Regression

Part 1

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Agenda

1. Modeling Conditional Variation
2. Adding Regression to the **Process**
3. Introducing the Data
4. Demonstrating Regressions

Regression & Conditional Analysis

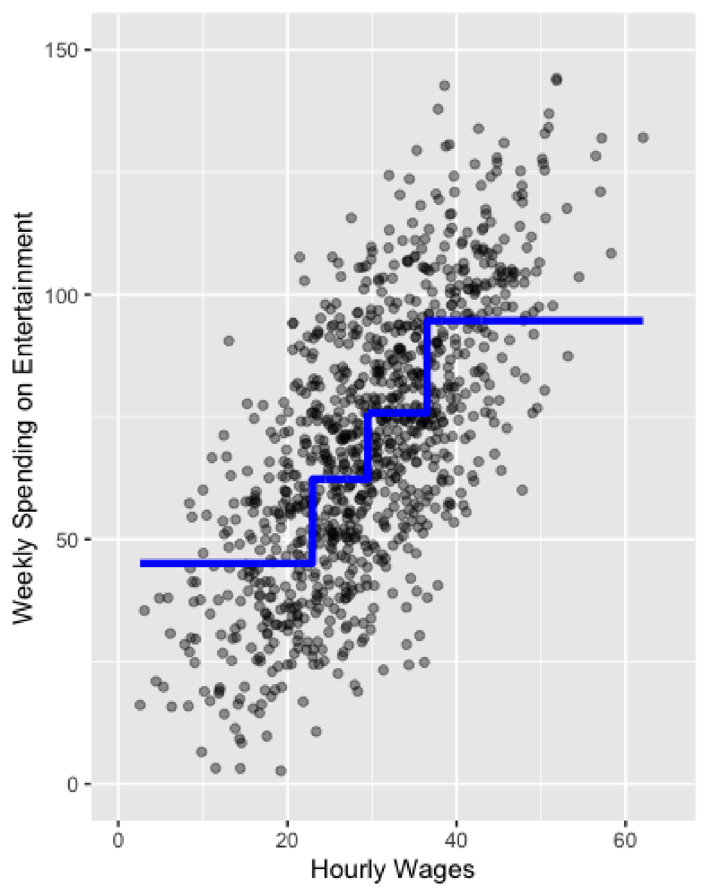
- Recall our discussion of **conditional** analysis
    - Conditional → **depends on**
    - Analyze with **conditional means**

Reminder of the **Process**

1. Determine variable **type**
    - I.e., categorical (ordered, unordered, binary) or continuous
    - In `R` terms: `chr`, `fct` for categorical, `dbl` for continuous
2. Type informs **univariate analysis**
    - I.e., histograms for continuous, barplots for categorical
3. Combination of types informs **conditional analysis**
    - Categorical X Categorical: proportions by categories (`geom_bar`)
    - Binary X Continuous: histograms by categories (`geom_histogram` / `geom_density`)
    - Categorical X Continuous: distributions by categories (`geom_boxplot` / `geom_violin`)
    - Continuous X Continuous: scatter plots (`geom_point`)

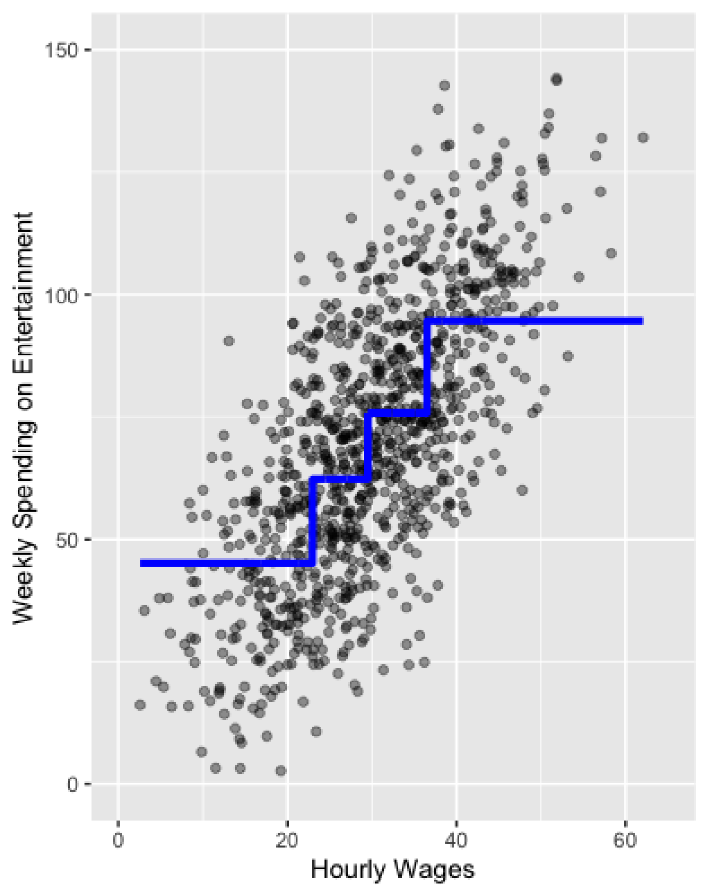
What is regression?

- Conditional means for continuous data



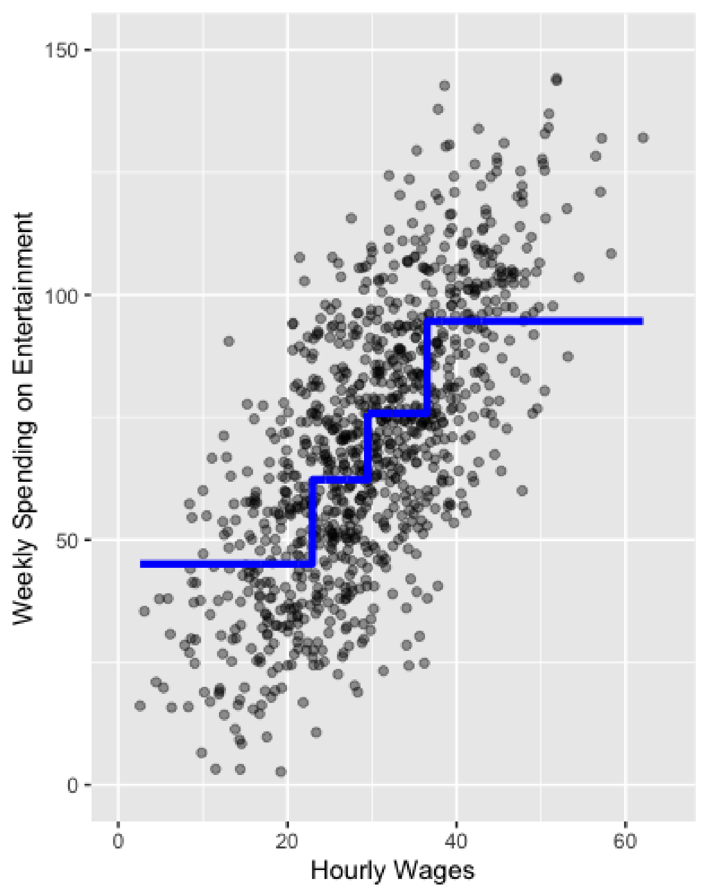
What is regression?

- People with hourly wages < $20 spend ~$50 on entertainment per week



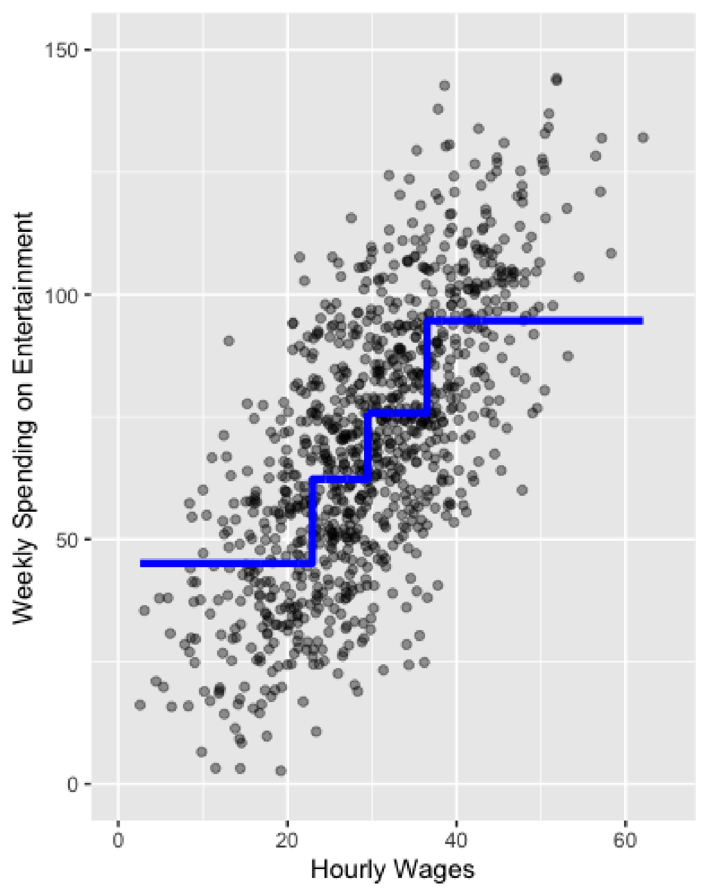
What is regression?

