Classification

Part 2

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Agenda

1. Introducing **logit**
2. Running logit
3. Evaluating logit

Logit Regression

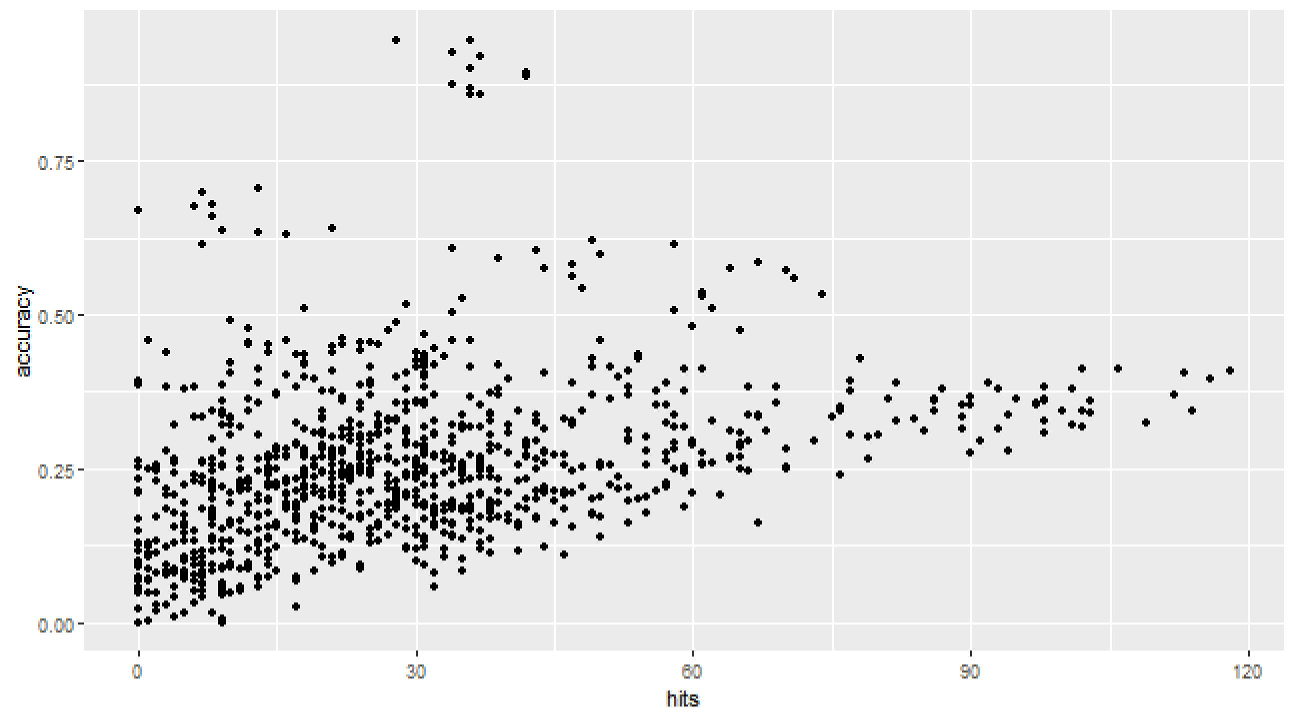
- A different **type** of **regression**
    - What do we mean by **type**?
- Let's take a step back

```
require(tidyverse)
require(scales)
fn <-
read_rds('https://github.com/jbisbee1/DS1000_F2024/raw/main/data/fn_cle
```

Regression Types

- "Linear" regression...why is it "linear"?

