Regression Part 1

Homework

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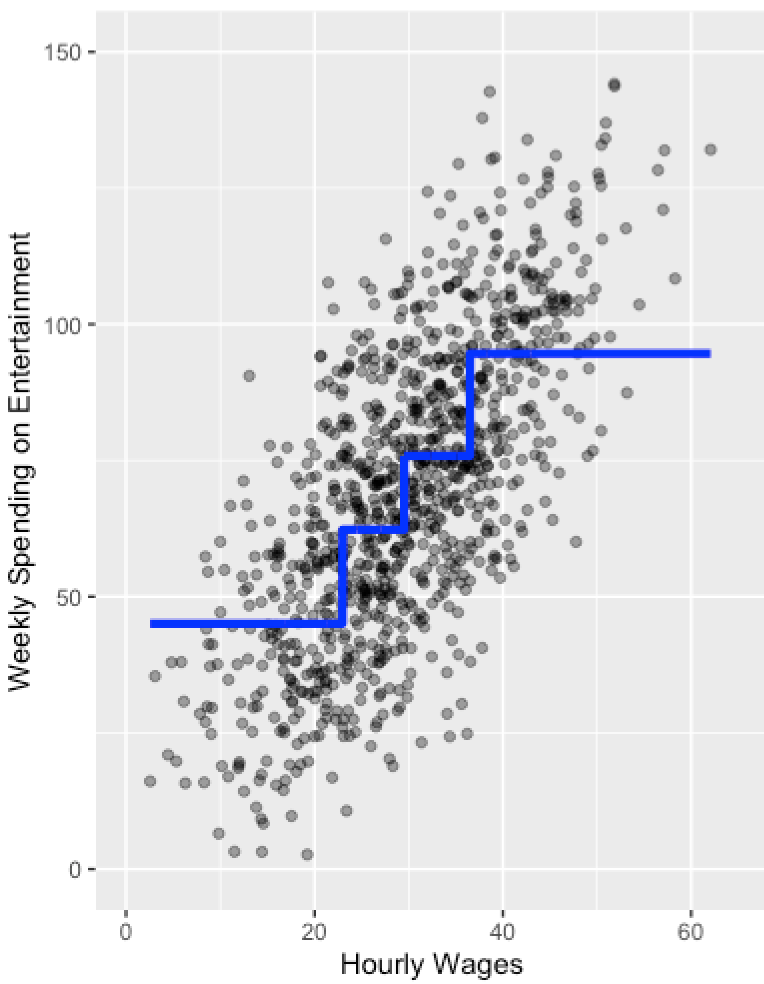
Due Date: 2024-07-15

Overview

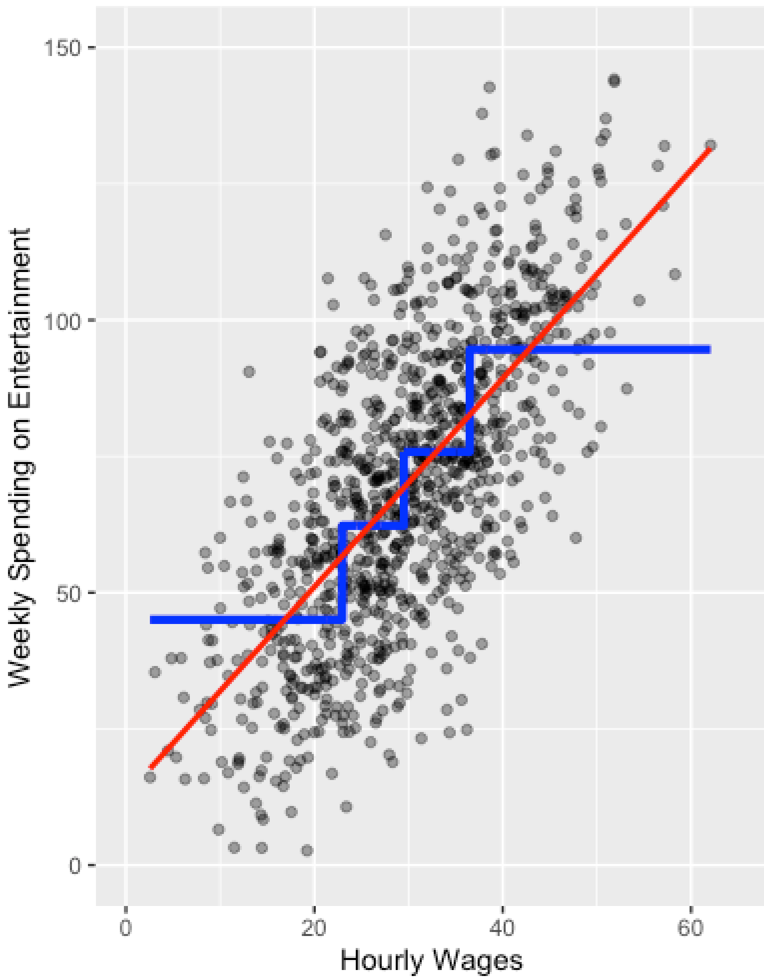
So far, we've been using just the simple mean to make predictions. Today, we'll continue using the simple mean to make predictions, but now in a complicated way. Before, when we calculated conditional means, we did so in certain "groupings" of variables. When we run linear regression, we no longer need to do so. Instead, linear regression allows us to calculate the conditional mean of the outcome at *every* value of the predictor. If the predictor takes on just a few values, then that's the number of conditional means that will be calculated. If the predictor is continuous and takes on a large number of values, we'll still be able to calculate the conditional mean at every one of those values.

As an example, consider the following plot of weekly spending on entertainment (movies, video games, streaming, etc) as a function of hourly wages.



If we're asked to summarize the relationship between wages and spending, we could use conditional means. That's what the blue line above does: for each of the four quartiles of hourly wages, it provides the average spending on entertainment. We could expand this logic and include both more levels of hourly wages and other variables with which to calculate the mean– for example, hourly wages and level of education. The problem is that this approach gives us lots of numbers back.

The graphic below fits a line (in red) to the data:



The line has the general function y=12+(2*x). We're saying that if hourly wages were 0, the person would be predicted to spend 12 on entertainment (who knows where they got it). As hourly wages go up, the line predicts that weekly spending goes up $2 for every $1 increase in hourly wages. The line we fit to this data summarizes the relationship using just two numbers: the intercept (value of the y when x=0) and slope– the amount that y increases as a function of x. We call the slope the *coefficient* on x.

The general model we posit for regression is as follows:

$$Y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \ldots \beta_k x_k + \epsilon$$

It's just a linear, additive model. Y increases or decreases as a function of x, with multiple x's included. ϵ is the extent to which an individual value is above or below the line created. β is the amount that Y increases or decreases for a one unit change in x. α (sometimes referred to as β_0) is the intercept that tells the value of Y when all the $x_1, \ldots, x_k$ are equal to zero. With the simplifying assumption of linearity, we can summarize what we know about the relationship between the independent and dependent variable in a very parsimonious way. The trade-off is that an assumption of linearity may not always be warranted.

Note that this general model is the empiricist's **theory**. We have to make choices about what $x_1, \ldots, x_k$ will be. For example, should we include wages and family structure into a model of spending on entertainment? Theoretically, we might think that, as your family gets bigger, you must spend more money on entertainment!

When we actually apply this model to data, we obtain **predictions** for both Y *and* each of the β parameters. We denote predicted values with a "hat": $\hat{Y}$, $\hat{\beta}_1$, etc. So how do we actually DO this?

College Data

We are going to go back to two previous datasets to get started. Our main focus will be on the college debt data from the beginning of the semester. Recall our previous work where we looked at the relationship between SAT scores and future earnings. All else equal, we expect that future earnings (Y) are positively related with SAT scores (X).

Let's load the data and packages first.

```
require(tidyverse)
require(plotly)
sc_debt <- read_rds('https://github.com/jbisbee1/ISP_Data_Science_2024/raw/main/data/sc_
debt.Rds')
```

Now let's look at our two variables of interest before we do anything else. Are these continuous? Categorical?

```
sc_debt %>%
  select(sat_avg,md_earn_wne_p6)
```

```
## # A tibble: 2,546 × 2
##    sat_avg md_earn_wne_p6
##      <int>          <int>
##  1     939          25200
##  2    1234          35100
##  3      NA          30700
##  4    1319          36200
##  5     946          22600
##  6    1261          37400
##  7      NA          23100
##  8      NA          33400
##  9    1082          30100
## 10    1300          39500
## # ℹ 2,536 more rows
```

As we can see, they both appear to be continuous measures, although we can see that many schools don't report the average SAT scores of their students. Let's use the `summary()` function to count how much missing data we are dealing with.