- People with hourly wages > $40 spend ~$95 on entertainment per week



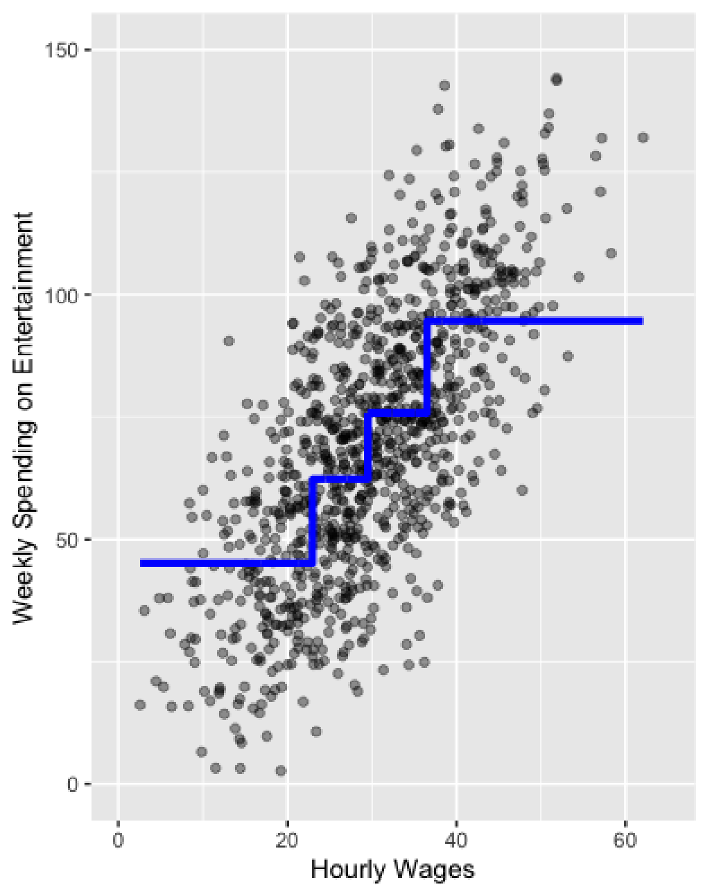
What is regression?

- Theory: the more you earn, the more you spend



What is regression?

- But **conditional means** make a lot of mistakes. Can we do better?



What is regression?

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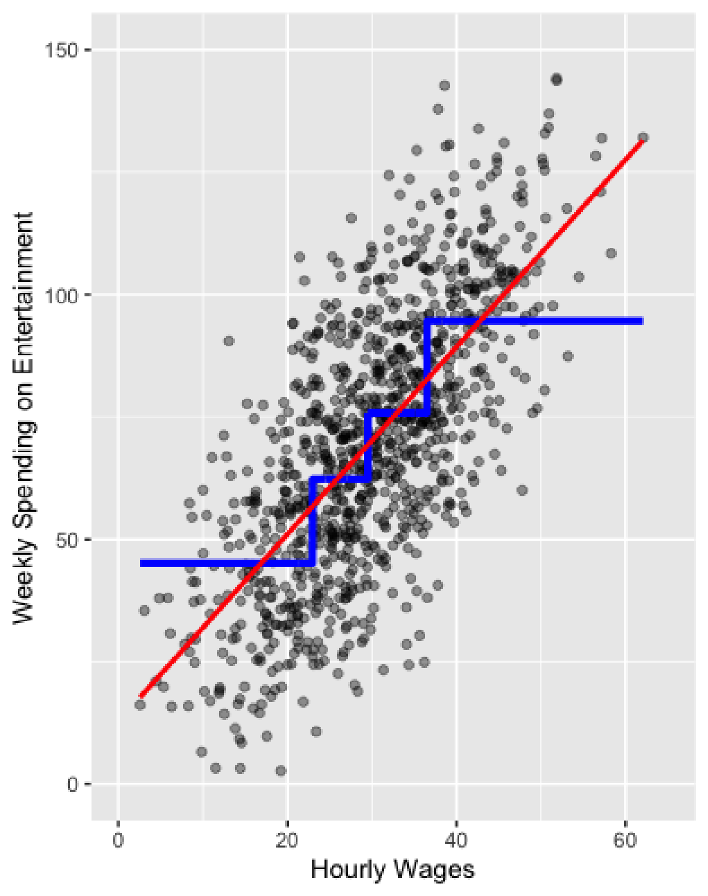
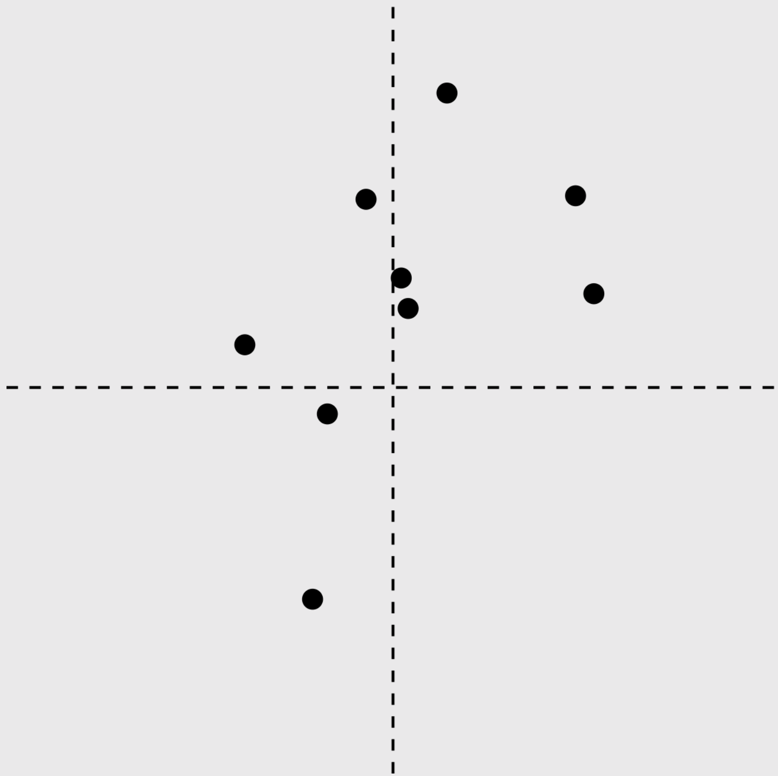


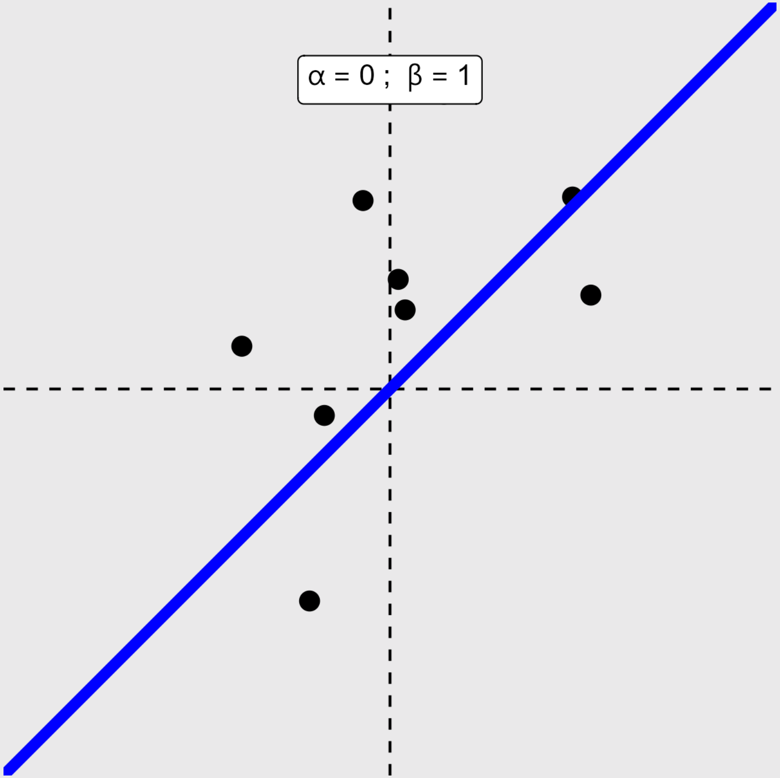
Regression

- Calculating a **line** that minimizes mistakes *for every observation*
    - NB: could be a curvey line! For now, just assume straight
- Recall from geometry how to graph a straight line
- $Y = a + bX$
    - a: the "intercept" (where the line intercepts the y-axis)
    - b: the "slope" (how much Y changes for each increase in X)
- (Data scientists use α and β instead of a and b b/c nerds)
- Regression analysis simply chooses the best line
    - "Best"?
    - The line that minimizes the mistakes (the **line of best fit**)