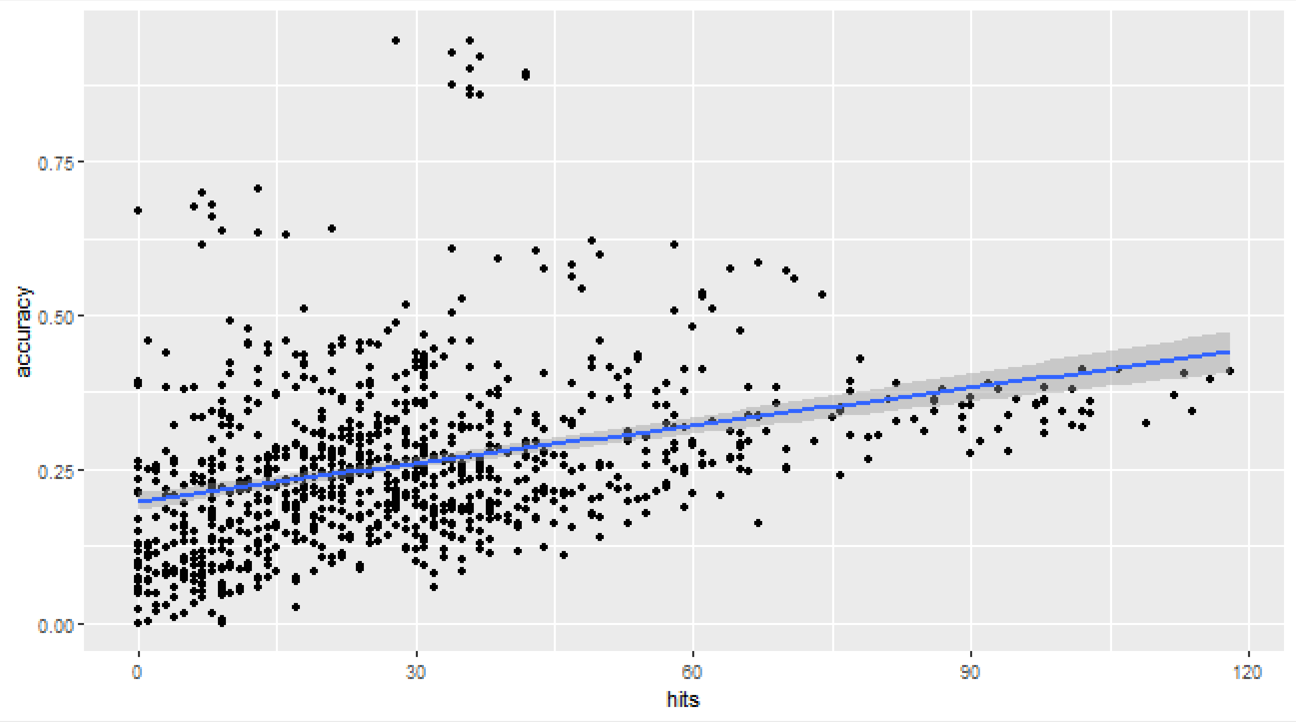
```
(p <- fn %>%
  ggplot(aes(x = hits,y = accuracy)) +
  geom_point())
```



Regression Types

- "Linear" regression...why is it "linear"?
- Because you can summarize it with a line!

