Clustering

Part 1

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Agenda

1. Structure in data
2. "Clustering"
3. Application

Structure

- Patterns in data
- Behind everything we've done thus far
    - Theory Testing: structure answers research question
    - Prediction: structure improves accuracy
- A third "camp" in data science: **Learning**

Learning

- No research question, no prediction goal
  - Just want to learn about **structure** of data
- Existing tools can do it
  - Run 1m regressions
  - Visualize a thousand variables
  - But these are *slow*
- This topic: letting **algorithms** learn for you!
  - Today: clustering

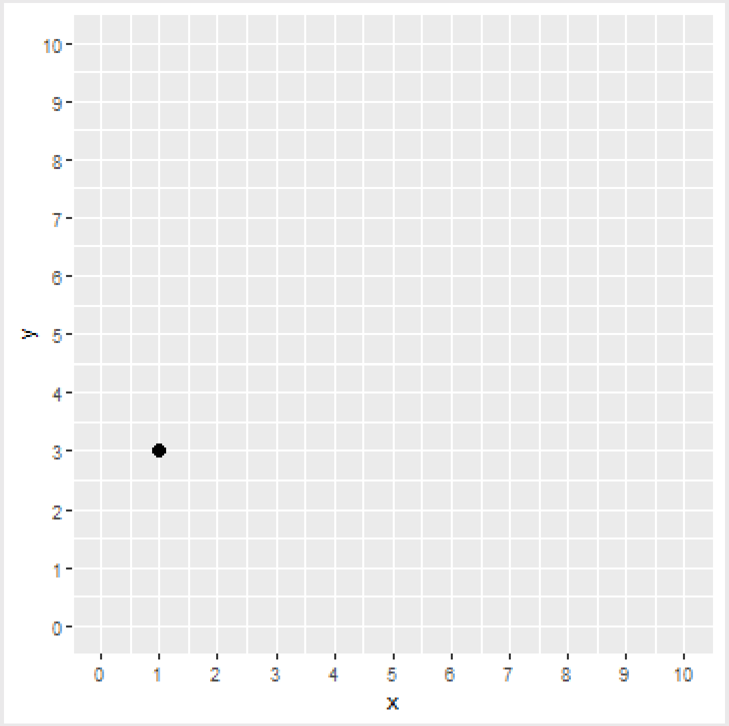
Clustering

- Identify observations that belong to groups
  - Similarities → group belonging
- Part of broader set of methods to identify underlying "structure"
  - Today: *k*-means clustering algorithm

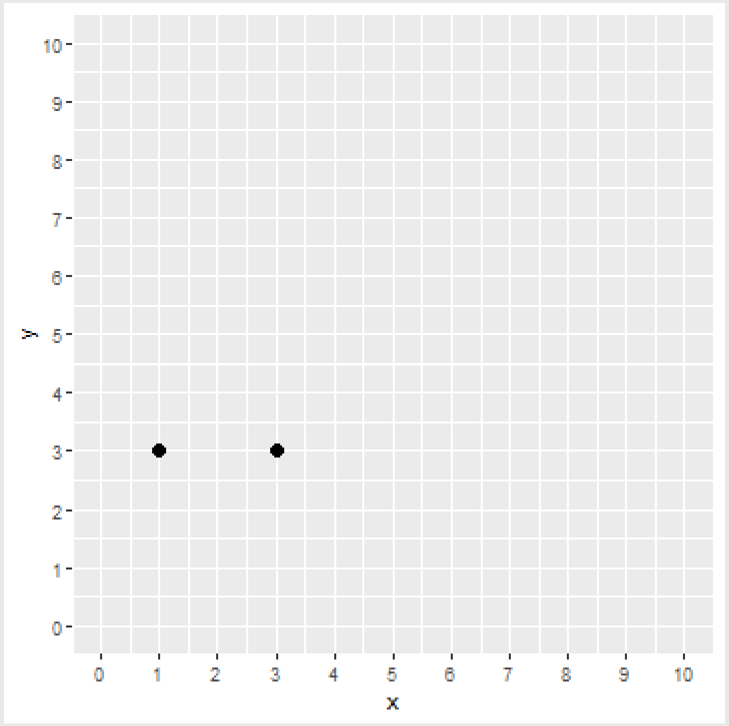
k-means Clustering

- *k*: number of clusters (i.e., groups)
- Algorithm assigns each observation to these $1 \ldots k$ groups
    1. Choose initial "centroids" at random
    2. Assign observations to each centroid based on "Euclidean distance"
    3. Calculate new centroid based on mean of each variable
    4. Repeat until assignments stabilize

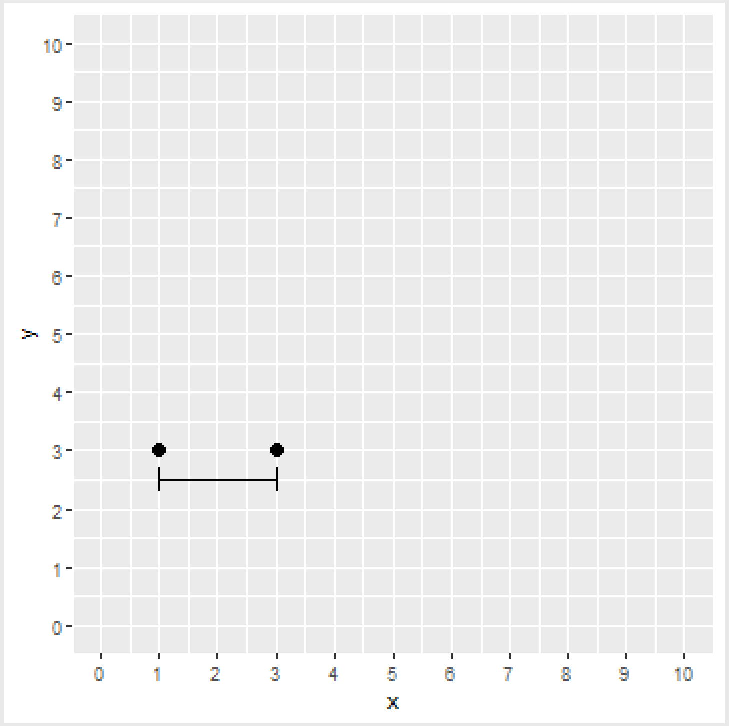
Euclidean Distance



Euclidean Distance



Euclidean Distance



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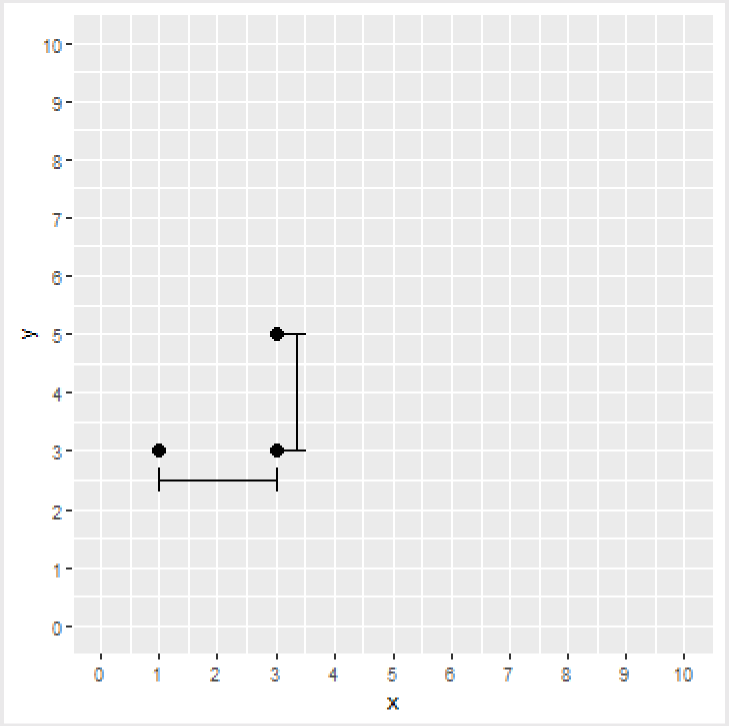

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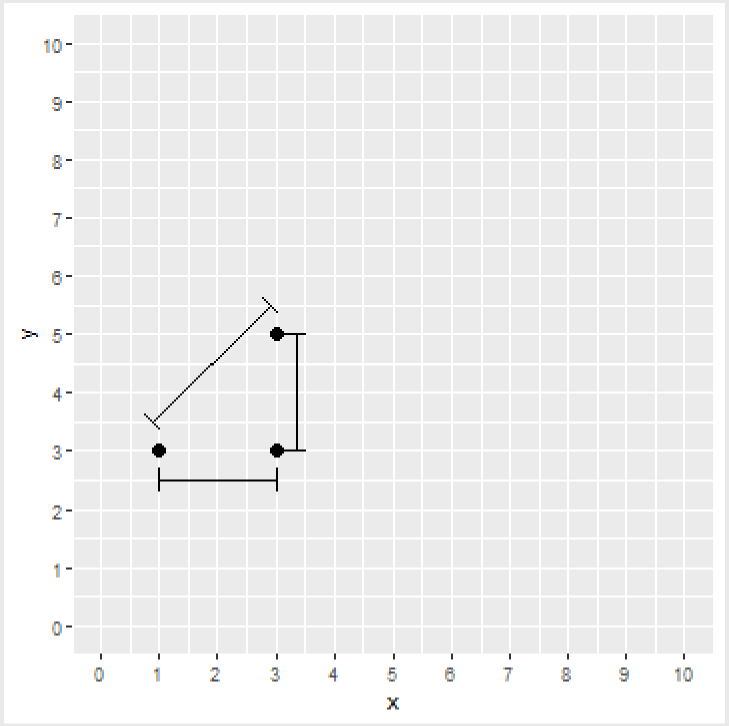
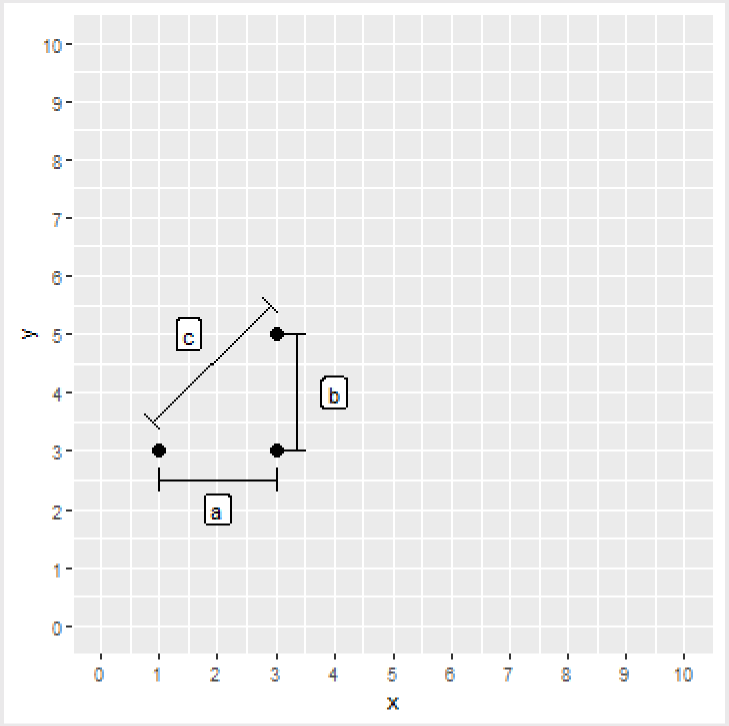


