Classification

Part 1

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Agenda

- 1. Classification
- 2. Fortnite gaming (i.e., Prof's desperate attempt to be relevant)

```
require(tidyverse)
fn <-
read_rds('https://github.com/jbisbee1/ISP_Data_Science_2024/raw/main/da</pre>
```

Definitions

- Classification: predicting the class of given data points via predictive modeling
 - Class: AKA targets, labels, or categories
 - \circ *Predictive Modeling*: Approximate mapping function f:X o Y
 - $\circ X$: predictor variables
 - $\circ Y$: outcome variable
 - ∘ *f*: ??

Mapping Functions

- We have already used a mapping functions!
- Linear Regression

$$\circ f: Y = \alpha + \beta X + \varepsilon$$

ullet Underlying idea: X contain information about Y

It is in the Y

- ullet If Y is continuous, we use OLS regression
- ullet If Y is **binary**, we use "logistic" regression (AKA "logit")
 - As always, this is a deep area of study for those interested
- ullet Today, using OLS for binary Y
 - Next few classes: replacing OLS regression with logit

Fortnite



Fortnite

- Goal is to win (i.e., be the last player alive)
- Professional e-sports teams want to maximize this probability
- RQ: How can we increase the number of victories?
- NB: we are moving out of the Research camp now, and into the Prediction world
 - We don't care so much about why a relationship exists, we just want to get accurate predictions
 - Theory can still help us, but want to start with the data to get our thinking started

The Data

glimpse(fn)

```
## Rows: 957
## Columns: 24
## $ placed
                           <dbl> 17, 41, 36, 28, 3, 15, 9, 29,...
## $ mental state
                           <chr> "sober", "sober", "high", "hi...
## $ eliminations
                           <dbl> 2, 0, 3, 1, 3, 0, 2, 3, 4, 1,...
## $ assists
                           <dbl> 0, 2, 0, 4, 2, 1, 2, 2, 0, 2,...
## $ revives
                           <dbl> 0, 0, 0, 0, 1, 0, 0, 0, 1, 0,...
## $ accuracy
                           <dbl> 0.19371429, 0.32400265, 0.336...
## $ hits
                           <dbl> 10, 17, 38, 22, 49, 4, 43, 14...
## $ head shots
                           <dbl> 1, 0, 0, 3, 18, 3, 2, 3, 13, ...
                           <dbl> 226, 370, 725, 266, 938, 148,...
## $ distance traveled
## $ materials gathered
                          <dbl> 0, 0, 0, 358, 305, 0, 1286, 1...
## $ materials used
                           <dbl> 0, 38, 0, 61, 234, 170, 195, ...
## $ damage taken
                           <dbl> 282, 203, 206, 262, 437, 151,...
## $ damage to players
                           <dbl> 372, 354, 206, 286, 823, 122,...
## $ damage to structures <dbl> 538, 1403, 260, 3841, 1470, 4...
                           <dbl> 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,...
## $ won
## $ player
                           <int> -5, -5, -5, -5, -5, -5, -5, -...
## $ gameId
                           <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10...
## $ startTime
                           <dttm> 2020-04-10 16:46:06, 2020-04...
```

The Data

- Start with the basics:
 - 1. What is the unit of analysis?
 - 2. Which variables are we interested in?

Prediction

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \dots + \varepsilon$$

- Y: victory (won)
- X:??
 - In prediction, we don't care about theory or research questions
 - Just want to maximize accuracy...which X's are the "best"?
 - \circ But theory can still help us make sensible choices about which X's to use
- Look at univariate & conditional relationships

The Data

• Outcome Y: won

```
require(scales)
fn %>%
  summarise(`Win %` = percent(mean(won)))
```

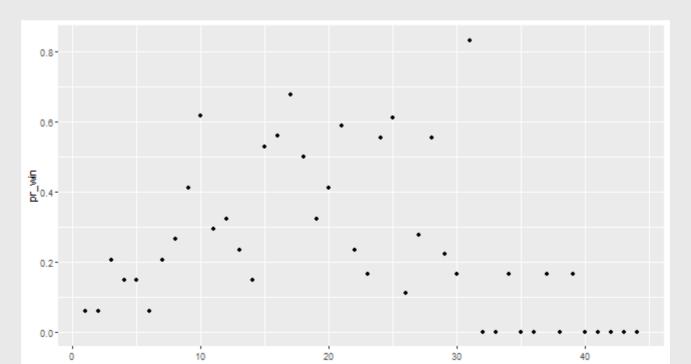
```
## # A tibble: 1 × 1
## `Win %`
## <chr>
## 1 30%
```

• Multivariate analysis?