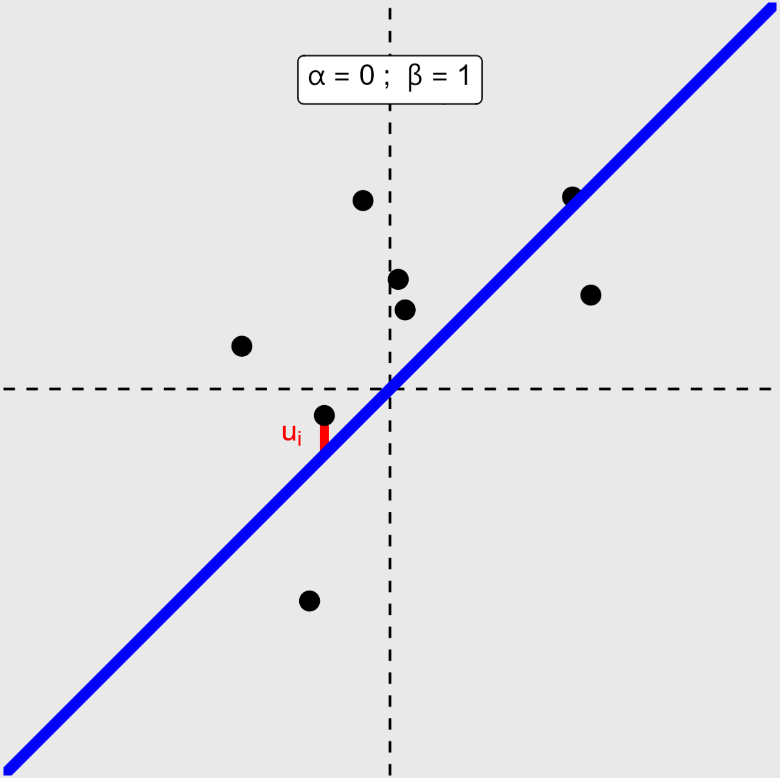
Linear Regression



Linear Regression

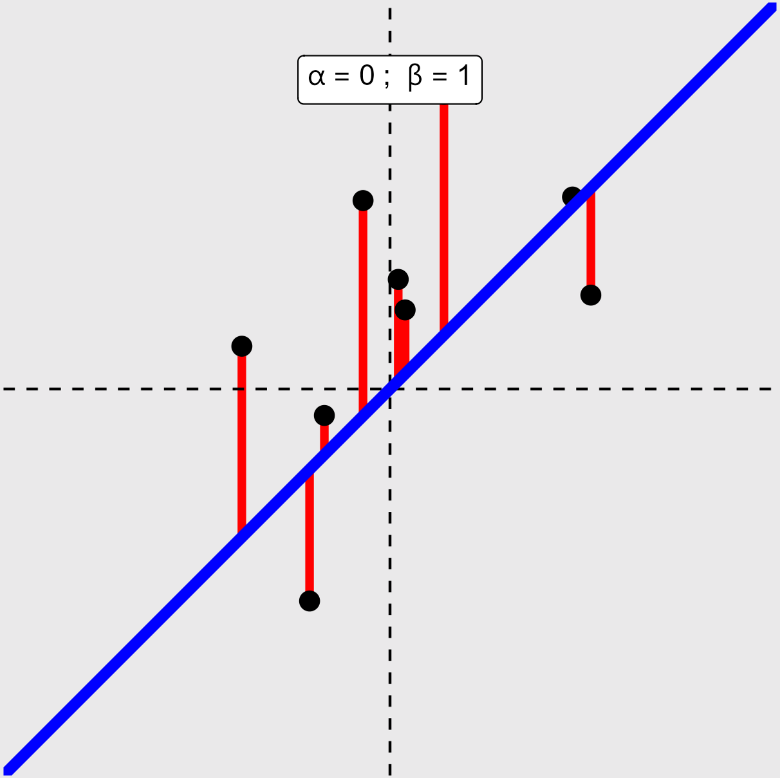


Linear Regression



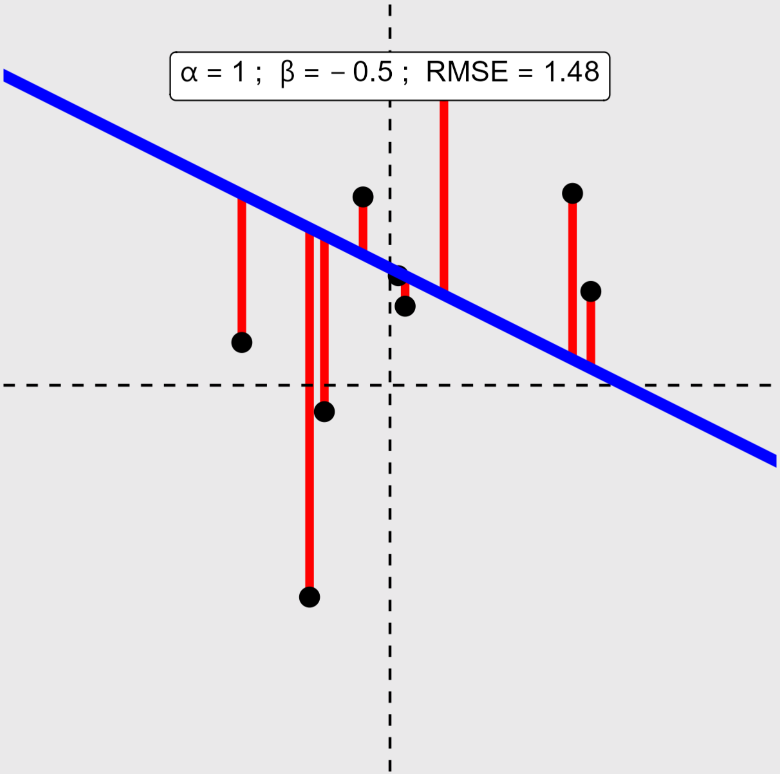
- **Error/Residual**: mistake made by a line
  - In math: $u_i = y_i - \hat{y}_i$
  - In English: difference between true outcome value (y_i) and prediction ($\hat{y}_i$)

Linear Regression



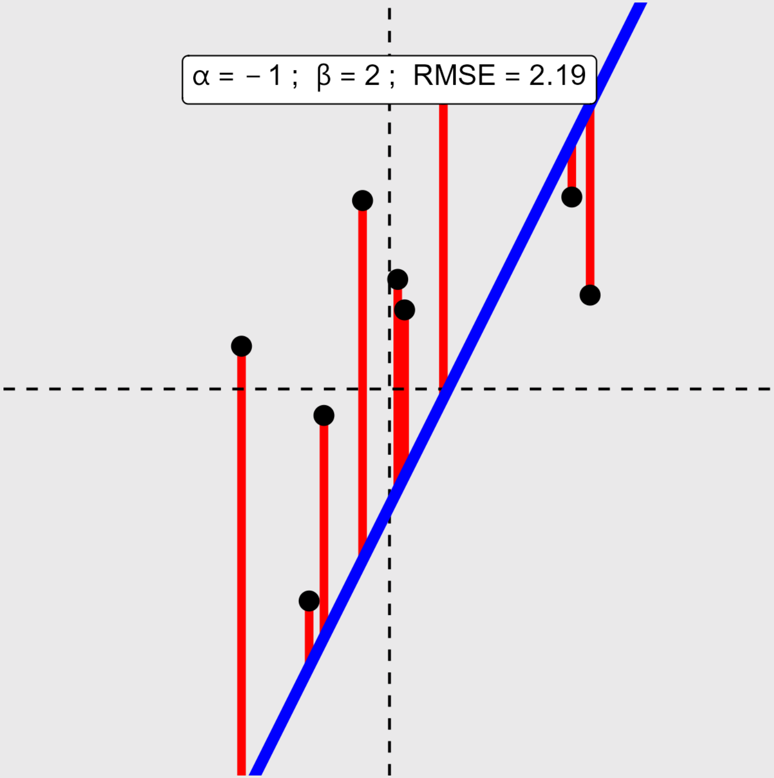
- Use **errors** to find **line of best fit**
- **RMSE** (**R**oot **M**ean **S**quared **E**rror)
  - Square the errors
  - Take their average
  - Take the square root
- **RMSE** = 1.23

Linear Regression



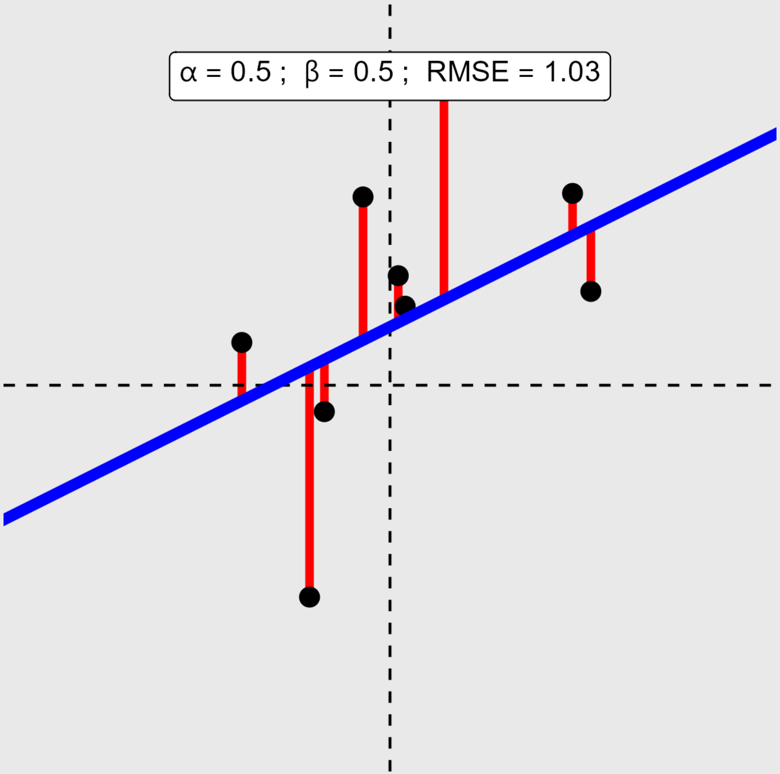
- Use **errors** to find **line of best fit**
- **RMSE** (**R**oot **M**ean **S**quared **E**rror)
    - Square the errors
    - Take their average
    - Take the square root
- **RMSE** = 1.48

