Regression

Part 2

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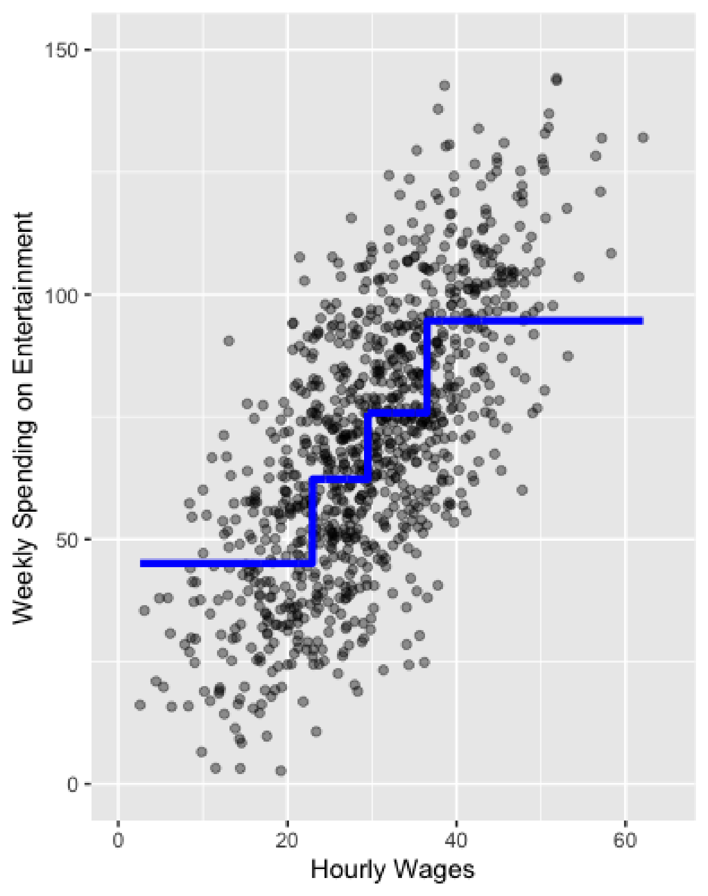
Agenda

1. Regression Recap
2. Logs and Skew
3. Evaluating a Regression
4. Introducing Cross Validation

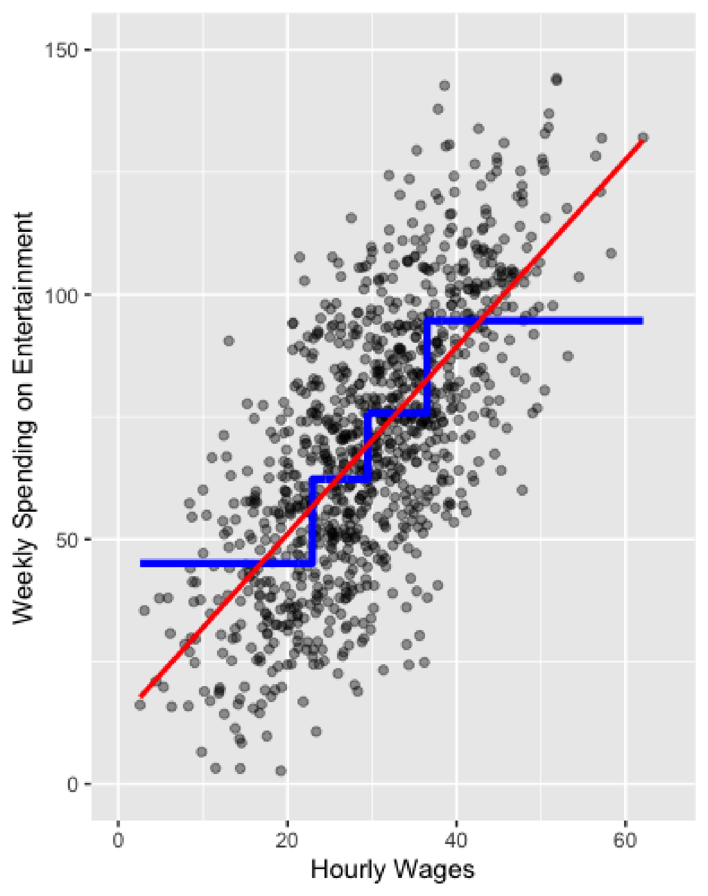
Regression Recap

- Regression very similar to **conditional means**



Regression Recap

- Regression very similar to **conditional means**



Regression Recap

- For this class, don't need to know how it happens
  - But the intuition is obvious
  - Given $Y = \alpha + \beta X$, just tweak α and β to reduce **errors**
  - Once you've minimized all the errors, you have the **line of best fit**

Evaluating Regression Results

- Understanding the **errors** helps us evaluate the model
- Define the errors $\varepsilon = Y - \hat{Y}$
  - True outcome values Y
  - Predicted outcome values $\hat{Y}$
- Useful to assess model performance

1. **Look** with univariate and multivariate visualization of the errors
2. Calculate the **RMSE**

Introducing the Data

- New dataset on **movies**
  - Download `mv.Rds` to your `data` folder and load to object `mv`
  - `require` `tidyverse`, and `plotly` packages

```
require(tidyverse)

mv <-
read_rds('https://github.com/jbisbee1/DS1000_S2024/raw/main/data/mv.Rds
```

RQ: Hollywood Finances

- Theory: the more a movie costs, the more it should make
  - If not, Hollywood would go out of business!
- Hypothesis: earnings (`gross`) and costs (`budget`) should be **positively** correlated
  - X: ?
  - Y: ?

