Regression

Part 3

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Agenda

1. Recap of Movie Analysis
2. Multiple Regression
3. Categorical Predictors

Recap of Movie Analysis

```r
require(tidyverse)

mv <-
read_rds('https://github.com/jbisbee1/DS1000_F2024/raw/main/data/mv.Rds
```

- Theory: the more a movie costs, the more it should make
  - If not, Hollywood would go out of business!
- X: budget
- Y: gross

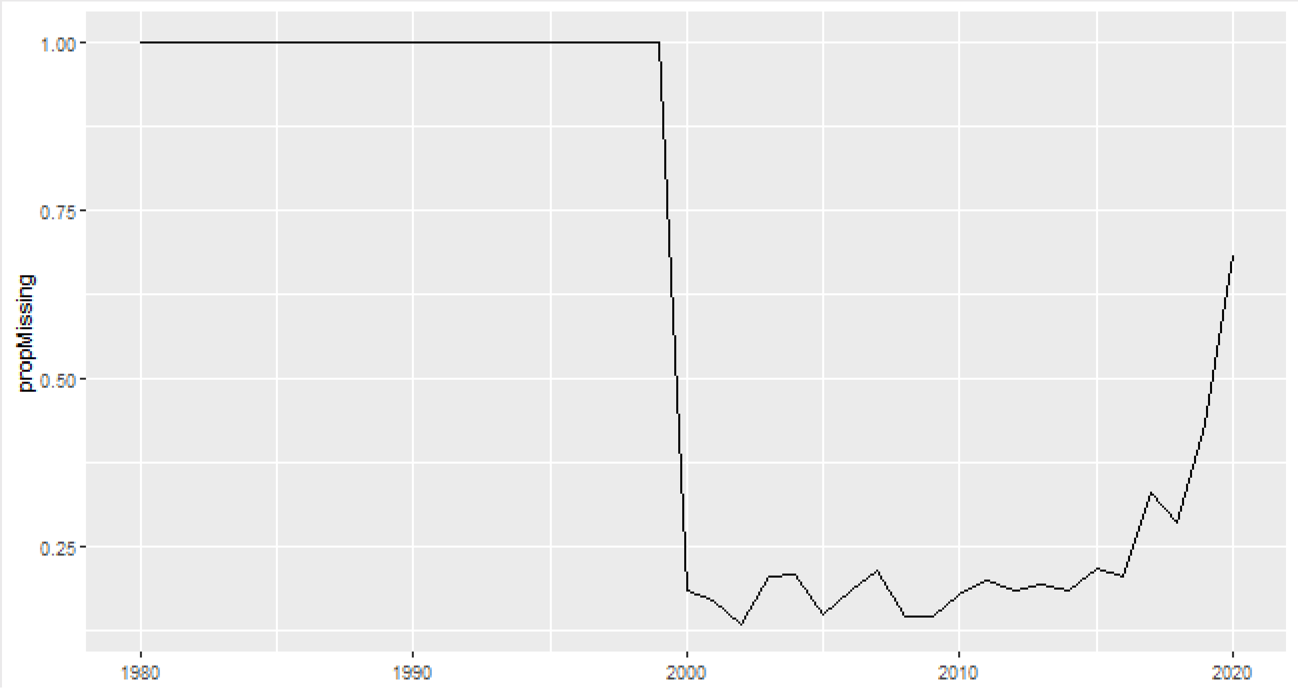
Step 1: Look

```
summary(mv %>% select(gross,budget))
```

```
##      gross                budget         
##  Min.   :7.140e+02   Min.   :     5172  
##  1st Qu.:1.121e+07   1st Qu.: 16865322  
##  Median :5.178e+07   Median : 37212044  
##  Mean   :1.402e+08   Mean   : 57420173  
##  3rd Qu.:1.562e+08   3rd Qu.: 77844746  
##  Max.   :3.553e+09   Max.   :387367903  
##  NA's   :3668        NA's   :4482
```

Step 1: Look

```
mv %>%
  mutate(missing = ifelse(is.na(gross) | is.na(budget),1,0)) %>%
  group_by(year) %>%
  summarise(propMissing = mean(missing)) %>%
  ggplot(aes(x = year,y = propMissing)) +
  geom_line()
```