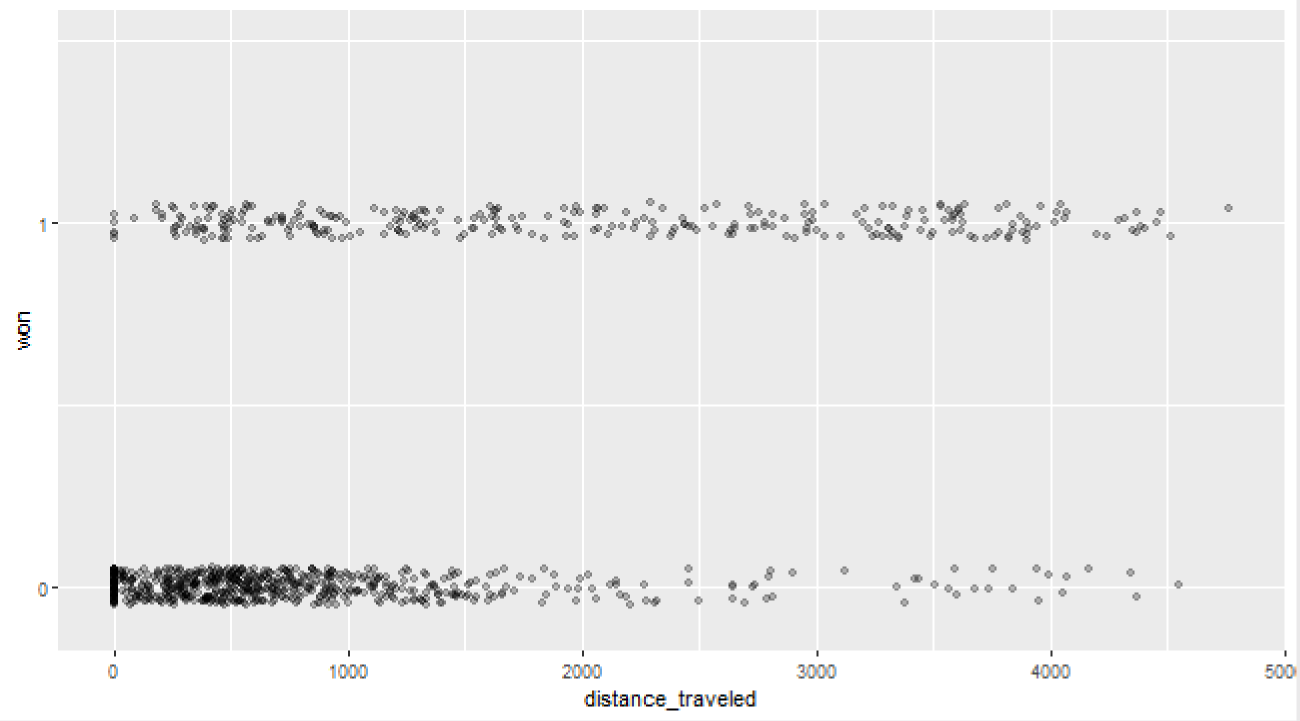
```
p + geom_smooth(method = 'lm')
```



Regression Types

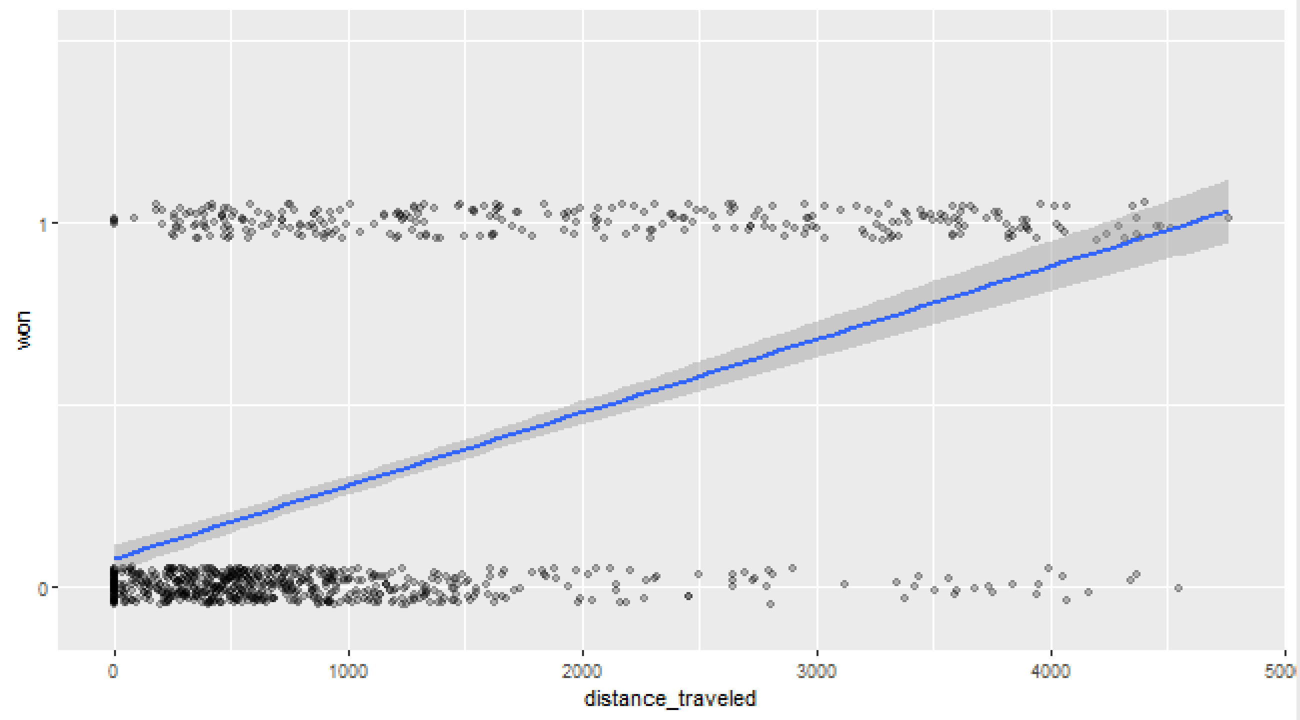
- But what if the outcome is binary?

```
(p <- fn %>% ggplot(aes(x = distance_traveled,y = won)) +
   scale_y_continuous(breaks = c(0,1),limits = c(-.1,1.5)) +
  geom_jitter(width = .01,height = .05,alpha = .25))
```



Regression Types

- But what if the outcome is binary?
- Lines seem too clumsy
  - If `1` = won, how can you go higher?