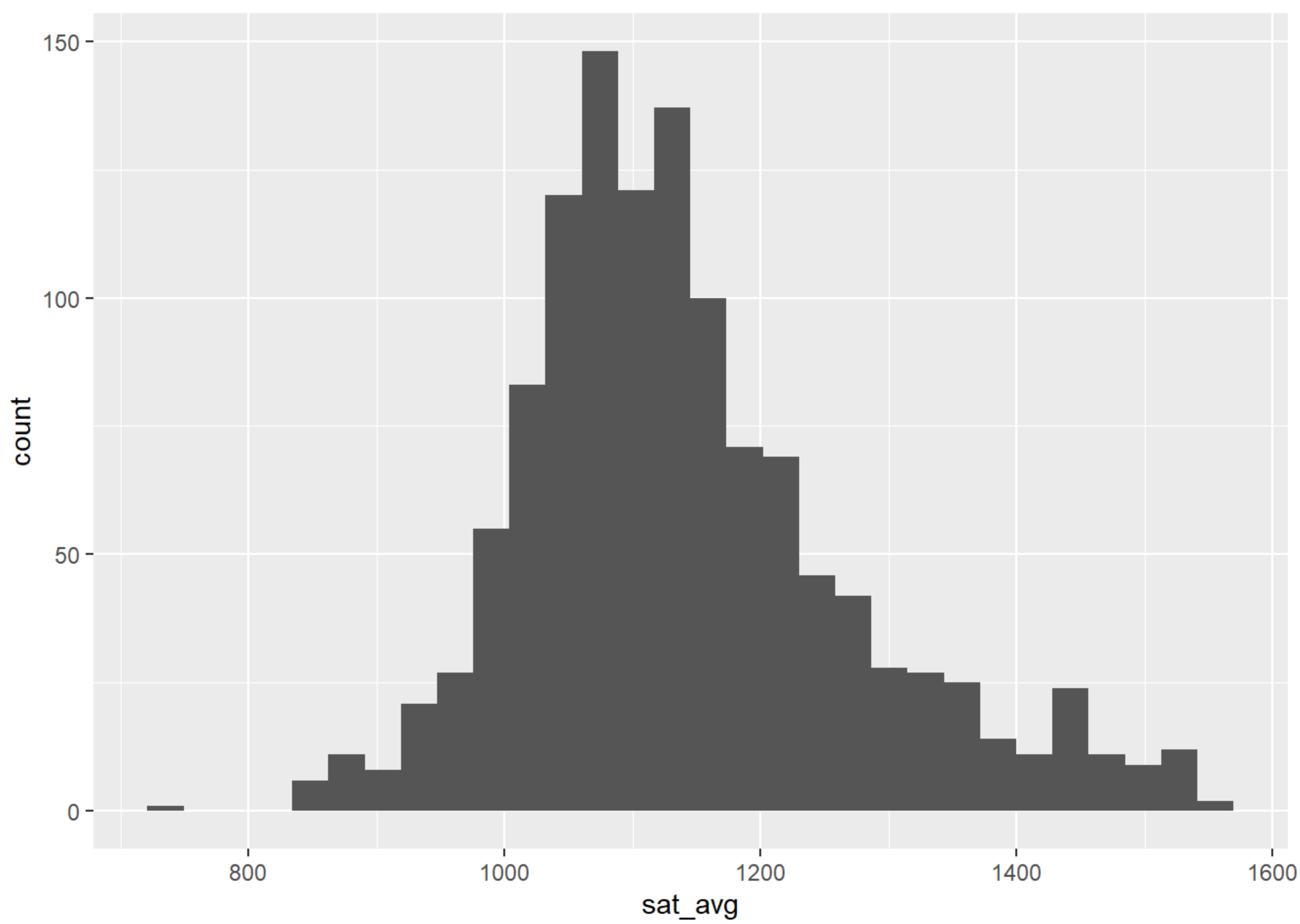
```
summary(sc_debt %>% select(sat_avg,md_earn_wne_p6))
```

```
##     sat_avg     md_earn_wne_p6
##  Min.   : 737   Min.   : 10600
##  1st Qu.:1053   1st Qu.: 26100
##  Median :1119   Median : 31500
##  Mean   :1141   Mean   : 33028
##  3rd Qu.:1205   3rd Qu.: 37400
##  Max.   :1557   Max.   :120400
##  NA's   :1317   NA's   :240
```

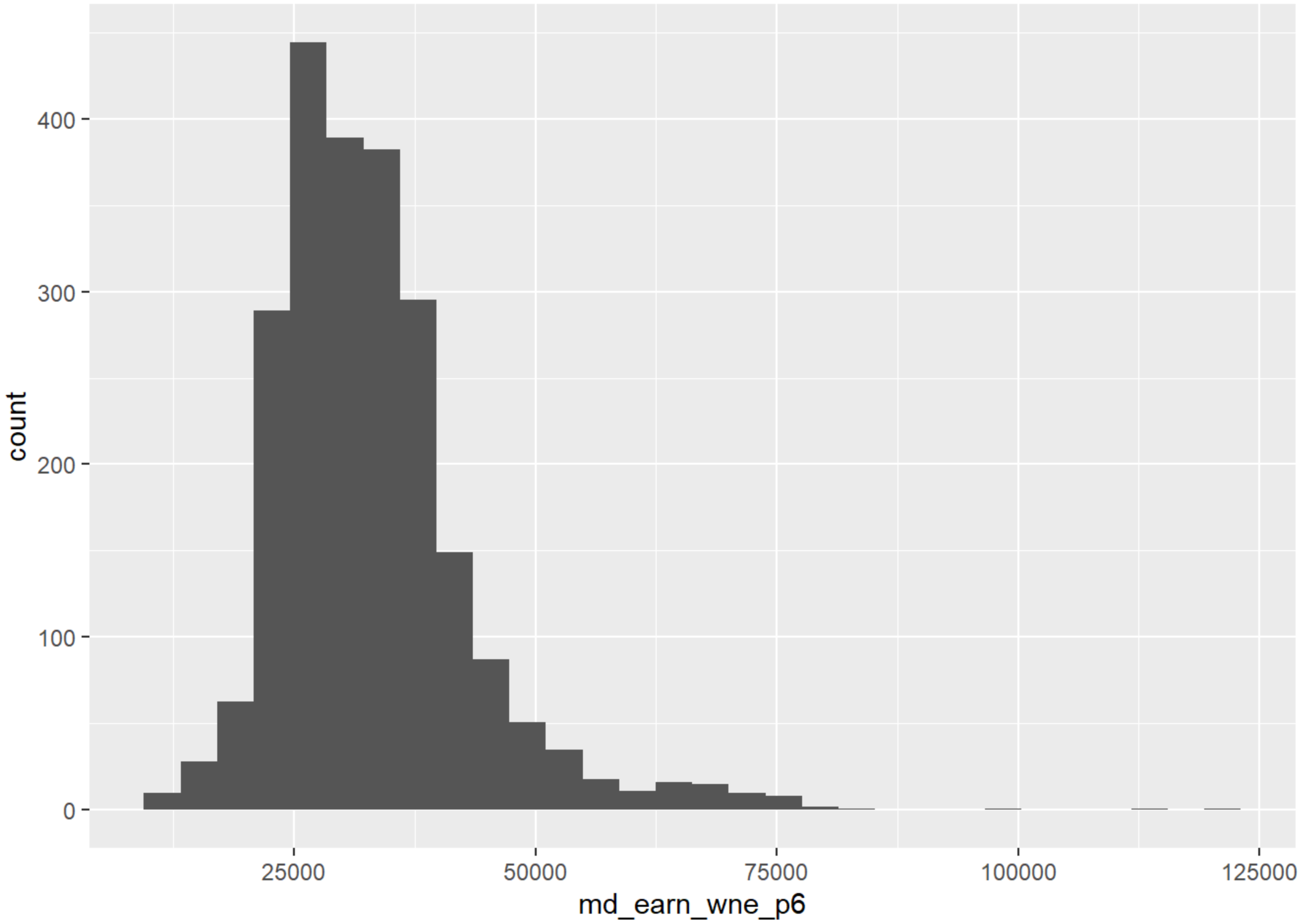
240 schools don’t report the future earnings of their recent graduates, and a whopping 1,317 schools don’t report SAT scores!

Let’s now look at both variables with univariate plots.

```
sc_debt %>%
  ggplot(aes(x = sat_avg)) +
  geom_histogram()
```



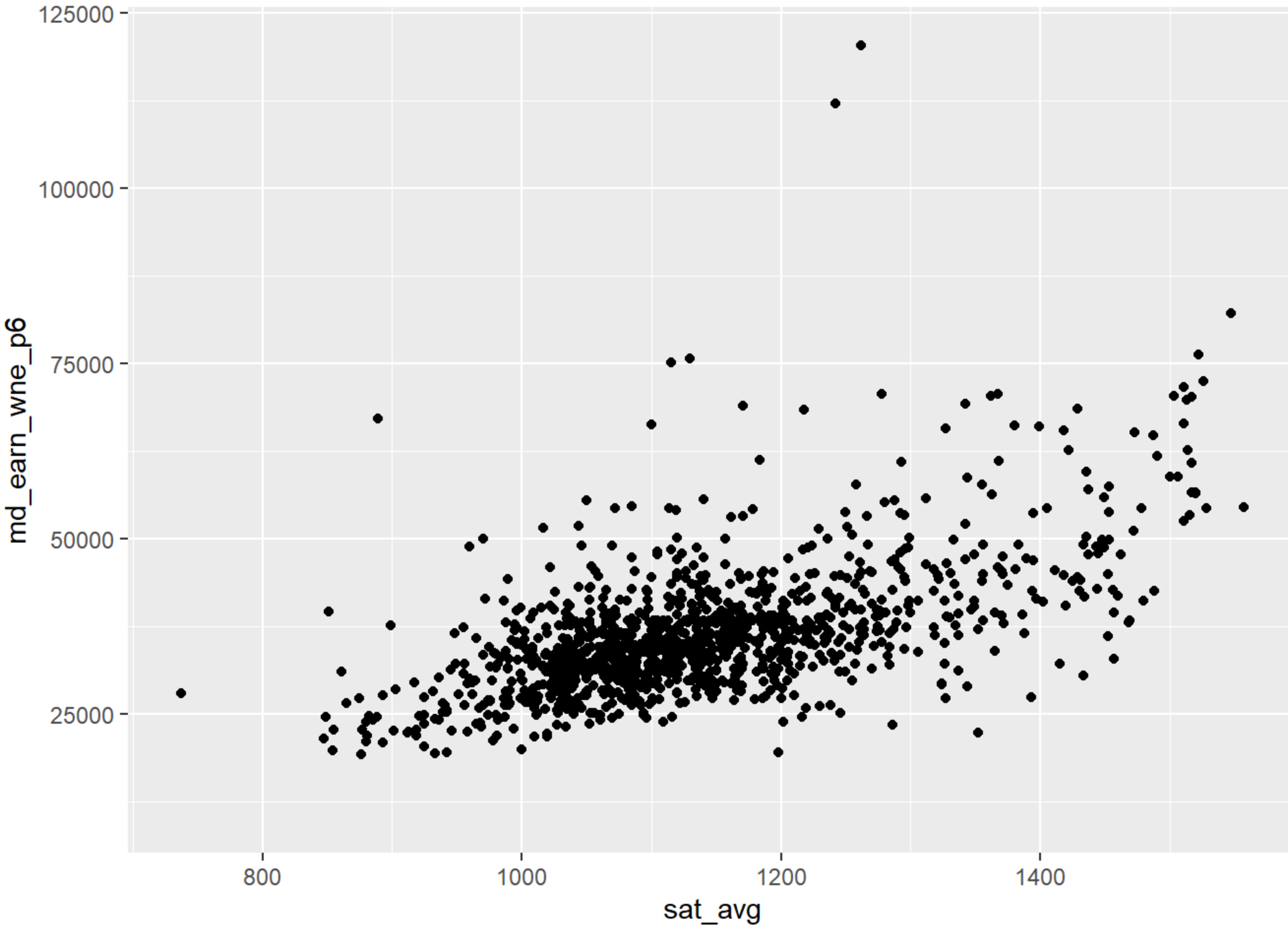

```
sc_debt %>%
  ggplot(aes(x = md_earn_wne_p6)) +
  geom_histogram()
```



Both variables are mildly skewed, but not enough for us to worry about transforming them.

Now let's plot them as a scatter plot.