Some quick wrangling

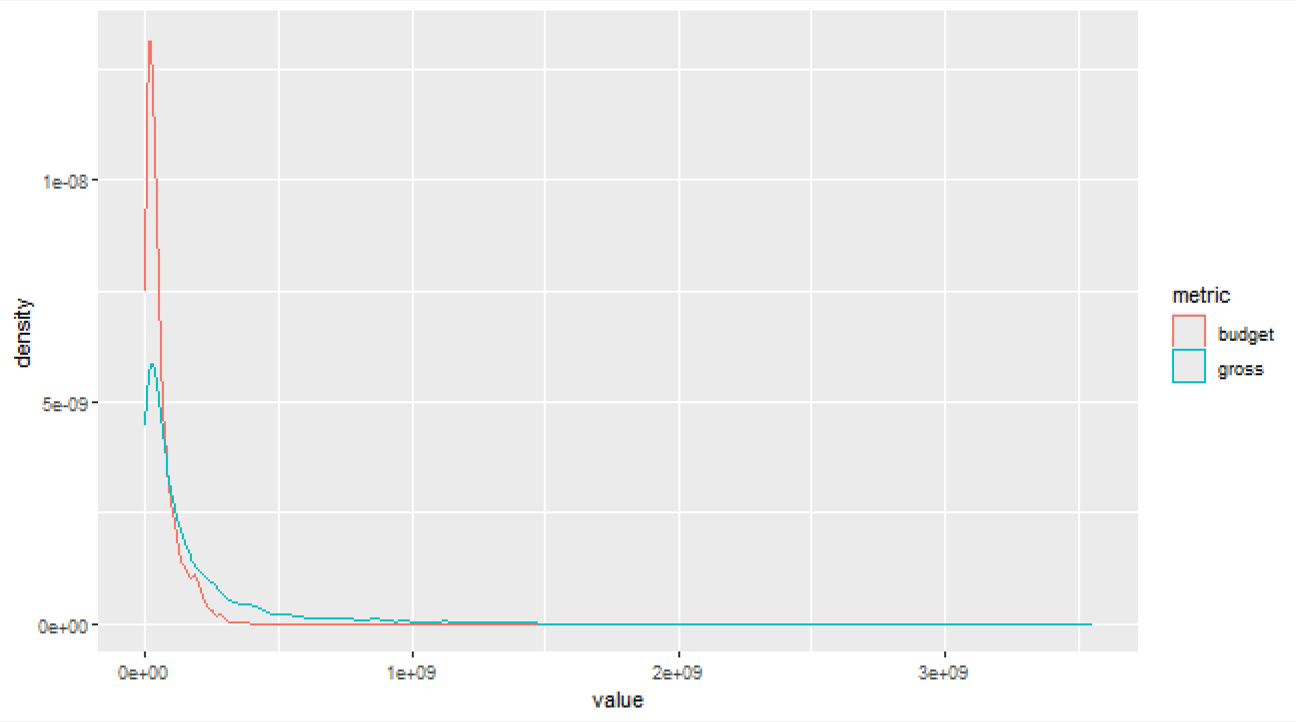
```
mv <- mv %>%
  drop_na(gross,budget)

mv %>%
  select(gross,budget) %>%
  glimpse()
```

```
## Rows: 3,179
## Columns: 2
## $ gross  <dbl> 73677478, 53278578, 723586629, 11490339, 62…
## $ budget <dbl> 93289619, 10883789, 160147179, 6996721, 139…
```

Step 2: Univariate Viz

```
mv %>%
  select(title,gross,budget) %>%
  gather(metric,value,-title) %>%
  ggplot(aes(x = value,color = metric)) +
  geom_density()
```



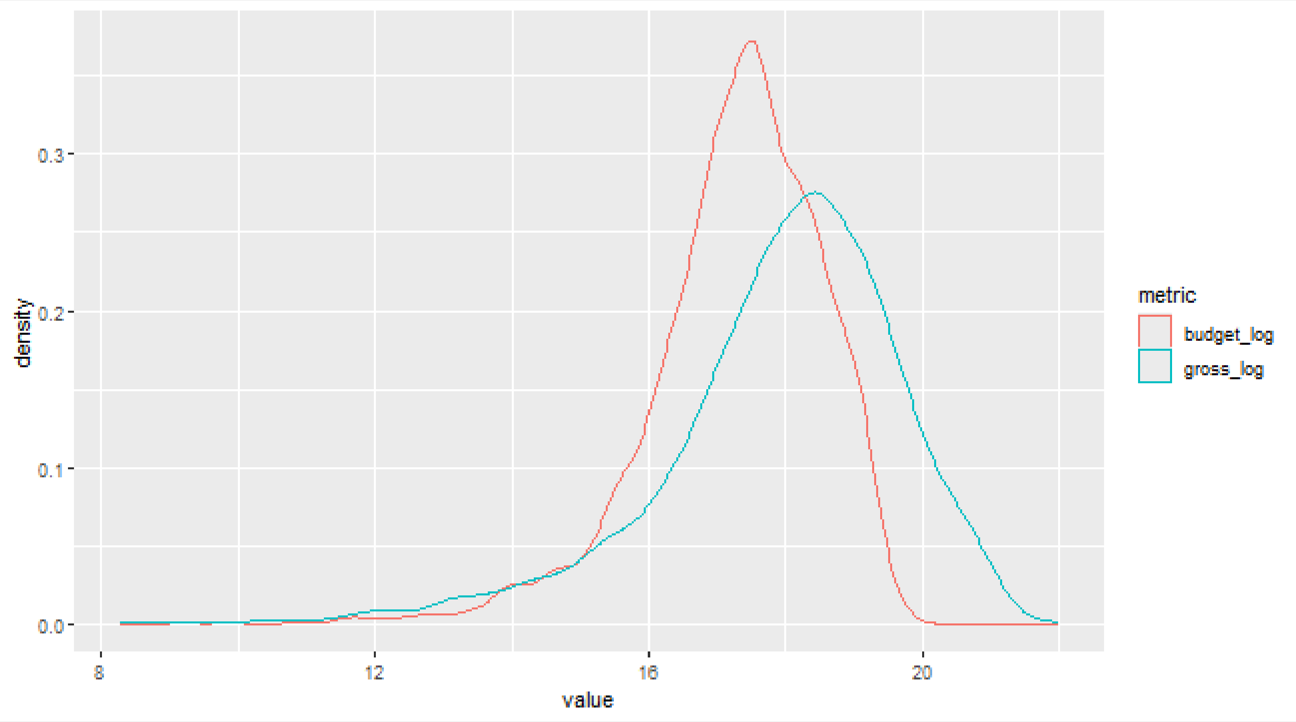
More Wrangling?