Follow the process: Look

- TONS of missingness!

```
summary(mv %>% select(gross,budget))
```

```
##      gross                budget
##  Min.   :7.140e+02   Min.   :     5172
##  1st Qu.:1.121e+07   1st Qu.: 16865322
##  Median :5.178e+07   Median : 37212044
##  Mean   :1.402e+08   Mean   : 57420173
##  3rd Qu.:1.562e+08   3rd Qu.: 77844746
##  Max.   :3.553e+09   Max.   :387367903
##  NA's   :3668        NA's   :4482
```

Missingness

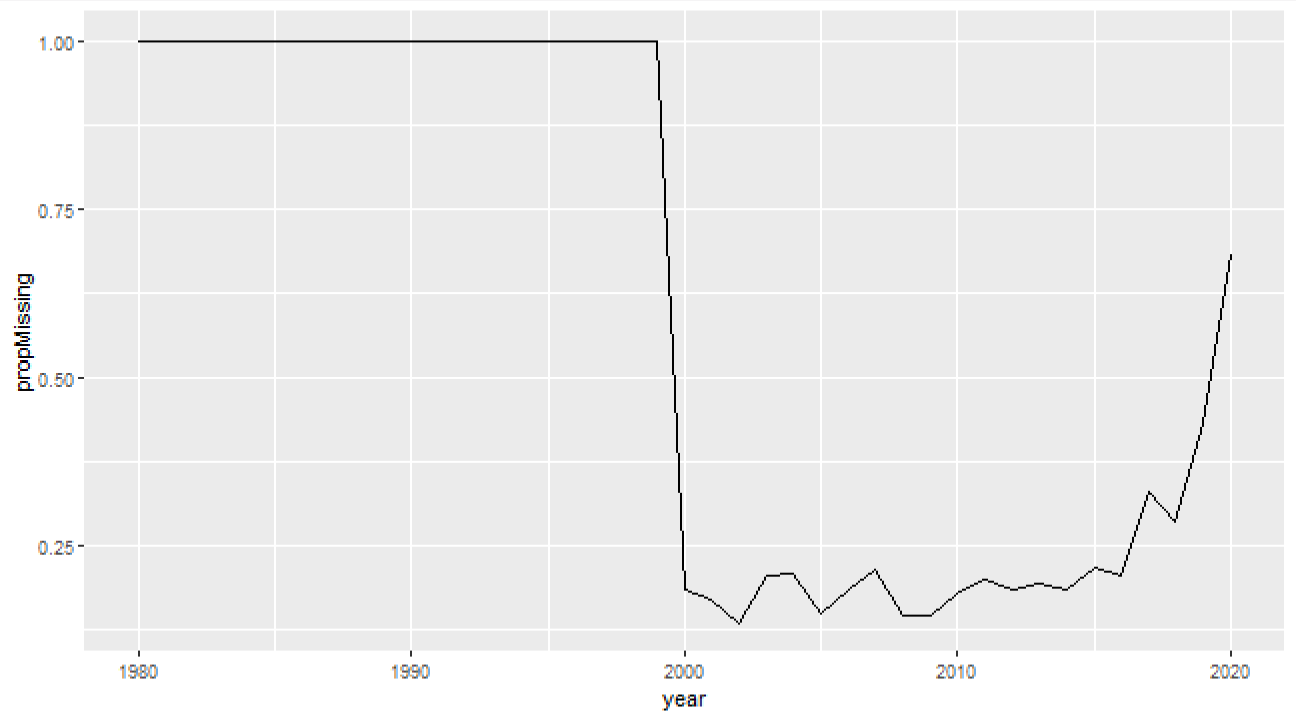
- What does this mean for "generalizability"
    - "Generalizability": Do our results with these data speak to other data?

```
p <- mv %>%
  mutate(missing = ifelse(is.na(gross) | is.na(budget),1,0)) %>%
  group_by(year) %>%
  summarise(propMissing = mean(missing)) %>%
  ggplot(aes(x = year,y = propMissing)) +
  geom_line()
```

Missingness

- We can only speak to post-2000s Hollywood!

```
p
```



Follow the process: Look

- What **type** of variables are earnings (`gross`) and costs (`budget`)?

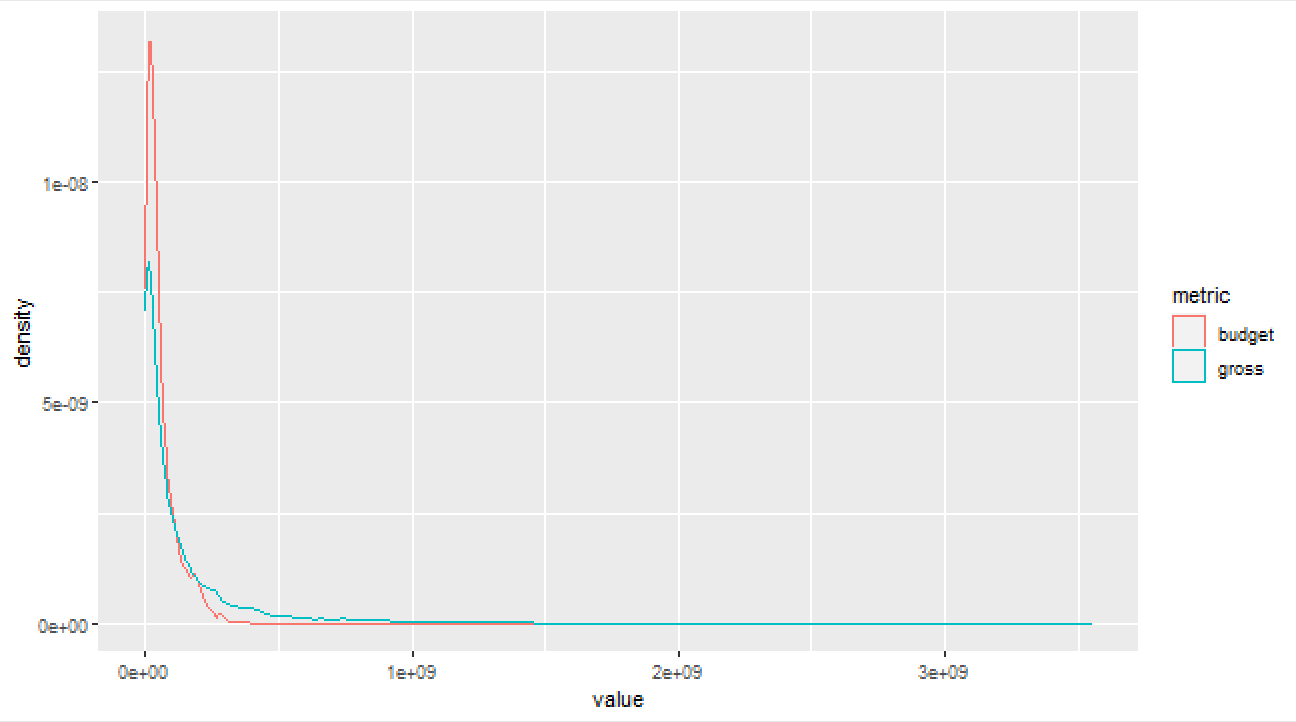
```
mv %>%
  drop_na(gross,budget) %>%
  select(gross,budget) %>% glimpse()
```

```
## Rows: 3,179
## Columns: 2
## $ gross  <dbl> 73677478, 53278578, 723586629, 11490339, 62…
## $ budget <dbl> 93289619, 10883789, 160147179, 6996721, 139…
```

- Looks like continuous measures to me!