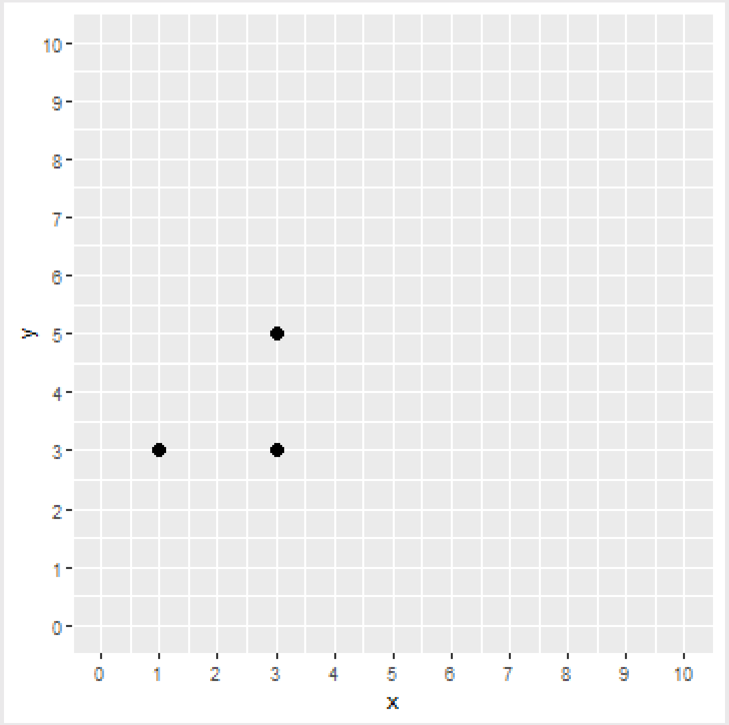
Euclidean Distance



- $c^2 = a^2 + b^2 \rightarrow c = \sqrt{a^2 + b^2}$
- $a^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ & $b^2 = (x_3 - x_2)^2 + (y_3 - y_2)^2$
- General: $\sqrt{\sum_i (q_i - p_i)^2}$

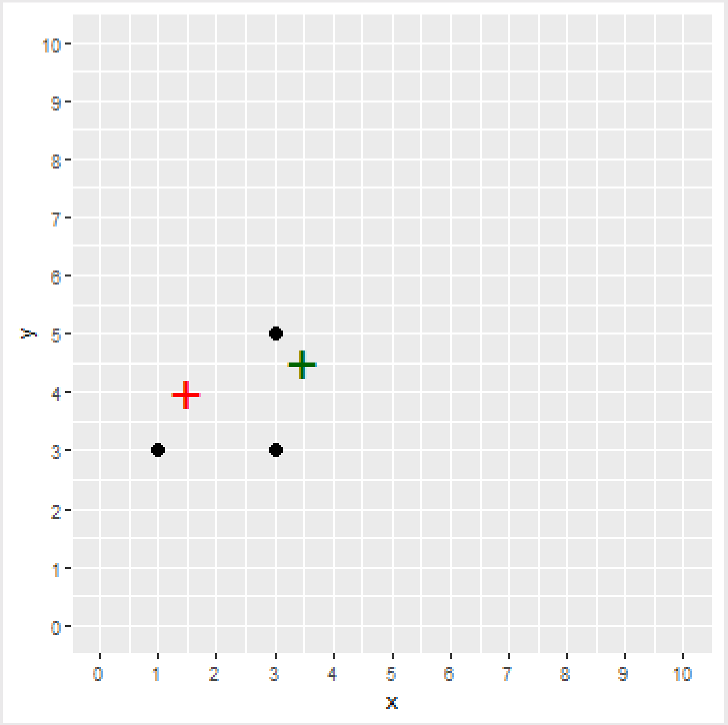
Centroids

- The center of some data



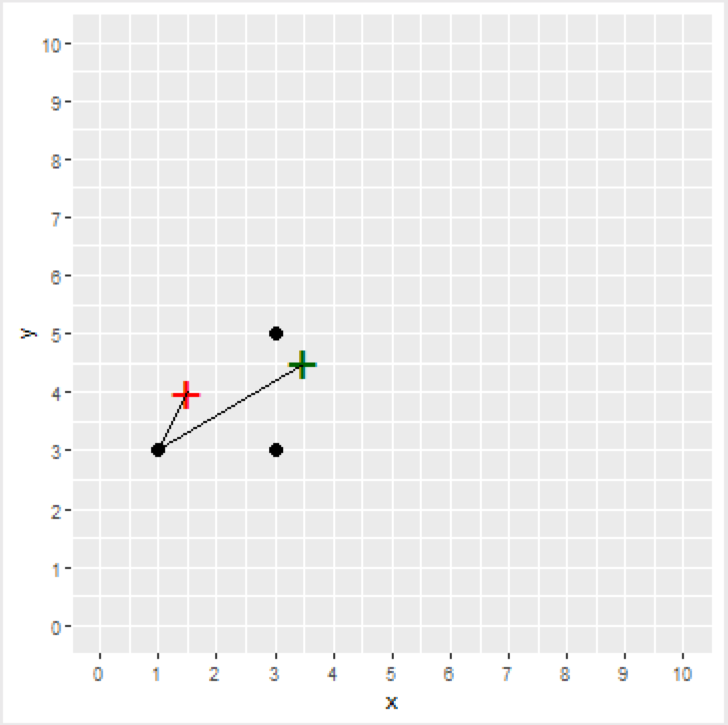
Centroids

- Initially chosen at random by the algorithm



Cluster Assignment

- Calculate Euclidean Distance for each observation



Cluster Assignment

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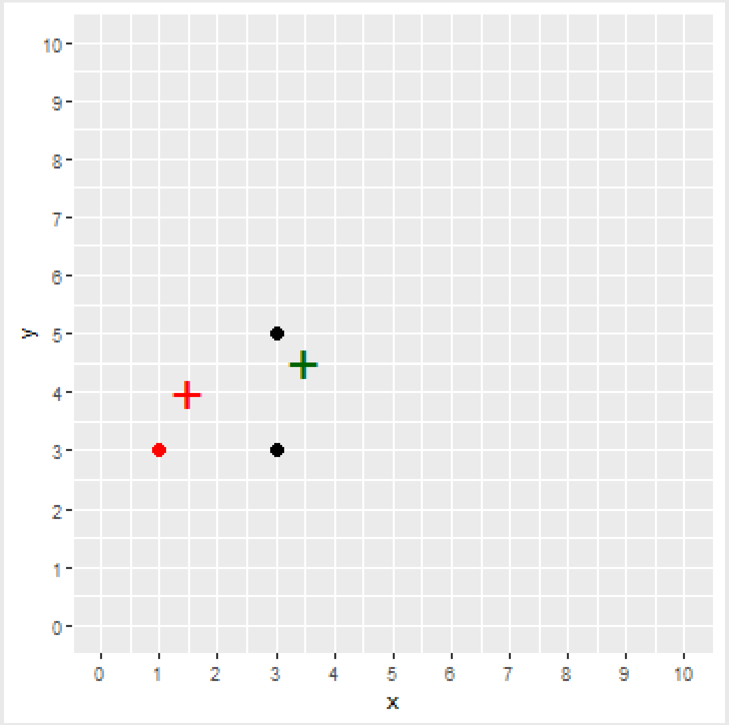

Cluster Assignment

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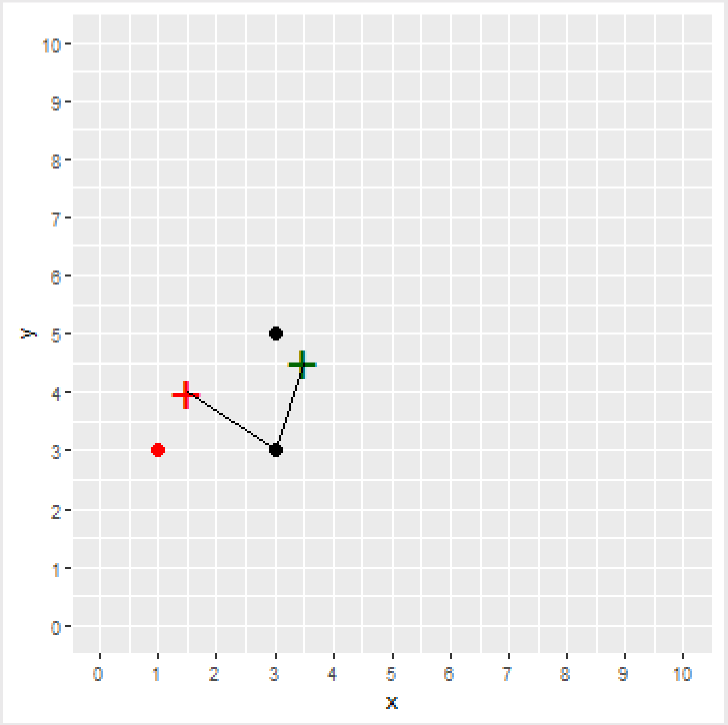

Cluster Assignment

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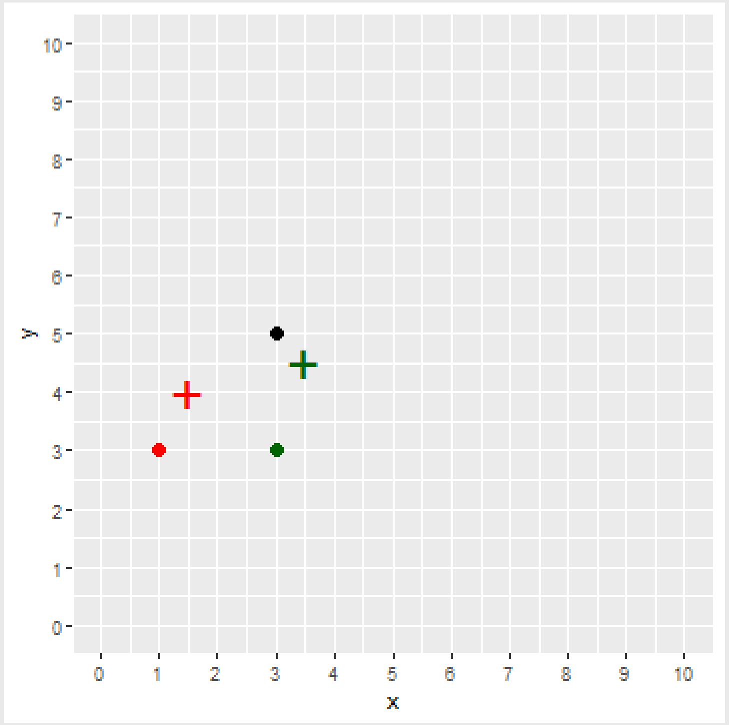

