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# Theoretical Limits in Constraining Tidal Quality Factors of Binary Stars

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## ABSTRACT

## 1. INTRODUCTION

We aim to place theoretical limits on the constraints we can get on $\mathcal{Q}$ and $\tau$ using tidal equillibrium models, and disentangle different sources of uncertainty:

- **Observational:** To what degree is the limitation on the uncertainty of $\mathcal{Q}$ and $\tau$ due to a limitation of observational (e.g. ability to observe a parameter, data precision, sample size), which might be improved by better data? That is, how does the accuracy and precision of our observations affect the derived uncertainty of $\mathcal{Q}$ and $\tau$ and which observational constraints matter most when it comes to inferring $\mathcal{Q}$ and $\tau$? What types of systems are most promising for constraining $\mathcal{Q}$ and $\tau$? Limitation of sample size or limitation of constraints? How do constraints from synchronization (rotation period + eccentricity + orbital period) compare to constraints from circularization (eccentricity + orbital period)? Smaller sample/more constraints vs. larger sample/less constraints

- **Model:** To what degree to inherent degeneracies/pathologies in the model formulation contribute to uncertainty? Is this a problem that is *well posed* for (Bayesian) inference–are the observables sensitive enough to your parameters of interest?

- **Hypothesis:** To what degree does our hypothesis fail to reproduce reality? (comparison between simulated data uncertainties and real data uncertainties; model comparison between CTL, CPL). Can theoretical limits to the uncertainties on $\mathcal{Q}$ and $\tau$ derived on *simulated data* give us context to intepret uncertainties on *real data*? Given enough free parameters, it can become possible for a model to reproduce any range of outcomes. How in this situation, can a model be invalidated?

## 2. METHODS

### 2.1. *Tidal Evolution Model*

### 2.2. *Stellar Evolution + Magnetic Braking Model*

### 2.3. *Global Sensitivity Analysis*

### 2.4. *Markov Chain Monte Carlo*

### 2.5. *Observational Constraints*

**Model initial conditions and observational constraints:**

| | Model Input | Prior | Observational Constraint | Good Unc |
|---|---|---|---|---|
| $M_1$ | primary mass [$M_\odot$] | $\mathcal{N}(m, s)$ | kepler solution (lc eclipse + rvs) | 0.001 |
| $M_2$ | secondary mass [$M_\odot$] | $\mathcal{N}(m, s)$ | kepler solution (lc eclipse + rvs) | 0.001 |
| $P_{\mathrm{rot1},i}$ | pri init rotation period [days] | $log\mathcal{N}(m, s)$ | dist in young open clusters | |
| $P_{\mathrm{rot2},i}$ | sec init rotation period [days] | $log\mathcal{N}(m, s)$ | dist in young open clusters | |
| $P_{\mathrm{orb},i}$ | init orbital period [days] | $\mathcal{U}(4.0, 10.0)$ | uninformed | |
| $e_i$ | init eccentricity | $\mathcal{U}(0, 0.5)$ | uninformed | |
| $age$ | system age [yr] | $\mathcal{N}(m, s)$ | open cluster age | 10% |
| $\varepsilon_{1,i}$ | pri init obliquity [deg] | $\mathcal{U}(0, 30)$ | uninformed | |
| $\varepsilon_{2,i}$ | sec init obliquity [deg] | $\mathcal{U}(0, 30)$ | uninformed | |
| $\mathcal{Q}_1$ | pri tidal phase lag | $\mathcal{U}(4, 9)$ | uninformed | |
| $\mathcal{Q}_2$ | sec tidal phase lag | $\mathcal{U}(4, 9)$ | uninformed | |
| $\tau_1$ | pri tidal time lag [log(s)] | $\mathcal{U}(-4, 2)$ | uninformed | |
| $\tau_2$ | sec tidal time lag [log(s)] | $\mathcal{U}(-4, 2)$ | uninformed | |