- Univariate visualization higlights significant **skew** in both measures
  - Most movies don't cost a lot and don't make a lot
  - But there are a few blockbusters that pull the density way out
- Let's **wrangle** two new variables that take the log of these skewed measures
  - Logging transforms skewed measures to more "normal" measures

```
mv <- mv %>%
  mutate(gross_log = log(gross),
         budget_log = log(budget))
```

Step 2: Univariate Viz

```
mv %>%
  select(title,gross_log,budget_log) %>%
  gather(metric,value,-title) %>%
  ggplot(aes(x = value,color = metric)) +
  geom_density()
```

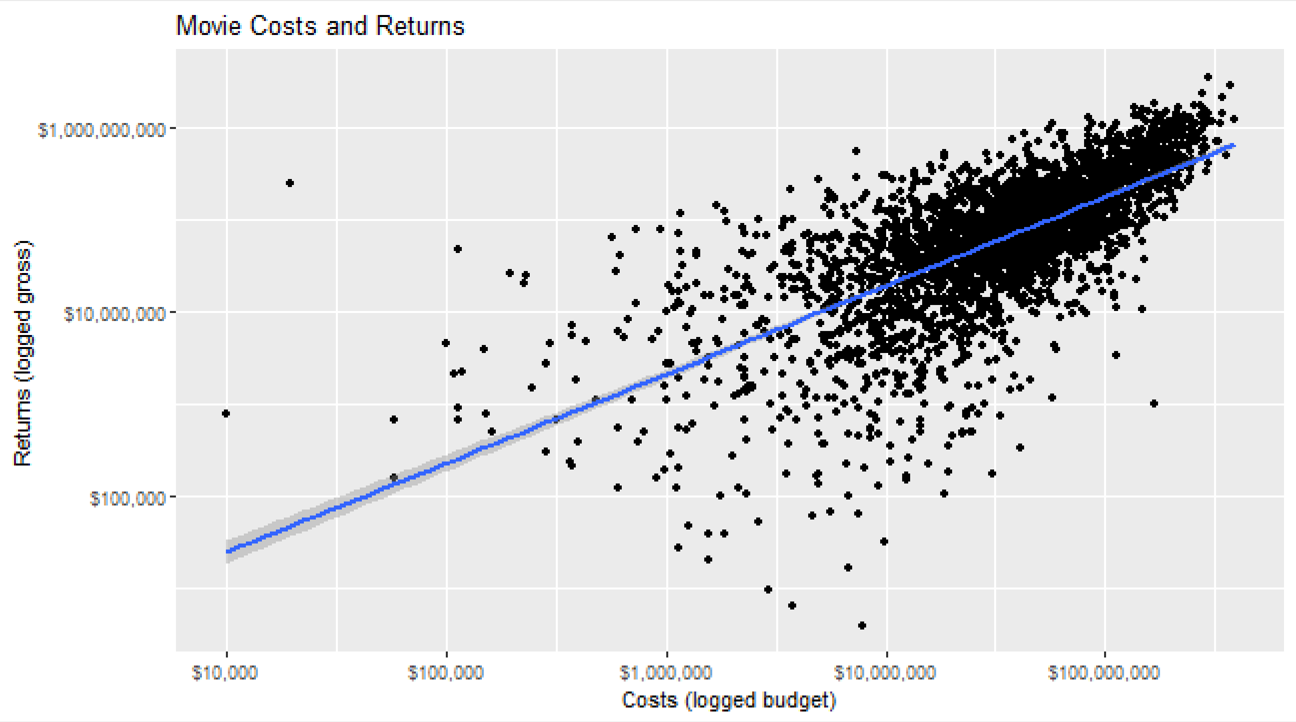


Step 3: Multivariate Viz

```
pClean <- mv %>%
  ggplot(aes(x = budget,y = gross)) +
  geom_point() +
  scale_x_log10(labels = scales::dollar) +
  scale_y_log10(labels = scales::dollar) +
  labs(title = "Movie Costs and Returns",
       x = "Costs (logged budget)",
       y = "Returns (logged gross)")
```

Step 3: Multivariate Viz

```
pClean + geom_smooth(method = 'lm')
```



Step 4: Regression!

```
m <- lm(gross_log ~ budget_log,data = mv)
summary(m)
```

```
##
## Call:
## lm(formula = gross_log ~ budget_log, data = mv)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.2672 -0.6354  0.1648  0.7899  8.5599
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.26107    0.30953   4.074 4.73e-05 ***
## budget_log   0.96386    0.01786  53.971  < 2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.281 on 3177 degrees of freedom
## Multiple R-squared:  0.4783,    Adjusted R-squared:  0.4782
## F-statistic:  2913 on 1 and 3177 DF,  p-value: < 2.2e-16
```