Logit

- Theory: binary outcomes are **proxies** for some **latent** measure
  - Binary outcome `won`: either placed first or did not
  - Latent outcome `placed`: continuous measure
  - Might also imagine **ability**: continuous measure
- The higher your `ability`, the more likely you are to win
- Logit regression: model the `ability`
  - What is `ability` actually?
  - Probability of winning: $Pr(won)$
- Part of a broader class of models called "generalized linear model" (GLM)

$$Pr(y = 1|x) = G(\alpha + \beta X)$$

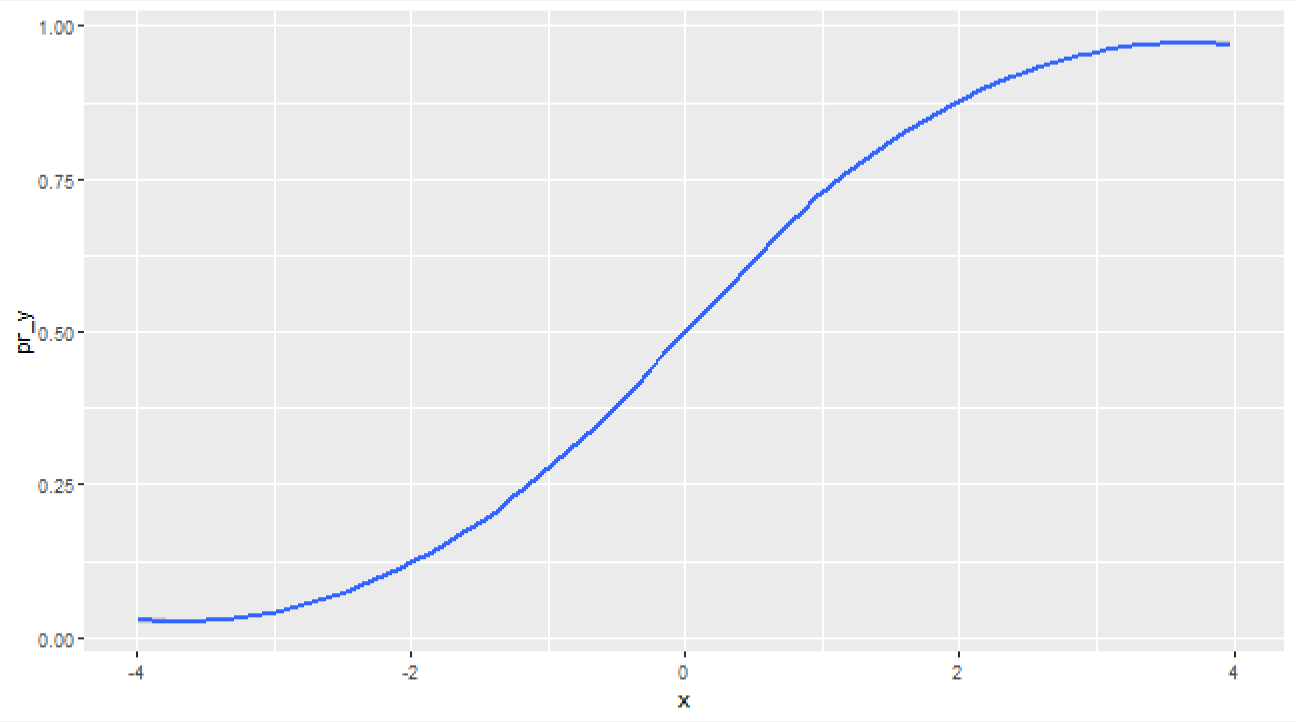
GLMs

- $Pr(y = 1|x) = G(\alpha + \beta X)$
- Does this look familiar?
- Linear regression: $Y = \alpha + \beta X$
  - Outcome: $Y \rightarrow Pr(y = 1|x)$
  - Mapping: $\alpha + \beta X \rightarrow G(\alpha + \beta X)$
- G is the "link function"
  - Transforms values of $\alpha + \beta X$ into **probabilities**
- Logistic function: specific type of link function

$$G(x) = \frac{1}{1 + exp(-x)}$$

Logistic Function

```
x <- runif(100,-4,4)
pr_y <- 1/(1 + exp(-x))
as_tibble(pr_y = pr_y,x = x) %>%
  ggplot(aes(x = x,y = pr_y)) +
  geom_smooth()
```