Cluster Assignment

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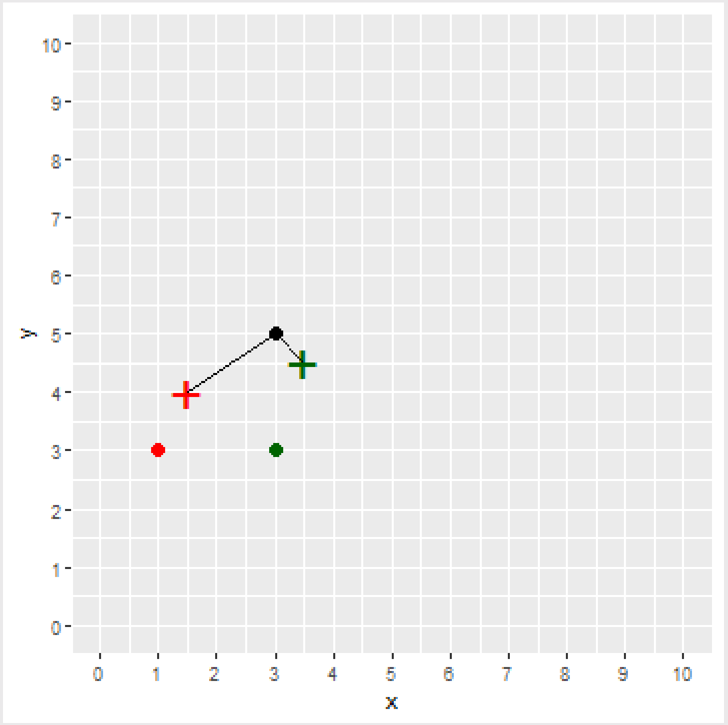
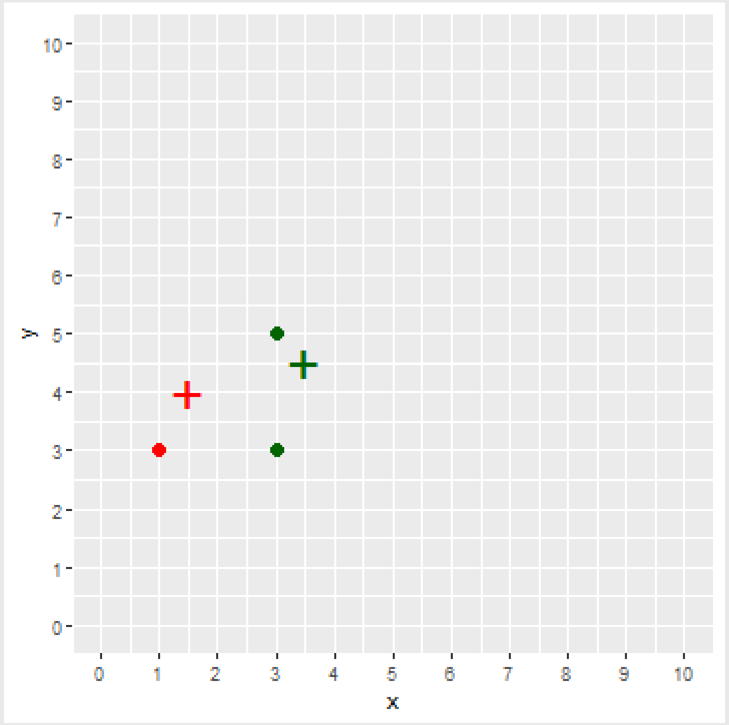


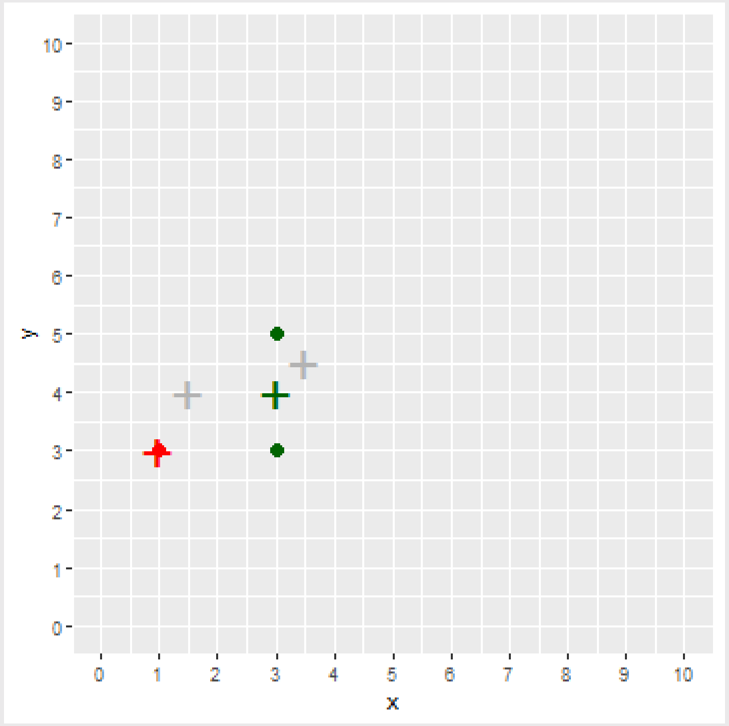
Cluster Assignment

- Calculate Euclidean Distance for each observation



Recalculate Centroids

- Set new centroids to mean of x and y among members



- A simulation

Clustering to Learn about MCs

```
library(tidyverse)
dat <-
read_csv('https://raw.githubusercontent.com/jbisbee1/DS1000_F2024/main/
glimpse(dat)
```

```
## Rows: 445
## Columns: 22
## $ congress                          <dbl> 97, 97, 97, 97, 97, …
## $ chamber                           <chr> "President", "House"…
## $ icpsr                             <dbl> 99907, 10717, 10721,…
## $ state_icpsr                       <dbl> 99, 41, 41, 41, 41, …
## $ district_code                     <dbl> 0, 2, 1, 4, 3, 5, 7,…
## $ state_abbrev                      <chr> "USA", "AL", "AL", "…
## $ party_code                        <dbl> 200, 200, 200, 100, …
## $ occupancy                         <dbl> 0, 0, 0, 0, 0, 0, 0,…
## $ last_means                        <dbl> 0, 1, 1, 1, 1, 1, 1,…
## $ bioname                           <chr> "REAGAN, Ronald Wils…
## $ bioguide_id                       <chr> NA, "D000326", "E000…
## $ born                              <dbl> 1911, 1925, 1928, 19…
## $ died                              <dbl> 2004, 2008, 2019, 20…
## $ nominate_dim1                     <dbl> 0.692, 0.398, 0.177,…
## $ nominate_dim2                     <dbl> -0.713, -0.057, 0.16…
```

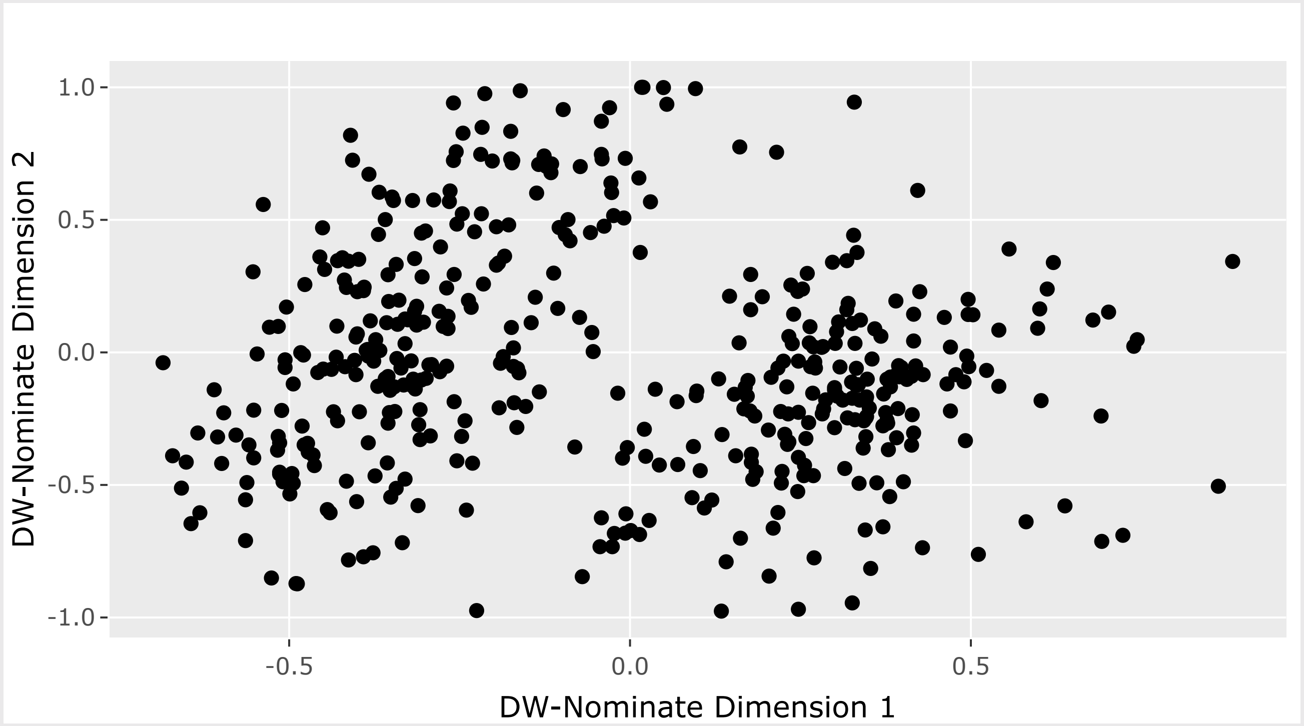
DW-NOMINATE

- DW-NOMINATE is a measure of how frequently different legislators vote together
- Often interpreted as "ideology"
- Two-dimensions:
  1. Standard left-right ideology (size of gov, redistribution, etc.)
  2. Second dimension changes, but typically **salient social issues**
- Can k-means clustering help us learn about legislators?

97th Congress (1981-1983)

```r
require(scales)
library(plotly)
gg <- dat %>%
  ggplot(aes(x = nominate_dim1,y = nominate_dim2,
             text = bioname)) +
  geom_point() +
  labs(x = 'DW-Nominate Dimension 1',
       y = 'DW-Nominate Dimension 2')
```

97th Congress (1981-1983)



Intuition Check

- Can we see some clusters?
  - What do we think these are?
- Let's try estimating k-means!
- Function `kmeans(x,centers,iter.max,nstart)`
  - `x` is the data (only select the columns of interest!)
  - `centers` is the number of centroids
  - `iter.max` maximum amount of "steps"
  - `nstart` how many times to re-estimate

Clustering on Ideology

- First, some light wrangling (convert numeric party code to character)

```r
datClust <- dat %>%
  mutate(party = ifelse(party_code == 200,'R',
                        ifelse(party_code == 100,'D','I'))) %>%
  mutate(nameParty = paste0(bioname,' (',party,')')) %>%
  select(nominate_dim1,nominate_dim2,nameParty) %>% drop_na()
```

Clustering on Ideology

- Second, estimate `kmeans()` function

```
m <- kmeans(x = datClust %>% select(nominate_dim1,nominate_dim2),
            centers = 2)

m
```

```
## K-means clustering with 2 clusters of sizes 214, 231
## 
## Cluster means:
##   nominate_dim1 nominate_dim2
## 1     0.2607991    -0.2443271
## 2    -0.3019524     0.1529177
## 
## Clustering vector:
##   [1] 1 1 1 2 2 2 2 1 1 1 2 2 2 1 2 2 1 1 2 2 1 2 2 2 1 2 1
##  [28] 2 2 1 1 2 1 1 2 2 2 2 1 2 1 2 2 1 2 2 1 1 1 2 1 1 1 1
##  [55] 1 2 1 1 2 1 2 1 2 2 1 1 2 1 1 2 1 2 2 1 2 2 2 2 2 2 1
##  [82] 1 2 2 2 2 1 1 2 2 2 2 1 2 2 2 2 1 2 2 2 1 1 1 1 2 2 2
## [109] 1 1 2 1 1 1 1 1 2 1 2 2 2 1 1 1 1 2 1 2 1 1 1 2 2 1 2
## [136] 1 1 1 2 2 1 1 1 1 1 1 2 1 1 1 2 2 1 2 2 1 1 2 2 2 1 2
## [163] 1 2 1 1 1 2 1 1 2 2 1 2 2 2 2 1 2 1 2 2 1 2 2 2 2 2 1
```

Clustering on Ideology

- Easier to see output with the help of `tidymodels` package

```
require(tidymodels)
tidy(m)
```

```
## # A tibble: 2 × 5
##   nominate_dim1 nominate_dim2  size withinss cluster
##           <dbl>         <dbl> <int>    <dbl> <fct>  
## 1         0.261        -0.244   214     33.4 1      
## 2        -0.302         0.153   231     48.7 2      
```

- First two columns are the **locations** of the centroids
- `size` is the number of observations associated with each group
- `withinss` is the **errors** each centroid makes