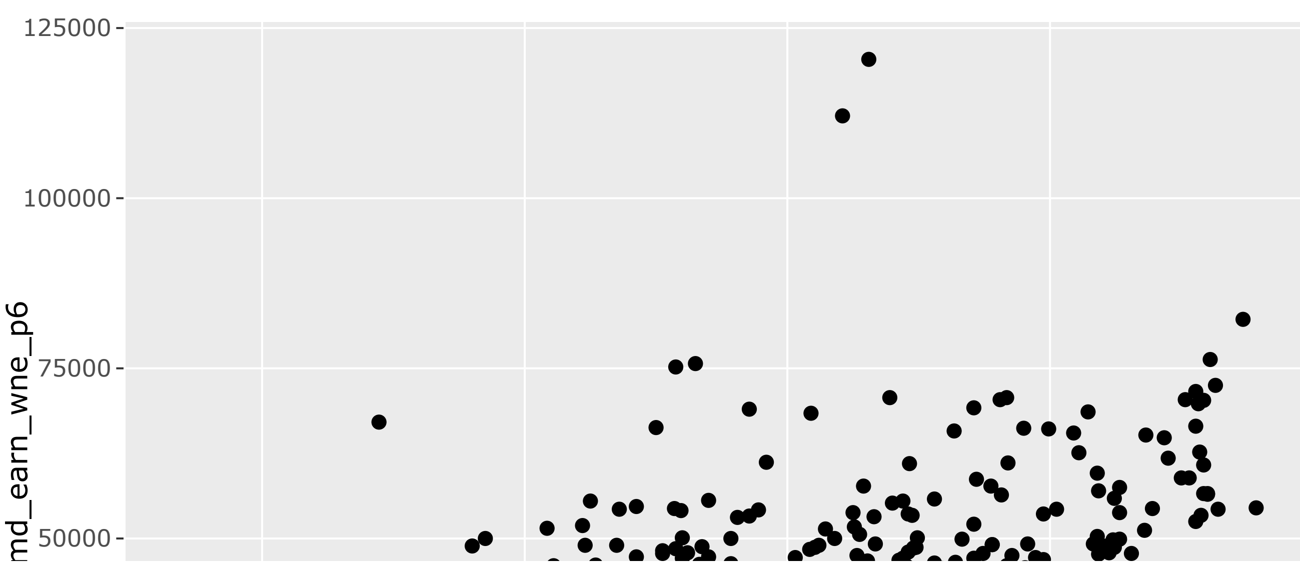
```
sc_debt %>%
  ggplot(aes(x = sat_avg,y = md_earn_wne_p6)) +
  geom_point()
```

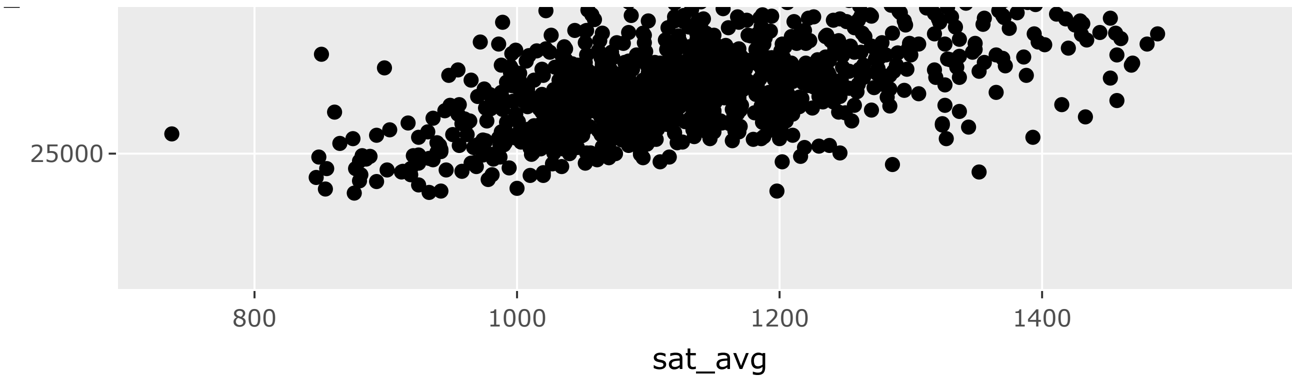


As we can see just by looking at the data, there is a positive relationship, where schools with higher SAT averages produce recent graduates with higher median earnings. However, note that there are a few outliers. Let's use `plotly` to help is identify these outliers.

```
p <- sc_debt %>%
  ggplot(aes(x = sat_avg,y = md_earn_wne_p6,text = instnm)) +
  geom_point()

ggplotly(p)
```

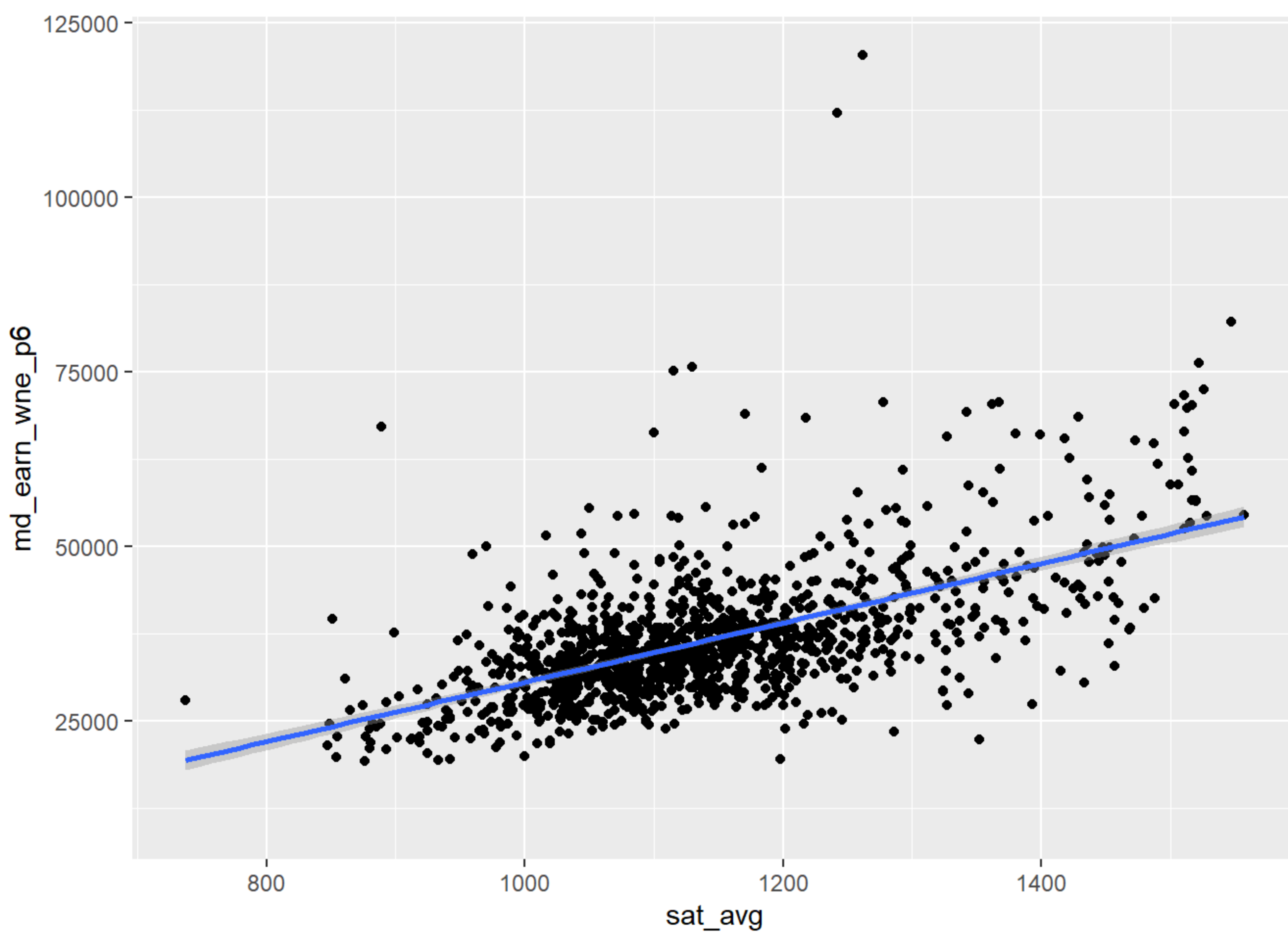




The University of Health Sciences and Pharmacy in St. Louis produces really wealthy recent graduates, despite having a middle-of-the-pack SAT score! (Why might this be?)

Now let's overlay the line of best fit using `geom_smooth(method = 'lm')`.

```
sc_debt %>%
  ggplot(aes(x = sat_avg,y = md_earn_wne_p6)) +
  geom_point() +
  geom_smooth(method = 'lm')
```



This line tells us that, on average, schools with average SAT scores of 1,000 produce recent graduates with incomes of roughly $30,000 annually.

Similarly, schools with average SAT scores of 1,400 produce graduates with median incomes of roughly $45,000.

The Linear Regression Model

Under the hood, `ggplot` is using a specific function to draw this line called the `lm()` function, which stands for "linear model". The function is choosing values of α and β that minimize the errors for every school in the dataset. We can access this function directly ourselves!

The `lm()` function takes two inputs that we must define. The first is the `formula`, and the second is the `data`.

The `formula` is literally just us writing the regression equation from above, but in terms `R` can understand. It takes the format of `Y ~ X`, which means $Y = \alpha + \beta X$. (We don't need to tell `R` about α and β in the `formula`…it calculates these values for us.)

In our setting, we want to set Y to `md_earn_wne_p6` and X to `sat_avg`, just like we drew them on the axes in our plot. Thus our formula becomes `md_earn_wne_p6 ~ sat_avg`. We also need to tell `R` which data we are using, in this case our `sc_debt` object. We then save the entire regression model to an object, that I'll call `model_earn_sat`

```
model_earn_sat <- lm(formula = md_earn_wne_p6 ~ sat_avg,data = sc_debt)
```

We can then look at the model results using the `summary()` function.

```
summary(model_earn_sat)
```

```
##
## Call:
## lm(formula = md_earn_wne_p6 ~ sat_avg, data = sc_debt)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -23239  -4311   -852   2893  78695
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -12053.87    1939.80  -6.214 7.12e-10 ***
## sat_avg         42.60       1.69  25.203  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7594 on 1196 degrees of freedom
##   (1348 observations deleted due to missingness)
## Multiple R-squared:  0.3469, Adjusted R-squared:  0.3463
## F-statistic: 635.2 on 1 and 1196 DF,  p-value: < 2.2e-16
```

The model gives us a lot of information. The results tell us that the `(Intercept)` (which is the same as α in our theory equation, and $\hat{\alpha}$ in our results) is equal to -12054.87, and that the `sat_avg` (which is the same as β in our theory equation, and $\hat{\beta}$ in our results) is equal to 42.60.