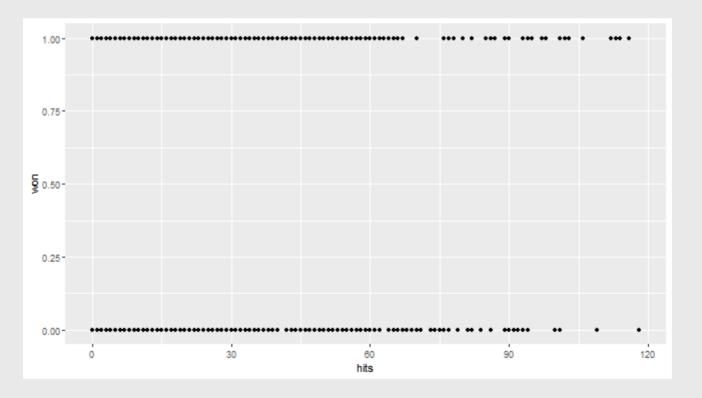
```
fn %>%
  group_by(mental_state) %>%
  summarise(pr_win = mean(won))
```

```
fn %>%
  group_by(gameIdSession) %>%
  summarise(pr_win = mean(won))
```

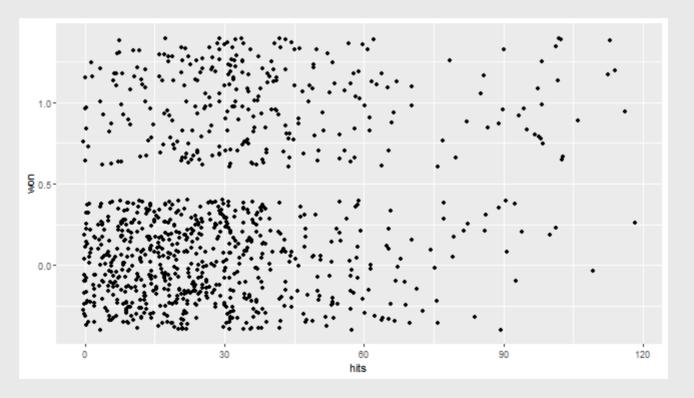
```
## # A tibble: 44 × 2
##
     gameIdSession pr win
##
               <int> <dbl>
##
                   1 0.0588
##
                   2 0.0588
##
                   3 0.206
##
                   4 0.147
##
                   5 0.147
##
                   6 0.0588
##
                   7 0.206
##
                   8 0.265
##
                   9 0.412
##
                  10 0.618
## # i 34 more rows
```



```
fn %>%
  ggplot(aes(x = hits,y = won)) +
  geom_point()
```



```
fn %>%
  ggplot(aes(x = hits,y = won)) +
  geom_jitter()
```

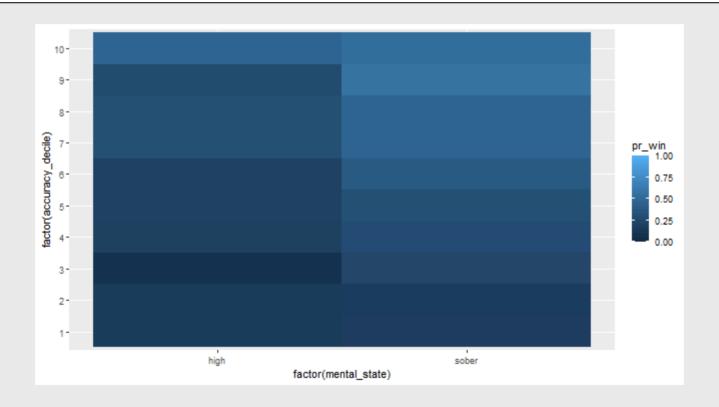


Heatmaps

- Look at 3-dimensions of data
 - Done this before by tweaking fill, color, or size
- geom tile(): create a heatmap

Heatmaps

p



Simplest Predictions

• Remember: regression is just fancier conditional means

```
fn <- fn %>%
  mutate(hits_decile = ntile(hits,n=10)) %>% # Bin hits by decile
(10%)
  group_by(hits_decile,mental_state) %>% # Calculate average winning
by mental state and accuracy
  mutate(prob_win = mean(won)) %>% # use mutate() instead of
summarise() to avoid collapsing the data
  mutate(pred_win = ifelse(prob_win > .5,1,0)) %>% # If the
probability is greater than 50-50, predict a win
  ungroup()
```

Simplest Predictions

Conditional means

```
fn %>%
  group_by(won,pred_win) %>%
  summarise(nGames=n(),.groups = 'drop')
```

- How good is this? Think about the underlying goal...we want a model that accurately predicts whether a game is won or not
- The won column is the truth...it tells us whether the game was won or not
- The pred_win column is our prediction

Accuracy

- What is "accuracy"?
 - Proportion "correct" predictions
- For a binary outcome, "accuracy" has two dimensions
 - Proportion of correct 1s: Sensitivity
 - Proportion of correct 0s: Specificity