**Model final conditions and observational constraints:**

| | Model Output | Likelihood | Observational Constraint | Good Unc |
|---|---|---|---|---|
| $P_{\mathrm{rot1},f}$ | pri final rotation period [days] | $\mathcal{N}(m, s)$ | lc autocorrelation function | 0.1 |
| $P_{\mathrm{rot2},f}$ | sec final rotation period [days] | $\mathcal{N}(m, s)$ | spectroscopic $v \sin i$ | 0.1 |
| $P_{\mathrm{orb},f}$ | final orbital period [days] | $\mathcal{N}(m, s)$ | lc lomb scargle | $10^{-5}$ |
| $e_f$ | final eccentricity | $\mathcal{N}(m, s)$ | lc eclipse + rvs | 0.001 |
| $R_{1,f}$ | pri final radius [$R_\odot$] | $\mathcal{N}(m, s)$ | stellar models + photometry | 0.01 |
| $R_{2,f}$ | sec final radius [$R_\odot$] | $\mathcal{N}(m, s)$ | eclipse shape + pri radius | 0.01 |
| $L_{1,f}$ | pri final lumniosity [$L_\odot$] | $\mathcal{N}(m, s)$ | stellar models + photometry | 0.1 |
| $L_{2,f}$ | sec final lumniosity [$L_\odot$] | $\mathcal{N}(m, s)$ | stellar models + photometry | 0.1 |
| $T_{\mathrm{eff1},f}$ | pri final temperature [K] | $\mathcal{N}(m, s)$ | stellar models + spectra | |
| $T_{\mathrm{eff2},f}$ | sec final temperature [K] | $\mathcal{N}(m, s)$ | stellar models + spectra | |

## 3. RESULTS

- sensitivity analysis (full 9 parameters + age) - CTL, CPL, CTL + STELLAR, CPL + STELLAR

  free parameters: $M_1$, $M_2$, $\varepsilon_{1,i}$, $\varepsilon_{2,i}$, $P_{\mathrm{rot1},i}$, $P_{\mathrm{rot2},i}$, $e_i$, $P_{\mathrm{orb},i}$, $\mathcal{Q}$

  computation time: ∼30 min eqtide only, ∼1 day for eqtide + stellar

- sensitivity analysis (6 parameters + age) - CTL, CPL, CTL + STELLAR, CPL + STELLAR

  free parameters: $M_1$, $M_2$, $P_{\mathrm{rot1},i}$, $P_{\mathrm{rot2},i}$, $e_i$, $\mathcal{Q}$ (without obliquity, prior for porb based on final porb, ecc)

  computation time: ∼30 min eqtide only, ∼1 day for eqtide + stellar

- MCMC recovery test on synthetic data (4 parameters + age)

  free parameters: $P_{\mathrm{rot1},i}$, $P_{\mathrm{rot2},i}$, $e_i$, $\mathcal{Q}$ (fix masses, without obliquity, prior for porb based on final porb, ecc)

  computation time (per MCMC run): ∼ few hours for eqtide only, ∼ few weeks for eqtide + stellar

- 1D likelihood tests

  free parameters: $\mathcal{Q}$

- MCMC recovery test on real data (4 parameters + age)?

    if there's an open cluster system in a tide-sensitive age range

    comparison with the synthetic data tests may be able to tell us how much posterior uncertainty is due to which input parameters/observational uncertainties/degeneracies?