Step 5.1: Univariate Viz of Errors

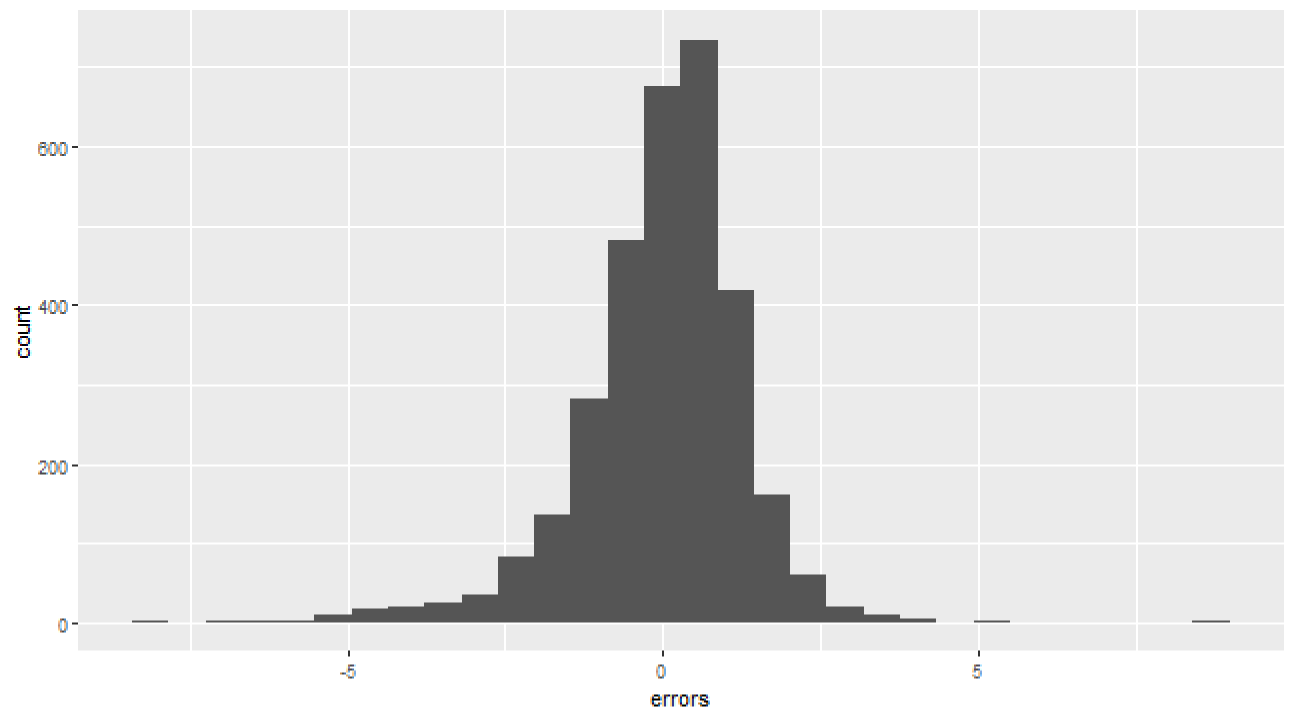
- Errors $\varepsilon = Y - \hat{Y}$
  - In `R`, can also get them via `resid()` function

```
mv %>%
  mutate(errors_manual = gross_log - predict(m),
         errors_resid = resid(m))
```

```
## # A tibble: 3,179 × 24
##    title   rating genre  year released score  votes director
##    <chr>   <chr>  <chr> <dbl> <chr>    <dbl>  <dbl> <chr>
##  1 Almost… R      Adve…  2000 Septemb…   7.9 2.6 e5 Cameron…
##  2 Americ… R      Come…  2000 April 1…   7.6 5.14e5 Mary Ha…
##  3 Gladia… R      Acti…  2000 May 5, …   8.5 1.4 e6 Ridley …
##  4 Requie… Unrat… Drama  2000 Decembe…   8.3 7.86e5 Darren …
##  5 Memento R      Myst…  2000 May 25,…   8.4 1.20e6 Christo…
##  6 Cast A… PG-13  Adve…  2000 Decembe…   7.8 5.42e5 Robert …
##  7 Scary … R      Come…  2000 July 7,…   6.2 2.38e5 Keenen …
##  8 The Pe… PG-13  Acti…  2000 June 30…   6.4 1.6 e5 Wolfgan…
##  9 Coyote… PG-13  Come…  2000 August …   5.7 1.08e5 David M…
## 10 X-Men   PG-13  Acti…  2000 July 14…   7.4 5.82e5 Bryan S…
## # ℹ 3,169 more rows
```

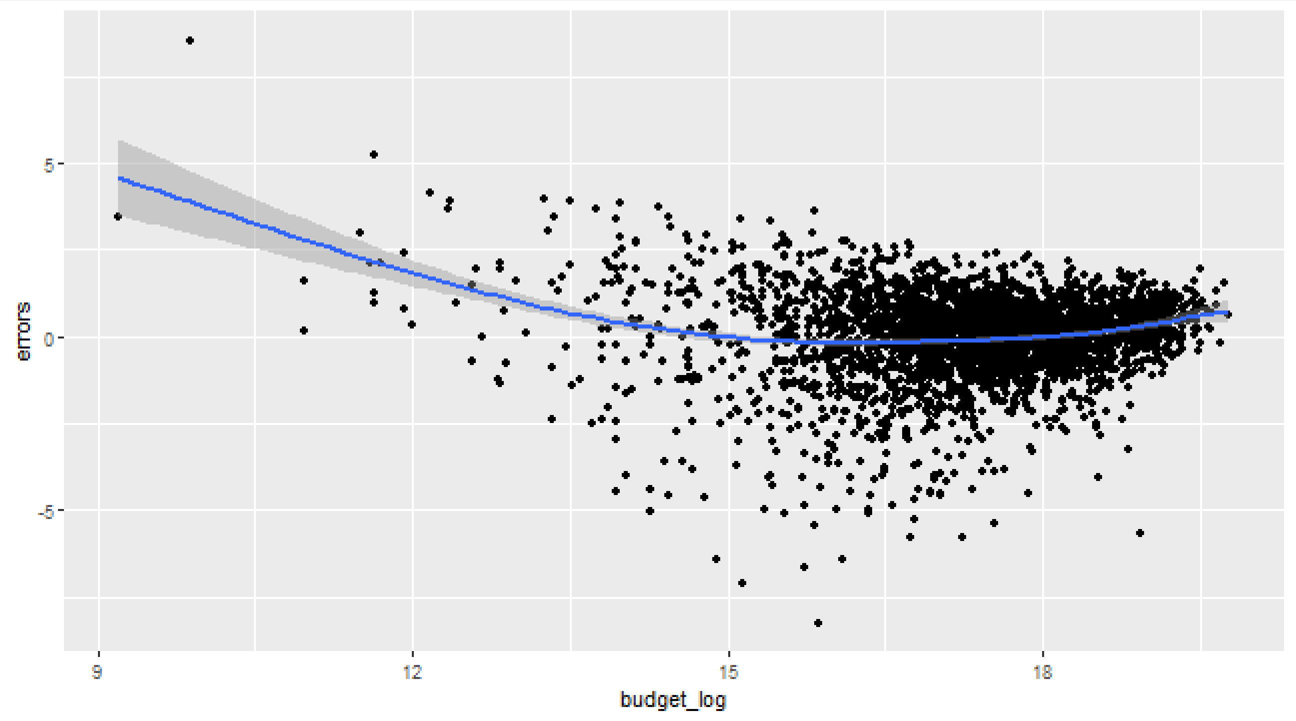
Step 5.1: Univariate Viz of Errors

```
mv %>%
  ggplot(aes(x = errors)) +
  geom_histogram()
```