Accuracy

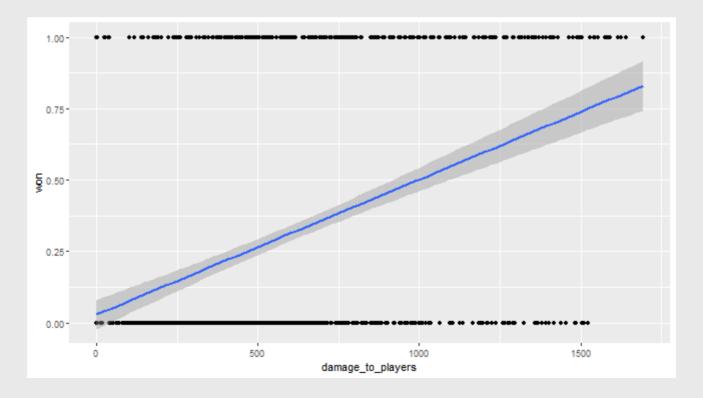
```
(sumTab <- fn %>%
  group_by(won) %>%
  mutate(total_games = n()) %>%
  group_by(won,pred_win,total_games) %>%
  summarise(nGames=n(),.groups = 'drop') %>%
  mutate(prop = nGames / total_games))
```

```
## # A tibble: 4 × 5
##
     won pred win total games nGames
                                  prop
          <dbl>
                 <int> <int> <dbl>
##
   <dbl>
## 1
                       666
                              625 0.938
## 2
                    666 41 0.0616
                       291 241 0.828
## 3
                       291 50 0.172
## 4
```

- Overall accuracy: (625+50) / (666+291) = 71%
- But we are doing **great** at predicting losses (94%)...
- ...and terribly at predicting wins (17%)

Regression

```
fn %>%
  ggplot(aes(x = damage_to_players,y = won)) +
  geom_point() +
  geom_smooth(method = 'lm')
```



Regression

- Binary outcome variable!
 - A linear regression is not the best solution
 - \circ Predictions can exceed support of Y
- But it can still work! linear probability model

```
mLM <- lm(won ~ hits + accuracy + mental_state,fn)</pre>
```

Linear Regression

require(broom) # broom package makes it easy to read regression
output

```
## Loading required package: broom
```

```
tidy(mLM) %>% # This would be the same as summary(mLM)
mutate_at(vars(-term), function(x) round(x,5))
```

Linear Regression

```
mLM <- lm(won ~ scale(hits) + scale(accuracy) + mental_state,fn)
tidy(mLM)</pre>
```

```
fn %>%
  summarise_at(vars(hits,accuracy),function(x) round(sd(x),1))
```

Evaluating Predictions

```
mLM <- lm(won ~ hits + accuracy + mental state + damage taken +
head shots + gameIdSession,fn)
fn %>%
  mutate(preds = predict(mLM)) %>%
  mutate(predBinary = ifelse(preds > .5,1,0)) %>%
  select(won,predBinary,preds)
```

```
## # A tibble: 957 × 3
##
   won predBinary preds
   <dbl> <dbl> <dbl> <dbl>
##
## 1
                  0 0.320
##
                    0 0.239
##
                    0 0.193
##
                    0 0.285
## 5
                    0 0.148
##
                    0 0.175
##
                    0 0.258
##
         0
                    0 0.115
##
         0
                    0 0.239
## 10
                    0 0.0982
  # i 947 more rows
```

Evaluating Predictions

```
(sumTab <- fn %>%
  mutate(pred_win = ifelse(predict(mLM) > .5,1,0)) %>%
  group_by(won) %>%
  mutate(total_games = n()) %>%
  group_by(won,pred_win,total_games) %>%
  summarise(nGames=n(),.groups = 'drop') %>%
  mutate(prop = percent(nGames / total_games)) %>%
  ungroup() %>%
  mutate(accuracy = percent(sum((won == pred_win)*nGames) /
  sum(nGames))))
```

```
## # A tibble: 4 × 6
##
      won pred win total games nGames prop accuracy
                     <int> <int> <chr> <chr> <</pre>
##
    <dbl>
             <dbl>
## 1
                                 615 92% 71%
                           666
                          666 51 8% 71%
## 2
## 3 <u>1</u>
                 0
                          291 226 78% 71%
## 4
                          291
                                  65 22%
                                          71%
```

Evaluating Predictions

- Overall accuracy is just the number of correct predictions (either 0 or 1) out of all possible
 - Is 71% good?
 - What would the dumbest guess be? Never win! 70%
- Might also want to care about just 1s
 - Sensitivity: Predicted wins / actual wins = 22%
- Also might care about just 0s
 - Specificity: Predicted losses / actual losses = 92%

• Shifting the threshold for 0 or 1 prediction can matter

```
fn %>%
  mutate(pred_win = ifelse(predict(mLM) > .4,1,0)) %>%
  group_by(won) %>%
  mutate(total_games = n()) %>%
  group_by(won,pred_win,total_games) %>%
  summarise(nGames=n(),.groups = 'drop') %>%
  mutate(prop = percent(nGames / total_games)) %>%
  ungroup() %>%
  mutate(accuracy = percent(sum((won == pred_win)*nGames) /
  sum(nGames)))
```

• Shifting the threshold for 0 or 1 prediction can matter

```
fn %>%
  mutate(pred_win = ifelse(predict(mLM) > .7,1,0)) %>%
  group_by(won) %>%
  mutate(total_games = n()) %>%
  group_by(won,pred_win,total_games) %>%
  summarise(nGames=n(),.groups = 'drop') %>%
  mutate(prop = percent(nGames / total_games)) %>%
  ungroup() %>%
  mutate(accuracy = percent(sum((won == pred_win)*nGames) /
  sum(nGames)))
```

```
## # A tibble: 4 × 6
##
      won pred win total games nGames prop accuracy
   <dbl> <dbl>
##
                     <int> <int> <chr> <chr>
## 1
                         666
                                663 99.5% 70%
## 2
                         666
                                  3 0.5% 70%
## 3
                0
                         291 280 96.2% 70%
                         291
                                11 3.8% 70%
## 4
```