2. Univariate Visualization

```
mv %>%
  select(title,gross,budget) %>%
  gather(metric,value,-title) %>%
  ggplot(aes(x = value,color = metric)) +
  geom_density()
```



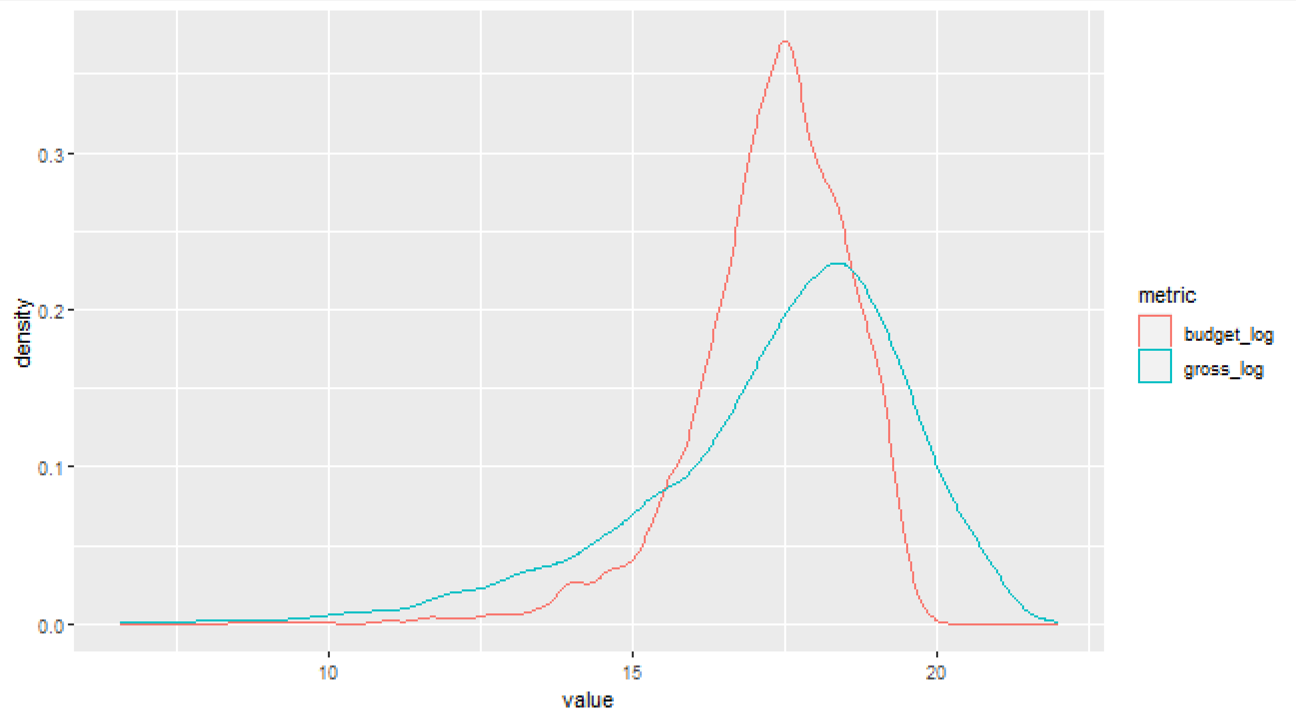
Log and Skew

- Univariate visualization higlights significant **skew** in both measures
  - Most movies don't cost a lot and don't make a lot, but there are a few blockbusters that pull the density way out
- Let's **wrangle** two new variables that take the log of these skewed measures
  - Logging transforms skewed measures to more "normal" measures
  - This is helpful for regression!

```
mv <- mv %>%
  mutate(gross_log = log(gross),
         budget_log = log(budget))
```

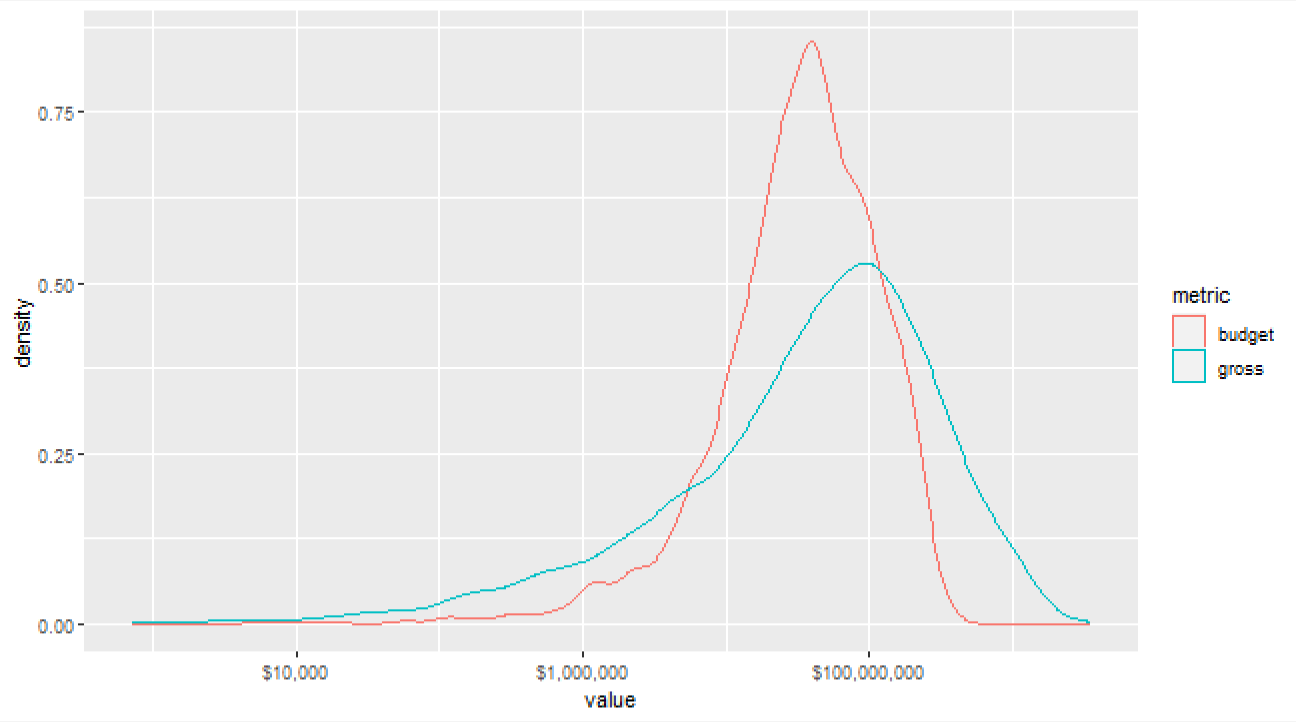
2. Univariate Visualization

```
mv %>%
  select(title,gross_log,budget_log) %>%
  gather(metric,value,-title) %>%
  ggplot(aes(x = value,color = metric)) +
  geom_density()
```