Step 5.2: Multivariate Viz of Errors

```
mv %>%
  ggplot(aes(x = budget_log,y = errors)) +
  geom_point() +
  geom_smooth()
```



Step 5.3: Cross Validated RMSE

```
set.seed(123)
rmseBudget <- NULL
for(i in 1:100) {
  inds <- sample(1:nrow(mv),size = round(nrow(mv)/2),replace = F)

  train <- mv %>% slice(inds)
  test <- mv %>% slice(-inds)

  mTrain <- lm(gross_log ~ budget_log,train)

  test$preds <- predict(mTrain,newdata = test)

  rmse <- sqrt(mean((test$gross_log - test$preds)^2,na.rm=T))
  rmseBudget <- c(rmseBudget,rmse)
}

mean(rmseBudget)
```

```
## [1] 1.279899
```

Thinking like a scientist

- Our previous model predicted `gross` as a function of `budget`
- Theoretically, is this sensible?
  1. Bigger budgets → famous actors → mass appeal → more tickets
  2. Bigger budgets → advertising money → mass appeal → more tickets
- But what if the movie is just...not good?

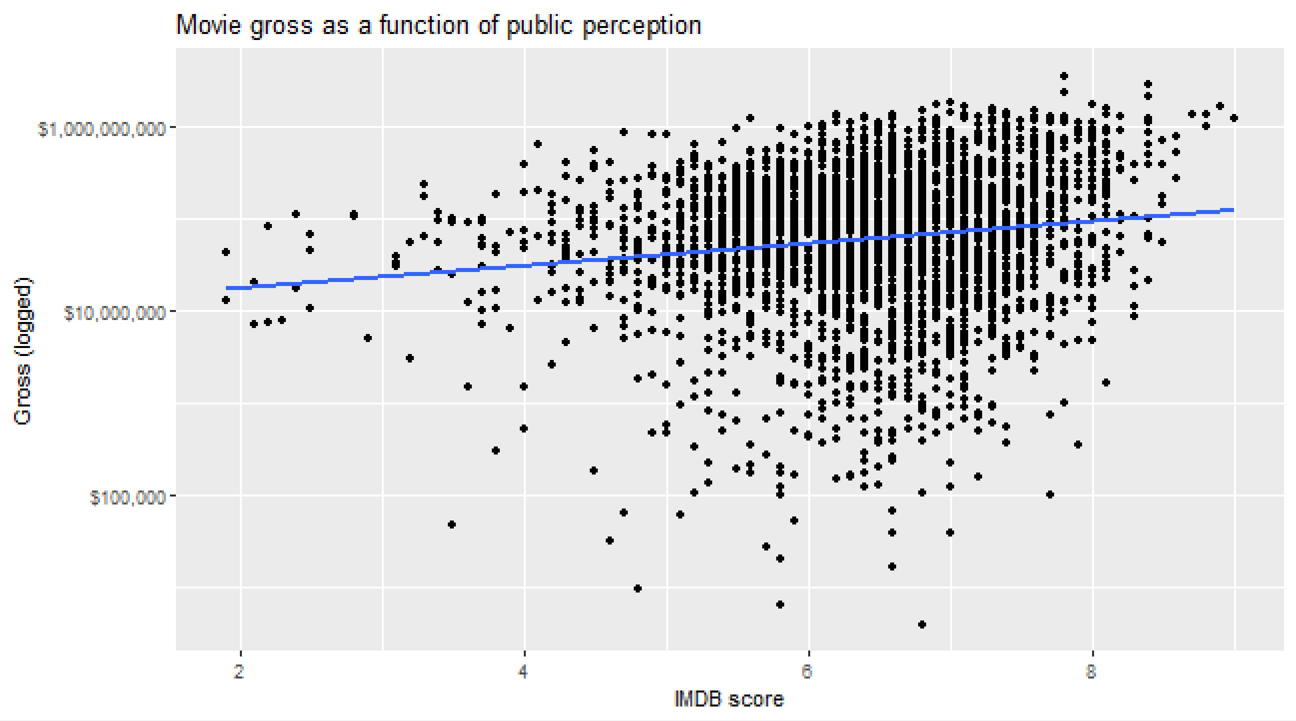
Alternative Theory

- Good movies make more money
  - Theory: good movies → recommendations → more tickets
- Predict gross with IMDB rating (`score`)

```
pIMDB <- mv %>%
  ggplot(aes(x = score,y = gross)) +
  geom_point() +
  labs(title = "Movie gross as a function of public perception",
       x = "IMDB score",
       y = "Gross (logged)") +
  scale_y_log10(label = scales::dollar) +
  geom_smooth(method = 'lm',se = F)
```