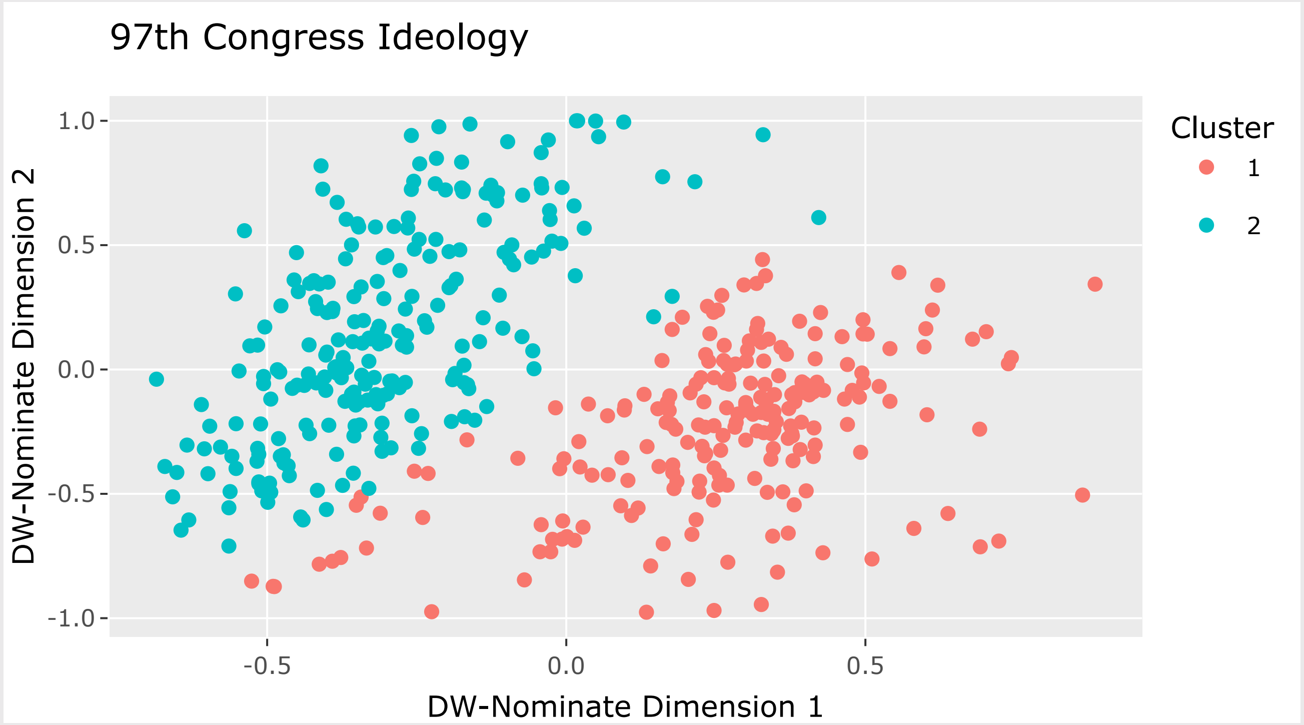
Clustering on Ideology

- Third, plot points and color by cluster

```
ggClust <- datClust %>%
  mutate(cluster = m$cluster) %>% # Add cluster to data
  ggplot(aes(x = nominate_dim1, y = nominate_dim2,
             color = factor(cluster),
             text=nameParty)) +
  geom_point() +
  labs(x = 'DW-Nominate Dimension 1',
       y = 'DW-Nominate Dimension 2',
       title="97th Congress Ideology",
       color = "Cluster")
```

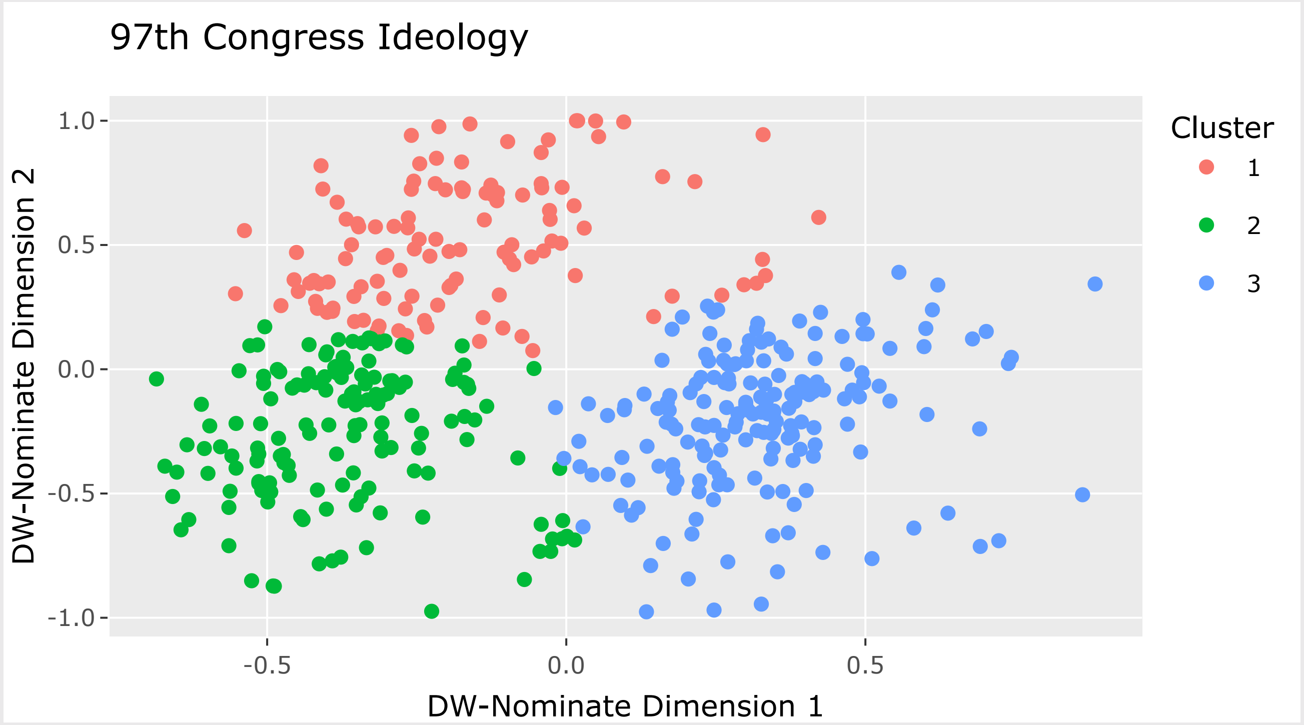
Clustering on Ideology

```
ggplotly(ggClust,tooltip = 'text')
```



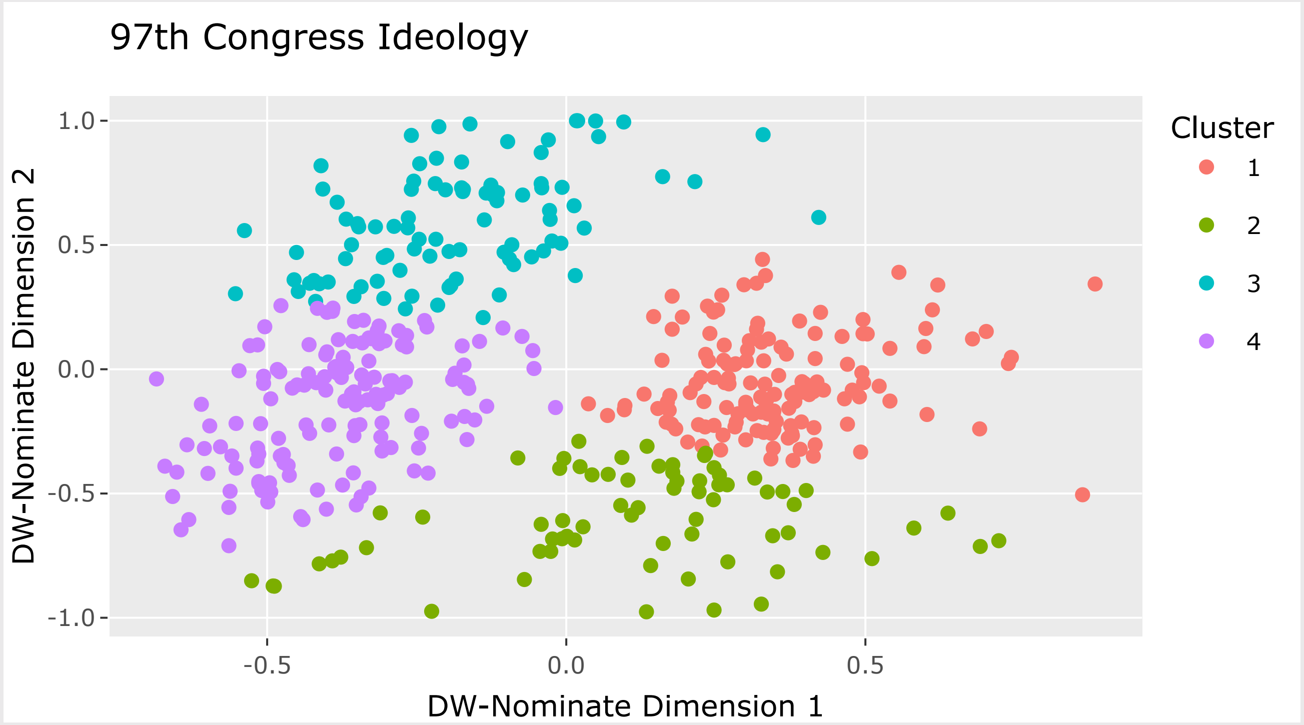
More Clusters

```r
m <- kmeans(x = datClust %>% select(nominate_dim1,nominate_dim2),
            centers = 3)
```



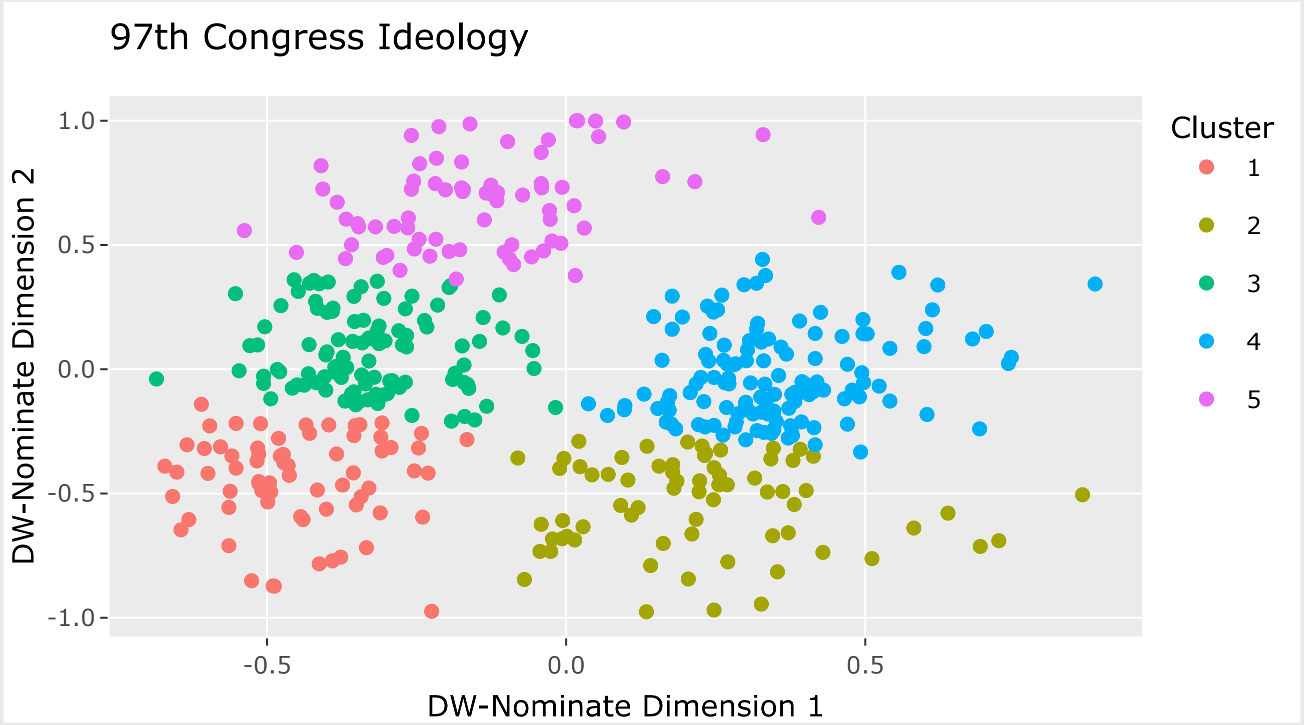
More Clusters

```r
m <- kmeans(x = datClust %>% select(nominate_dim1,nominate_dim2),
            centers = 4)
```