## 4. DISCUSSION

## 5. CONCLUSION

**Figure 1.** EQTIDE + STELLAR: CPL model

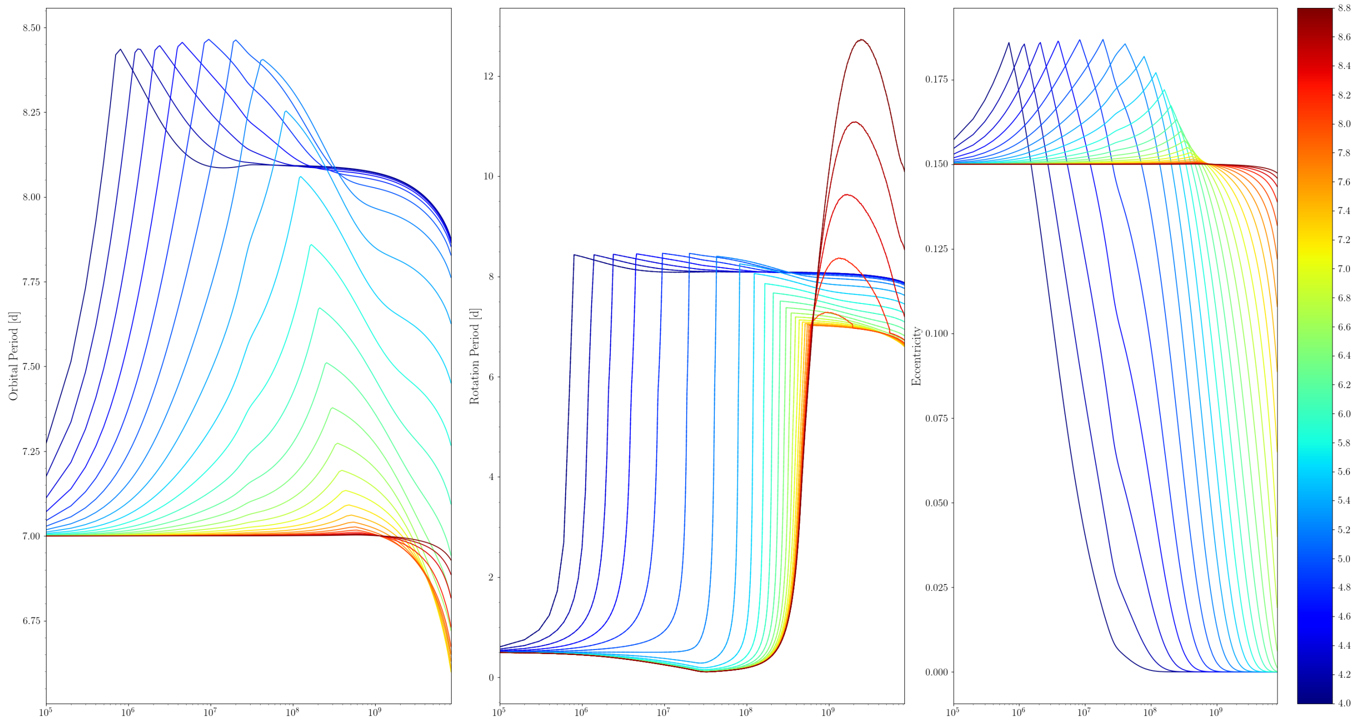