Alternative Model

```
pIMDB
```



Evaluating the Model

- Let's go straight to RMSE
    - We can have `R` calculate errors for us with `residuals()` command

```
m2 <- lm(gross_log ~ score,mv)
error <- residuals(m2)
(rmseScore <- sqrt(mean(error^2)))
```

```
## [1] 1.753146
```

- Even worse!

Multivariate Regression

- Recall that we can **model** our outcome with multiple **predictors**

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \cdots + \varepsilon$$

- How much better can we predict `gross` with **BOTH** `budget` and `score`?

```
m3 <- lm(gross_log ~ budget_log + score,mv)
error <- residuals(m3)
(rmseBudgScore <- sqrt(mean(error^2)))
```

```
## [1] 1.248817
```

Comparing Models

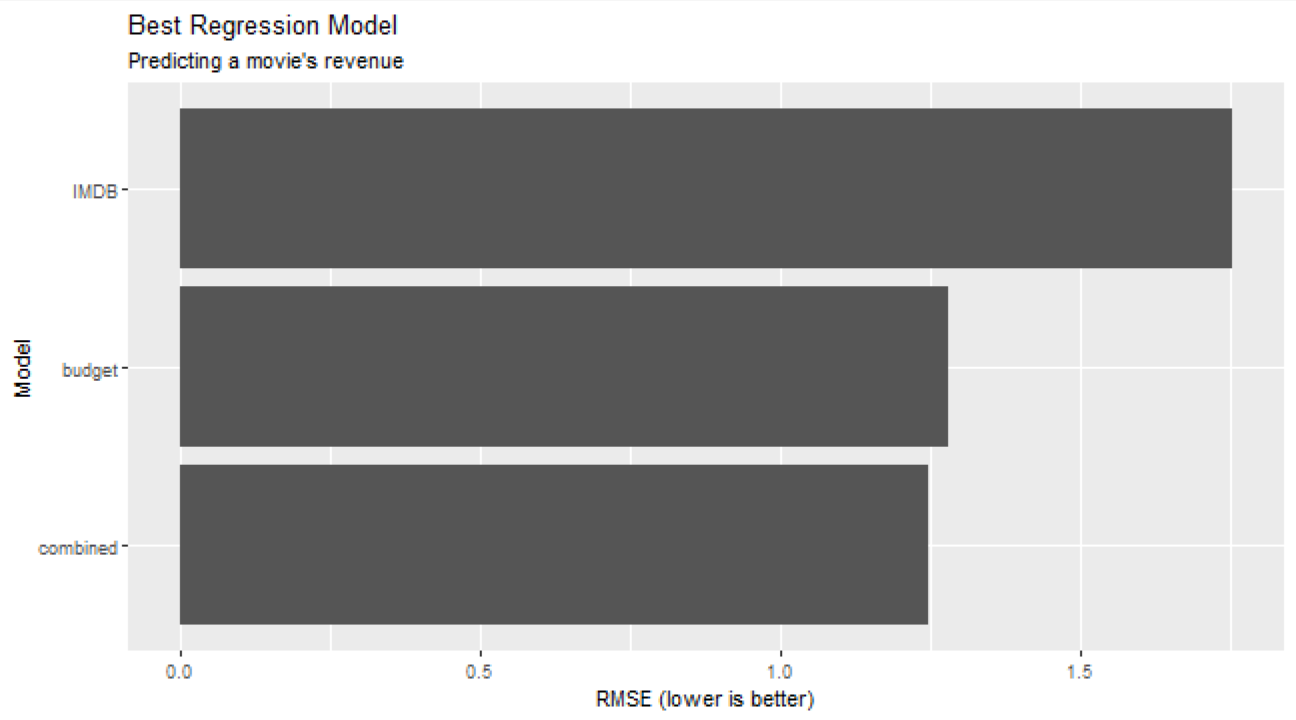
- Which model best predicts movie revenues?

```
p <- data.frame(budget = mean(rmseBudget),
            IMDB = rmseScore,
            combined = rmseBudgScore) %>%
  gather(model,rmse) %>%
  ggplot(aes(x = reorder(model,rmse),y = rmse)) +
  geom_bar(stat = 'identity') +
  labs(title = "Best Regression Model",
       subtitle = "Predicting a movie's revenue",
       y = "RMSE (lower is better)",
       x = "Model") +
  coord_flip()
```

Comparing Models

- Which model best predicts movie revenues?

p



Why RMSE?

- Want to understand how good / bad our model is
- Can use it to compare models

Why RMSE?