NB: Could also use `ggplot`

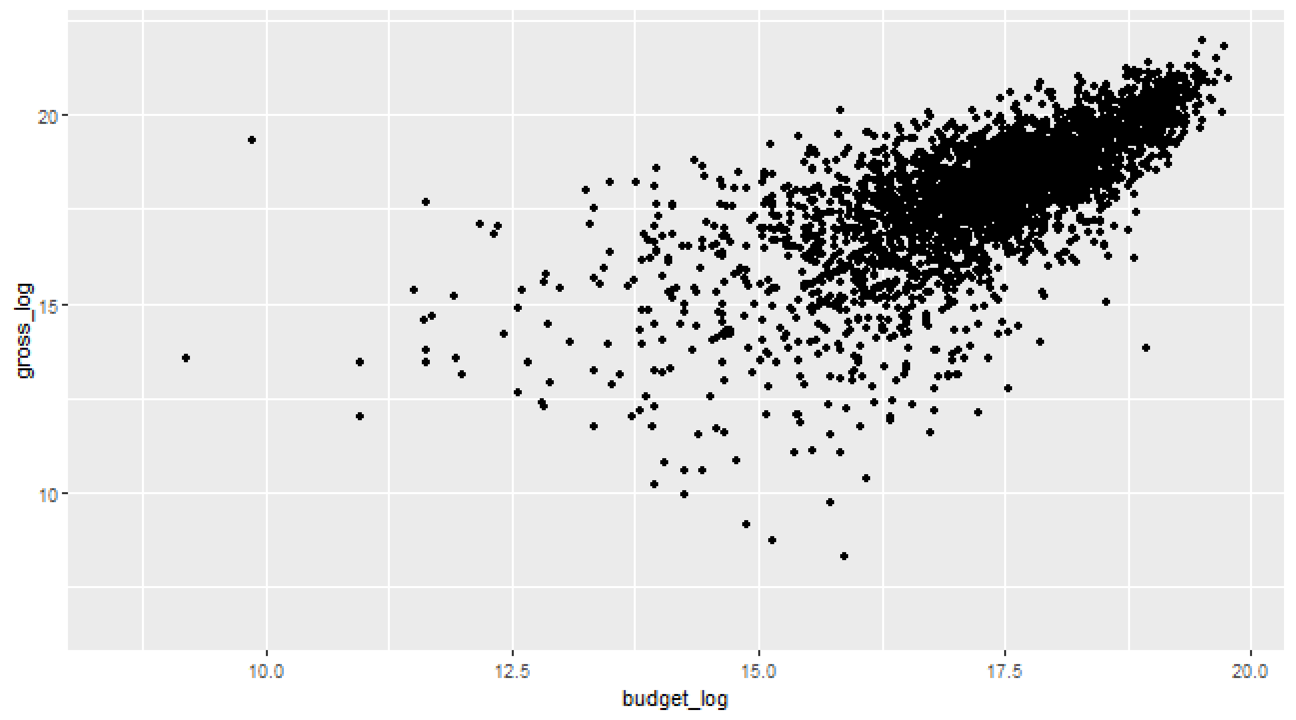
```
mv %>%
  select(title,gross,budget) %>%
  gather(metric,value,-title) %>%
  ggplot(aes(x = value,color = metric)) + geom_density() +
  scale_x_log10(labels = scales::dollar)
```



3. Conditional Analysis

- Continuous X continuous variables? Scatter with `geom_point()`!

```
mv %>%
  ggplot(aes(x = budget_log,y = gross_log)) +
  geom_point()
```