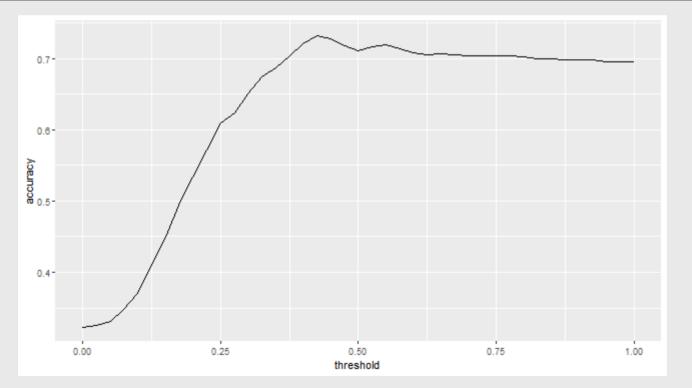
31 / 74

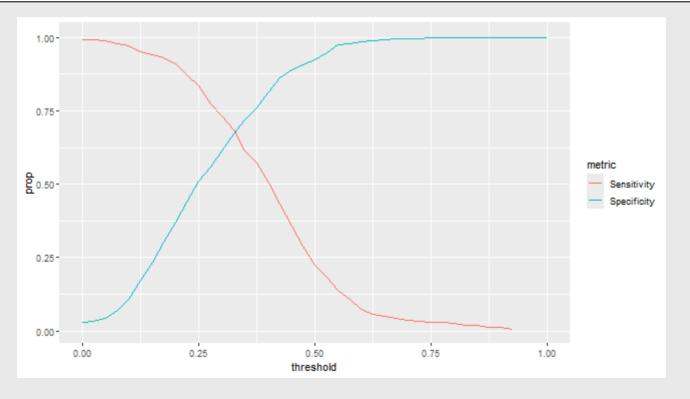
- We could keep trying different values until we hit on one that maximizes our accuracy
- But this is inefficient! Let's loop it instead!

```
toplot <- NULL
for(thresh in seq(0,1,by = .025)) {
  toplot <- fn %>%
  mutate(pred win = ifelse(predict(mLM) > thresh,1,0)) %>%
  group by(won) %>%
  mutate(total games = n()) %>%
  group by(won,pred win,total games) %>%
  summarise(nGames=n(),.groups = 'drop') %>%
  mutate(prop = nGames / total_games) %>%
  ungroup() %>%
  mutate(accuracy = sum((won == pred win)*nGames) / sum(nGames)) %>%
  mutate(threshold = thresh) %>%
    bind rows(toplot)
```

• We might only care about accuracy by itself (although this is a bit naive)

```
toplot %>%
  select(accuracy,threshold) %>%
  distinct() %>%
  ggplot(aes(x = threshold,y = accuracy)) +
  geom_line()
```



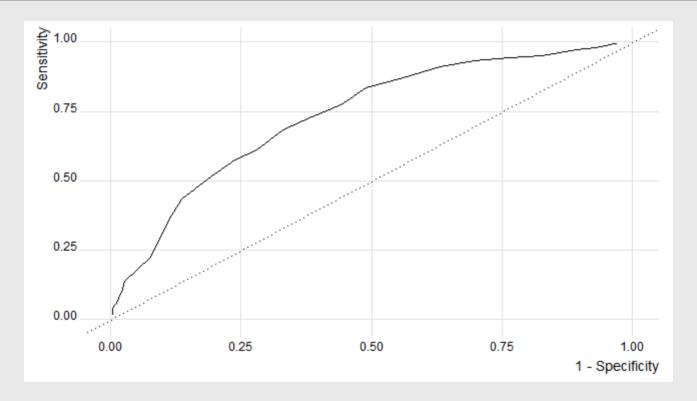


ROC Curve

- Receiver-Operator Characteristic (ROC) Curve
- Commonly used to evaluate classification methods
 - X-axis: 1-specificity
 - Y-axis: sensitivity

ROC Curve

р



• Better models have high levels of sensitivity **and** specificity at every threshold

AUC Measure

- Area Under the Curve (AUC)
 - A single number summarizing classification performance

AUC

- What is a "good" AUC?
 - We know it is bounded between 0 (i.e., it predicts everything perfectly wrong) and 1 (i.e., it predicts everything perfectly correct)
 - But typically we don't see AUC values less than 0.5 (why is this?)
- AUC can be interpreted like numeric grades at Vandy (and for this class)
 - 0.95+ is amazing
 - 0.9 0.95 is very good
 - 0.8-range is B-tier
 - 0.7-range is C-tier
 - 0.6-range is really bad
 - AUC values less than 0.6 are failing

Party time!