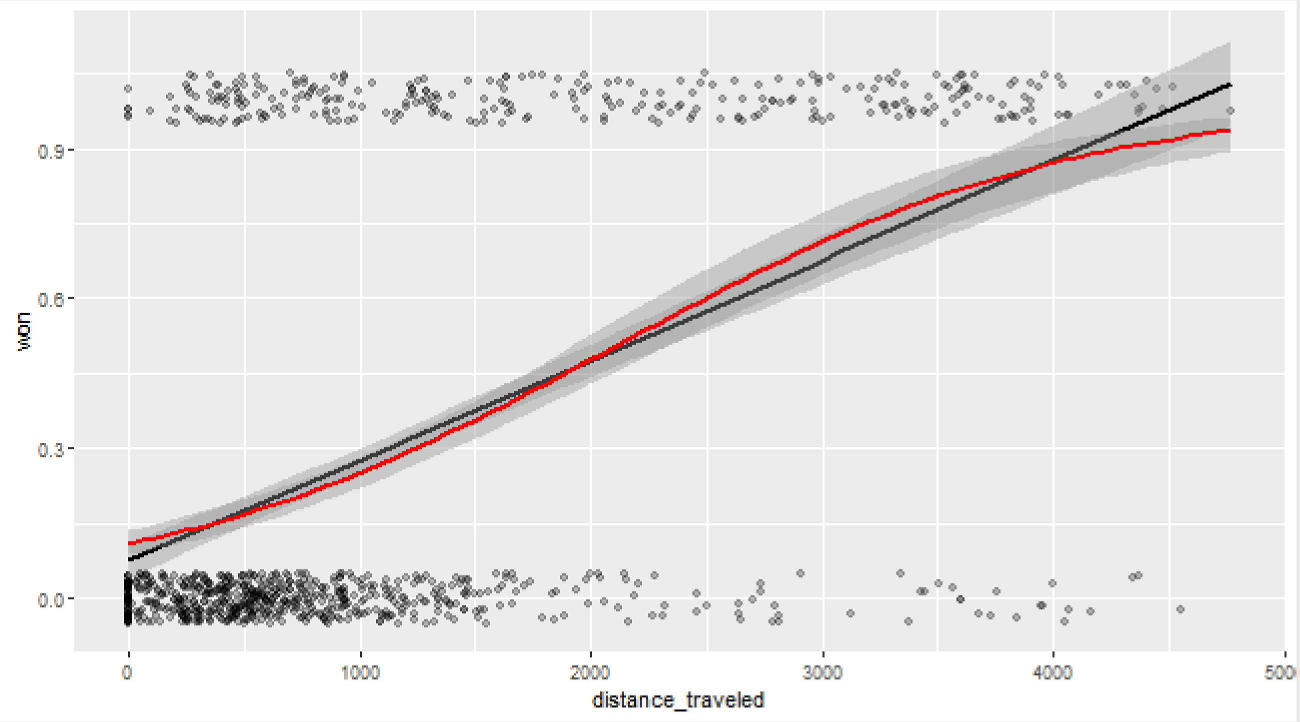


Logistic Function

- But what about real data like $\alpha + \beta X$?
- $G(X) = \frac{exp(\alpha + \beta X)}{1 + exp(\alpha + \beta X)}$
- We estimate this with `glm(formula,data,family)`
  - Note similarity to `lm(formula,data)`
- `family = binomial(link = "logit")`

Logistic Regression (logit)

```
fn %>% ggplot(aes(x = distance_traveled,y = won)) +
  geom_jitter(width = .01,height = .05,alpha = .25) +
  geom_smooth(method = 'lm',color = 'black') +
  geom_smooth(method = 'glm',color = 'red',
              method.args = list(family = binomial(link = 'logit')))
```