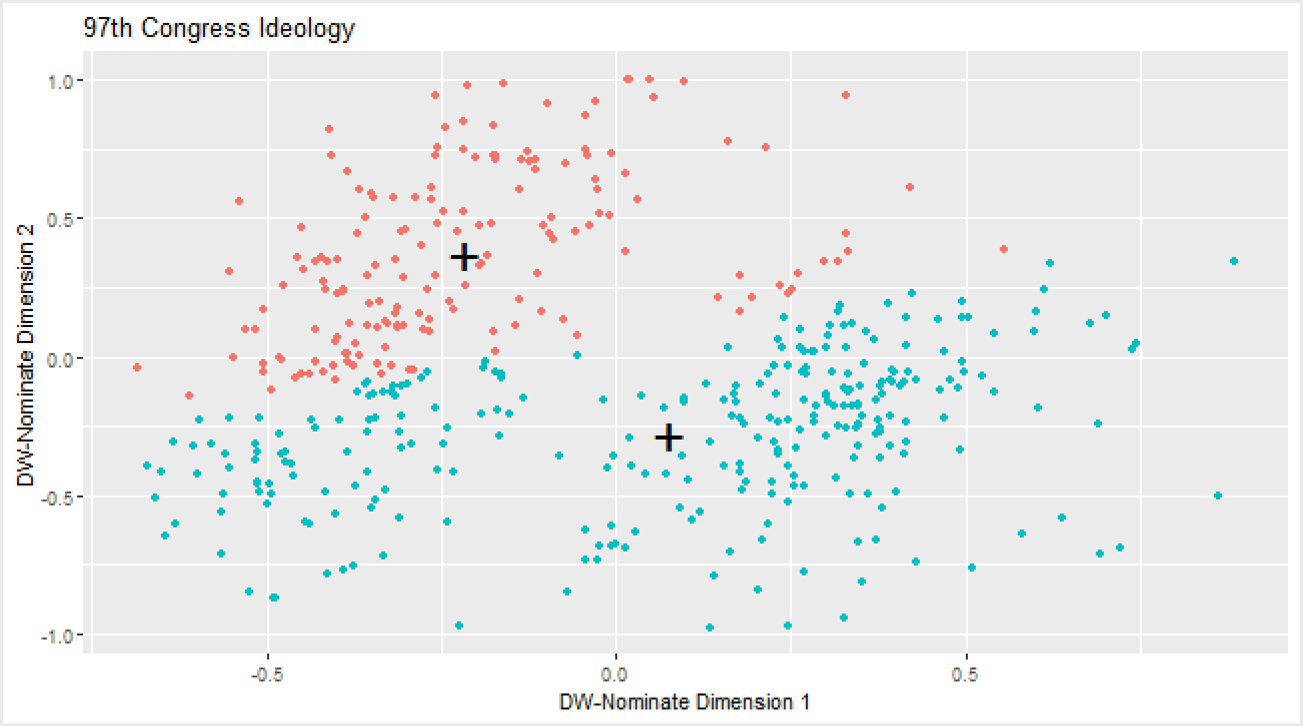
More Clusters

```r
m <- kmeans(x = datClust %>% select(nominate_dim1,nominate_dim2),
            centers = 5)
```



How many clusters?

- Recall from regression that we are interested in **errors**
- What are "errors" in the context of clustering?



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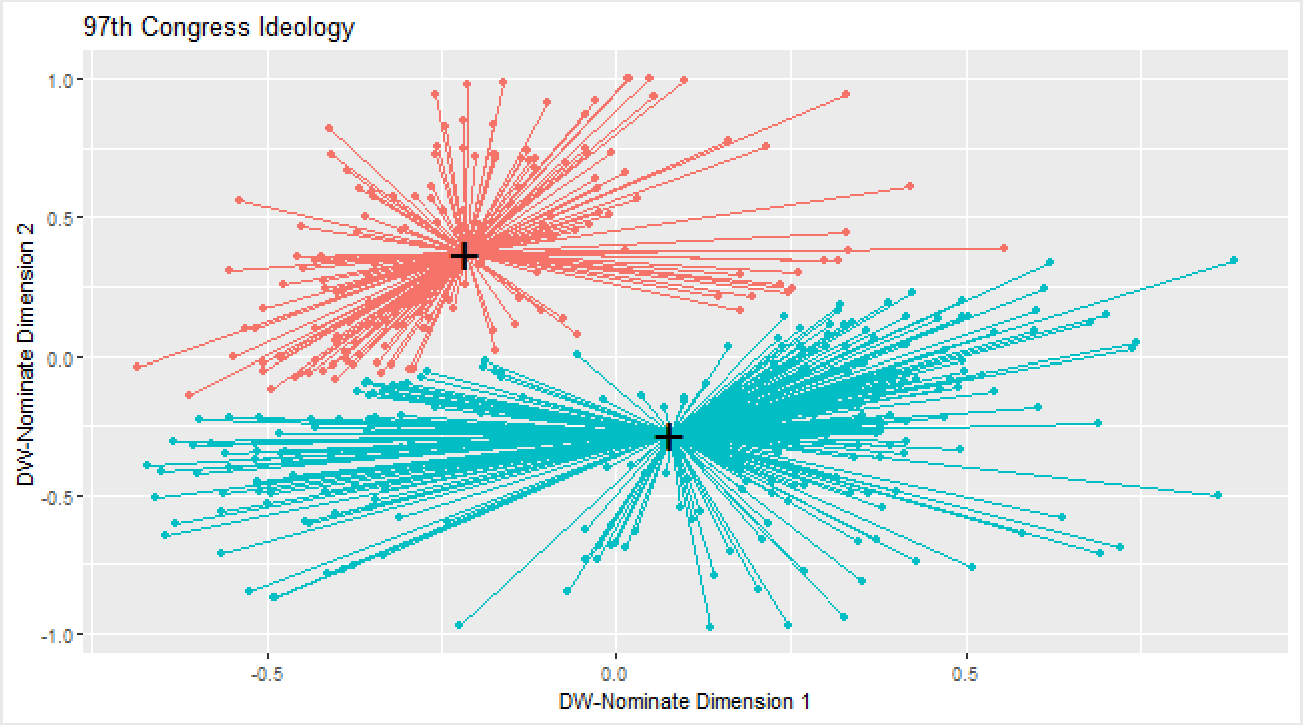


How many clusters?

- Recall from regression that we are interested in **errors**
- What are "errors" in the context of clustering?
- Just the sum of each observation's distance from its centroid!
  - Within Sum of Squares (**WSS**)

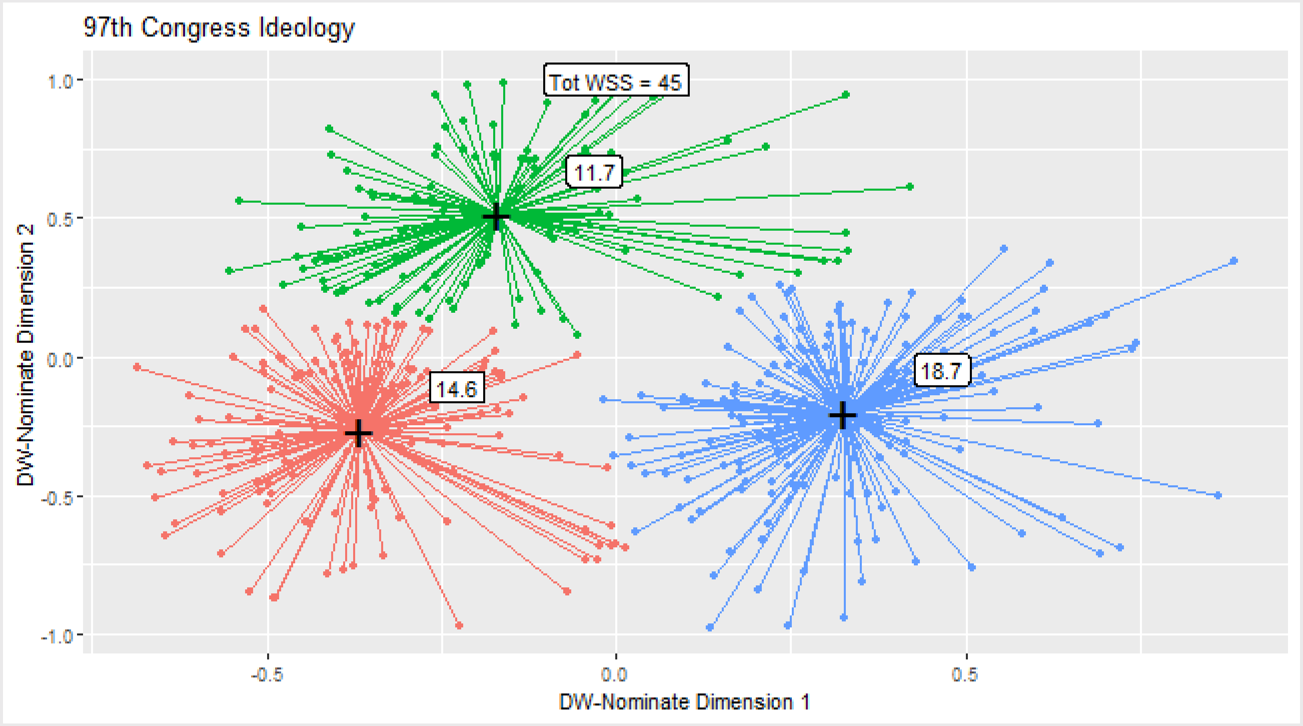
```
tidy(m)
```

```
## # A tibble: 2 × 5
##   nominate_dim1 nominate_dim2  size withinss cluster
##           <dbl>         <dbl> <int>    <dbl> <fct>
## 1        -0.216         0.370   166     23.7 1
## 2         0.0783       -0.281   279     58.0 2
```

So...how many?!