• Adding more variables / trying different combinations

Workflow

- 1. Train models
- 2. Predict models
- 3. Evaluate models

Train models

```
m1 <- lm(won ~ hits,fn)
m2 <- lm(won ~ hits + head_shots,fn)
m3 <- lm(won ~ hits + accuracy + head_shots,fn)
m4 <- lm(won ~ hits + accuracy + head_shots + mental_state,fn)
m5 <- lm(won ~ hits + accuracy + head_shots + mental_state +
distance_traveled,fn)
m6 <- lm(won ~ hits + accuracy + mental_state + head_shots +
distance_traveled + gameIdSession,fn)</pre>
```

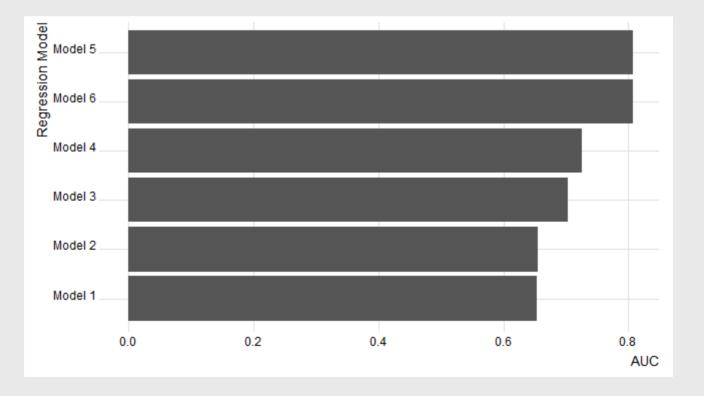
Predict models

Evaluate models

```
rocRes <- NULL
for(model in 1:6) {
  rocRes <- roc_auc(toEval,truth,paste0('m',model,'Preds')) %>%
    mutate(model = paste0('Model ',model)) %>%
    bind_rows(rocRes)
}
```

Evaluate models

```
rocRes %>%
  ggplot(aes(x = .estimate,y = reorder(model,.estimate))) +
  geom_bar(stat = 'identity') +
  ggridges::theme_ridges() + labs(x = 'AUC',y = 'Regression Model')
```



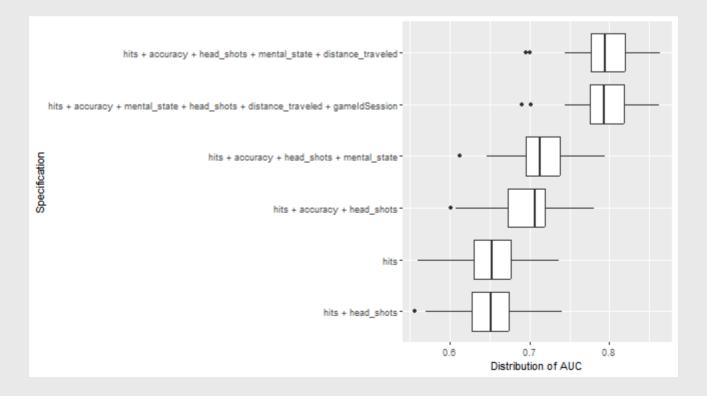
OVERFITTING

Cross validation to the rescue!

```
set.seed(123)
cvRes <- NULL
for(i in 1:100) {
 # Cross validation prep
 inds <- sample(1:nrow(fn), size = round(nrow(fn)*.8), replace = F)</pre>
 train <- fn %>% slice(inds)
 test <- fn %>% slice(-inds)
 # Training models
 m1 <- lm(won ~ hits,train)</pre>
 m2 <- lm(won ~ hits + head_shots,train)</pre>
 m3 <- lm(won ~ hits + accuracy + head_shots,train)
 m4 <- lm(won ~ hits + accuracy + head_shots + mental_state,train)</pre>
 m5 <- lm(won ~ hits + accuracy + head shots + mental state + distance traveled, train)
 m6 <- lm(won ~ hits + accuracy + mental state + head shots + distance traveled + gameIdSession,train)
 # Predicting models
 toEval <- test %>%
    mutate(m1Preds = predict(m1, newdata = test),
           m2Preds = predict(m2,newdata = test),
           m3Preds = predict(m3,newdata = test),
           m4Preds = predict(m4, newdata = test),
           m5Preds = predict(m5,newdata = test),
           m6Preds = predict(m6, newdata = test),
           truth = factor(won,levels = c('1','0')))
 # Evaluating models
 rocResBS <- NULL
 for(model in 1:6) {
    rocResBS <- roc_auc(toEval,truth,paste0('m',model,'Preds')) %>%
      mutate(model = as.character(get(paste0('m',model))$call$formula)[3]) %>%
      bind rows(rocResBS)
 cvRes <- rocResBS %>%
    mutate(bsInd = i) %>%
    bind_rows(cvRes)
```

Cross Validation AUC

```
cvRes %>%
  ggplot(aes(x = .estimate,y = factor(reorder(model,.estimate)))) +
  geom_boxplot() + labs(x = 'Distribution of AUC',y =
'Specification')
```