Linear Regression



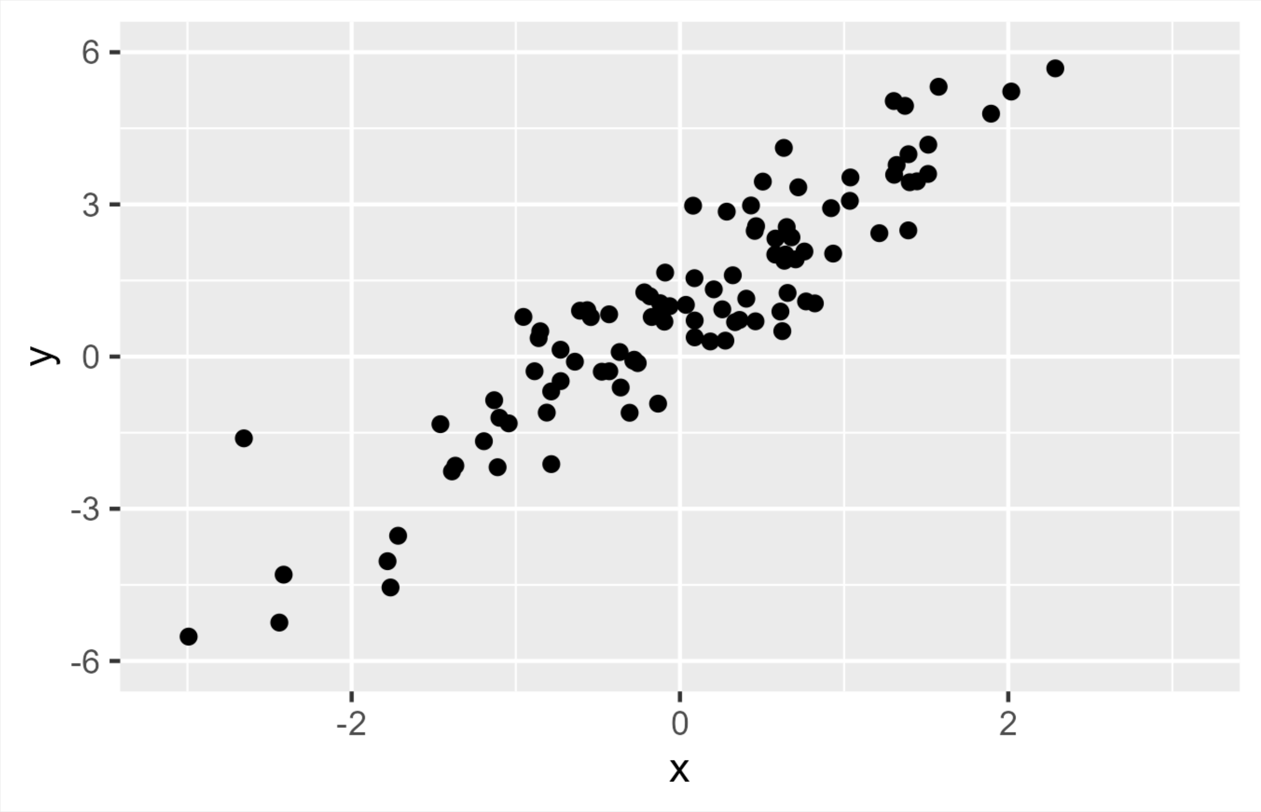
- Use **errors** to find **line of best fit**
- **RMSE** (**R**oot **M**ean **S**quared **E**rror)
  - Square the errors
  - Take their average
  - Take the square root
- **RMSE** = 2.19

Linear Regression

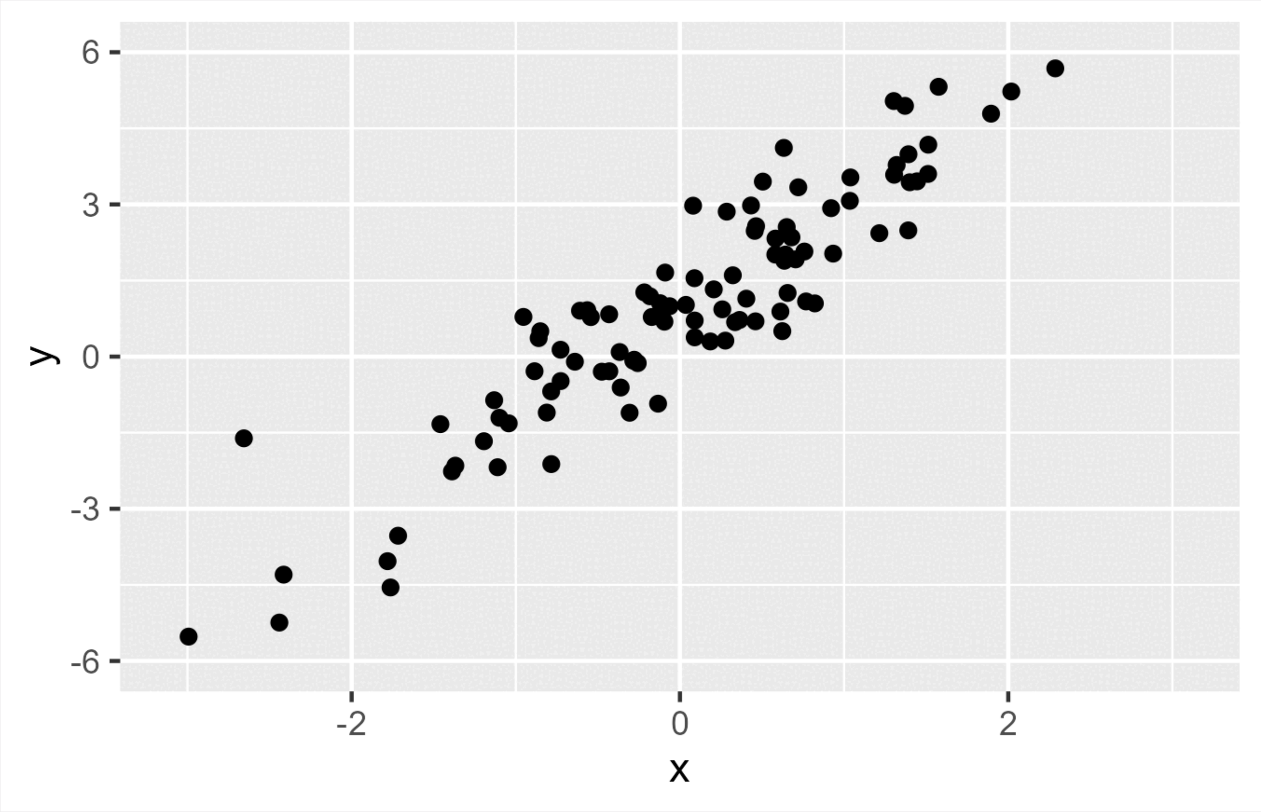


- Use **errors** to find **line of best fit**
- **RMSE** (**R**oot **M**ean **S**quared **E**rror)
  - Square the errors
  - Take their average
  - Take the square root
- **RMSE** = 1.03

Visual Intuition



Visual Intuition



Regression

- The line is substantively meaningful
- Red line on scatter plot of spending and wages: $Y = 12 + 2 * X$
- α tells us the value of Y when X is zero
  - People who don't make any money spend $12 per week on entertainment
- β tells us how much Y increases for each additional X
  - People spend an additional $2 per week for each additional $1 in hourly wages

Regression

- These are called **"linear models"**
  - **Not** because the line is straight (it might not be)
  - but because the components are additive ($\alpha + \beta X$)
- Can extend to multiple predictors (X 's)
  - $Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \cdots + \varepsilon$
  - X_1 might be wages and X_2 might be age (for example)
  - The final term ε measures how bad our mistakes are

Regression

- Let's demonstrate with the `debt` data

```
require(tidyverse)

debt <-
read_rds('https://github.com/jbisbee1/DS1000_S2024/raw/main/data/sc_deb

glimpse(debt)
```

```
## Rows: 2,546
## Columns: 16
## $ unitid        <int> 100654, 100663, 100690, 100706, 100…
## $ instnm        <chr> "Alabama A & M University", "Univer…
## $ stabbr        <chr> "AL", "AL", "AL", "AL", "AL", "AL",…
## $ grad_debt_mdn <int> 33375, 22500, 27334, 21607, 32000, …
## $ control       <chr> "Public", "Public", "Private", "Pub…
## $ region        <chr> "Southeast", "Southeast", "Southeas…
## $ preddeg       <chr> "Bachelor's", "Bachelor's", "Associ…
## $ openadmp      <int> 2, 2, 1, 2, 2, 2, 1, NA, 2, 2, 2, 1…
## $ adm_rate      <dbl> 0.9175, 0.7366, NA, 0.8257, 0.9690,…
## $ ccbasic       <int> 18, 15, 20, 16, 19, 15, 2, 22, 18, …
## $ sat_avg       <int> 939, 1234, NA, 1319, 946, 1261, NA,…
```

Research Camp

- Research Question: What is the relationship between SAT scores and median future earnings?
- Theory: Students with higher SAT scores work harder and have learned more. Employers reward these attributes with higher wages in the private market.
- Hypothesis: The relationship between SAT scores and future earnings should be positive.
    - **NB**: Important caveats to this simplistic theory!
    - Socioeconomic status: predicts both higher SAT scores and higher wages
    - **Correlation $\neq$ Causation**

Set Up

- Linking Theory to Data
- Our SAT scores are theorized to explain future earnings
  - Thus the SAT scores are the independent / explanatory / predictor variable X
  - And earnings are the dependent / outcome variable Y

Regression

- There is a simple recipe to follow
- And it is exactly how the syllabus for the class is designed!