Substantively, we interpret these as follows:

- If a school has an average SAT score of 0, its recent grads are predicted to have median earnings of -$12,053.87.
- For each additional point the school has in terms of average SAT scores, its recent grad earnings are predicted to increase by $42.60.

Obviously, the α value (the `(Intercept)`) is somewhat meaningless. There are no schools whose average SAT scores are zero…the minimum possible SAT score can't even be zero!

But the β value is very interesting and directly speaks to the research question, theory, and hypothesis! According to this model, each additional point on the SATs yields an increase in earnings of $42!

Reading a Regression Table

There are many other numbers in this table, including the Standard Error (`Std. Error`), the t-statistic (`t value`), and the p-value (`Pr(>|t|)`). It turns out that the t-statistic is just the coefficient (`Estimate`) divided by the standard error (`Std. Error`), and the p-value is a way of converting the t-statistic into a measure of uncertainty! You don't need to know these steps in detail for this course. However, you do need to remember that the p-value is just 1 minus the confidence level. The smaller the p-value, the **more** confident we are that the β coefficient is not zero (i.e., that there is a relationship).

In this case, the p-value is basically zero, meaning we are basically 100% confident that the relationship between `md_earn_wne_p6` and `sat_avg` is positive!

Another Example

Let's ask a different research question: what is the relationship between future earnings and the admissions rate?

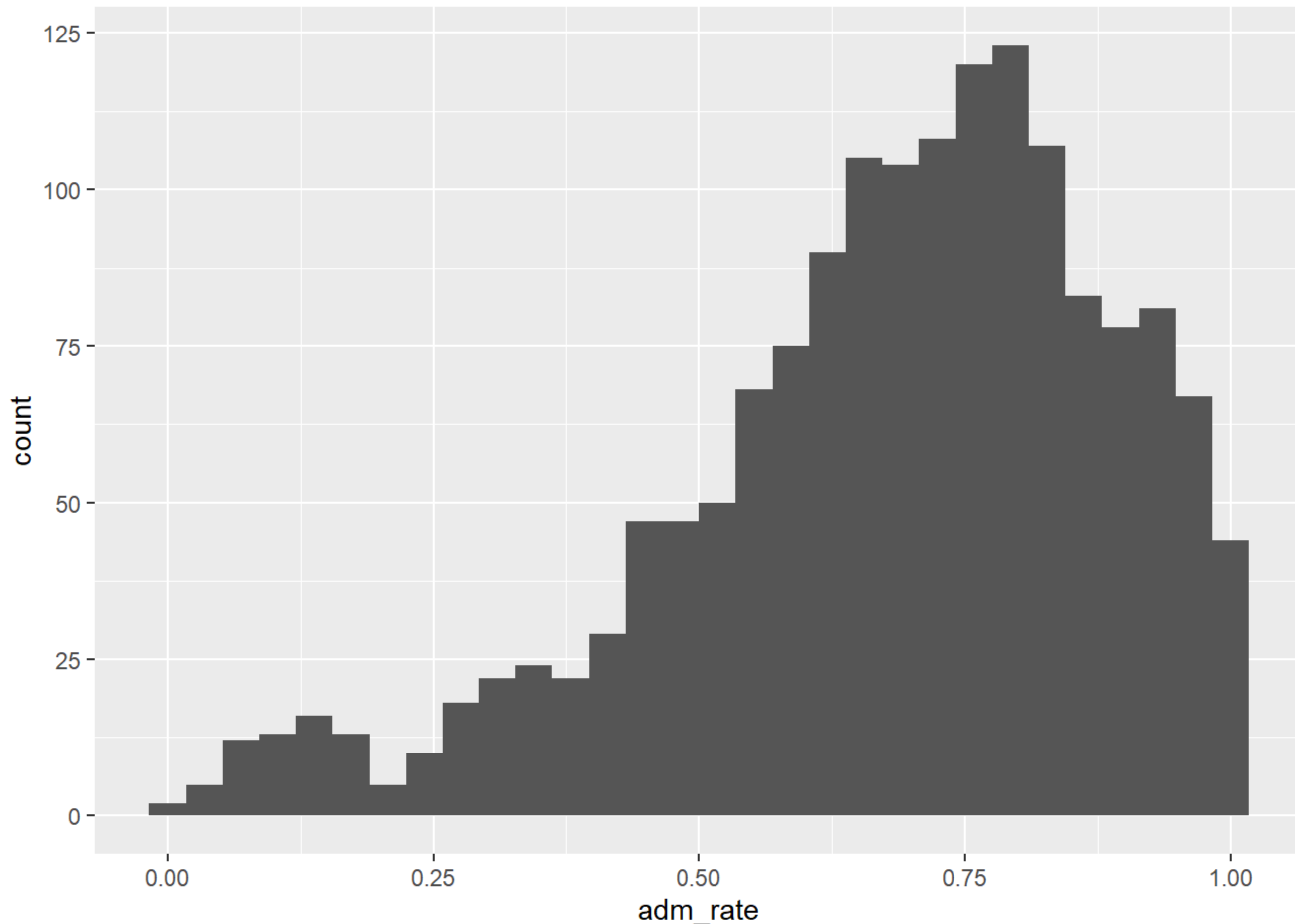
Theory: Can you come up with a theory? I would assume that more selective schools have more rigorous training, which then leads to wealthier recent graduates.

Hypothesis: What is your hypothesis? Mine is that the relationship between the admissions rate and future earnings is *negative*.

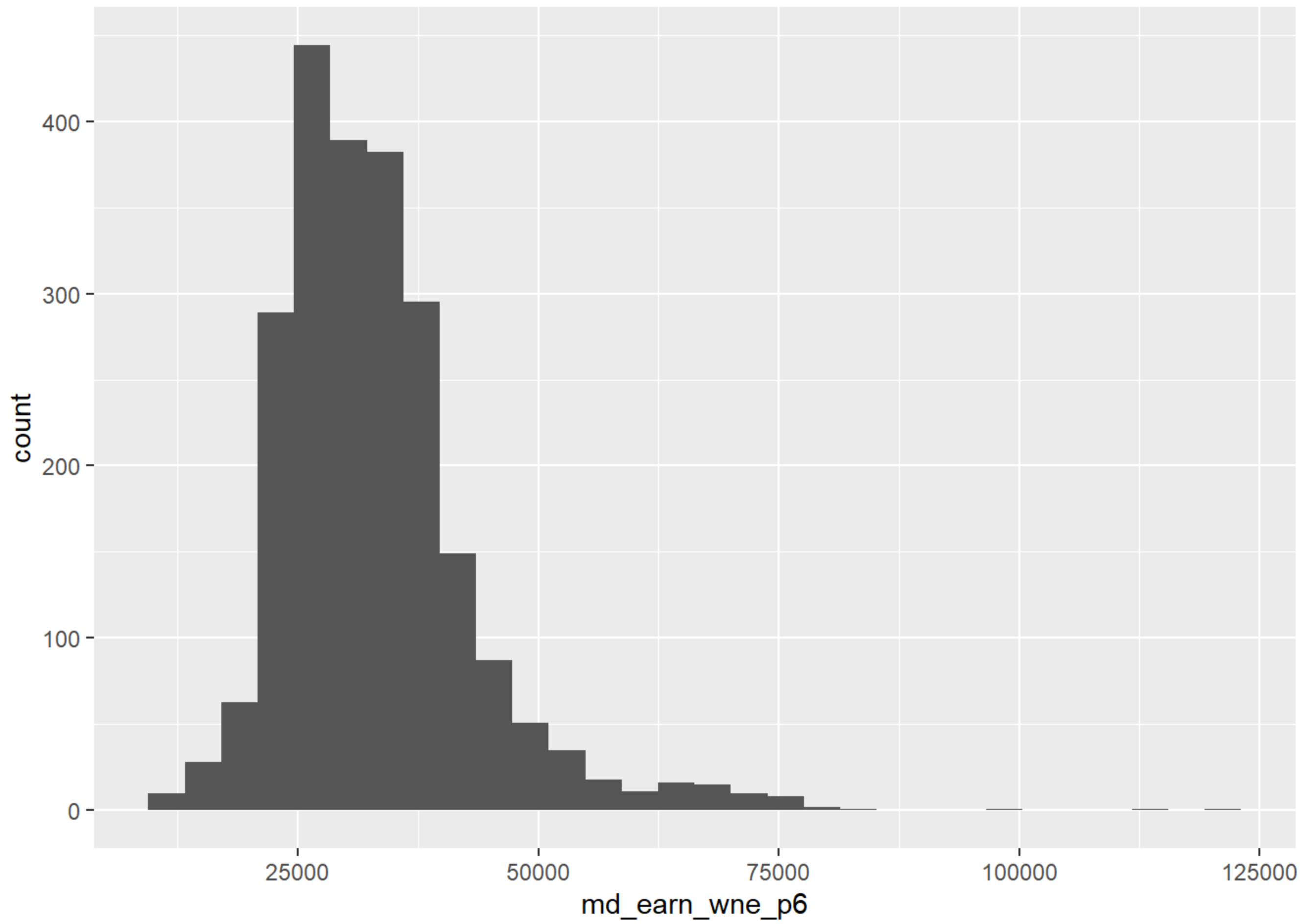
Let's test it!

First, we want to again look at both univariate and multivariate visualizations of our Y and X variables.

```
sc_debt %>%
  ggplot(aes(x = adm_rate)) +
  geom_histogram()
```




```
sc_debt %>%
  ggplot(aes(x = md_earn_wne_p6)) +
  geom_histogram()
```



As above, there is some mild skew, but nothing to be too worried about.

Let's also check on missingness, just so we know how many schools we **don't** have in our data.