Conclusion

- Classification is just a type of prediction
 - We used linear regression
 - But there are much fancier algorithms out there
- After the break:
 - A *slightly* fancier algorithm: logistic regression
 - How to use the models to achieve the team's goals

BREAK

Agenda

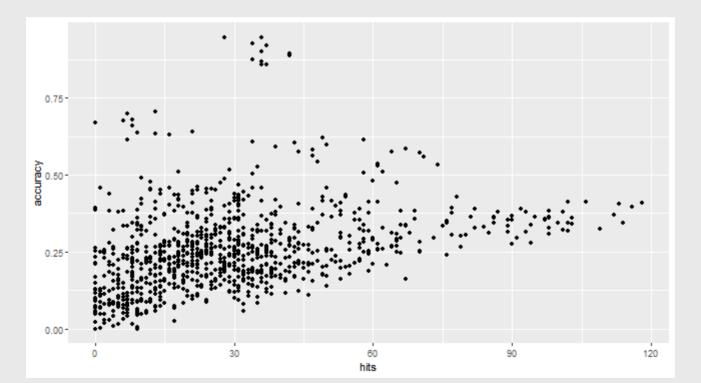
- 1. Introducing logit
- 2. Running logit
- 3. Evaluating logit

Logit Regression

- A different type of regression
 - What do we mean by **type**?
- Let's take a step back

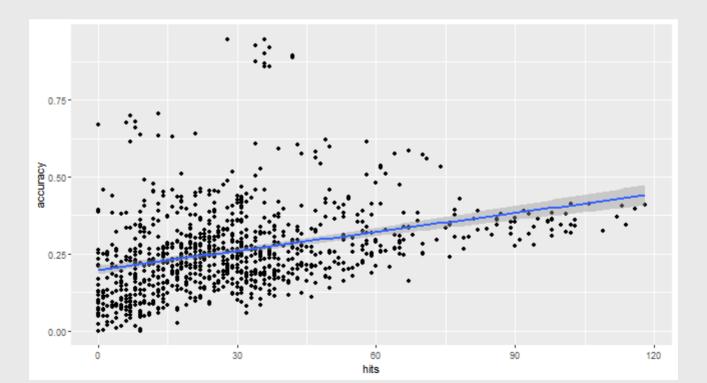
• "Linear" regression...why is it "linear"?

```
(p <- fn %>%
  ggplot(aes(x = hits,y = accuracy)) +
  geom_point())
```



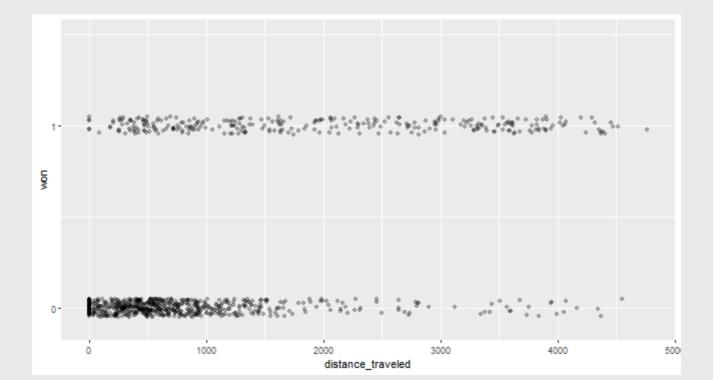
- "Linear" regression...why is it "linear"?
- Because you can summarize it with a line!

```
p + geom_smooth(method = 'lm')
```

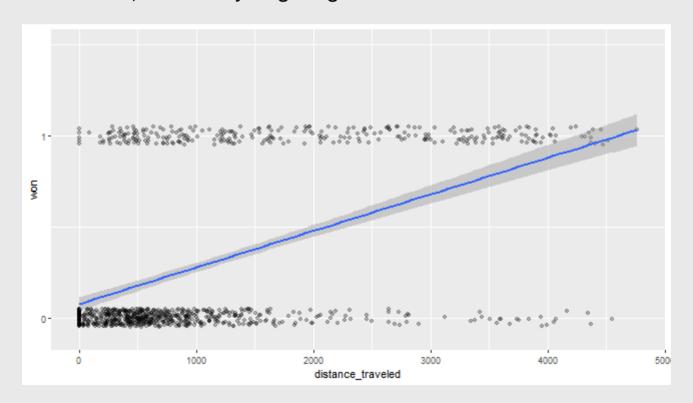


But what if the outcome is binary?

```
(p <- fn %>% ggplot(aes(x = distance_traveled,y = won)) +
   scale_y_continuous(breaks = c(0,1),limits = c(-.1,1.5)) +
   geom_jitter(width = .01,height = .05,alpha = .25))
```



- But what if the outcome is binary?
- Lines seem too clumsy
 - ∘ If 1 = won, how can you go higher?