Logistic Regression (logit)

```
# Train model
mLogit <- glm(formula = won ~ distance_traveled,data = fn,family =
binomial(link = 'logit'))

# Predict model
fn <- fn %>%
  mutate(prob_won = predict(mLogit,type = 'response')) %>%
  mutate(pred_won = ifelse(prob_won > .5,1,0))

# Evaluate model
eval <- fn %>%
  group_by(won) %>%
  mutate(total_games = n()) %>%
  group_by(won,pred_won,total_games) %>%
  summarise(nGames=n(),.groups = 'drop') %>%
  mutate(prop = nGames / total_games) %>%
  ungroup() %>%
  mutate(accuracy = percent(sum((won == pred_won)*nGames) /
sum(nGames)))
```

Logistic Regression (logit)

```
eval
```

```
## # A tibble: 4 × 6
##     won pred_won total_games nGames   prop accuracy
##   <dbl>    <dbl>       <int>  <int>  <dbl> <chr>
## 1     0        0         666    620 0.931  78%
## 2     0        1         666     46 0.0691 78%
## 3     1        0         291    163 0.560  78%
## 4     1        1         291    128 0.440  78%
```

Logistic Regression (logit)

- Can also calculate ROC Curve and AUC

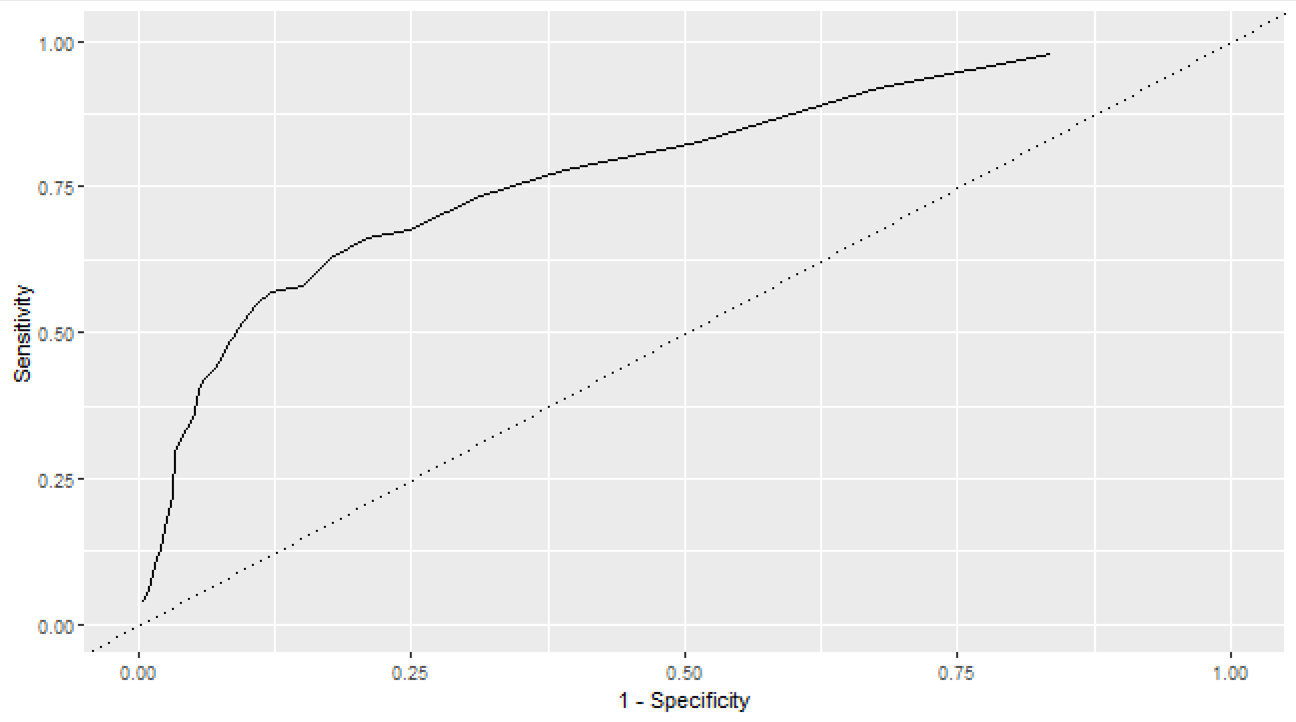
```
toplot <- NULL
for(thresh in seq(0,1,by = .025)) {
  toplot <- fn %>%
    mutate(pred_won = ifelse(predict(mLogit,type = 'response') >
thresh,1,0)) %>%
    group_by(won) %>%
    mutate(total_games = n()) %>%
    group_by(won,pred_won,total_games) %>%
    summarise(nGames=n(),.groups = 'drop') %>%
    mutate(prop = nGames / total_games) %>%
    ungroup() %>%
    mutate(threshold = thresh) %>%
    bind_rows(toplot)
}
```