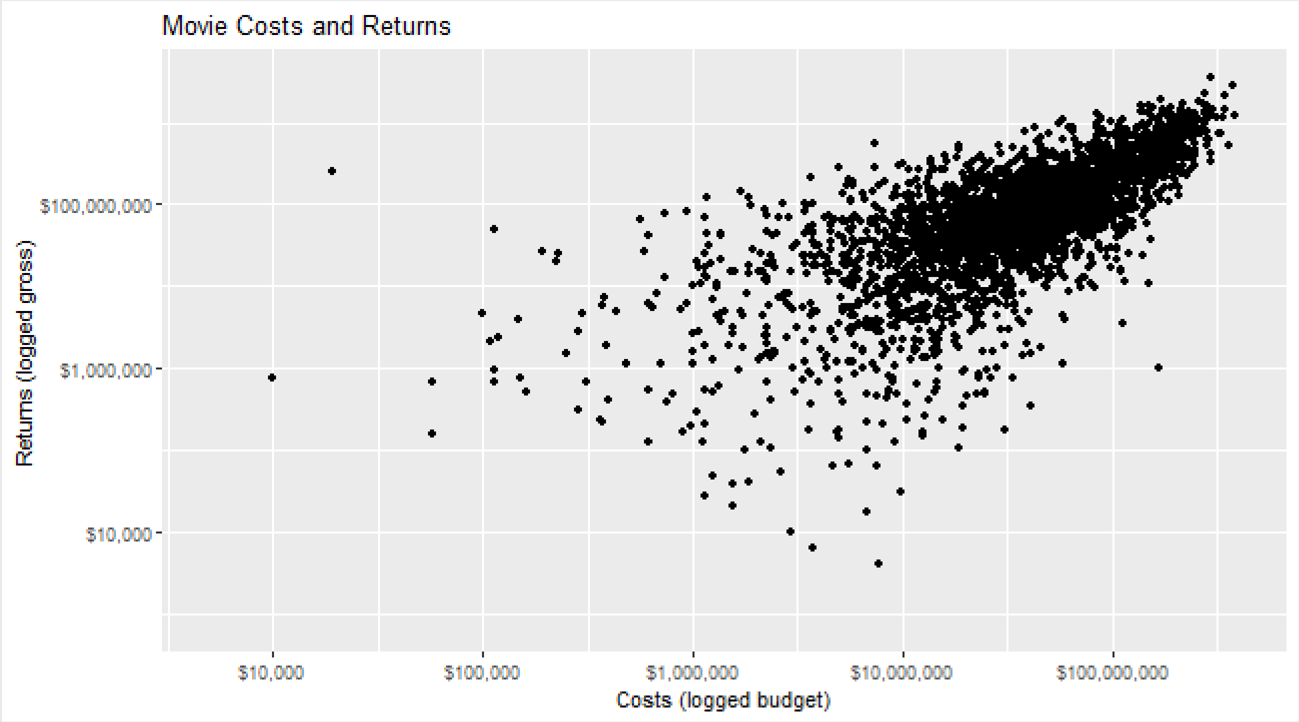
3. Conditional Analysis

- Why did I choose to put budget on the x-axis and gross on the y-axis?
  - Reveals assumption about **causality**
- (BTW, I know I've been violating the tenets of data viz for several slides now. Let's fix that.)

```
pClean <- mv %>%
  ggplot(aes(x = budget,y = gross)) +
  geom_point() +
  scale_x_log10(labels = scales::dollar) +
  scale_y_log10(labels = scales::dollar) +
  labs(title = "Movie Costs and Returns",
       x = "Costs (logged budget)",
       y = "Returns (logged gross)")
```

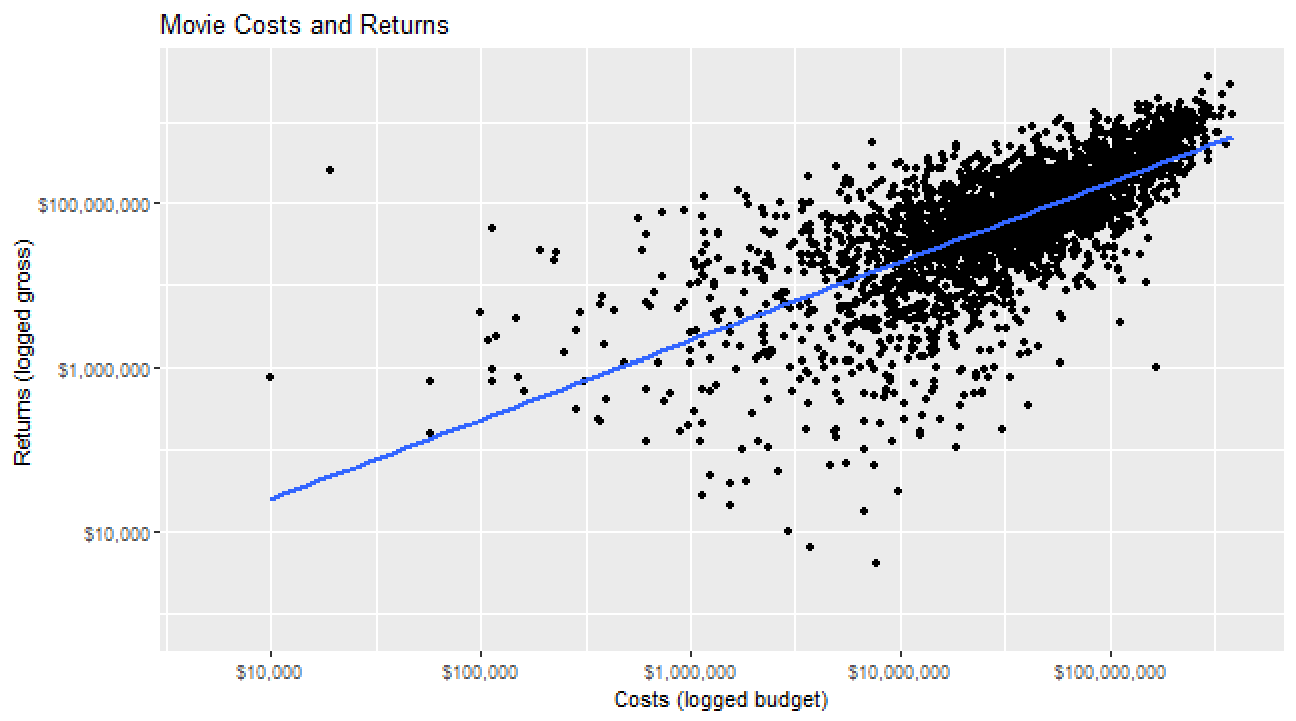
3. Conditional Analysis

```
pClean
```



4. Regression!

```
pClean +
  geom_smooth(method = 'lm',se = F)
```



4. Regression!

```
m <- lm(gross_log ~ budget_log,data = mv)
summary(m)
```

```
##
## Call:
## lm(formula = gross_log ~ budget_log, data = mv)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.2672 -0.6354  0.1648  0.7899  8.5599
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.26107    0.30953   4.074 4.73e-05 ***
## budget_log   0.96386    0.01786  53.971  < 2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.281 on 3177 degrees of freedom
##   (4494 observations deleted due to missingness)
## Multiple R-squared:  0.4783,    Adjusted R-squared:  0.4782
```

Interpreting with Logs

- For the α coefficient, when the budget is \$1, the movie makes $3.53

```
exp(1.26107)
```

```
## [1] 3.529196
```

- For the β coefficient, it depends on where the logged variable appears:

  1. `log(Y) ~ X`: 1 unit change in X → `(exp(b)-1)*100`% change in Y
  2. `Y ~ log(X)`: 1% increase in X → `b/100` unit change in Y
  3. `log(Y) ~ log(X)`: 1% increase in X → `b`% change in Y

- In our example, a 1% increase in the budget corresponds to a 0.96% increase in gross

Evaluation

- Every regression line makes mistakes
  - If they didn't, they wouldn't be good at **reducing complexity**!
- How bad do ours look?
  - How should we begin to answer this question!?
- Are there patterns to the mistakes?
  - We **overestimate** gross for movies that cost between $1m and $10m
  - These are the "indies"
  - We also **underestimate** gross to the "blockbusters"
- Why?