- Want to choose a number of clusters that reduces the total **WSS**

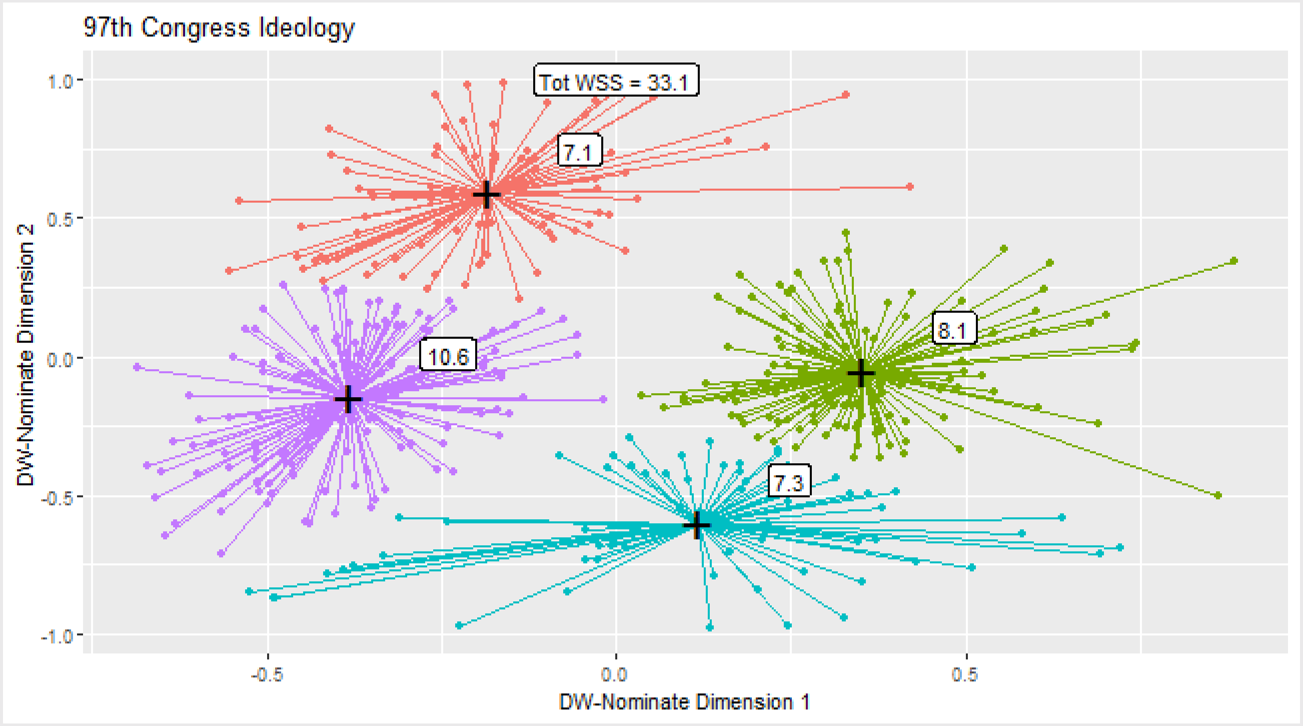
```
m.cluster <- datClust %>%
  select(-nameParty) %>% # Same as selecting two dimensions
  kmeans(centers = 3)
```



So...how many?!

- Want to choose a number of clusters that reduces the total **wss**

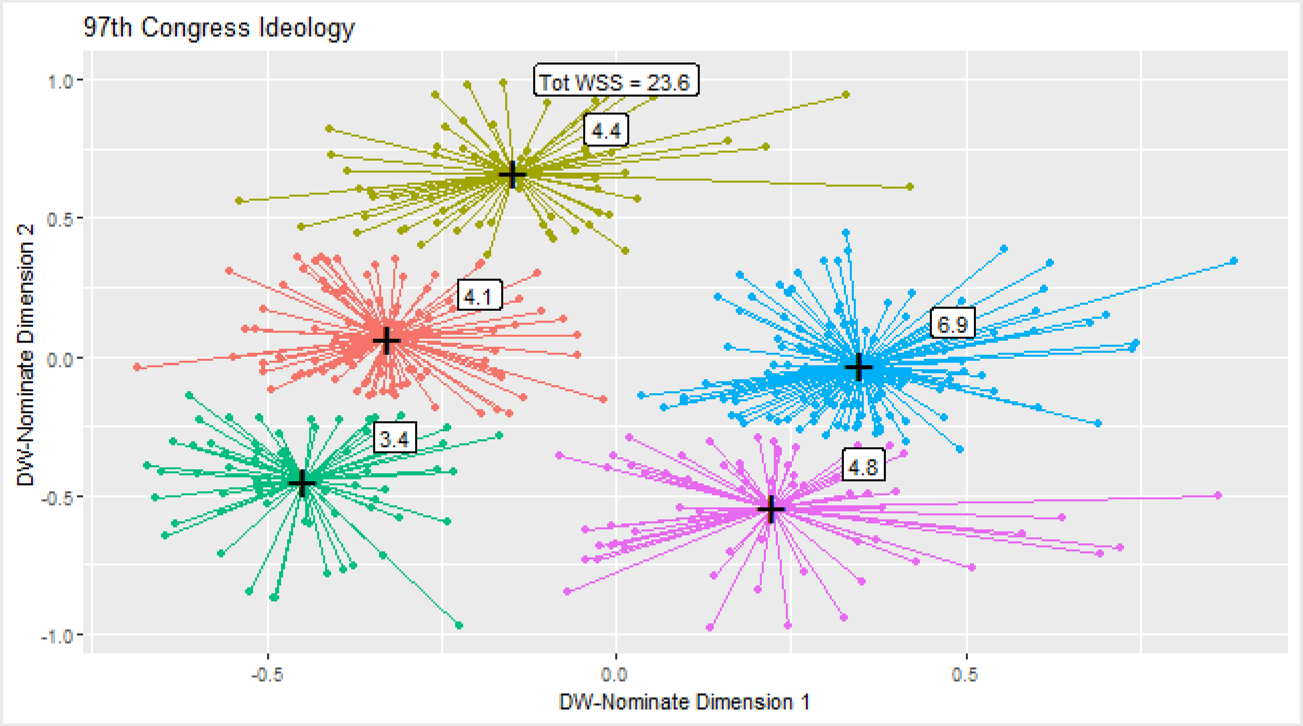
```
m.cluster <- datClust %>%
  select(-nameParty) %>%
  kmeans(centers = 4)
```



So...how many?!

- Want to choose a number of clusters that reduces the total **wss**

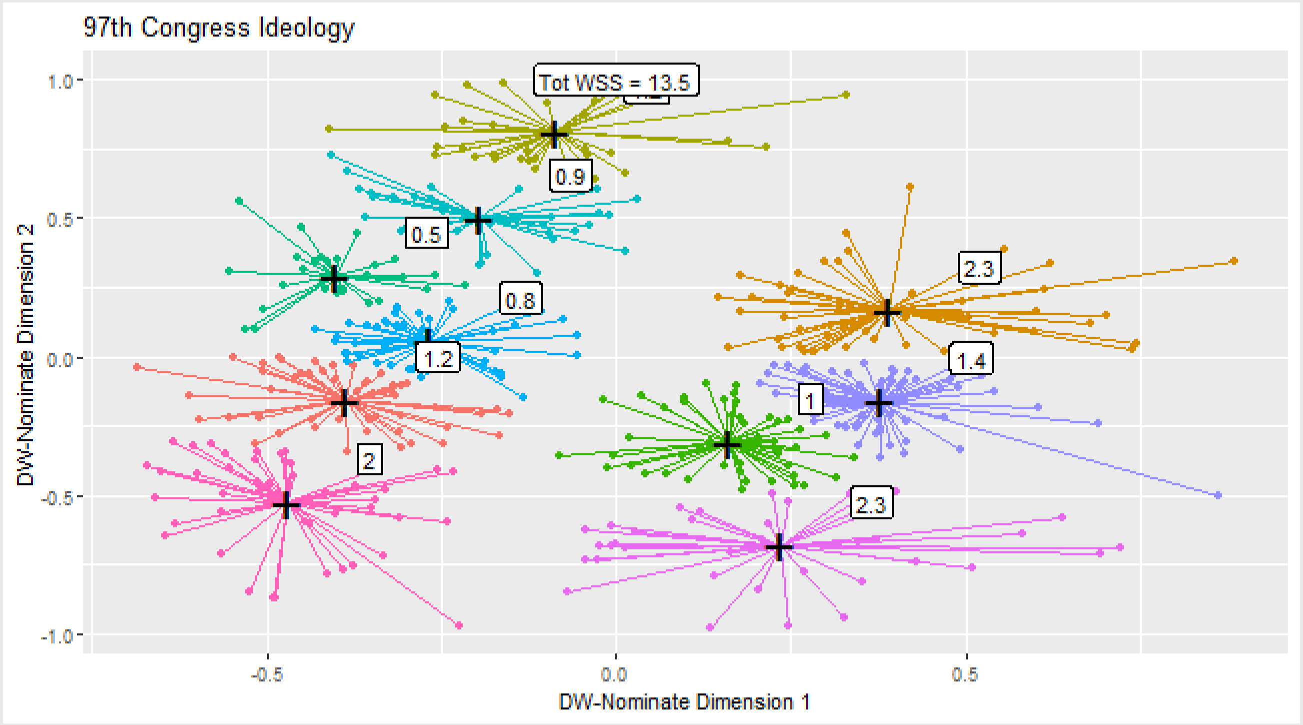
```
m.cluster <- datClust %>%
  select(-nameParty) %>%
  kmeans(centers = 5)
```



So...how many?!

- Want to choose a number of clusters that reduces the total **wss**

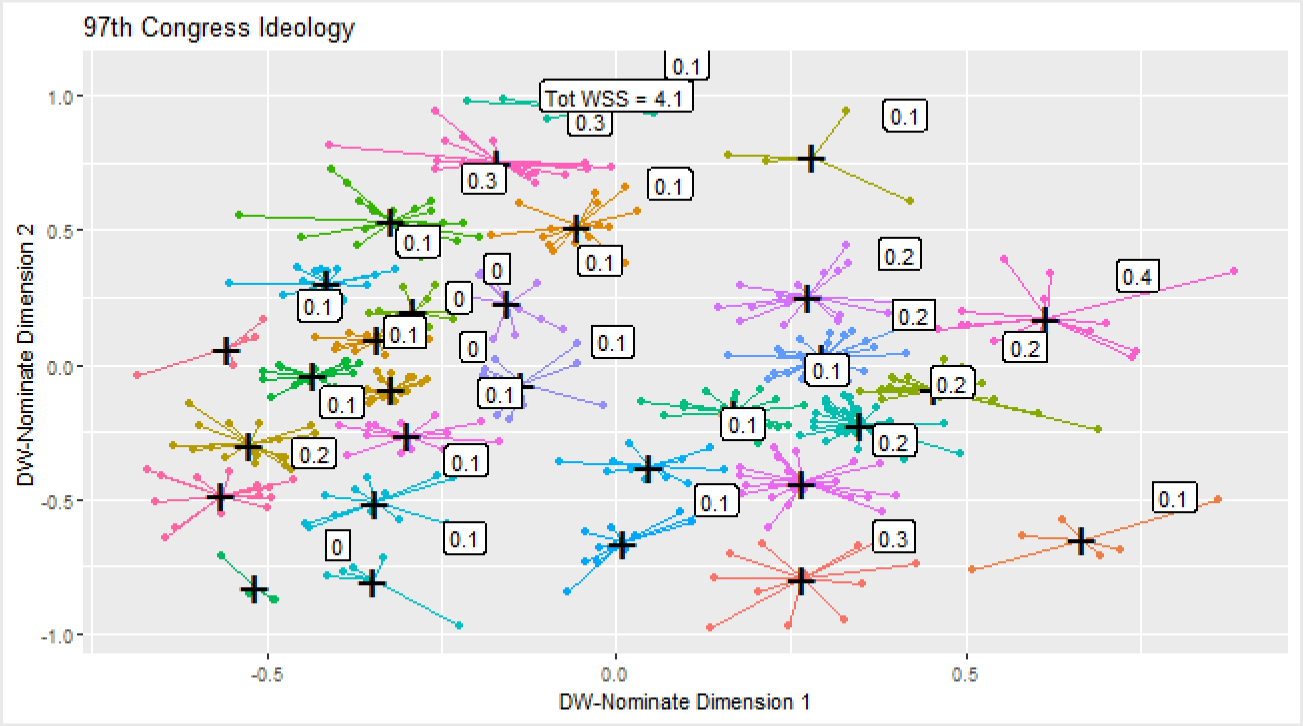
```
m.cluster <- datClust %>%
  select(-nameParty) %>%
  kmeans(centers = 10)
```



So…how many?!