1. Look at your data to identify missingness (**Wrangling: Lecture 5**)
2. **Univariate** visualization of your variables (**Lecture 6**)
3. **Multivariate** visualization of your variables (**Lectures 7-10**)
4. **Regression** (today)
5. Evaluation of **errors** (next lecture)

Step 1: Look

- Why worry about **missingness**?

1. **Substantive:** external validity

2. **Technical:** cross validation won't work! (Wednesday's lecture)

```
summary(debt %>% select(sat_avg,md_earn_wne_p6))
```

```
##     sat_avg     md_earn_wne_p6
##  Min.   : 737   Min.   : 10600
##  1st Qu.:1053   1st Qu.: 26100
##  Median :1119   Median : 31500
##  Mean   :1141   Mean   : 33028
##  3rd Qu.:1205   3rd Qu.: 37400
##  Max.   :1557   Max.   :120400
##  NA's   :1317   NA's   :240
```

Step 2: Univariate Viz

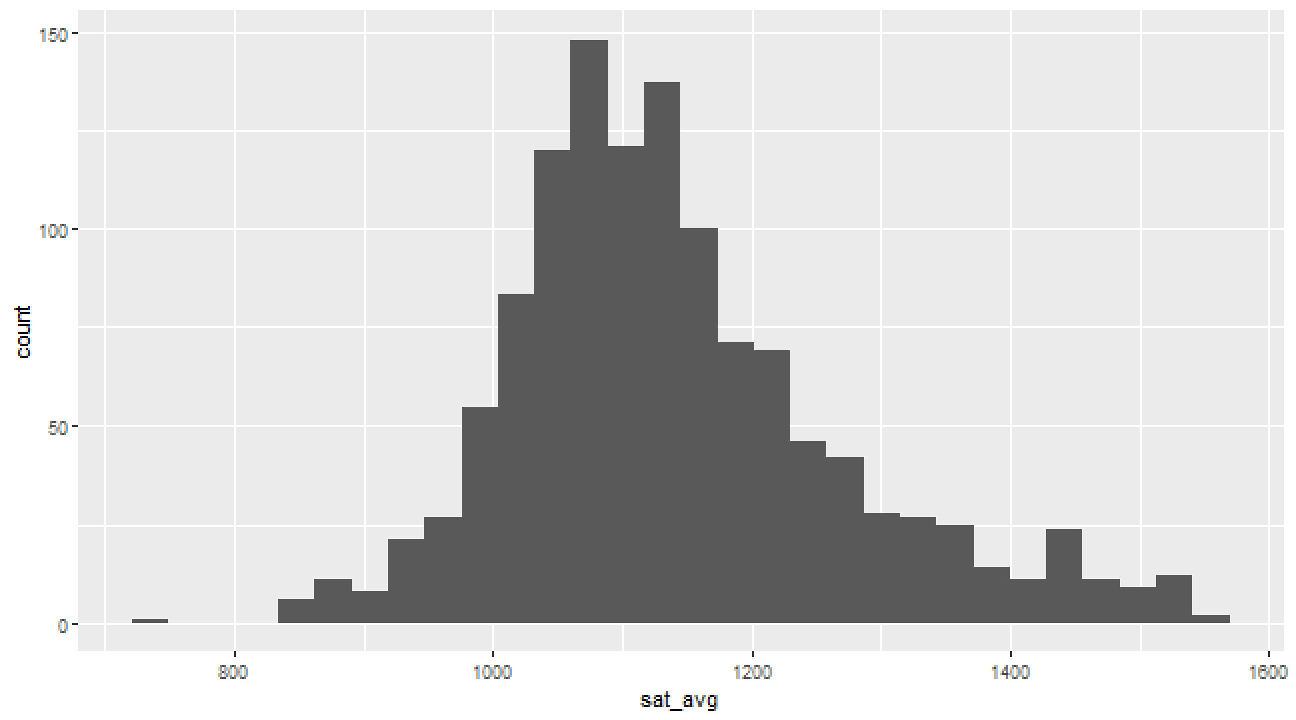
- Why visualize both Y and X?

1. **Substantive:** *See* which units you are talking about
2. **Technical:** Adjust for *skew*

Step 2: Univariate Viz

- Why visualize both Y and X?

```
debt %>%
  ggplot(aes(x = sat_avg)) +
  geom_histogram()
```



Step 2: Univariate Viz

- Why visualize both Y and X?

```
debt %>%
  ggplot(aes(x = md_earn_wne_p6)) +
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```
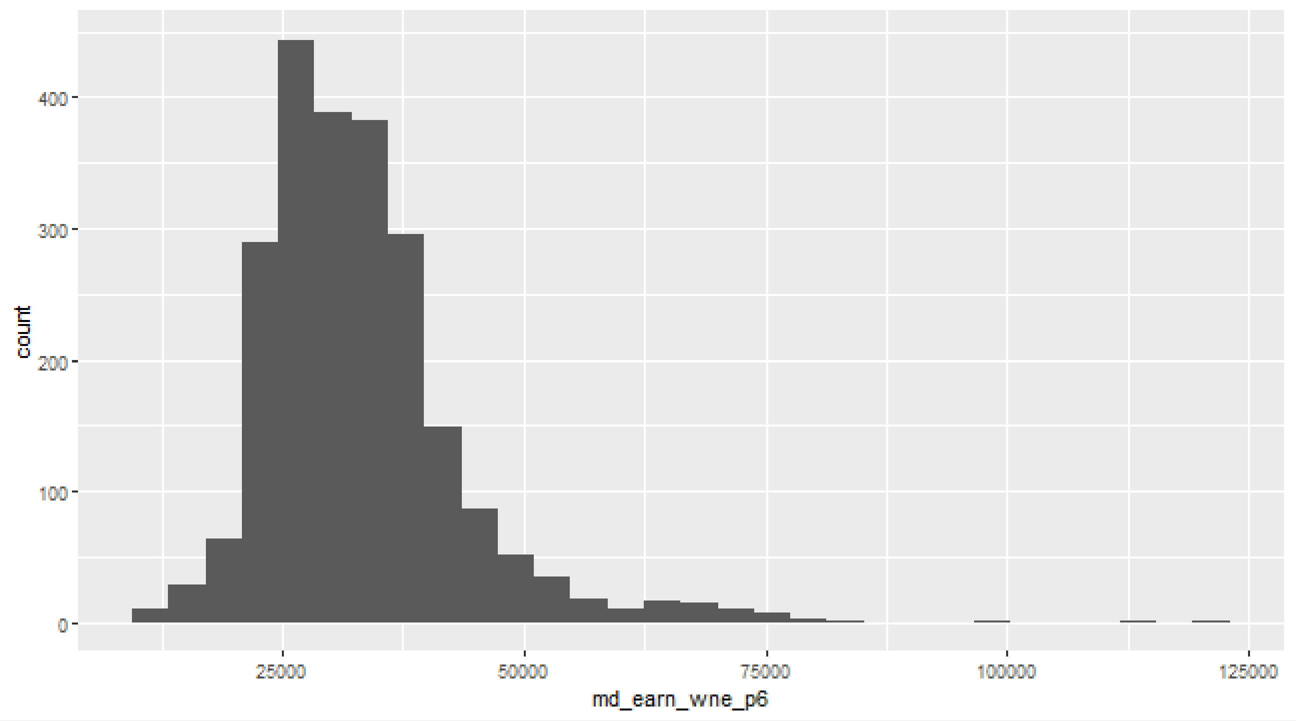
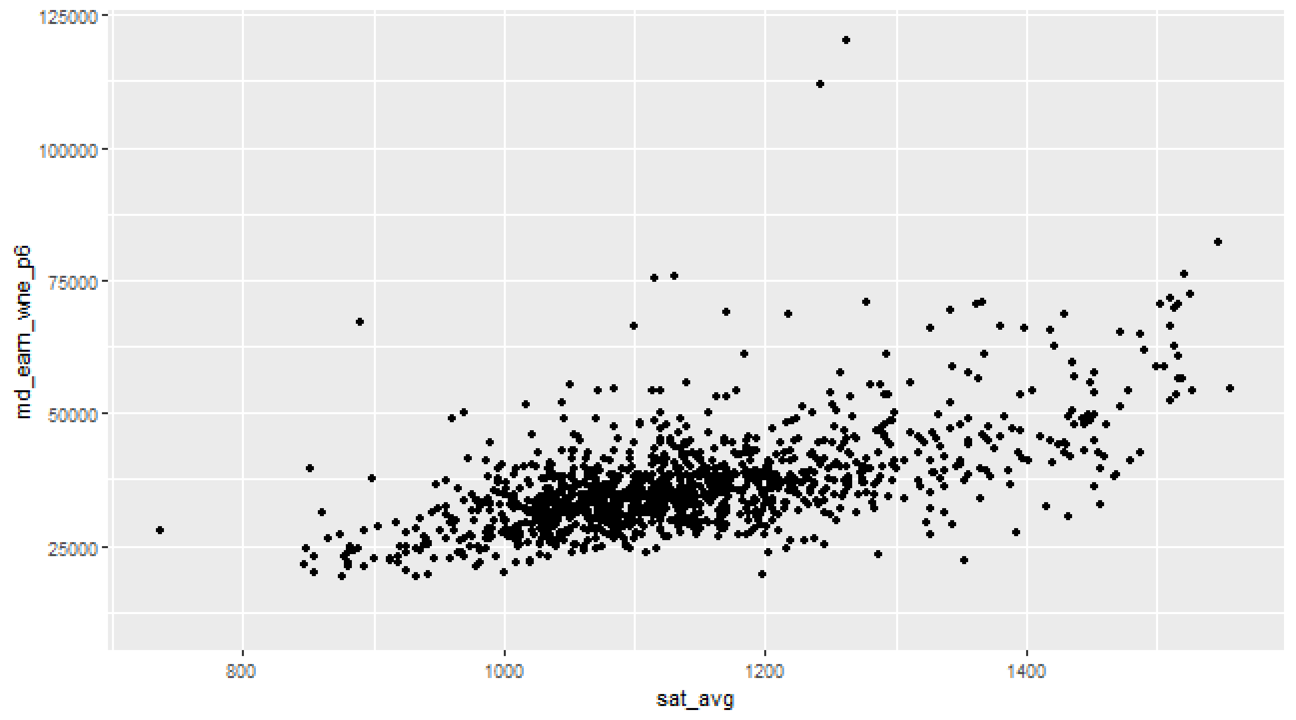



Step 3: Multivariate

- Eyeball the relationship first!

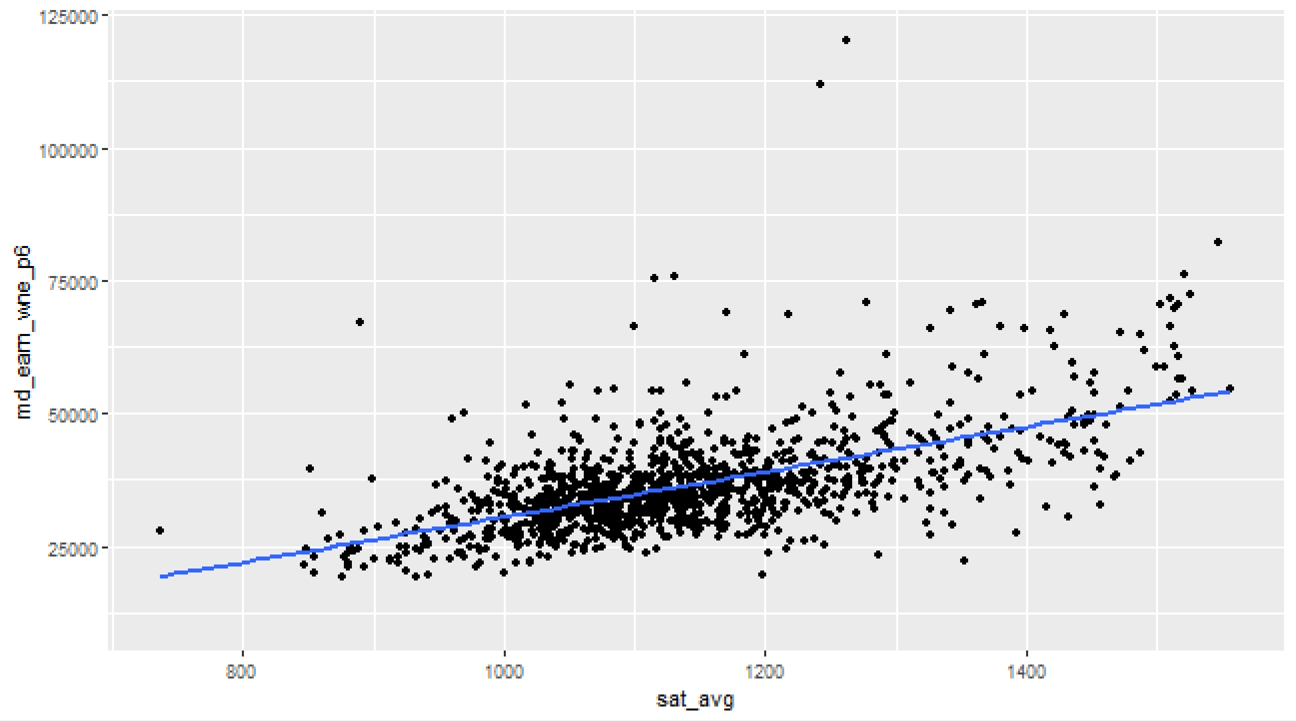
```
debt %>%
  ggplot(aes(x = sat_avg,y = md_earn_wne_p6)) +
  geom_point()
```



Step 3: Multivariate Viz

- Adding regression line