- Do we improve our model with `score`?

```
set.seed(123)
bsRes <- NULL
for(i in 1:100) {
  inds <- sample(1:nrow(mv),size = round(nrow(mv)/2),replace = F)
  train <- mv %>% slice(inds)
  test <- mv %>% slice(-inds)

  mB <- lm(gross_log ~ budget_log,train)
  mS <- lm(gross_log ~ score,train)
  mC <- lm(gross_log ~ budget_log + score,train)

  bsRes <- test %>%
    mutate(pB = predict(mB,newdata = test),
           pS = predict(mS,newdata = test),
           pC = predict(mC,newdata = test)) %>%
    summarise(Budget = sqrt(mean((gross_log - pB)^2,na.rm=T)),
              Score = sqrt(mean((gross_log - pS)^2,na.rm=T)),
              Combined = sqrt(mean((gross_log - pC)^2,na.rm=T))) %>%
    bind_rows(bsRes)
}
```

ASIDE: alternative code

- `sample_n()` and `anti_join()`

```
set.seed(123)
bsRes <- NULL
for(i in 1:100) {
  train <- mv %>%
    sample_n(size = round(nrow(.)*.8),replace = F)
  test <- mv %>%
    anti_join(train)

  mB <- lm(gross_log ~ budget_log,train)
  mS <- lm(gross_log ~ score,train)
  mC <- lm(gross_log ~ budget_log + score,train)

  bsRes <- test %>%
    mutate(pB = predict(mB,newdata = test),
           pS = predict(mS,newdata = test),
           pC = predict(mC,newdata = test)) %>%
    summarise(Budget = sqrt(mean((gross_log - pB)^2,na.rm=T)),
              Score = sqrt(mean((gross_log - pS)^2,na.rm=T)),
              Combined = sqrt(mean((gross_log - pC)^2,na.rm=T))) %>%
    bind_rows(bsRes)
}
```

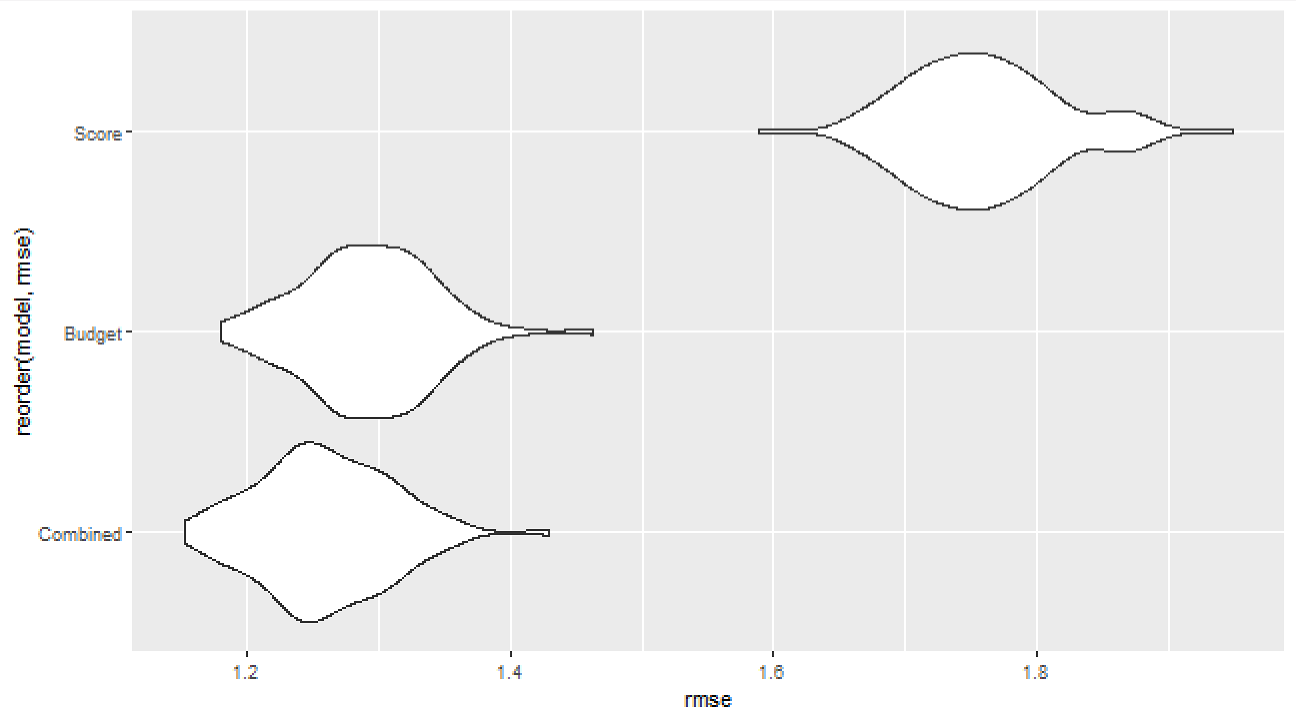
Why RMSE?

```
bsRes %>%
  summarise_all(mean,na.rm=T)
```

```
## # A tibble: 1 × 3
##   Budget Score Combined
##    <dbl> <dbl>    <dbl>
## 1   1.29  1.76     1.26
```

Visualizing

```
bsRes %>%
  gather(model,rmse) %>%
  ggplot(aes(x = rmse,y = reorder(model,rmse))) +
  geom_violin()
```



Categorical Data

- Thus far, only using continuous variables
- But we can do regression with categorical data too!
- The Bechdel Test: 3 questions of a movie

  1. Does it have two women in it?
  2. Who talk to each other?
  3. About something other than a man?

```
mv %>%
  count(bechdel_score)
```

```
## # A tibble: 5 × 2
##   bechdel_score     n
##           <dbl> <int>
## 1             0   141
## 2             1   526
## 3             2   206
## 4             3  1185
## 5            NA  1121
```