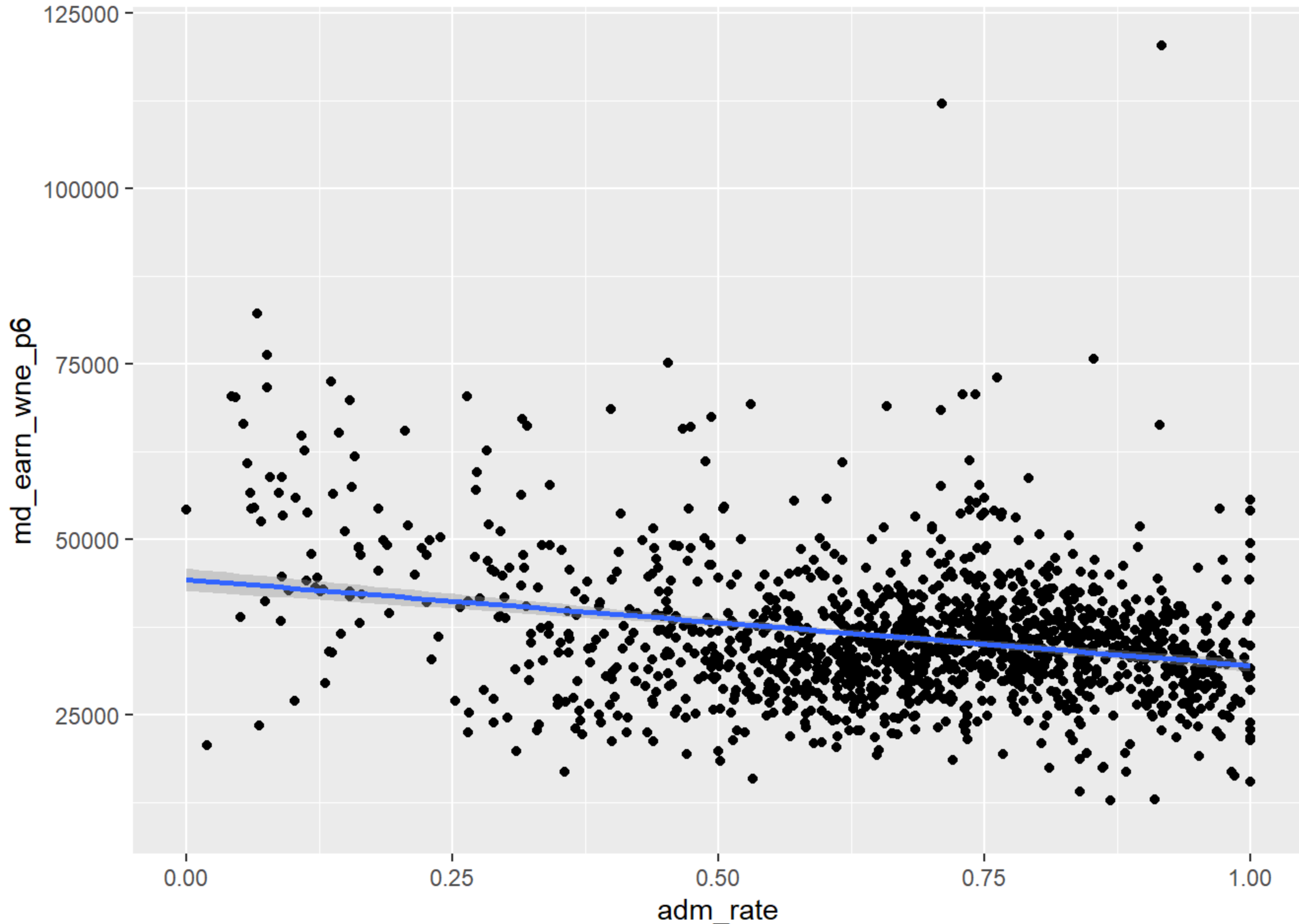
```
summary(sc_debt %>% select(md_earn_wne_p6,adm_rate))
```

```
##  md_earn_wne_p6      adm_rate
##  Min.   : 10600   Min.   :0.0000
##  1st Qu.: 26100   1st Qu.:0.5668
##  Median : 31500   Median :0.7114
##  Mean   : 33028   Mean   :0.6791
##  3rd Qu.: 37400   3rd Qu.:0.8333
##  Max.   :120400   Max.   :1.0000
##  NA's   :240      NA's   :958
```

As above, 240 schools don't report recent graduate earnings, and almost 1,000 don't report on their admissions rate! This is an important aspect of the data to recognize, since it limits how generalizable our conclusions might be.

Now let's plot the multivariate relationship.

```
sc_debt %>%
  ggplot(aes(x = adm_rate,y = md_earn_wne_p6))+
  geom_point() +
  geom_smooth(method = 'lm')
```



We find some evidence of a downward sloping relationship, although it is less steep than what we saw with SAT scores.

Now let's run the regression again. What do you conclude? How confident are you in this conclusion?

```
# INSERT CODE HERE
```

Motivation: predicting movie revenues

"Nobody knows anything…… Not one person in the entire motion picture field knows for a certainty what's going to work. Every time out it's a guess and, if you're lucky, an educated one."

-William Goldman, screenwriter of The Princess Bride and All the President's Men.

Next, we're going to see if we can predict which movies will bring in more money. Predicting movie revenues is known to be a very difficult problem, as some movies vastly outperform expectations, while other (very expensive) movies flop badly. Unlike other areas of the economy, it's not always easy to know which characteristics of movies are associated with higher gross revenues. Nevertheless, we shall persist!

It's typical for an investor group to have a model to understand the range of performance for a given movie. Investors want to know what range of return they might expect for an investment in a given movie. We'll try and get started on just such a model.

The Data

Load in libraries.

```
library(tidyverse)
library(plotly)
library(scales)
```

Load in data.

```
mv<-read_rds("https://github.com/jbisbee1/ISP_Data_Science_2024/raw/main/data/mv.Rds")%
>%
  filter(!is.na(budget))

glimpse(mv)
```

```
## Rows: 3,191
## Columns: 20
## $ title         <chr> "Almost Famous", "American Psycho", "Gladiator", "Requie…
## $ rating        <chr> "R", "R", "R", "Unrated", "R", "PG-13", "R", "PG-13", "P…
## $ genre         <chr> "Adventure", "Comedy", "Action", "Drama", "Mystery", "Ad…
## $ year          <dbl> 2000, 2000, 2000, 2000, 2000, 2000, 2000, 2000, 2000, 20…
## $ released      <chr> "September 22, 2000 (United States)", "April 14, 2000 (U…
## $ score         <dbl> 7.9, 7.6, 8.5, 8.3, 8.4, 7.8, 6.2, 6.4, 5.7, 7.4, 6.4, 7…
## $ votes         <dbl> 260000, 514000, 1400000, 786000, 1200000, 542000, 238000…
## $ director      <chr> "Cameron Crowe", "Mary Harron", "Ridley Scott", "Darren …
## $ writer        <chr> "Cameron Crowe", "Bret Easton Ellis", "David Franzoni", …
## $ star          <chr> "Billy Crudup", "Christian Bale", "Russell Crowe", "Elle…
## $ country       <chr> "United States", "United States", "United States", "Unit…
## $ budget        <dbl> 93289619, 10883789, 160147179, 6996721, 13993443, 139934…
## $ gross         <dbl> 73677478, 53278578, 723586629, 11490339, 62266278, 66800…
## $ company       <chr> "Columbia Pictures", "Am Psycho Productions", "Dreamwork…
## $ runtime       <dbl> 122, 101, 155, 102, 113, 143, 88, 130, 100, 104, 130, 16…
## $ id            <dbl> 877, 64, 1163, 2050, 52, 3795, 5544, 9301, 1093, 101, 68…
## $ imdb_id       <chr> "0181875", "0144084", "0172495", "0180093", "0209144", "…
## $ bechdel_score <dbl> 3, 3, 0, 3, 1, 2, 3, 2, 3, 1, 2, 3, 3, 0, NA, NA, 1, NA,…
## $ boxoffice_a   <dbl> 50586063, 23431686, 291849463, 5652546, 39717849, 363257…
## $ language      <chr> "English, French", "English, Spanish, Cantonese", "Engli…
```

This data comes from the Internet Movie Data Based, with contributions from several other sources.

Name	Description
title	Film Title
rating	MPAA Rating
genre	Genre: Adventure, Action etc
year	Year
released	Date released

Name	Description
score	IMDB Score
votes	Votes on IMDB
director	Director
writer	Screenwriter (top billed)
star	Top billed actor
country	Country where produced
runtime	Running time in minutes
id	Alternate ID
imdb_id	IMDB unique ID
bechdel_score	1=two women characters, 2=they speak to each other, 3= about something other than a man, 0=none of the above
box_office_a	Box office take
language	Languages spoken in the movie
gross	Gross revenue

Can we make money in the movie business?

There are a couple of ways we can answer this question. First of all, let's look at gross minus budget (I'm avoiding the word profit because movie finance is deeply weird (https://en.wikipedia.org/wiki/Hollywood_accounting)).

```
mv%>%
  mutate(gross_less_budget=gross-budget)%>%
  summarize(dollar(mean(gross_less_budget,na.rm=TRUE)))
```

```
## # A tibble: 1 × 1
##   `dollar(mean(gross_less_budget, na.rm = TRUE))`
##   <chr>
## 1 $113,277,345
```

Looks good! But wait a second, some movies must lose money right? Let's see how many that is:

```
mv%>%
  mutate(gross_less_budget=gross-budget)%>%
  mutate(made_money=ifelse(gross_less_budget>0,1,0))%>%
  group_by(made_money)%>%
  drop_na()%>%
  count()%>%
  ungroup()%>%
  mutate(prop=n/sum(n))
```

```
## # A tibble: 2 × 3
##   made_money     n  prop
##        <dbl> <int> <dbl>
## 1          0   336 0.170
## 2          1  1645 0.830
```

Oof, so something like 18 percent of movies in this dataset lost money. How much did they lose?

```
mv%>%
  mutate(gross_less_budget=gross-budget)%>%
  mutate(made_money=ifelse(gross_less_budget>0,1,0))%>%
  group_by(made_money)%>%
  summarize(dollar(mean(gross_less_budget)))%>%
  drop_na()
```