```
debt %>%
  ggplot(aes(x = sat_avg,y = md_earn_wne_p6)) +
  geom_point() + geom_smooth(method = 'lm',se = F)
```



Step 3: Multivariate Viz

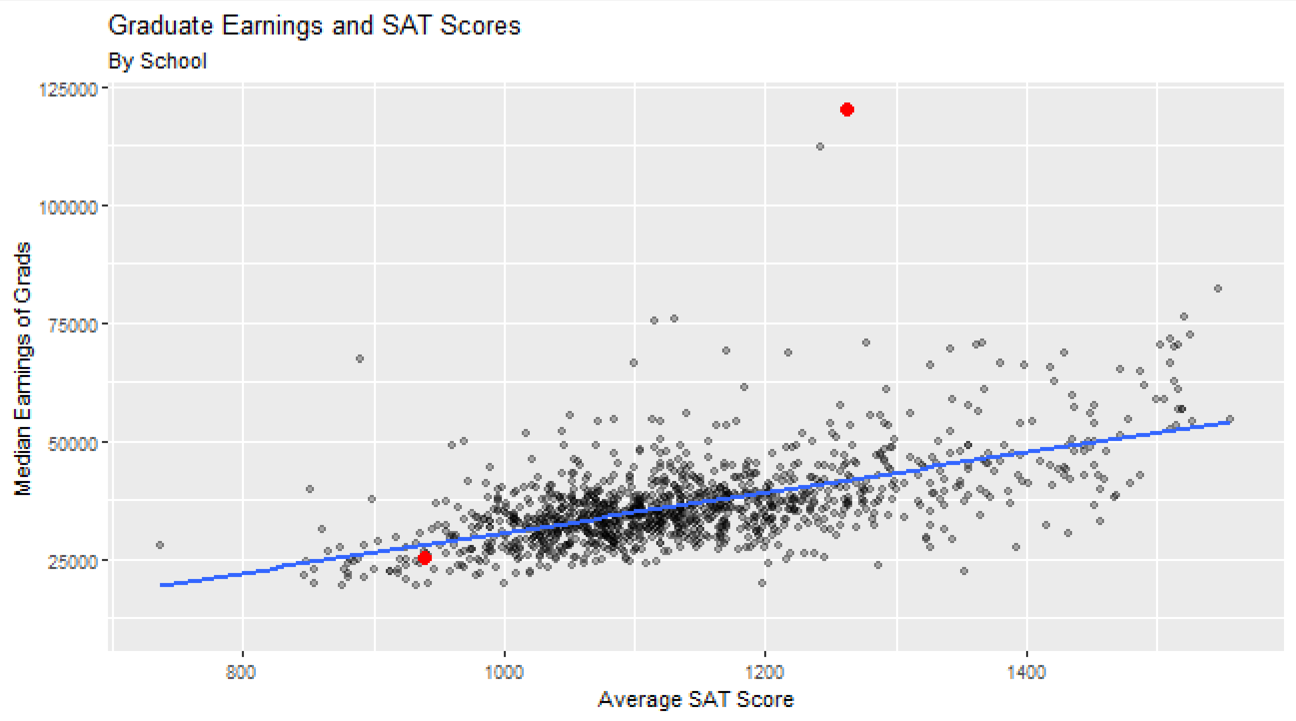
- Let's focus on two schools

```
toplot <- debt %>%
  mutate(hl = ifelse(unitid %in% c(100654,179265),'hl','none')) #
Choosing two examples
p2 <- toplot %>%
  ggplot(aes(x = sat_avg, y = md_earn_wne_p6,color = hl,group =
1,alpha = hl)) +
  geom_point(data = toplot %>% filter(hl == 'none')) +
  geom_point(data = toplot %>% filter(hl == 'hl'),size =3) +
  scale_alpha_manual(values = c(1,.3)) +
  scale_color_manual(values = c('red','black')) +
  geom_smooth(method = 'lm',se = F) +
  theme(legend.position = 'none') +
  labs(title = "Graduate Earnings and SAT Scores",
       subtitle = "By School",
       x = "Average SAT Score",
       y = "Median Earnings of Grads")
```

Step 3: Multivariate Viz

- Adding regression line