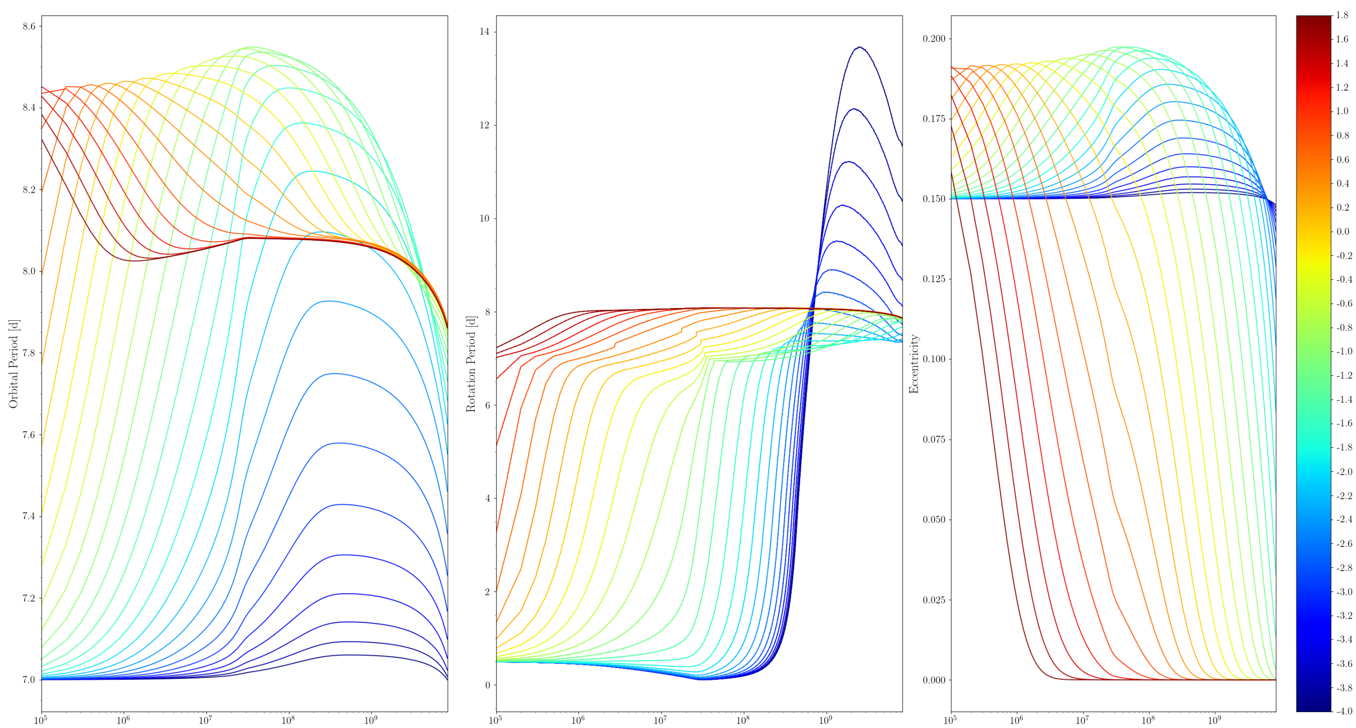
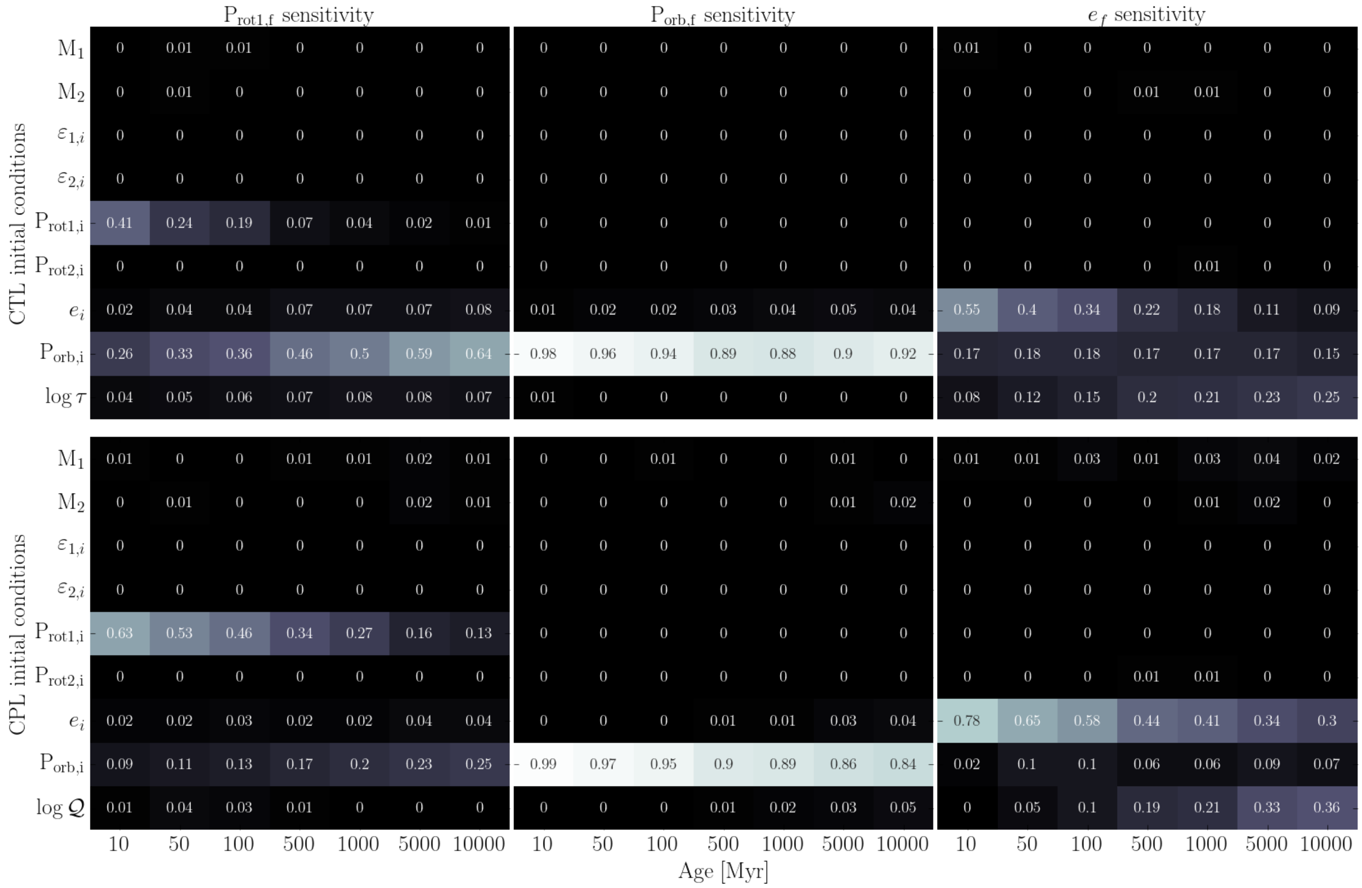