Understanding Regression Lines

- Regression lines choose α and β to minimize mistakes
    - Mistakes (aka "errors" or "residuals") are captured in the ε term
    - We can apply the **process** to these!

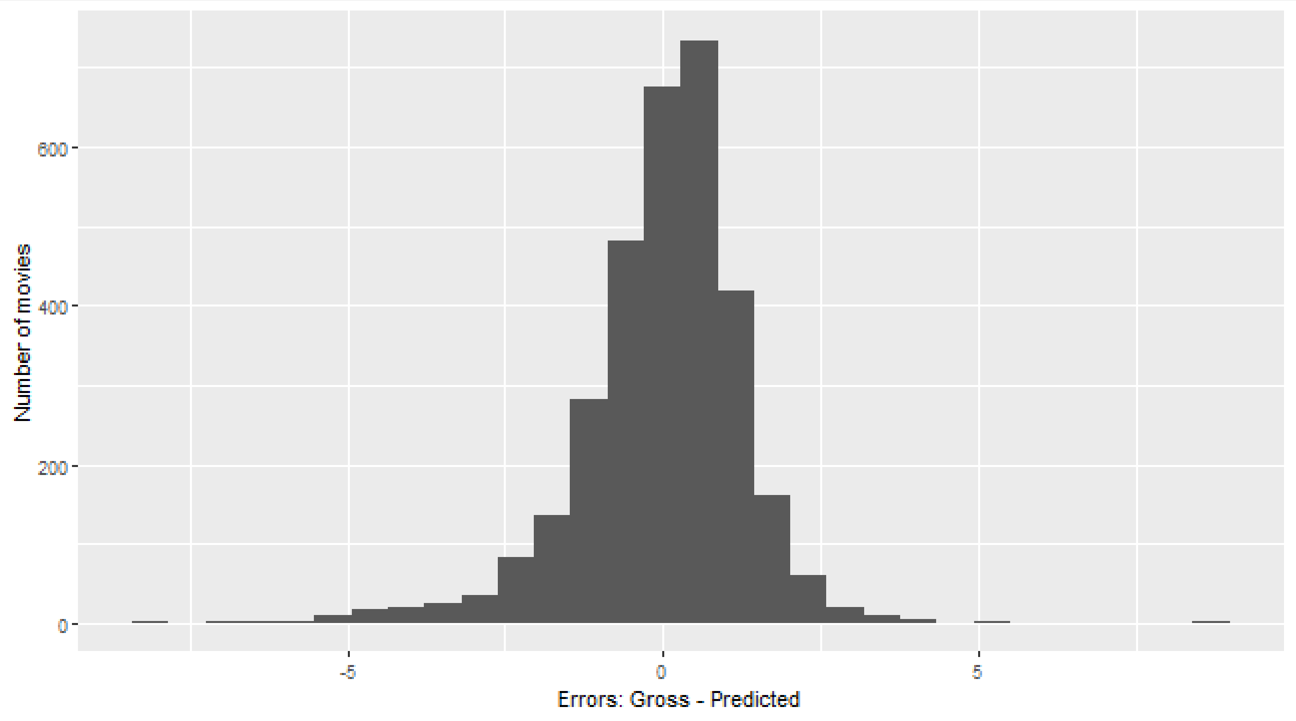
```
# Wrangle data to drop missingness!
mv_analysis <- mv %>% drop_na(gross_log,budget_log)
m <- lm(gross_log ~ budget_log,data = mv_analysis)
mv_analysis$predictions <- predict(m)
mv_analysis$errors <- mv_analysis$gross_log - mv_analysis$predictions

summary(mv_analysis$errors)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -8.2672 -0.6354  0.1648  0.0000  0.7899  8.5599
```

Univariate Viz of Errors

```
mv_analysis %>%
  ggplot(aes(x = errors)) +
  geom_histogram() +
  labs(x = 'Errors: Gross - Predicted',y = 'Number of movies')
```