```
p2
```



Step 3: Multivariate Viz

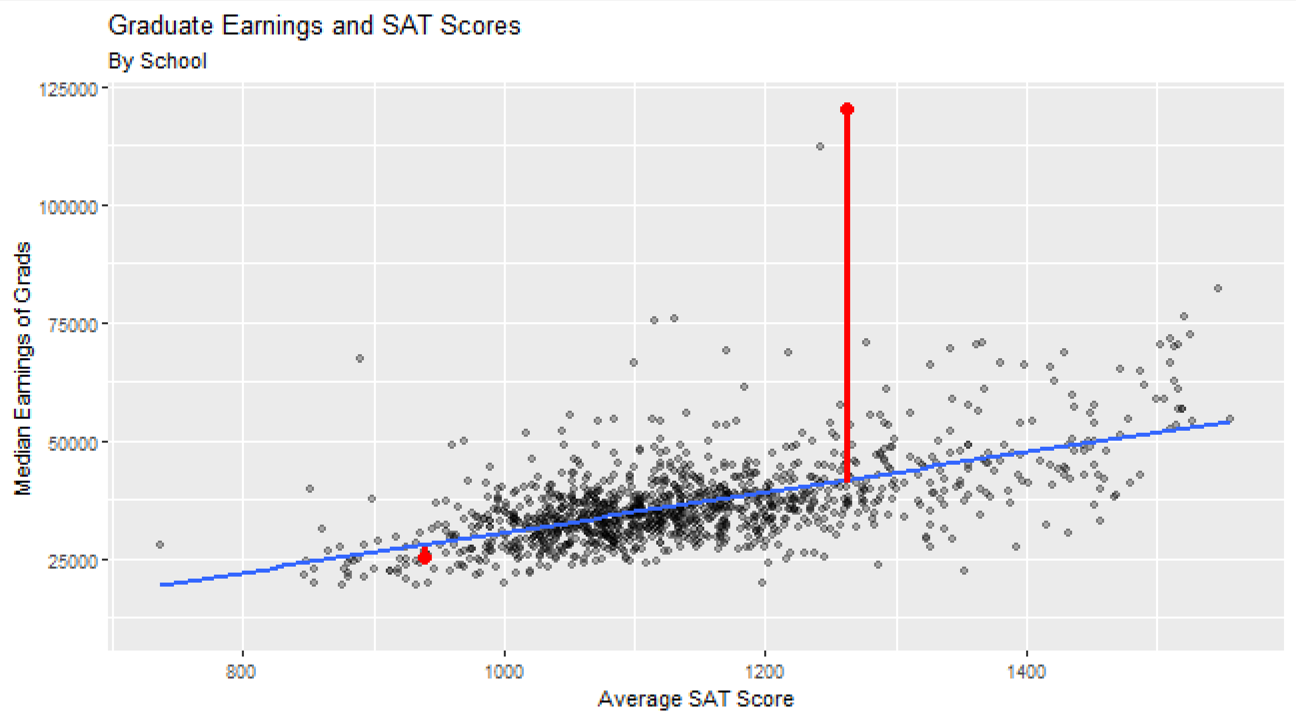
- Defining ε

```
p3 <- toplot %>%
  ggplot(aes(x = sat_avg, y = md_earn_wne_p6,color = hl,group =
1,alpha = hl)) +
  geom_point(data = toplot %>% filter(hl == 'none')) +
  geom_point(data = toplot %>% filter(hl == 'hl'),size =3) +
  scale_alpha_manual(values = c(1,.3)) +
  scale_color_manual(values = c('red','black')) +
  geom_smooth(method = 'lm',se = F) +
  annotate(geom = 'segment',
           x = toplot %>% filter(hl == 'hl') %>% .$sat_avg,
           y = toplot %>% filter(hl == 'hl') %>% .$md_earn_wne_p6,
           xend = toplot %>% filter(hl == 'hl') %>% .$sat_avg,
           yend = c(27500,41000),color = 'red',lwd = 1.2) +
  theme(legend.position = 'none') +
  labs(title = "Graduate Earnings and SAT Scores",
       subtitle = "By School",
       x = "Average SAT Score",
       y = "Median Earnings of Grads")
```

Step 3: Multivariate Viz

- Measuring errors