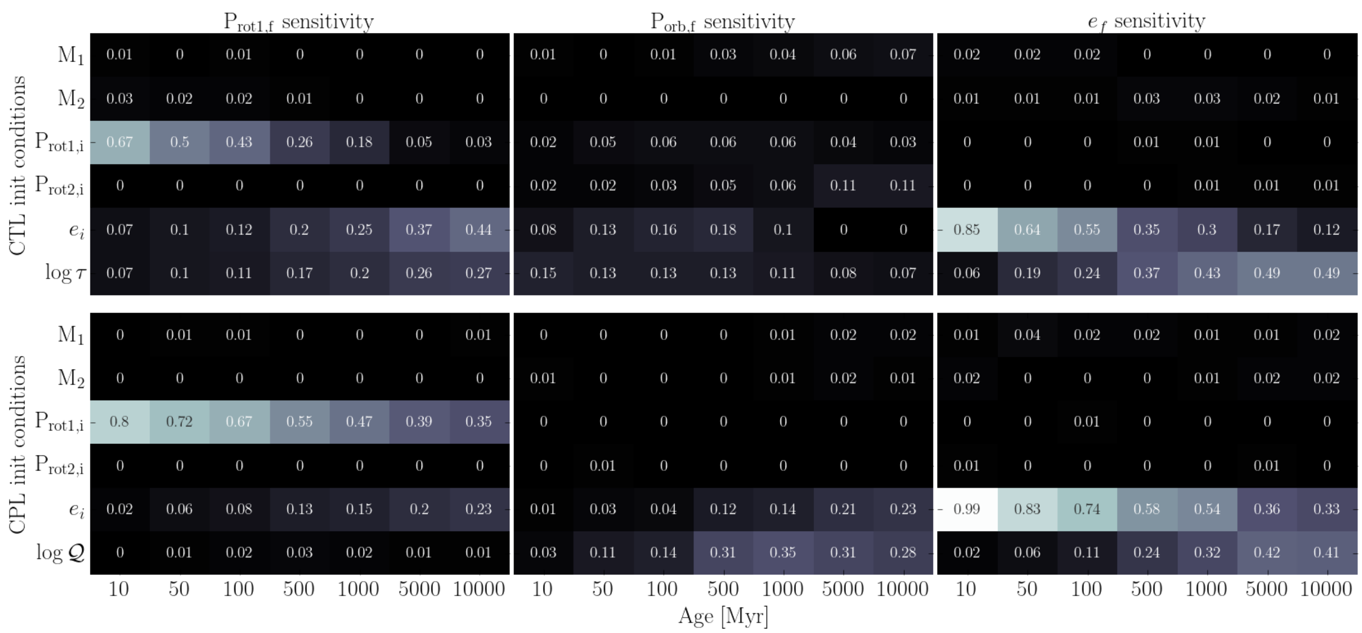

**Figure 2.** EQTIDE + STELLAR: CTL model




# APPENDIX



$P_{rot1,f}$ sensitivity
$P_{orb,f}$ sensitivity
$e_f$ sensitivity
CTL initial conditions
$M_1$ 0 0.01 0.01 0 0 0 0 0 0 0 0 0 0 0 0.01 0 0 0 0 0 0
$M_2$ 0 0.01 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.01 0.01 0 0
$\varepsilon_{1,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$\varepsilon_{2,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$P_{rot1,i}$ 0.41 0.24 0.19 0.07 0.04 0.02 0.01 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$P_{rot2,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.01 0 0
$e_i$ 0.02 0.04 0.04 0.07 0.07 0.07 0.08 0.01 0.02 0.02 0.03 0.04 0.05 0.04 0.55 0.4 0.34 0.22 0.18 0.11 0.09
$P_{orb,i}$ 0.26 0.33 0.36 0.46 0.5 0.59 0.64 0.98 0.96 0.94 0.89 0.88 0.9 0.92 0.17 0.18 0.18 0.17 0.17 0.17 0.15
$\log\tau$ 0.04 0.05 0.06 0.07 0.08 0.08 0.07 0.01 0 0 0 0 0 0 0.08 0.12 0.15 0.2 0.21 0.23 0.25
CPL initial conditions
$M_1$ 0.01 0 0 0.01 0.01 0.02 0.01 0 0 0.01 0 0 0.01 0 0.01 0.01 0.03 0.01 0.03 0.04 0.02
$M_2$ 0 0.01 0 0 0 0.02 0.01 0 0 0 0 0 0.01 0.02 0 0 0 0 0.01 0.02 0
$\varepsilon_{1,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$\varepsilon_{2,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$P_{rot1,i}$ 0.63 0.53 0.46 0.34 0.27 0.16 0.13 0 0 0 0 0 0 0 0 0 0 0 0 0 0
$P_{rot2,i}$ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.01 0.01 0 0
$e_i$ 0.02 0.02 0.03 0.02 0.02 0.04 0.04 0 0 0 0.01 0.01 0.03 0.04 0.78 0.65 0.58 0.44 0.41 0.34 0.3