Logit

- Theory: binary outcomes are **proxies** for some **latent** measure
 - Binary outcome won: either placed first or did not
 - Latent outcome placed: continuous measure
 - Might also imagine ability: continuous measure
- The higher your ability, the more likely you are to win
- Logit regression: model the ability
 - What is ability actually?
 - \circ Probability of winning: Pr(won)
- Part of a broader class of models called "generalized linear model" (GLM)

$$Pr(y = 1|x) = G(\alpha + \beta X)$$

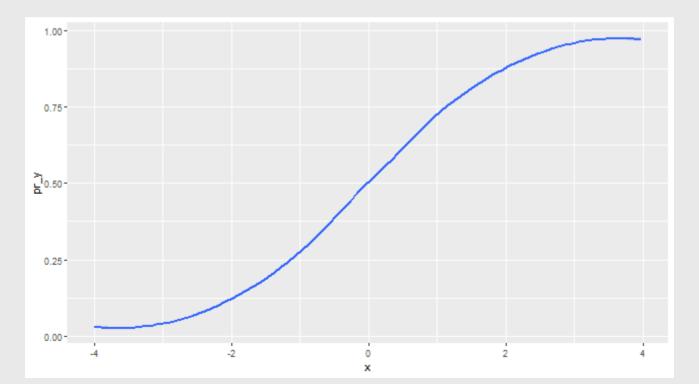
GLMs

- $Pr(y=1|x) = G(\alpha + \beta X)$
- Does this look familiar?
- Linear regression: $Y = \alpha + \beta X$
 - \circ Outcome: $Y \rightarrow Pr(y=1|x)$
 - \circ Mapping: $\alpha + \beta X \rightarrow G(\alpha + \beta X)$
- ullet G is the "link function"
 - \circ Transforms values of lpha+eta X into **probabilities**
- Logistic function: specific type of link function

$$G(x) = \frac{1}{1 + exp(-x)}$$

Logistic Function

```
x <- runif(100,-4,4)
pr_y <- 1/(1 + exp(-x))
as_tibble(pr_y = pr_y,x = x) %>%
    ggplot(aes(x = x,y = pr_y)) +
    geom_smooth()
```

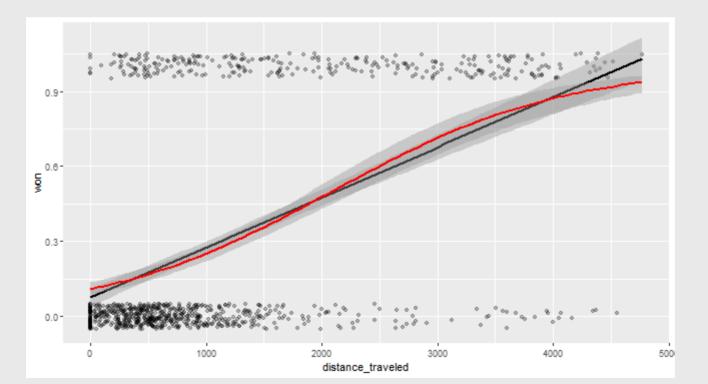


Logistic Function

• But what about real data like $\alpha + \beta X$?

•
$$G(X) = rac{exp(lpha + eta X)}{1 + exp(lpha + eta X)}$$

- We estimate this with glm(formula, data, family)
 - Note similarity to lm(formula, data)
- family = binomial(link = "logit")



```
# Train model.
mLogit <- glm(formula = won ~ distance traveled,data = fn,family =</pre>
binomial(link = 'logit'))
# Predict model.
fn <- fn %>%
  mutate(prob won = predict(mLogit, type = 'response')) %>%
  mutate(pred won = ifelse(prob won > .5,1,0))
# Fvaluate model.
eval <- fn %>%
  group by(won) %>%
  mutate(total games = n()) %>%
  group by(won,pred won,total games) %>%
  summarise(nGames=n(),.groups = 'drop') %>%
  mutate(prop = nGames / total games) %>%
  ungroup() %>%
  mutate(accuracy = percent(sum((won == pred won)*nGames) /
sum(nGames)))
```

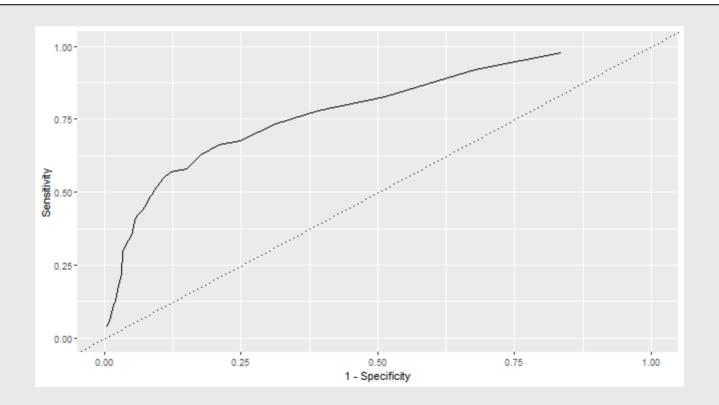
```
eval
```

```
## # A tibble: 4 × 6
##
      won pred won total games nGames prop accuracy
                     <int> <int> <dbl> <chr>
   <dbl>
          <dbl>
##
## 1
                          666
                                620 0.931
                                          78%
                0
                         666 46 0.0691 78%
## 2
## 3
                0
                         291 163 0.560 78%
## 4
                         291 128 0.440 78%
```