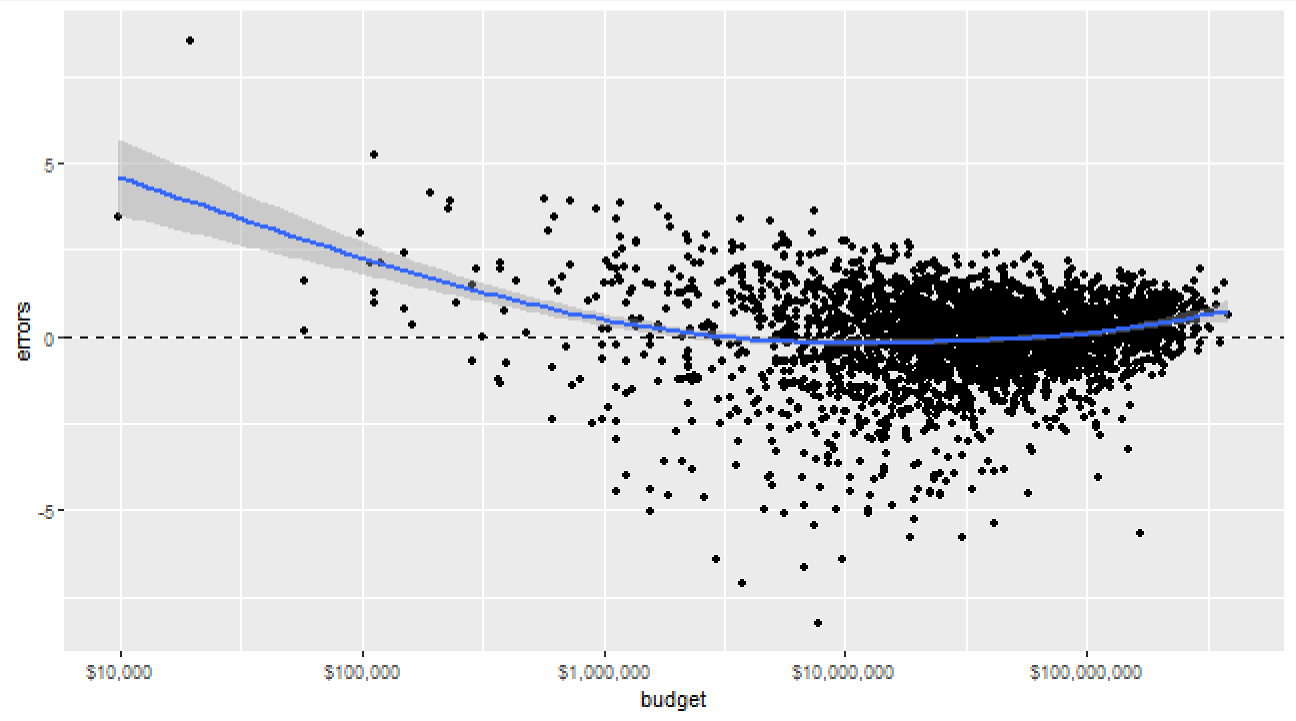


Univariate Viz of Errors

- Note that they are on average zero
  - Don't feel too proud! Mean 0 error is baked into the method
  - More concerned about **skew**...there is evidence of overestimating
- Can we do more? **Conditional Analysis**
  - Conditional on the x-axis?
  - Conditional on the **predictor** (the X variable)

Multivariate Viz of Errors

```
mv_analysis %>%
  ggplot(aes(x = budget,y = errors)) +
  geom_point() + geom_hline(yintercept = 0,linetype = 'dashed') +
  scale_x_log10(label = scales::dollar) + geom_smooth()
```



Multivariate Viz of Errors

- Ideal is where errors are unrelated to predictor
  - I.e., predictor and errors should be unrelated
  - This **should** appear as a rectangular cloud of points around zero
- This is not the case for us!
  - Evidence of a U-shape → underpredict low and high budgets, overpredict middle budgets
- Ergo, our model is **not great**!
  - Could add additional predictors X_2, X_3, etc.
  - Next lecture!

RMSE

- Univariate / Multivariate visualization of errors is **important**
- But we want to summarize model quality in a simpler way
- **RMSE**: summarizes model performance with a *single number*
  - Useful for comparing multiple models to each other

RMSE

- **E**rror (ε): actual outcome (Y_i) - predicted outcome ($\hat{Y}_i$)
  - The "distance" between the data and the model
- **S**quared: ε^2
  1. Makes all values positive
  2. Exaggerates the presence of larger errors
- **M**ean: average these squared errors
- **R**oot: take their square root (un-exaggerate)

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}$$

RMSE

- **E**rror (ε): actual outcome (Y_i) - predicted outcome ($\hat{Y}_i$)
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$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\underbrace{Y_i - \hat{Y}_i}_{\varepsilon})^2}$$

RMSE

- **E**rror (ε): actual outcome (Y_i) - predicted outcome ($\hat{Y}_i$)
  - The "distance" between the data and the model
- **S**quared: ε^2
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$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\underbrace{(\varepsilon)^2}_{S}}$$

RMSE

- **E**rror (ε): actual outcome (Y_i) - predicted outcome ($\hat{Y}_i$)
    - The "distance" between the data and the model
- **S**quared: ε^2
    1. Makes all values positive
    2. Exaggerates the presence of larger errors
- **M**ean: average these squared errors
- **R**oot: take their square root (un-exaggerate)

$$RMSE = \sqrt{\underbrace{\frac{1}{n}\sum_{i=1}^{n}}_{M}(SE)}$$

RMSE

- **E**rror (ε): actual outcome (Y_i) - predicted outcome ($\hat{Y}_i$)
  - The "distance" between the data and the model
- **S**quared: ε^2
  1. Makes all values positive
  2. Exaggerates the presence of larger errors
- **M**ean: average these squared errors
- **R**oot: take their square root (un-exaggerate)

$$RMSE = \sqrt{(MSE)}$$

RMSE

- RMSE is a **single measure that summarizes model performance**

```
e <- mv_analysis$gross_log - mv_analysis$predictions
se <- e^2
mse <- mean(se)
rmse <- sqrt(mse)
# Or
(rmseBudget <- sqrt(mean(mv_analysis$errors^2)))
```

```
## [1] 1.280835
```

- Is this good?

Predicting with uncertainty

- Say we're talking to investors about a new movie that costs $10m
  - How do we plug 10m into our model?

```
summary(m)$coefficients
```

```
##                Estimate Std. Error   t value     Pr(>|t|)
## (Intercept) 1.2610666 0.30952898  4.074147 4.73126e-05
## budget_log  0.9638585 0.01785871 53.971323 0.00000e+00
```

- $\hat{Y}_i = \alpha + \beta * X$
  - $\alpha = 1.26$ and $\beta = 0.96$
  - where $\hat{Y}_i$ is predicted gross (log) and X is $10m budget (log)

```
pred_gross_log <- 1.26 + 0.96*log(1e7)
```

Predicted Gross

- Again, convert back out of logged values with `exp()`

```
scales::dollar(exp(pred_gross_log))
```

```
## [1] "$18,501,675"
```

- Cool! We'll make $8.5m!
  - But we know our model isn't perfect
  - Need to adjust for it's errors via **RMSE**

Incorporating RMSE

- Simple idea: add and subtract RMSE from this prediction

```
pred_gross_log_ub <- 1.26 + 0.96*log(1e7) + rmseBudget
pred_gross_log_lb <- 1.26 + 0.96*log(1e7) - rmseBudget
scales::dollar(exp(c(pred_gross_log_ub,pred_gross_log_lb)))
```

```
## [1] "$66,599,457" "$5,139,861"
```

- So we'll either make a $56m profit or we'll lose almost $5m?
- **CONCLUSION PART 2**: maybe our model isn't very good?

