 0.01 0 0.01 0.01 0.03 0.01 0.03 0.04 0.02
M_2 0 0.01 0 0 0 0.02 0.01 0 0 0 0 0 0.01 0.02 0 0 0 0 0.01 0.02 0
$\varepsilon_{1,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$\varepsilon_{2,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$P_{rot1,i}$ 0.63 0.53 0.46 0.34 0.27 0.16 0.13 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$P_{rot2,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.01 0.01 0 0
e_i 0.02 0.02 0.03 0.02 0.02 0.04 0.04 0 0 0 0.01 0.01 0.03 0.04 0.78 0.65 0.58 0.44 0.41 0.34 0.3
$P_{orb,i}$ 0.09 0.11 0.13 0.17 0.2 0.23 0.25 0.99 0.97 0.95 0.9 0.89 0.86 0.84 0.02 0.1 0.1 0.06 0.06 0.09 0.07
$\log\mathcal{Q}$ 0.01 0.04 0.03 0.01 0 0 0 0 0 0 0.01 0.02 0.03 0.05 0 0.05 0.1 0.19 0.21 0.33 0.36
10 50 100 500 1000 5000 10000 10 50 100 500 1000 5000 10000 10 50 100 500 1000 5000 10000
Age [Myr]

REFERENCES



$P_{rot1,f}$ sensitivity
$P_{orb,f}$ sensitivity
e_f sensitivity
CTL_REDUCED initial conditions
CPL_REDUCED initial conditions
M_1
M_2
$P_{rot1,i}$
$P_{rot2,i}$
e_i
$\log \tau$
$\log \mathcal{Q}$
0.01 0 0.01 0 0 0 0 0.01 0 0.01 0.03 0.04 0.06 0.07 0.02 0.02 0.02 0 0 0 0
0.03 0.02 0.02 0.01 0 0 0 0 0 0 0 0 0 0 0.01 0.01 0.01 0.03 0.03 0.02 0.01
0.67 0.5 0.43 0.26 0.18 0.05 0.03 0.02 0.05 0.06 0.06 0.06 0.04 0.03 0 0 0 0.01 0.01 0 0
0 0 0 0 0 0 0 0.02 0.02 0.03 0.05 0.06 0.11 0.11 0 0 0 0 0.01 0.01 0.01
0.07 0.1 0.12 0.2 0.25 0.37 0.44 0.08 0.13 0.16 0.18 0.1 0 0 0.85 0.64 0.55 0.35 0.3 0.17 0.12
0.07 0.1 0.11 0.17 0.2 0.26 0.27 0.15 0.13 0.13 0.13 0.11 0.08 0.07 0.06 0.19 0.24 0.37 0.43 0.49 0.49
0 0.01 0.01 0 0 0 0.01 0 0 0 0 0.01 0.02 0.02 0.01 0.04 0.02 0.02 0.01 0.01 0.02
0 0 0 0 0 0 0 0.01 0 0 0 0.01 0.02 0.01 0.02 0 0 0 0.01 0.02 0.02
0.8 0.72 0.67 0.55 0.47 0.39 0.35 0 0 0 0 0 0 0 0 0 0.01 0 0 0 0
0 0 0 0 0 0 0 0 0.01 0 0 0 0 0 0.01 0 0 0 0 0.01 0
0.02 0.06 0.08 0.13 0.15 0.2 0.23 0.01 0.03 0.04 0.12 0.14 0.21 0.23 0.99 0.83 0.74 0.58 0.54 0.36 0.33
0 0.01 0.02 0.03 0.02 0.01 0.01 0.03 0.11 0.14 0.31 0.35 0.31 0.28 0.02 0.06 0.11 0.24 0.32 0.42 0.41
10 50 100 500 1000 5000 10000 10 50 100 500 1000 5000 10000 10 50 100 500 1000 5000 10000
Age [Myr]