Logistic Regression (logit)

```
p <- toplot %>%
  mutate(metric = ifelse(won == 1 & pred_won == 1,'Sensitivity',
                               ifelse(won == 0 & pred_won ==
0,'Specificity',NA))) %>%
  drop_na(metric) %>%
  select(prop,metric,threshold) %>%
  spread(metric,prop) %>%
  arrange(desc(Specificity),Sensitivity) %>%
  ggplot(aes(x = 1-Specificity,y = Sensitivity)) +
  geom_line() +
  xlim(c(0,1)) + ylim(c(0,1)) +
  geom_abline(slope = 1,intercept = 0,linetype = 'dotted')
```

Logistic Regression (logit)

```
p
```



Logistic Regression (logit)

```
require(tidymodels)
roc_auc(data = fn %>%
  mutate(prob_won = predict(mLogit,type = 'response'),
         truth = factor(won,levels = c('1','0'))) %>%
  select(truth,prob_won),truth,prob_won)
```

```
## # A tibble: 1 × 3
##   .metric .estimator .estimate
##   <chr>   <chr>          <dbl>
## 1 roc_auc binary         0.782
```

Comparing Models

- Two big questions in prediction:

    1. Do I have the correct predictors X?

    2. Do I have the best model?

- Two types of outcomes (thus far)

    1. Continuous Y: use **RMSE**

    2. Binary Y: use **AUC**

- Let's determine the best model from the following:

    - X: (1) `distance_traveled + mental_state` vs. (2) `distance_traveled + mental_state + hits`

    - Model: (1) conditional means vs. (2) `lm` vs. (3) `glm`

Comparing Models

- Conditional means - simple X

```
results <- NULL

# Train & Predict
toEval <- fn %>%
  mutate(distDec = ntile(distance_traveled,n = 10)) %>%
  group_by(distDec,mental_state) %>%
  mutate(prob_won = mean(won),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob_won)

# Evaluate
results <- roc_auc(data = toEval,truth,prob_won) %>%
  mutate(model = 'CM',
         predictors = 'Simple') %>%
  bind_rows(results)
```

Comparing Models

- Conditional means - complex X

```
# Train & Predict
toEval <- fn %>%
  mutate(distDec = ntile(distance_traveled,n = 10),
         hitsDec = ntile(hits,n = 10)) %>%
  group_by(distDec,hitsDec,mental_state) %>%
  mutate(prob_won = mean(won),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob_won)

# Evaluate
results <- roc_auc(data = toEval,truth,prob_won) %>%
  mutate(model = 'CM',
         predictors = 'Complex') %>%
  bind_rows(results)
```

Comparing Models

- Linear regression (`lm`) - simple X

```
# Train
m <- lm(won ~ distance_traveled + mental_state,fn)

# Predict
toEval <- fn %>%
  mutate(prob_won = predict(m),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob_won)

# Evaluate
results <- roc_auc(data = toEval,truth,prob_won) %>%
  mutate(model = 'LM',
         predictors = 'Simple') %>%
  bind_rows(results)
```

Comparing Models

- Linear regression (`lm`) - complex X

```
# Train
m <- lm(won ~ distance_traveled + mental_state + hits,fn)

# Predict
toEval <- fn %>%
  mutate(prob_won = predict(m),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob_won)

# Evaluate
results <- roc_auc(data = toEval,truth,prob_won) %>%
  mutate(model = 'LM',
         predictors = 'Complex') %>%
  bind_rows(results)
```

Comparing Models

- Logit regression (`glm`) - simple X

```
# Train
m <- glm(won ~ distance_traveled + mental_state,fn,family =
binomial(link = 'logit'))

# Predict
toEval <- fn %>%
  mutate(prob_won = predict(m,type = 'response'),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob_won)

# Evaluate
results <- roc_auc(data = toEval,truth,prob_won) %>%
  mutate(model = 'GLM',
         predictors = 'Simple') %>%
  bind_rows(results)
```