- Want to choose a number of clusters that reduces the total **wss**

```
m.cluster <- datClust %>%
  select(-nameParty) %>%
  kmeans(centers = 30)
```



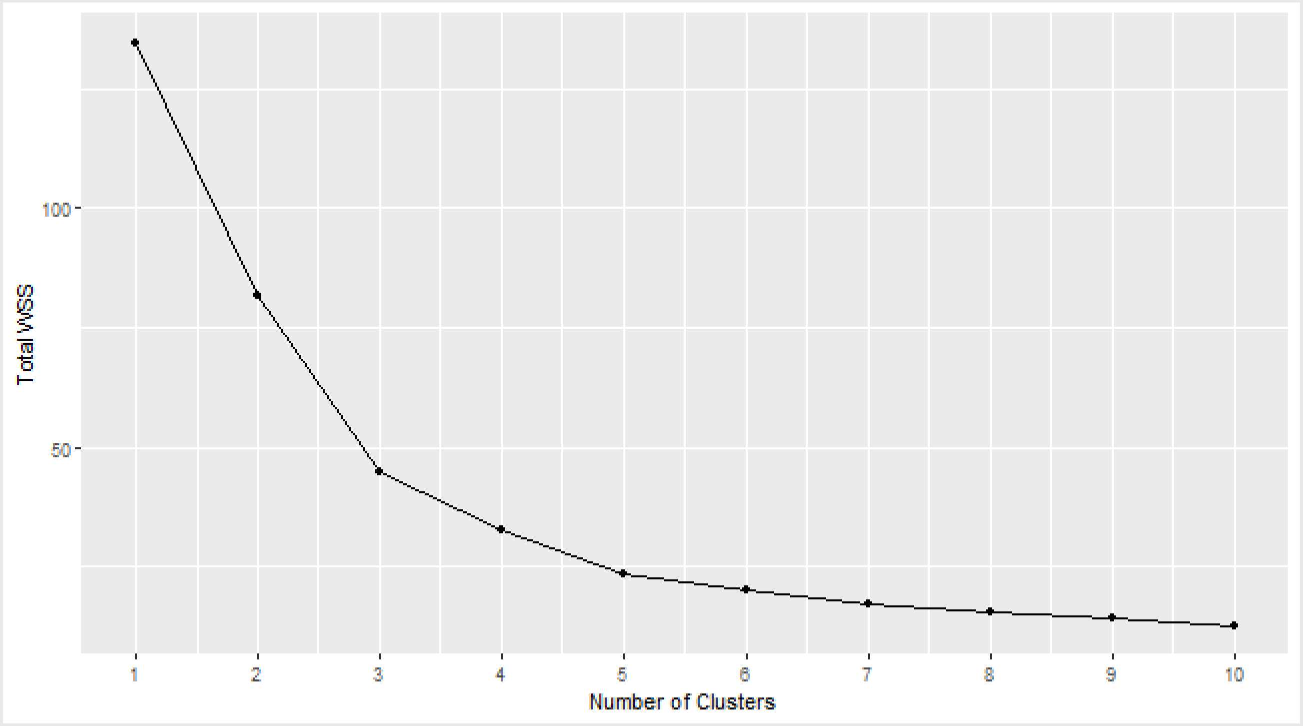
So...how many?!

- But there's a trade-off!
    - Accuracy versus **parsimony**
- Simple rule: look for the "elbow"

```
totWSS <- NULL
for(k in 1:10) {
  m.cluster <- datClust %>%
    select(-nameParty) %>%
    kmeans(centers = k,nstart = 25)
  totWSS <- data.frame(totWSS = m.cluster$tot.withinss,
                  k = k) %>%
    bind_rows(totWSS)
}
```

Looking for the "elbow"

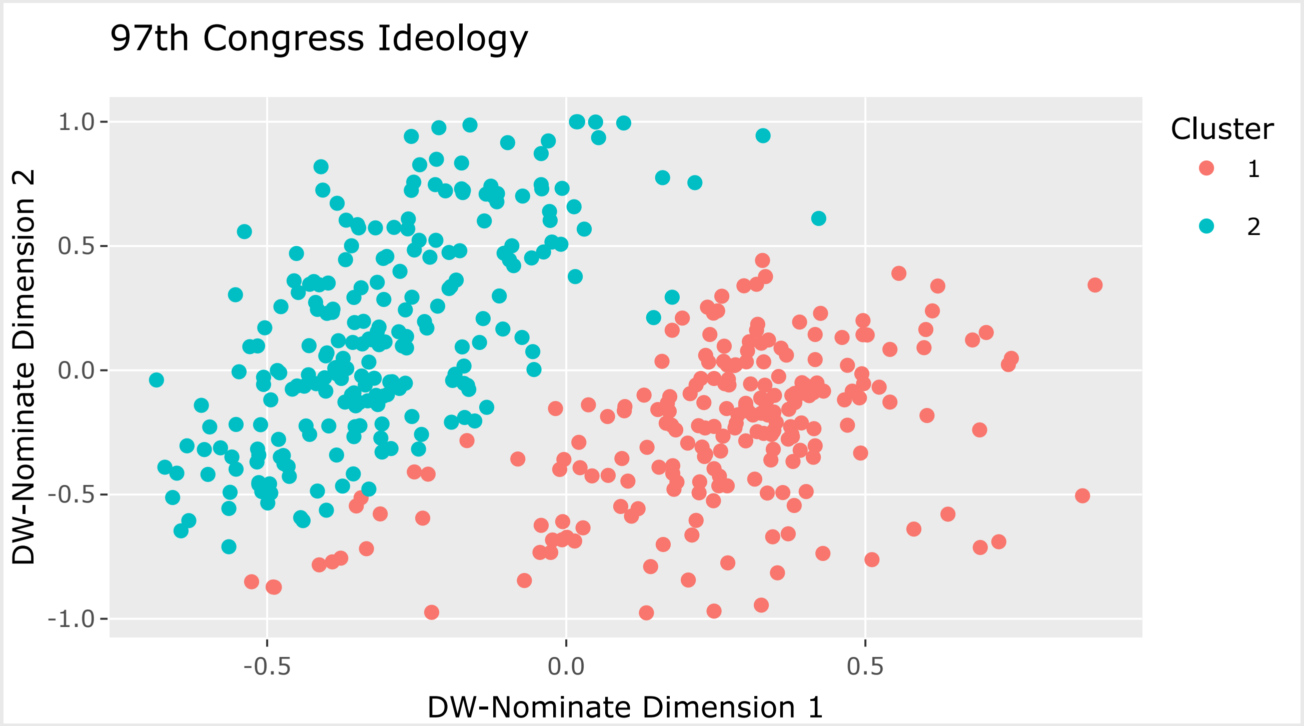
```
totWSS %>%
  ggplot(aes(x = k,y = totWSS)) +
  geom_line() + geom_point() +
  labs(x = 'Number of Clusters',y = 'Total WSS') +
  scale_x_continuous(breaks = 1:10)
```



Clustering on Ideology

- Note that we need to `set.seed()`! Centroid starting points are **random**!

```
m <- kmeans(x = datClust %>% select(nominate_dim1,nominate_dim2),
                    centers = 2)
```



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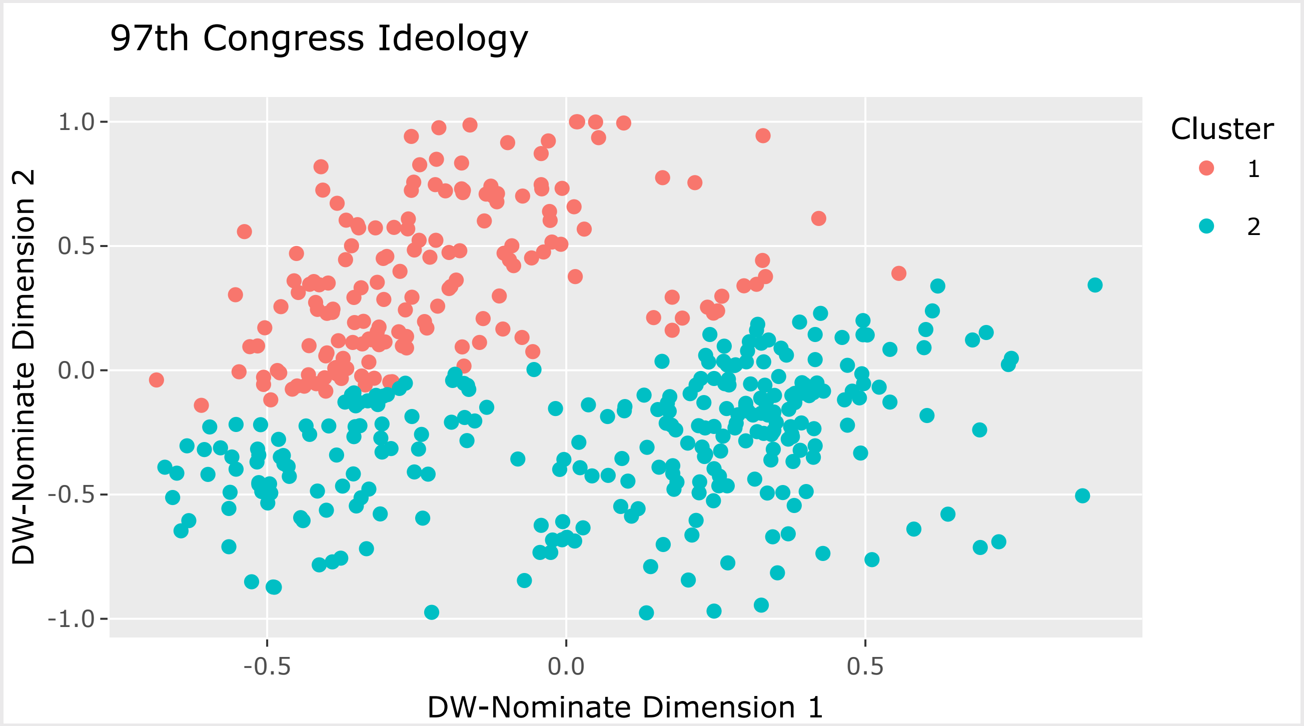
```



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- Note that we need to `set.seed()`! Centroid starting points are **random**!