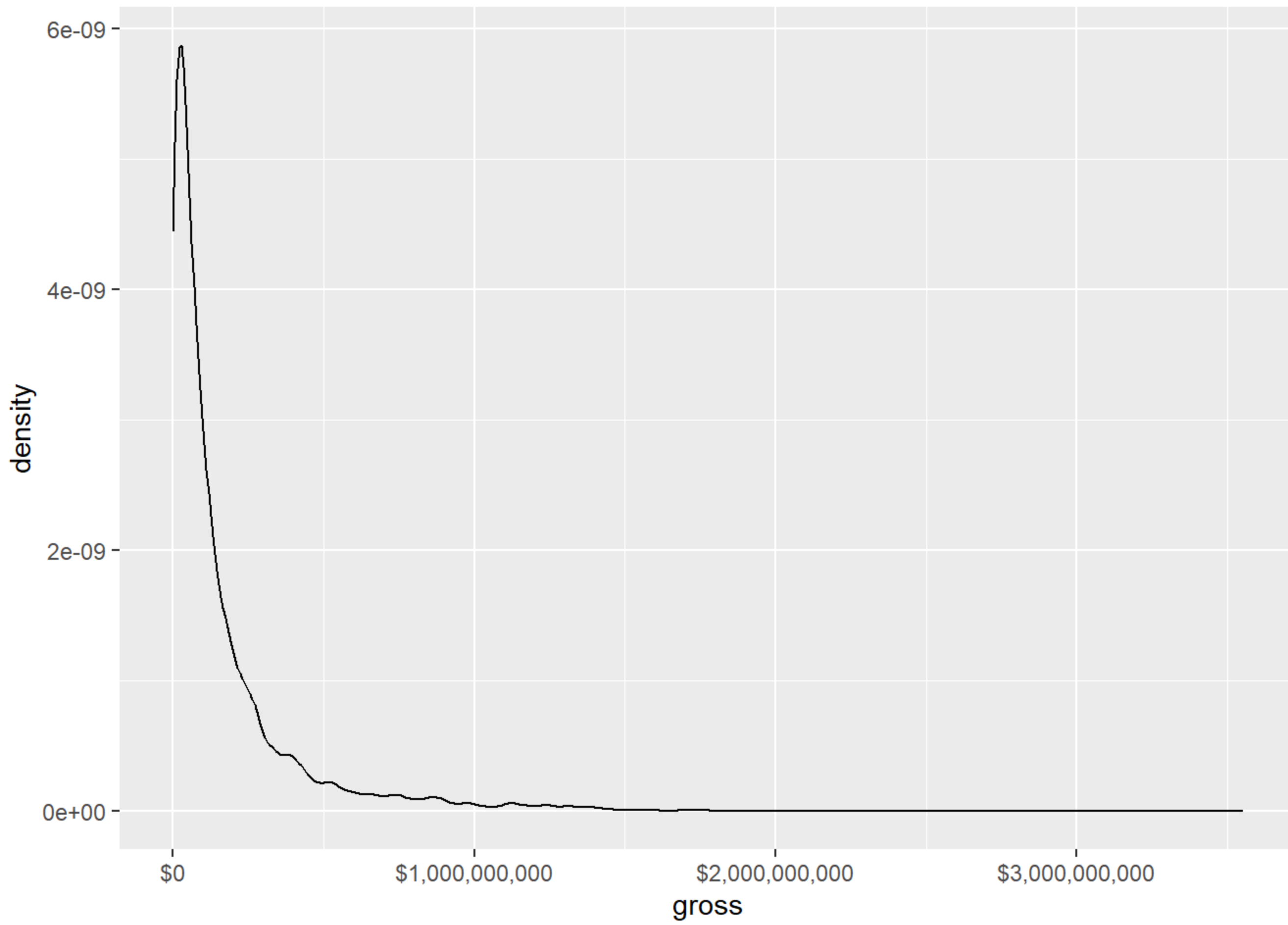
```
## # A tibble: 2 × 2
##   made_money `dollar(mean(gross_less_budget))`
##        <dbl> <chr>
## 1          0 -$16,705,383
## 2          1 $155,755,006
```

16 million on average. We better get this right! Okay, Let’s get started on our model.

Dependent Variable

Our dependent variable will be gross from the movie which is a measure of revenue from all sources: box office, streaming, dvds etc.

```
mv%>%
  ggplot(aes(gross))+
  geom_density()+
  scale_x_continuous(labels=dollar_format())
```

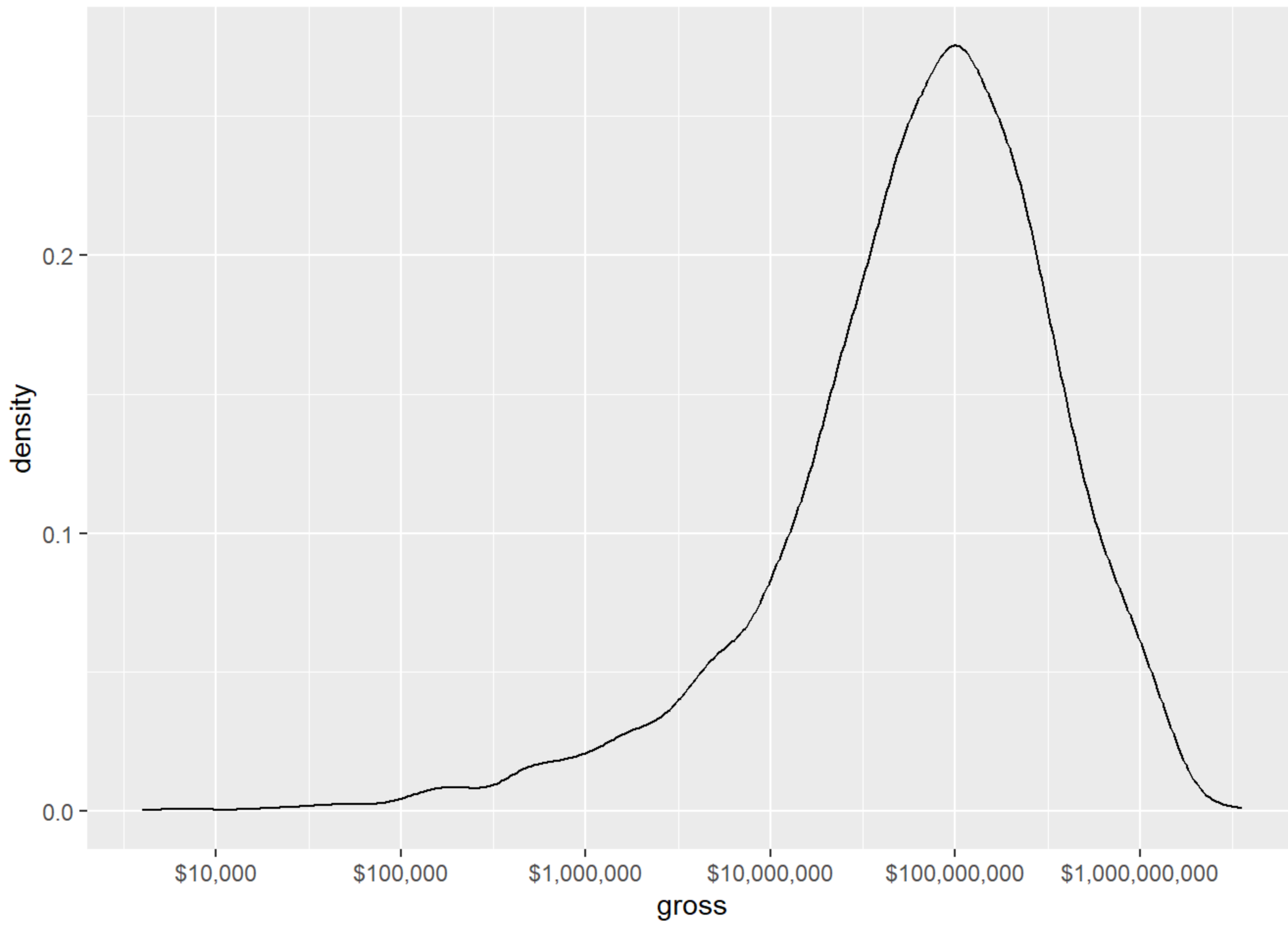


Well, that looks weird. Some movies make A LOT. Most, not so much. We need to work with this on an appropriate scale. When working with almost any data that has to do with money we're going to see things on an exponential scale. Our solution will be to transform this to be on a "log scale". Really what we're doing is taking the natural log of a number, which is the amount that Euler's constant e would need to be raised to in order to equal the nominal amount.

$$log_e(y) = x, \equiv e^x = y$$

Let's transform gross to be on a log scale. We can do this within ggplot to allow for some nicer labeling.

```
mv%>%
  ggplot(aes(x=gross))+
  geom_density()+
  scale_x_continuous(trans="log",
                     labels=dollar_format(),
                     breaks=breaks_log(n=6))
```



That looks somewhat better. Notice how the steps on the log scale imply very different amounts. Effectively what we're doing is changing our thinking to be about percent changes. A 10% increase in gross above 10,000 is 1000. A 10% increase in gross above 100,000 is 10,000. On a log scale, these are (sort of) equivalent steps. Transforming this on the front end will have some implications on the back end, but we'll deal with that in good time.