Comparing Models

- Logit regression (`glm`) - complex X

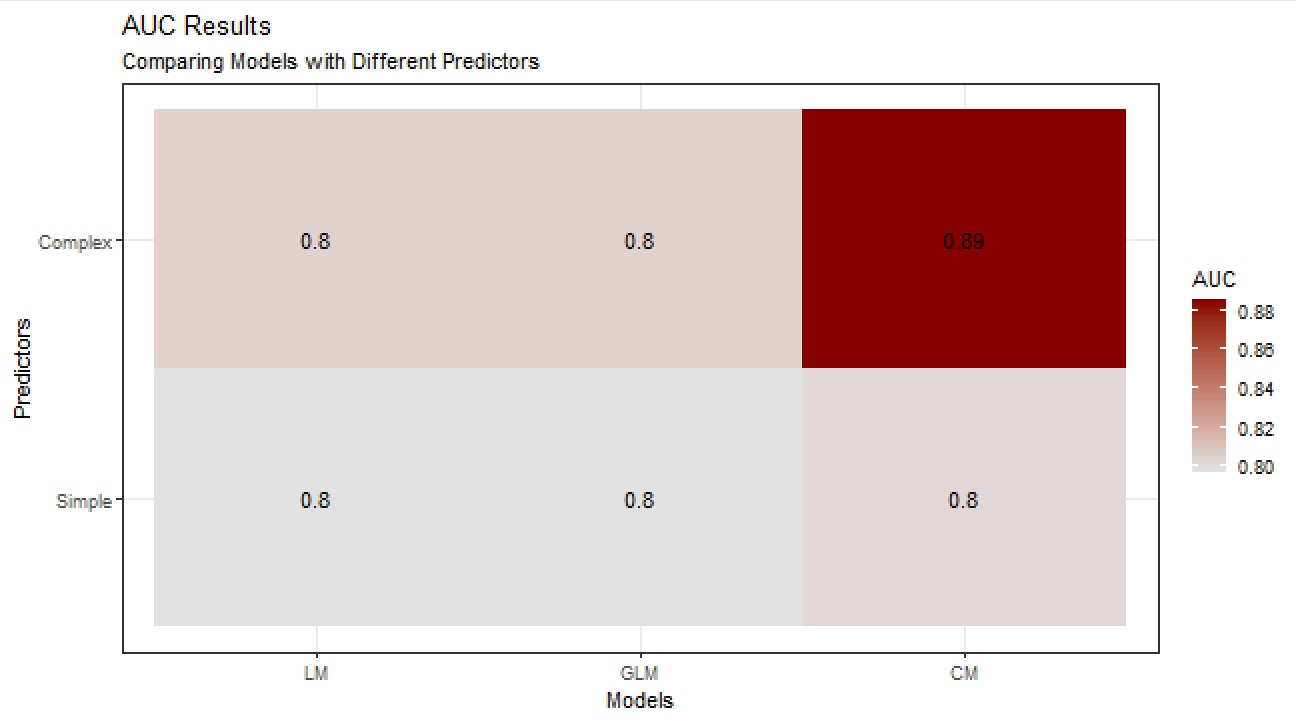
```
# Train
m <- glm(won ~ distance_traveled + mental_state + hits,fn,family =
binomial(link = 'logit'))

# Predict
toEval <- fn %>%
  mutate(prob_won = predict(m,type = 'response'),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob_won)

# Evaluate
results <- roc_auc(data = toEval,truth,prob_won) %>%
  mutate(model = 'GLM',
         predictors = 'Complex') %>%
  bind_rows(results)
```

Comparing Models

```
p <- results %>%
  ggplot(aes(x = reorder(model,.estimate),
             y = reorder(predictors,.estimate),
             fill = .estimate,label = round(.estimate,2))) +
  geom_tile() +
  scale_fill_continuous(low = 'grey90',high = 'darkred') +
  geom_text() +
  labs(title = 'AUC Results',
       subtitle = 'Comparing Models with Different Predictors',
       x = 'Models',y = 'Predictors',
       fill = 'AUC') +
  theme_bw()
```

Comparing Models

```
p
```



Conclusion

- Conditional means outperform regression models?
  - Yes: conditional means allow for cell-specific predictions
  - No: conditional means are more susceptible to **overfitting**
- How would you re-evaluate these models-X-predictors to account for overfitting?
- Go to Brightspace and take the **15th** quiz
- **Homework:**
  - Problem Set 8
  - HW 16