```
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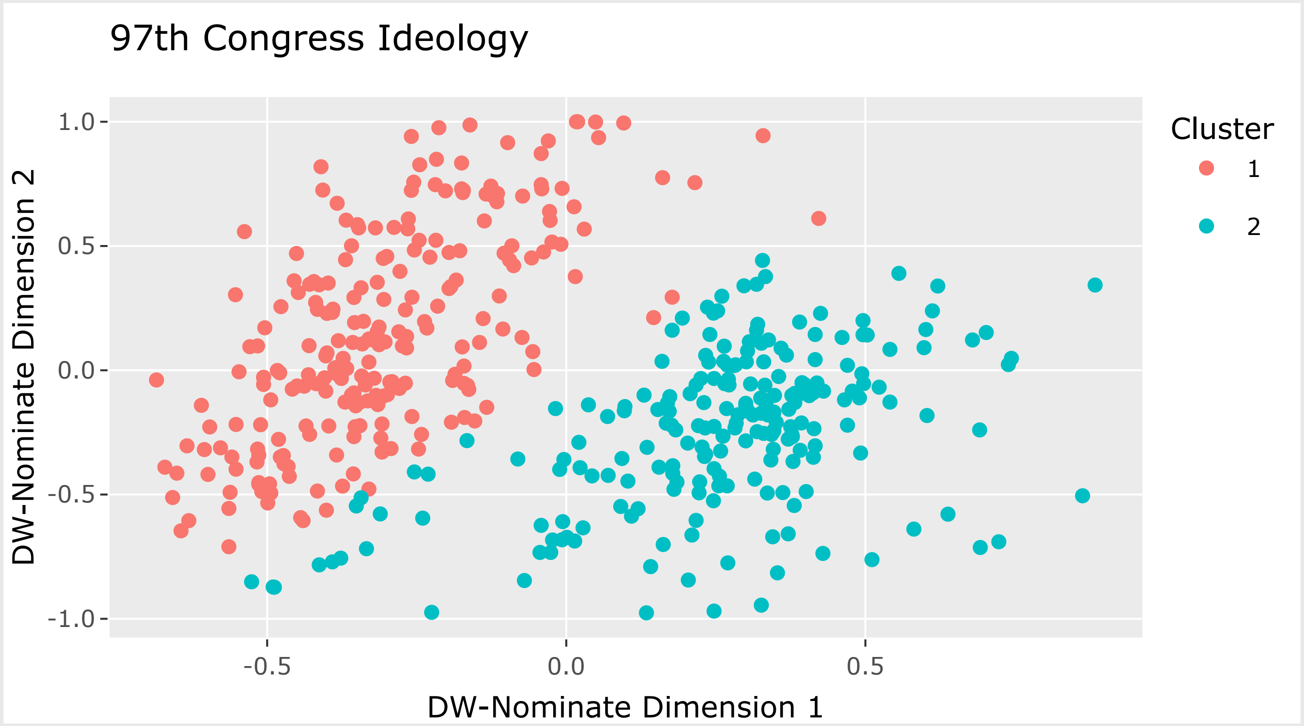
                    centers = 2)
```



Clustering on Ideology

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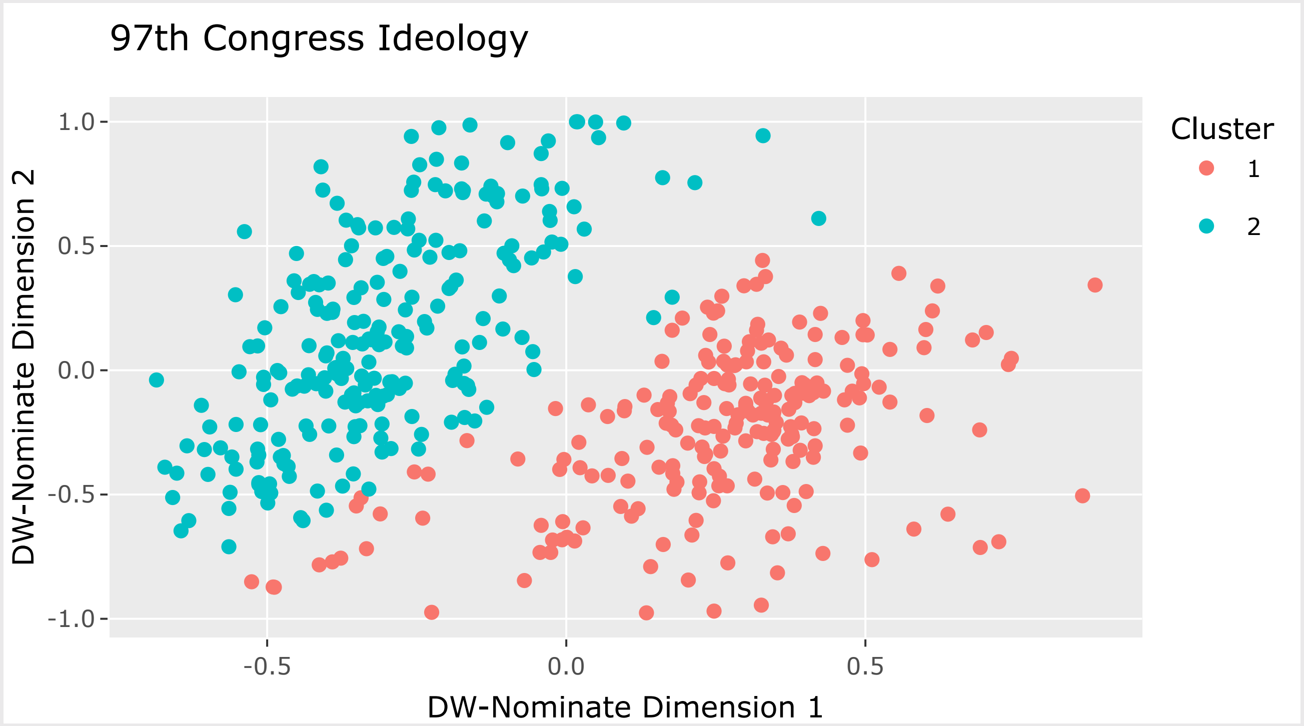
```
m <- kmeans(x = datClust %>% select(nominate_dim1,nominate_dim2),
                    centers = 2)
```



Clustering Randomness

- Can overcome with `nstart`
    - Attempts multiple initial centroids and chooses the "best"

```r
set.seed(42)
c1 <- kmeans(datClust %>% select(-nameParty),centers = 2,nstart = 25)

set.seed(123)
c2 <- kmeans(datClust %>% select(-nameParty),centers = 2,nstart = 25)

table(c1$cluster,c2$cluster)
```

```
##
##         1   2
##     1 166   0
##     2   0 279
```

Is Polarization Increasing?

- Compare 97th Congress (1981-1983) to 117th Congress (2021-2023)

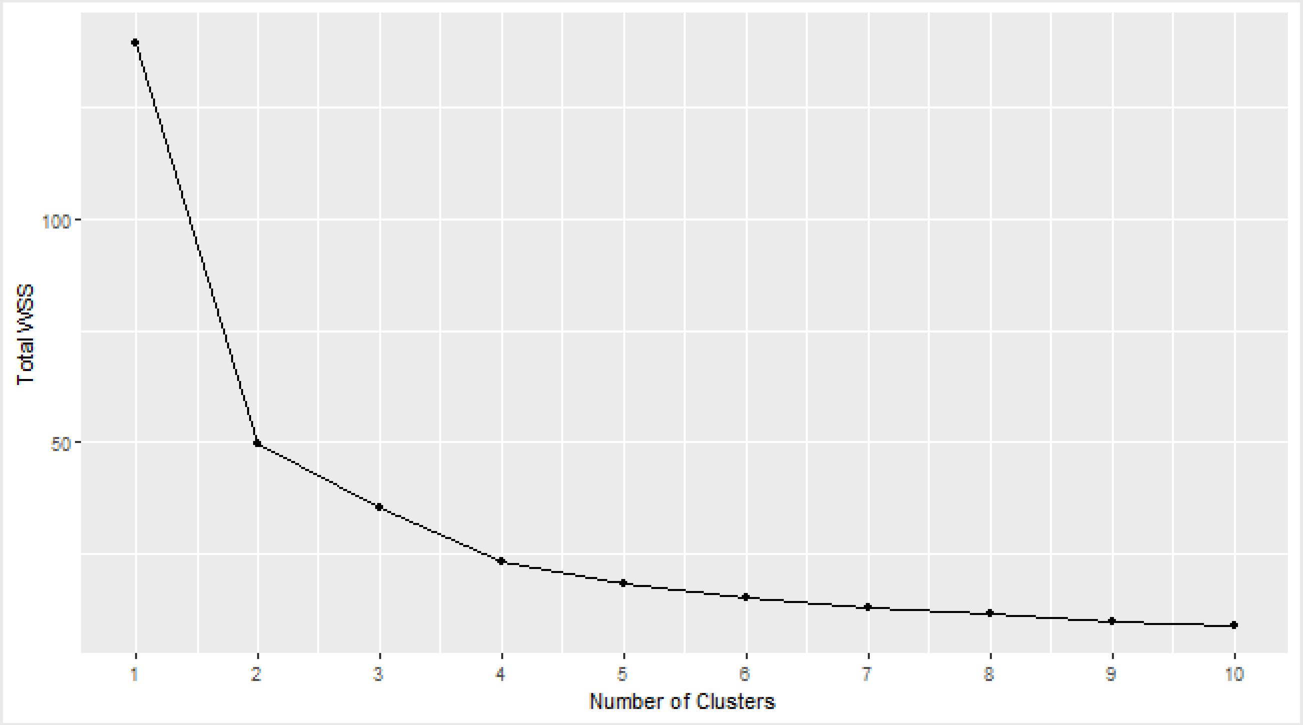
```
dat <-
read_csv('https://raw.githubusercontent.com/jbisbee1/DS1000_F2024/main/
datClust <- dat %>%
  mutate(party = ifelse(party_code == 200,'R',
                        ifelse(party_code == 100,'D','I'))) %>%
  mutate(nameParty = paste0(bioname,' (',party,')')) %>% # combine
name + party
  select(nominate_dim1,nominate_dim2,nameParty) %>% drop_na()
```

- Check for the "elbow"

```
totWSS <- NULL
for(k in 1:10) {
  m.cluster <- datClust %>%
    select(-nameParty) %>% kmeans(centers = k,nstart = 25)
  totWSS <- data.frame(totWSS = m.cluster$tot.withinss,k = k) %>%
    bind_rows(totWSS)
}
```

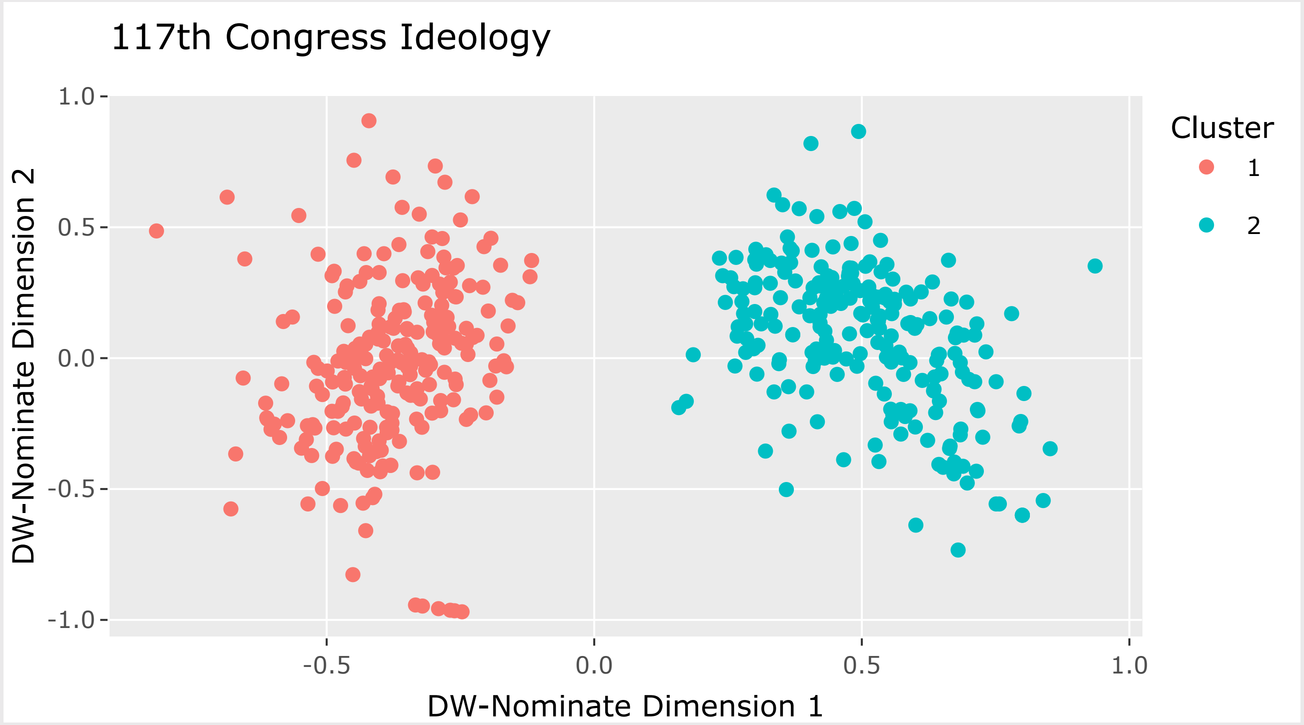
Check for the "elbow"

```
totWSS %>%
  ggplot(aes(x = k,y = totWSS)) +
  geom_line() + geom_point() +
  labs(x = 'Number of Clusters',y = 'Total WSS') +
  scale_x_continuous(breaks = 1:10)
```



Growing polarization?

```r
m <- kmeans(x = datClust %>% select(nominate_dim1,nominate_dim2),
            centers = 2)
```



Conclusion

- Clustering is part of a third camp of data science
  1. Theory Testing
  2. Prediction
  3. **Learning**
- Next lecture: clustering applied to **TEXT**
- Homework:
  - Problem set 9
  - Homework 18