Research Question

- Do movies that pass the Bechdel Test make more money?
  - Theory: Women are ~50% of the population. Movies that pass the test are more appealing to women.
  - Hypothesis: Movies that pass the test make more money.
- Wrangling: Let's turn the `bechdel_score` variable into a binary

```
mv <- mv %>%
  mutate(bechdel_bin = ifelse(bechdel_score == 3,1,0)) %>%
  mutate(bechdel_factor=recode_factor(bechdel_bin,
                                      `1`="Pass",
                                      `0`="Fail",
                                      ))
```

Regression

- We can add the binary factor to our regression

```
summary(lm(gross_log ~ bechdel_factor,mv))
```

```
##
## Call:
## lm(formula = gross_log ~ bechdel_factor, data = mv)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.8817 -0.7918  0.2253  1.0831  3.8225
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)        18.16844    0.04835 375.794   <2e-16 ***
## bechdel_factorFail  0.15969    0.07423   2.151   0.0316 *
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.664 on 2056 degrees of freedom
##   (1121 observations deleted due to missingness)
```

Regression

- Coefficient is positive
- What is the interpretation?
  - Movies that fail make more money...
  - ...than what?
  - Movies that pass the Bechdel Test
- Categorical variables are **always interpreted in relation to the hold-out category**!

Regression

- Movies that fail the test make more money!?
- **REMEMBER**: Correlation $\neq$ causation
  - What might explain this pattern?
  - Budgets in a sexist Hollywood!
  - Movies that fail the test get larger budgets
  - Budgets are positively associated with gross
- So we want to "control" for budget by adding it to our regression

```
mBechCtrl <- lm(gross_log ~ budget_log + bechdel_factor,mv)
```

Regression

```
summary(mBechCtrl)
```

```
##
## Call:
## lm(formula = gross_log ~ budget_log + bechdel_factor, data = mv)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.6325 -0.5305  0.1287  0.6792  7.9370
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)         2.30814    0.34497   6.691 2.85e-11 ***
## budget_log          0.92089    0.01993  46.199  < 2e-16 ***
## bechdel_factorFail -0.18795    0.05254  -3.577 0.000356 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.166 on 2055 degrees of freedom
##   (1121 observations deleted due to missingness)
## Multiple R-squared:  0.5106,    Adjusted R-squared:  0.5101
```

Regression

- Our hypothesis is supported!
- What about non-binary categorical variables?

```
mv %>%
  count(rating)
```

```
## # A tibble: 9 × 2
##   rating        n
##   <chr>     <int>
## 1 G            54
## 2 NC-17         6
## 3 Not Rated    34
## 4 PG          434
## 5 PG-13      1249
## 6 R          1388
## 7 TV-MA         2
## 8 Unrated       7
## 9 <NA>          5
```

Categorical

- Let's first remove rarely-occurring ratings

```
mvAnalysis <- mv %>%
  filter(!rating %in% c('Approved','TV-14','TV-MA','TV-PG','X'))
```

Categorical

```
summary(lm(gross_log ~ rating,mvAnalysis))
```

```
##
## Call:
## lm(formula = gross_log ~ rating, data = mvAnalysis)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.6749 -0.8189  0.1630  1.1082  5.2339
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       19.1818     0.2177  88.113  < 2e-16 ***
## ratingNC-17       -2.4483     0.6884  -3.556 0.000381 ***
## ratingNot Rated   -4.4322     0.3502 -12.655  < 2e-16 ***
## ratingPG          -0.3905     0.2308  -1.692 0.090784 .
## ratingPG-13       -0.7633     0.2224  -3.433 0.000605 ***
## ratingR           -1.9123     0.2219  -8.618  < 2e-16 ***
## ratingUnrated     -4.6564     0.6426  -7.246 5.38e-13 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Categorical

- Everything makes less money than the hold-out category!
  - "G"-rated movies are powered by children
- What if we wanted to compare to a different reference category?

```
mvAnalysis <- mvAnalysis %>%
  mutate(rating = factor(rating,
                         levels = c('R','PG-13','PG','G','Not
Rated')))

mRating2 <- lm(gross_log ~ rating,mvAnalysis)
```

Categorical

```
summary(mRating2)
```

```
##
## Call:
## lm(formula = gross_log ~ rating, data = mvAnalysis)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.6749 -0.8184  0.1610  1.1082  5.2339
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)       17.26952    0.04296 402.005   <2e-16 ***
## ratingPG-13        1.14905    0.06242  18.408   <2e-16 ***
## ratingPG           1.52178    0.08802  17.289   <2e-16 ***
## ratingG            1.91231    0.22199   8.614   <2e-16 ***
## ratingNot Rated -2.51988    0.27782  -9.070   <2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.6 on 3154 degrees of freedom
```

Cross Validation

- This is why `sample_n()` is useful

```
set.seed(123)
rmseRes_rating <- NULL
for(i in 1:100) {
  train <- mvAnalysis %>%
    group_by(rating) %>%
    sample_n(size = round(n()*.8),replace = F)
  test <- mvAnalysis %>% anti_join(train)

  m <- lm(gross_log ~ rating,train)
  rmseRes_rating <- test %>%
    mutate(preds = predict(m,newdata = test)) %>%
    summarise(rmse = sqrt(mean((gross_log - preds)^2,na.rm=T))) %>%
    bind_rows(rmseRes_rating)
}
rmseRes_rating %>%
  summarise(rmse = mean(rmse))
```

```
## # A tibble: 1 × 1
##    rmse
##   <dbl>
```

Quiz & Homework

- Go to Brightspace and take the **13th** quiz
    - The password to take the quiz is ####
- **Homework:**
    1. Homework 14 (due next week)