$P_{orb,i}$ 0.09 0.11 0.13 0.17 0.2 0.23 0.25 0.99 0.97 0.95 0.9 0.89 0.86 0.84 0.02 0.1 0.1 0.06 0.06 0.09 0.07
$\log\mathcal{Q}$ 0.01 0.04 0.03 0.01 0 0 0 0 0 0 0.01 0.02 0.03 0.05 0 0.05 0.1 0.19 0.21 0.33 0.36
10 50 100 500 1000 5000 10000 10 50 100 500 1000 5000 10000 10 50 100 500 1000 5000 10000
Age [Myr]

$P_{rot1,f}$ sensitivity
$P_{orb,f}$ sensitivity
$e_f$ sensitivity
CTL init conditions
$M_1$ 0.01 0 0.01 0 0 0 0 0.01 0 0.01 0.03 0.04 0.06 0.07 0.02 0.02 0.02 0 0 0 0
$M_2$ 0.03 0.02 0.02 0.01 0 0 0 0 0 0 0 0 0 0 0.01 0.01 0.01 0.03 0.03 0.02 0.01
$P_{rot1,i}$ 0.67 0.5 0.43 0.26 0.18 0.05 0.03 0.02 0.05 0.06 0.06 0.06 0.04 0.03 0 0 0 0.01 0.01 0 0
$P_{rot2,i}$ 0 0 0 0 0 0 0 0.02 0.02 0.03 0.05 0.06 0.11 0.11 0 0 0 0 0.01 0.01 0.01
$e_i$ 0.07 0.1 0.12 0.2 0.25 0.37 0.44 0.08 0.13 0.16 0.18 0.1 0 0 0.85 0.64 0.55 0.35 0.3 0.17 0.12
$\log\tau$ 0.07 0.1 0.11 0.17 0.2 0.26 0.27 0.15 0.13 0.13 0.13 0.11 0.08 0.07 0.06 0.19 0.24 0.37 0.43 0.49 0.49
CPL init conditions
$M_1$ 0 0.01 0.01 0 0 0 0.01 0 0 0 0 0.01 0.02 0.02 0.01 0.04 0.02 0.02 0.01 0.01 0.02
$M_2$ 0 0 0 0 0 0 0 0.01 0 0 0 0.01 0.02 0.01 0.02 0 0 0 0.01 0.02 0.02
$P_{rot1,i}$ 0.8 0.72 0.67 0.55 0.47 0.39 0.35 0 0 0 0 0 0 0 0 0 0.01 0 0 0 0
$P_{rot2,i}$ 0 0 0 0 0 0 0 0 0.01 0 0 0 0 0 0.01 0 0 0 0 0.01 0
$e_i$ 0.02 0.06 0.08 0.13 0.15 0.2 0.23 0.01 0.03 0.04 0.12 0.14 0.21 0.23 0.99 0.83 0.74 0.58 0.54 0.36 0.33
$\log\mathcal{Q}$ 0 0.01 0.02 0.03 0.02 0.01 0.01 0.03 0.11 0.14 0.31 0.35 0.31 0.28 0.02 0.06 0.11 0.24 0.32 0.42 0.41
10 50 100 500 1000 5000 10000 10 50 100 500 1000 5000 10000 10 50 100 500 1000 5000 10000
Age [Myr]


# REFERENCES