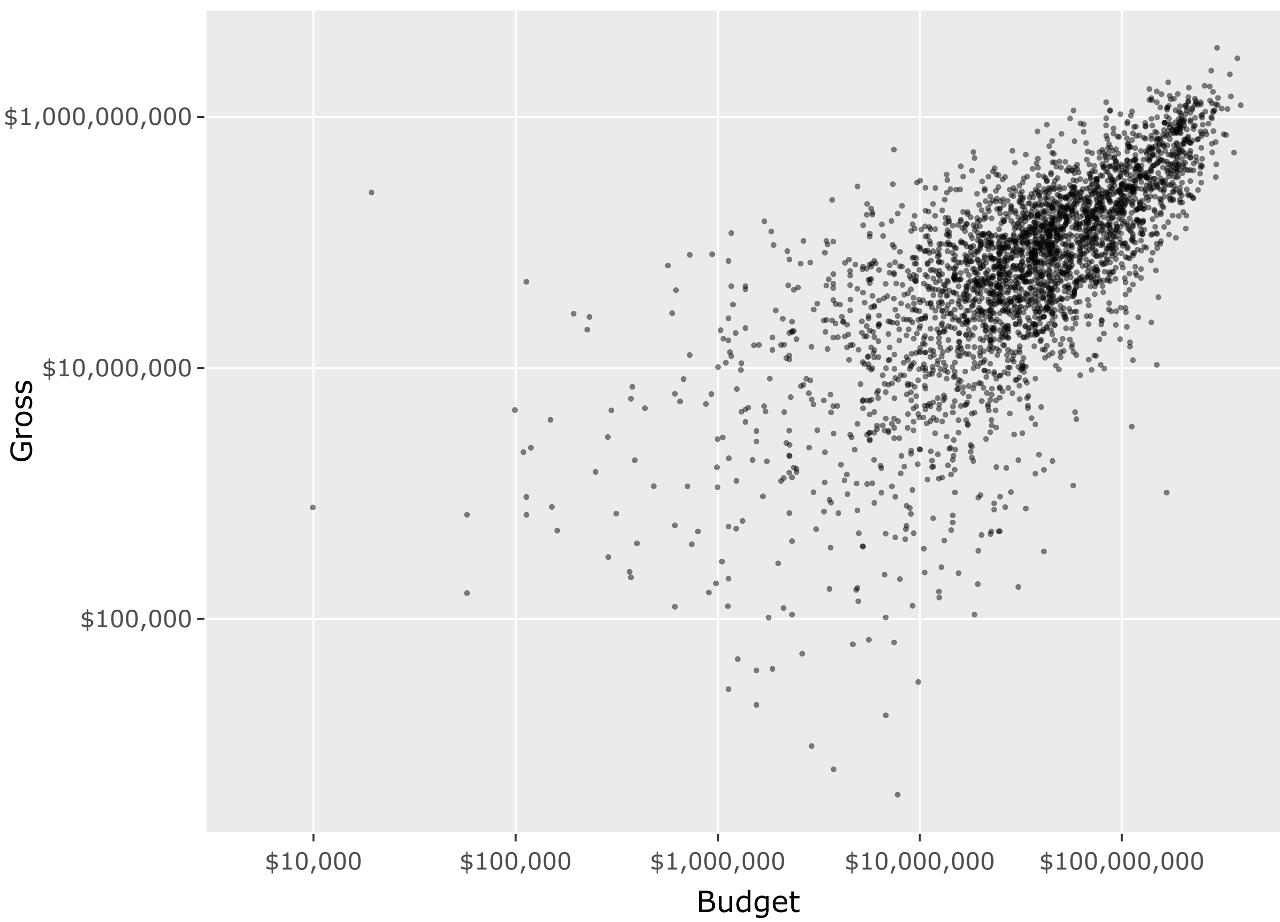
Gross as a Function of Budget

It's quite likely that gross revenues will be related to the budget, but how likely? Let's plot the relationship and find out.

```
gg<-mv%>%
  ggplot(aes(x=budget,y=gross,text=paste(title,"<br>",
                                          "Budget:", dollar(budget), "<br>",
                                          "Gross:" ,dollar(gross))))+
    geom_point(size=.25,alpha=.5)+

  scale_x_continuous(trans="log",
                     labels=label_dollar(),
                     breaks=log_breaks())+
   scale_y_continuous(trans="log",
                     labels=label_dollar(),
                     breaks=log_breaks())+
  xlab("Budget")+
  ylab("Gross")

ggplotly(gg,tooltip = "text")
```



You can navigate this plot and find out what the titles are. This is made possible by the ggplotly command, which uses the plotly library.

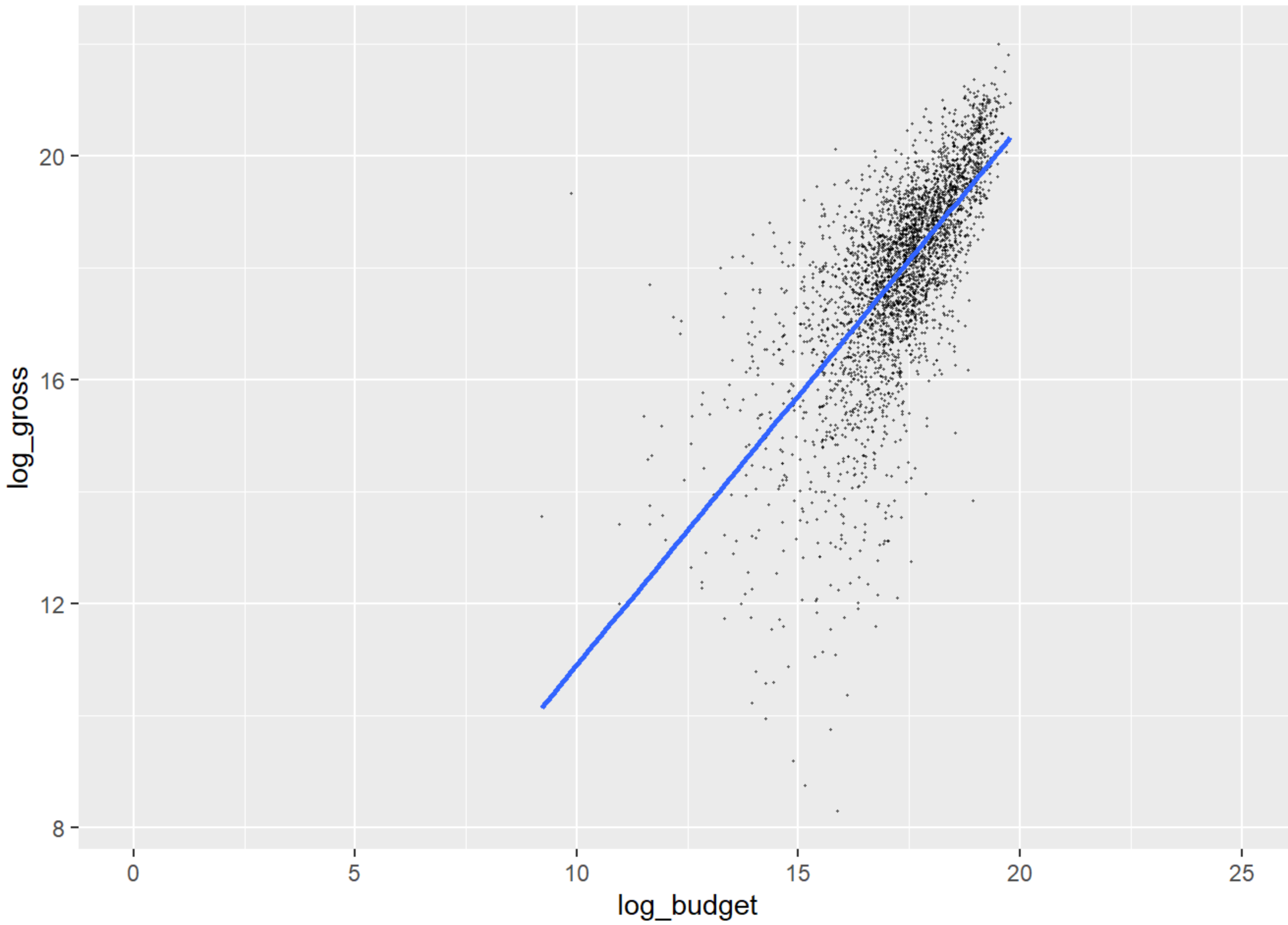
To make things easier, I’m going to create a new variable that is just the log of gross revenues

```
mv<-mv%>%
  mutate(log_gross=log(gross))
```

Basic Linear Model

This plot shows a line fit to the data, with the log of budget on the x axis and the log of gross budget on the y axis.

```
mv%>%
  mutate(log_budget=log(budget))%>%
  ggplot(aes(y=log_gross,x=log_budget))+
  geom_point(size=.25,alpha=.5)+
  geom_smooth(method="lm",se=FALSE)+
  xlim(0,25)
```



Does an assumption of linearity work in this case? Why or why not? Can we summarize this whole dataset in just two numbers?

Note that the `geom_smooth(method = 'lm')` command is running the model for us! Recall from above where we defined the **theoretical** regression as $Y = \alpha + \beta_1 X + \epsilon$. The figure above is now calculating $\hat{Y} = \hat{\alpha} + \hat{\beta}_1 X + \hat{\epsilon}$.

How is it doing this? It is using the "linear model" regression function that is included in `R`! This function takes the form `lm(formula,data)` where `formula` is the regression equation and `data` is just the data. For example, if `Y` is logged gross and `X` is logged budget, we would write: `lm(formula = log_gross ~ log_budget,data = mv)`. We can save this model to an object `m1` with the assignment operator `<-` and then look at the results with `summary()`.

```
m1 <- lm(log_gross ~ log_budget,mv %>%
           mutate(log_gross = log(gross),log_budget = log(budget)))

summary(m1)
```

```
##
## Call:
## lm(formula = log_gross ~ log_budget, data = mv %>% mutate(log_gross = log(gross),
##     log_budget = log(budget)))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.2672 -0.6354  0.1648  0.7899  8.5599
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.26107    0.30953   4.074 4.73e-05 ***
## log_budget   0.96386    0.01786  53.971  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.281 on 3177 degrees of freedom
##   (12 observations deleted due to missingness)
## Multiple R-squared:  0.4783, Adjusted R-squared:  0.4782
## F-statistic:  2913 on 1 and 3177 DF,  p-value: < 2.2e-16
```

The results tell us that the `(Intercept)` (which is the same as α in our theory equation, and $\hat{\alpha}$ in our results) is equal to 1.26, and that the `log_budget` (which is the same as β_1 in our theory equation, and $\hat{\beta}_1$ in our results) is equal to 0.96. Thus, for every one unit increase in the logged budget, there is a little less than a 1 unit increase in the gross.

When we are comparing the relationship between a logged outcome and a logged predictor, we can interpret the coefficient as a percent change. Specifically, we would say that a one percent change in the budget corresponds to a 0.96 percent change in gross. For the full breakdown of how to talk about regression interpretations when it comes to logged data, see this website: https://sites.google.com/site/curtiskephart/ta/econ113/interpreting-beta (https://sites.google.com/site/curtiskephart/ta/econ113/interpreting-beta).

Evaluating a Model

The `summary()` function tells us the predicted values for the right hand side of the regression equation: $\hat{\alpha}$ and $\hat{\beta}_1$. But we will also want to know the left-hand side: how good are our predictions for $\hat{Y}$? To answer this question, we want to measure the “errors” that our model makes. In other words, how off are our predicted values compared to the true values?

There are two ways to answer this question. The first is to just **look** at the errors, both as a univariate visualization and as a comparison between the errors and the predictor. The second is to calculate something called the **R**oot **M**ean **S**quared **E**rror, or **RMSE**.

But in either case, we need to calculate the errors themselves first, which are defined as the difference between the true values of Y and the predicted values, denoted $\hat{Y}$. To calculate $\hat{Y}$, we can run the `predict()` function on the model object.

```
predY <- predict(m1)
predY %>% head()
```

```
##        1        2        3        4        5        6
## 18.94904 16.87826 19.46990 16.45239 17.12049 19.33986
```

These values are exactly corresponding to the original data, meaning we can add them as a new column to our dataset in order to evaluate how good our model is. The more similar the predicted values to the actual values of the movie gross, the better our model!