Introducing Cross Validation

- We ran a model on the full data and calculated the RMSE
- But this approach risks "overfitting"
  - Overfitting is when we get a model that happens to do well on our specific data, but isn't actually that useful for predicting elsewhere.
  - "Elsewhere": Other periods, other movies, other datasets
- Theory: Why care about **external validity**?
  - What is the point of measuring relationship if they don't generalize?

Introducing Cross Validation

- In order to avoid overfitting, we want to "train" our model on one part of the data, and then "test" it on a different part of the data.
    - Model "can't see" the test data → better way to evaluate performance
- Cross Validation: randomly split our data into a train set and test set
    - *Similar to bootstrapping*

Introducing Cross Validation (CV)

```
set.seed(1021)
# Create list of row numbers at random
inds <- sample(1:nrow(mv_analysis),
               size = round(nrow(mv_analysis)/2),
               replace = F)

# Use slice(inds) to get training data
train <- mv_analysis %>%
  slice(inds)

# Use slice(-inds) to get test data
test <- mv_analysis %>%
  slice(-inds)
```

- We now have two datasets of roughly the same number of observations!

CV to Calculate RMSE

- We want to estimate a model based on the **test** data
- And evaluate RMSE based on the **train** data

```
m2 <- lm(gross_log ~ budget_log,train)

# predict() function on a new dataset
test$preds <- predict(m2,newdata = test)

# Now calculate RMSE on the new dataset
e <- test$gross_log - test$preds
se <- e^2
mse <- mean(se,na.rm=T)
rmse <- sqrt(mse)
rmse
```

```
## [1] 1.28959
```

CV to Calculate RMSE

- We did worse with CV! This is a *feature*
  - We are not being overconfident
  - We are avoiding "overfitting"
- Want to do this many times (like bootstrapping)

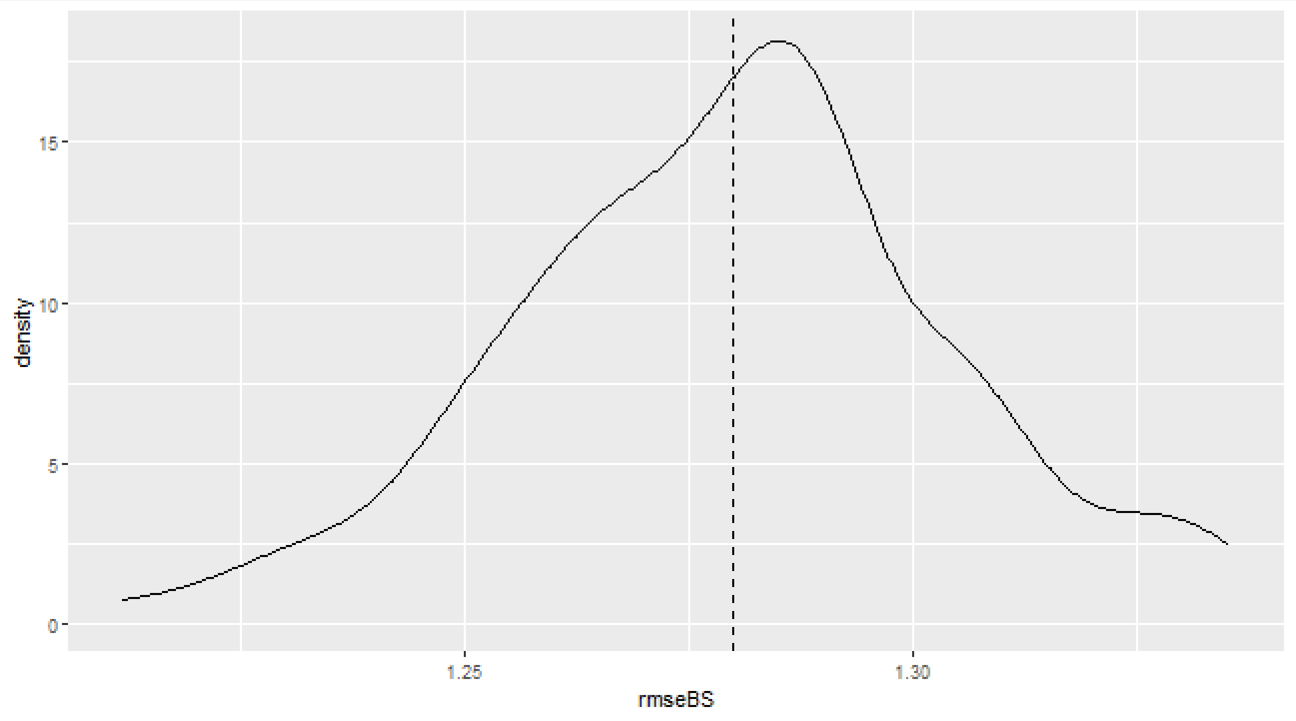
CV to Calculate RMSE

```
set.seed(123)
bsRes <- NULL
for(i in 1:100) {
  inds <- sample(1:nrow(mv_analysis),
                 size = round(nrow(mv_analysis)/2),
                 replace = F)

  train <- mv_analysis %>% slice(inds)
  test <- mv_analysis %>% slice(-inds)

  mTrain <- lm(gross_log ~ budget_log,train)

  test$preds <- predict(mTrain,newdata = test)

  rmse <- sqrt(mean((test$gross_log - test$preds)^2,na.rm=T))
  bsRes <- c(bsRes,rmse)
}

mean(bsRes)
```

```
## [1] 1.279899
```

CV to Calculate RMSE

```
data.frame(rmseBS = bsRes) %>%
  ggplot(aes(x = rmseBS)) +
  geom_density() +
  geom_vline(xintercept = mean(bsRes),linetype = 'dashed')
```



Cross Validation

- In this example, we used a 50-50 split
- Often, data scientists prefer an 80-20 split
    - **Improves** the model (80% of the data is more to learn from)...
    - ...but still **protects** against overfitting

```
inds <- sample(1:nrow(mv_analysis),
               size = round(nrow(mv_analysis)*.8),
               replace = F)
```

Quiz & Homework

- Go to Brightspace and take the **12th** quiz
  - The password to take the quiz is ####
- **Homework:**
  1. Work through ds1000_hw_12.Rmd
  2. Problem Set 7 (due Friday)