```
p3
```



The Data Scientist's Trade-off

- Those mistakes seem pretty big!
- Why not use a curvier line?

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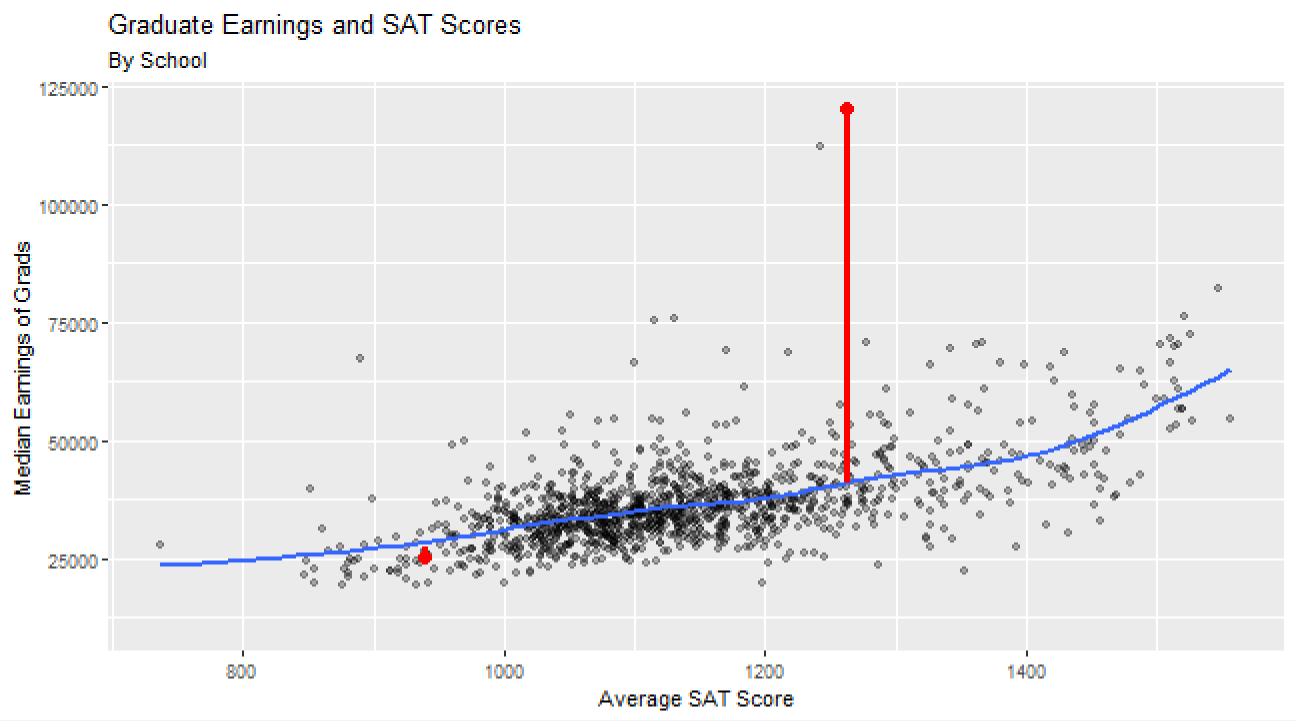

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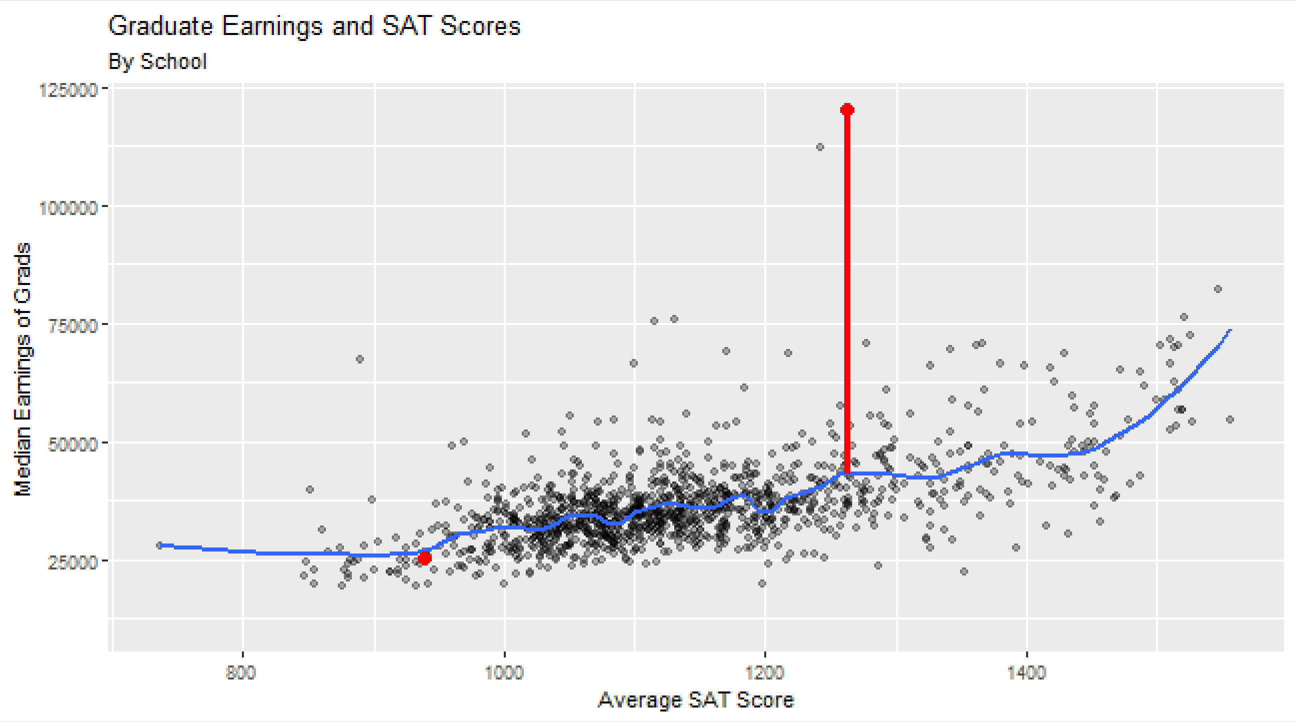

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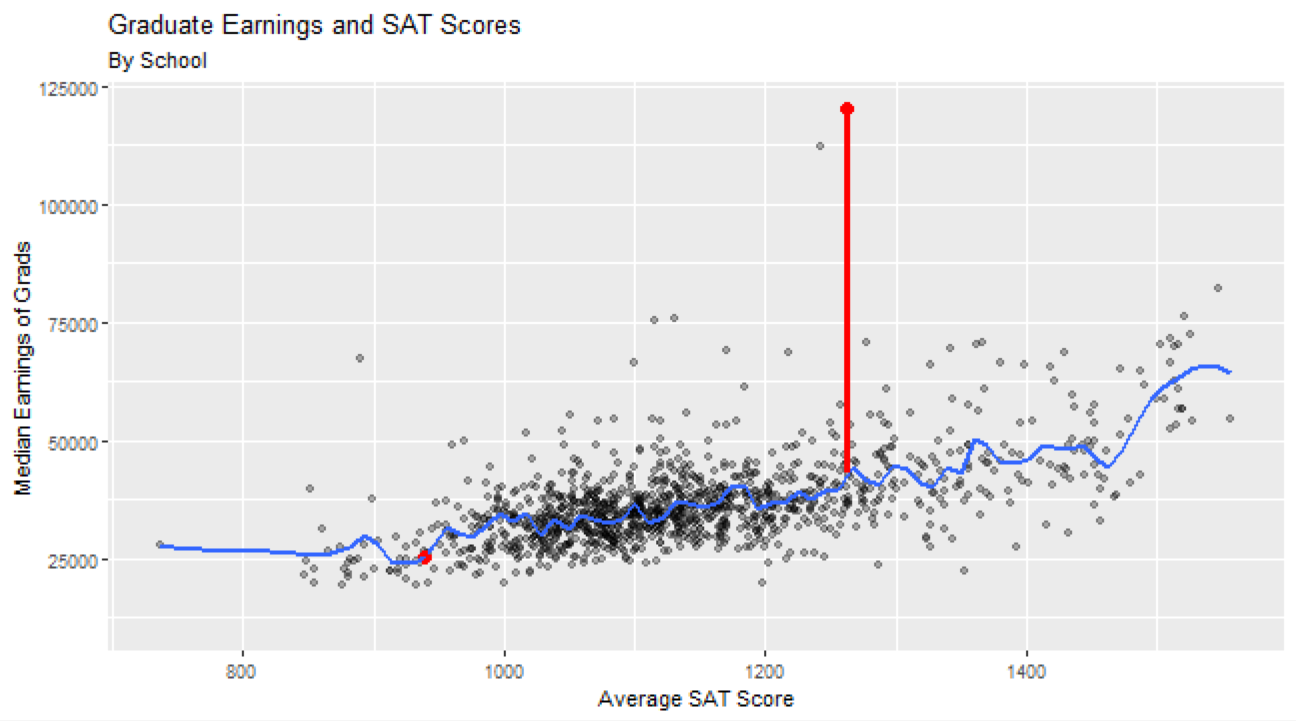

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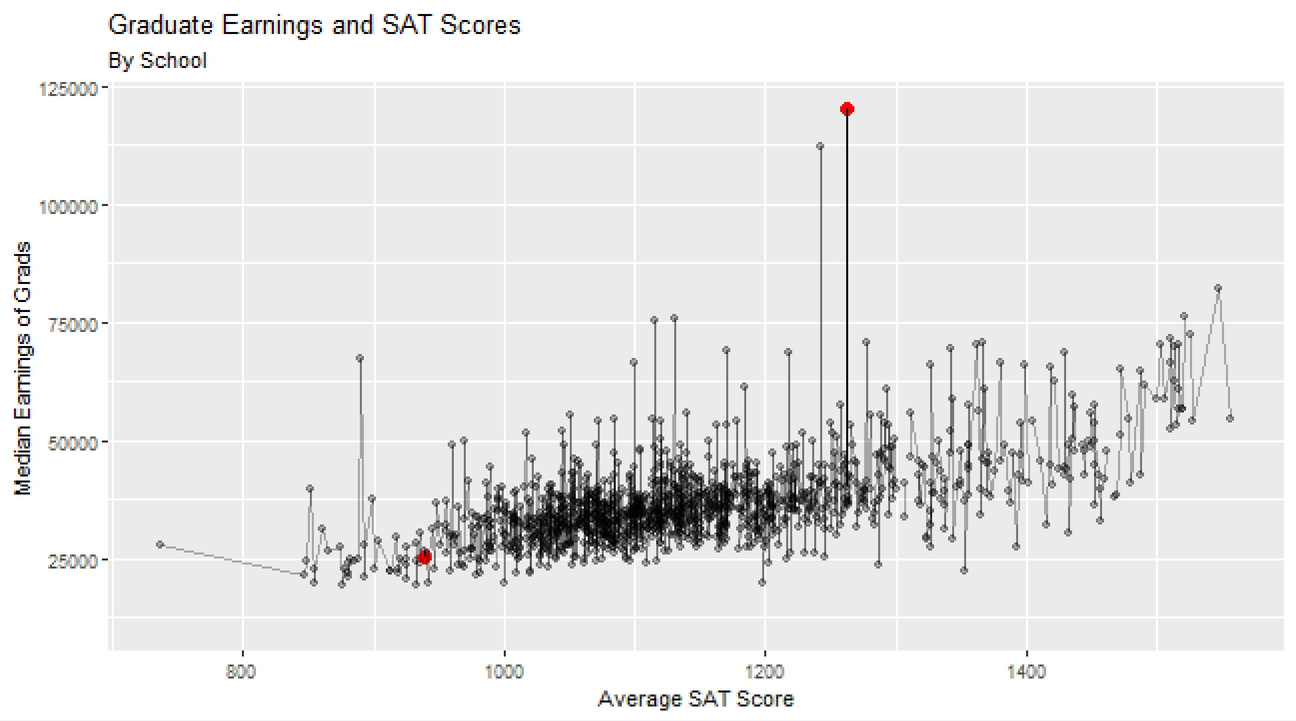

The Data Scientist's Trade-off

- Want to **reduce complexity**
- But also want to be **accurate**
- What is the right answer?
  - It depends on your theory and the data
  - It is context-dependent
- And this is still only using *linear regression models*!
  - This is a deep area of study, for those interested

Step 4: Regression

- Introducing the `lm(formula,data)` function
- Two inputs to care about:
  - `formula`: Code for $Y = \alpha + \beta X$
  - `data`: What is the data we are using?
- `formula` is written as `Y ~ X`
  - `R` will calculate α and β for us
  - Just need to tell it what is Y (`md_earn_wne_p6`) and X (`sat_avg`)
  - The tilde (`~`) is `R`'s version of the equals sign
- Save the model to an object

```
model_earn_sat <- lm(formula = md_earn_wne_p6 ~ sat_avg,data = debt)
```

Step 4: Interpretation

- What is in this object?
- The regression results! Look at them with `summary()`

```
summary(model_earn_sat)
```

```
##
## Call:
## lm(formula = md_earn_wne_p6 ~ sat_avg, data = debt)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -23239  -4311   -852   2893  78695
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -12053.87    1939.80  -6.214 7.12e-10 ***
## sat_avg         42.60       1.69  25.203  < 2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

Step 4: Interpretation

- Starting with the first column called `Estimate`
- 1st Row `(Intercept)` is α: the predicted value of Y when X is zero
  - Schools with average SAT scores of 0 produce graduates who earn -$12,053.87
  - Sensible?
- 2nd Row `sat_avg` is the β: the increase in Y when X increases by one
  - For each unit increase in the average SAT score, recent graduates earn $42.60 more
  - Sensible?

Step 4: Interpretation

- Other 3 columns?
  - `Std. Error` is the "standard error"
  - `t value` is the "t-statistic"
  - `Pr(>|t|)` is the "p-value"
- t-statistic = Estimate / standard error
- p-value = function(t-statistic)
  - Only really need to remember the p-value for this course
  - This is 1 minus confidence
  - The lower the p-value, the more confident we are that the `Estimate` is not zero

Step 4: Interpretation

```
summary(model_earn_sat)
```

```
##
## Call:
## lm(formula = md_earn_wne_p6 ~ sat_avg, data = debt)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -23239  -4311   -852   2893  78695
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -12053.87    1939.80  -6.214 7.12e-10 ***
## sat_avg         42.60       1.69  25.203  < 2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7594 on 1196 degrees of freedom
##   (1348 observations deleted due to missingness)
## Multiple R-squared:  0.3469,    Adjusted R-squared:  0.3463
## F-statistic: 635.2 on 1 and 1196 DF,  p-value: < 2.2e-16
```

Another Example

- We will come back to the RMSE next class
- For now, let's try with a different research question!
- What is the relationship between admissions and future earnings?
    - Theory: More selective schools are more prestigious
    - Hypothesis: There should be a negative relationship between the admissions rate and future earnings

Do It Together!

1. Look at the data and acknowledge missingness
2. Univariate visualization of X and Y
3. Multivariate visualization of X and Y
4. Regression

Quiz & Homework

- Go to Brightspace and take the **11th** quiz
  - The password to take the quiz is ####
- **Homework:**
  1. Work through ds1000_hw_12.Rmd
  2. Problem set 7 (last before the midterm!)