Can also calculate ROC Curve and AUC

```
toplot <- NULL
for(thresh in seq(0,1,by = .025)) {
  toplot <- fn %>%
    mutate(pred_won = ifelse(predict(mLogit,type = 'response') >
  thresh,1,0)) %>%
    group_by(won) %>%
    mutate(total_games = n()) %>%
    group_by(won,pred_won,total_games) %>%
    summarise(nGames=n(),.groups = 'drop') %>%
    mutate(prop = nGames / total_games) %>%
    ungroup() %>%
    mutate(threshold = thresh) %>%
    bind_rows(toplot)
}
```

р



- Two big questions in prediction:
 - 1. Do I have the correct predictors X?
 - 2. Do I have the best model?
- Two types of outcomes (thus far)
 - 1. Continuous Y: use **RMSE**
 - 2. Binary Y: use **AUC**
- Let's determine the best model from the following:
 - X: (1) distance_traveled + mental_state vs. (2)
 distance_traveled + mental_state + hits
 - Model: (1) conditional means vs. (2) 1m vs. (3) g1m

ullet Conditional means - simple X

```
results <- NULL
# Train & Predict
toFval <- fn %>%
 mutate(distDec = ntile(distance traveled, n = 10)) %>%
  group by(distDec,mental state) %>%
  mutate(prob won = mean(won),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob won)
# Fvaluate
results <- roc auc(data = toEval,truth,prob won) %>%
  mutate(model = 'CM',
         predictors = 'Simple') %>%
  bind rows(results)
```

ullet Conditional means - complex X

```
# Train & Predict
toFval <- fn %>%
  mutate(distDec = ntile(distance traveled, n = 10),
         hitsDec = ntile(hits,n = 10)) %>%
  group by(distDec,hitsDec,mental state) %>%
  mutate(prob won = mean(won),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob won)
# Fvaluate
results <- roc auc(data = toEval,truth,prob won) %>%
  mutate(model = 'CM',
         predictors = 'Complex') %>%
  bind rows(results)
```

• Linear regression (1m) - simple X

```
# Train
m <- lm(won ~ distance traveled + mental state,fn)</pre>
# Predict
toEval <- fn %>%
  mutate(prob won = predict(m),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob won)
# Fvaluate
results <- roc auc(data = toEval,truth,prob won) %>%
  mutate(model = 'LM',
         predictors = 'Simple') %>%
  bind rows(results)
```

• Linear regression (1m) - complex X

```
# Train
m <- lm(won ~ distance traveled + mental state + hits,fn)</pre>
# Predict
toEval <- fn %>%
  mutate(prob won = predict(m),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob won)
# Fvaluate
results <- roc auc(data = toEval,truth,prob won) %>%
  mutate(model = 'LM',
         predictors = 'Complex') %>%
  bind rows(results)
```

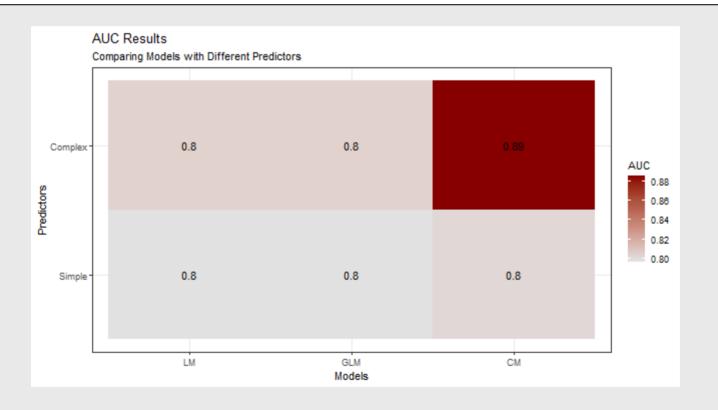
ullet Logit regression (glm) - simple X

```
# Train
m <- glm(won ~ distance traveled + mental state,fn,family =</pre>
binomial(link = 'logit'))
# Predict
toFval <- fn %>%
  mutate(prob won = predict(m, type = 'response'),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob won)
# Fvaluate
results <- roc auc(data = toEval,truth,prob won) %>%
  mutate(model = 'GLM',
         predictors = 'Simple') %>%
  bind rows(results)
```

• Logit regression (glm) - complex X

```
# Train
m <- glm(won ~ distance traveled + mental state + hits,fn,family =
binomial(link = 'logit'))
# Predict
toFval <- fn %>%
  mutate(prob won = predict(m, type = 'response'),
         truth = factor(won,levels = c('1','0'))) %>%
    ungroup() %>%
    select(truth,prob won)
# Fvaluate
results <- roc auc(data = toEval,truth,prob won) %>%
  mutate(model = 'GLM',
         predictors = 'Complex') %>%
  bind rows(results)
```

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Conclusion

- Conditional means outperform regression models?
 - Yes: conditional means allow for cell-specific predictions
 - No: conditional means are more susceptible to overfitting
- How would you re-evaluate these models-X-predictors to account for overfitting?