NB: the number of predictions is not the same as the total number of rows in our data. This is because several of the rows contained missing data in either the outcome (`gross`) or in the predictor (`budget`). We can either create a new dataset that removes these missing values ahead of time, or we can use the information contained in the `m1` object which tells us which rows were dropped due to missing data.

```
mvEval1 <- mv %>%
  slice(-m1$na.action) %>% # Drop any observations that were not used in the regression
  mutate(pred_gross = predY,
         log_gross = log(gross),
         log_budget = log(budget)) %>%
  select(title,log_gross,pred_gross,log_budget)

mvEval1 %>%
  head()
```

```
## # A tibble: 6 × 4
##   title               log_gross pred_gross log_budget
##   <chr>                   <dbl>      <dbl>      <dbl>
## 1 Almost Famous            18.1       18.9       18.4
## 2 American Psycho          17.8       16.9       16.2
## 3 Gladiator                20.4       19.5       18.9
## 4 Requiem for a Dream      16.3       16.5       15.8
## 5 Memento                  17.9       17.1       16.5
## 6 Cast Away                20.3       19.3       18.8
```

Now that we have our true Y (`log_gross`) and our predicted $\hat{Y}$ (`pred_gross`), we can calculate the errors (denoted ε) by just subtracting the predicted values from the true values.

$$\varepsilon = Y - \hat{Y}$$

```
mvEval1 <- mvEval1 %>%
  mutate(errors = log_gross - pred_gross)
mvEval1 %>%
  head()
```

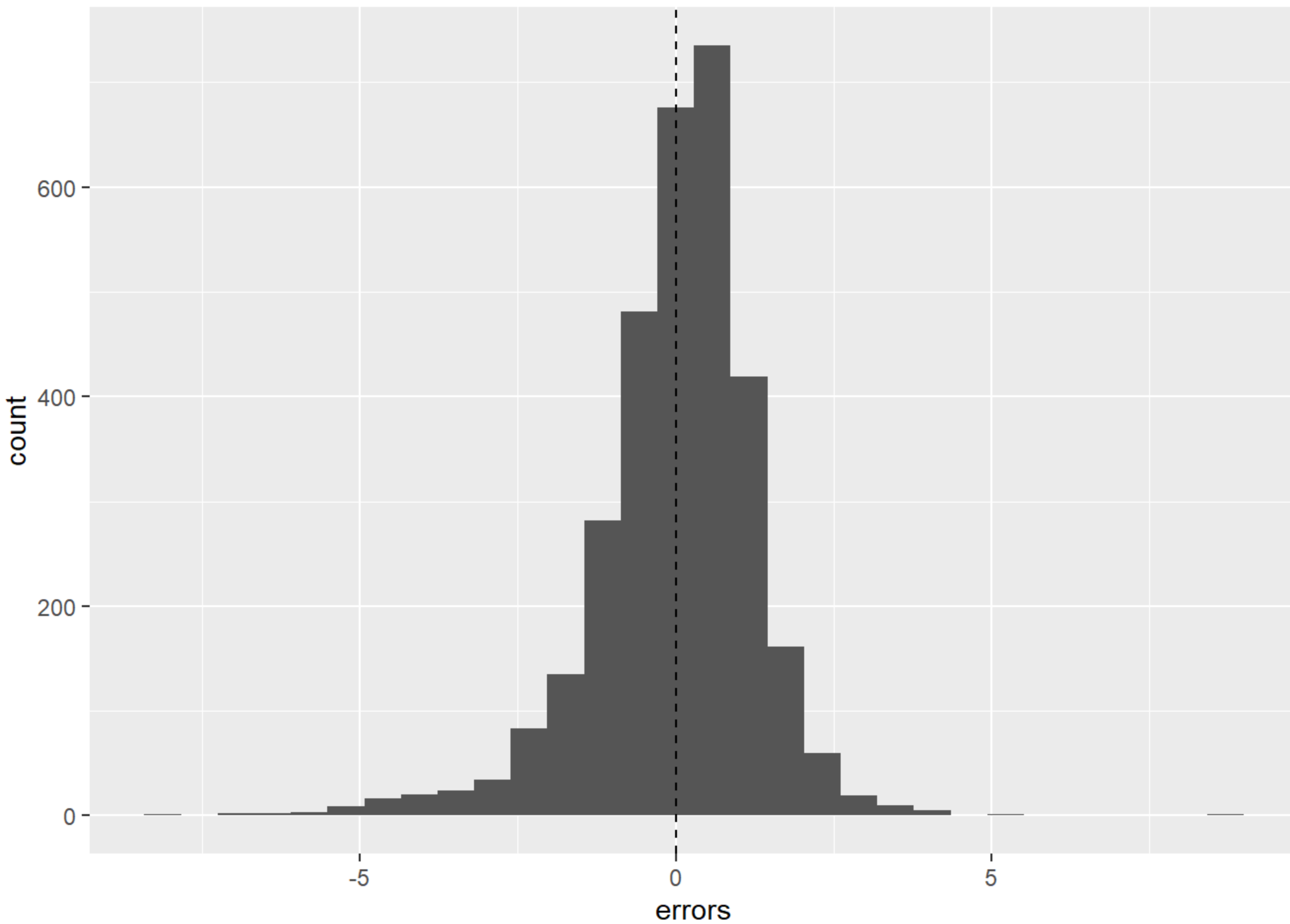
```
## # A tibble: 6 × 5
##   title                log_gross pred_gross log_budget errors
##   <chr>                    <dbl>      <dbl>      <dbl>  <dbl>
## 1 Almost Famous             18.1       18.9       18.4 -0.834
## 2 American Psycho           17.8       16.9       16.2  0.913
## 3 Gladiator                 20.4       19.5       18.9  0.930
## 4 Requiem for a Dream       16.3       16.5       15.8 -0.195
## 5 Memento                   17.9       17.1       16.5  0.826
## 6 Cast Away                 20.3       19.3       18.8  0.980
```

As we can see, sometimes our model **overpredicts** the log gross, which corresponds to the negative errors for Almost Famous and Requiem for a Dream. In other cases, it **underpredicts** the log cross, corresponding to positive errors for American Psycho, Gladiator, Memento, and Cast Away.

Visualizing Errors

The first thing to do is to just look at these errors. We do this in two ways. First we just plot them as a histogram or density. By construction, the errors will be centered around zero. However, a “good” model will have errors that are symmetrically distributed, taking the appearance of a bell curve. As we can see, our model does a pretty decent job. There is very mild skew, where we **overpredict** more than we **underpredict** (see that the distribution goes further out to the left below zero than to the right). Nevertheless, the distribution looks pretty good overall.

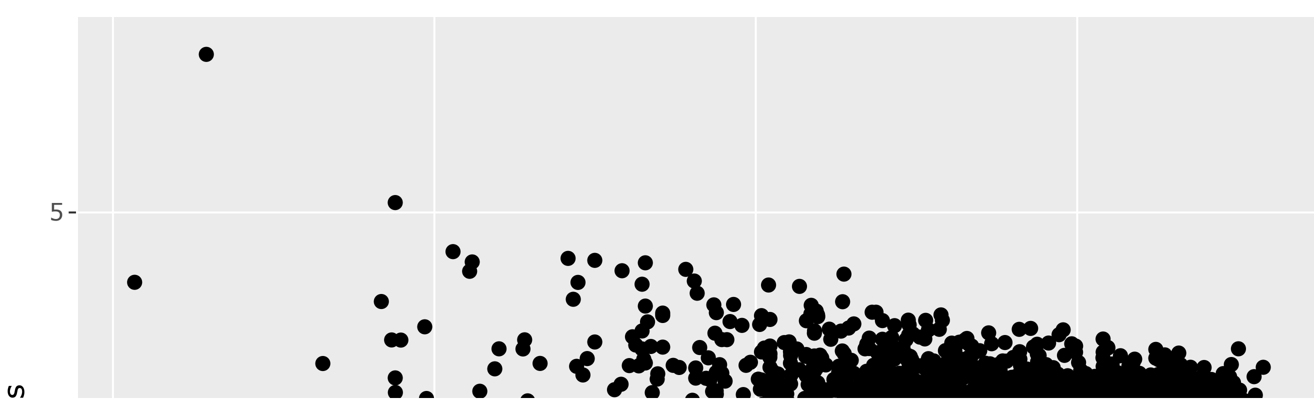
```
mvEval1 %>%
  ggplot(aes(x = errors)) +
  geom_histogram() +
  geom_vline(xintercept = 0,linetype = 'dashed')
```

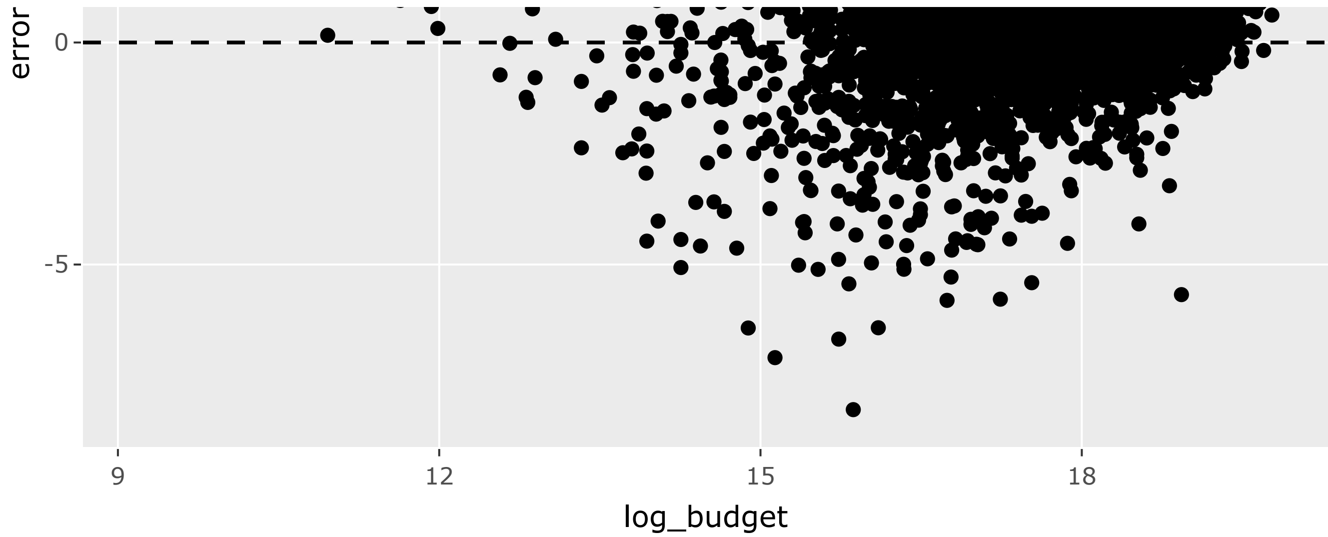


The second thing to do is to visualize the errors as a scatter plot, where the errors are on the y-axis and the X predictor (in this case, `log_budget`) is on the x-axis. Again, the center of this plot will be around zero on the y-axis by design. In this plot, a "good" model will look like a rectangular cloud of errors, where we miss above and below roughly equally, regardless of the budget. We can add a `geom_smooth()` to help visualize the performance. Ideally, we want the line to be flat and zero.

```
p <- mvEval1 %>%
  ggplot(aes(x = log_budget,y = errors,text = title)) +
  geom_point() +
  geom_smooth() +
  geom_hline(yintercept = 0,linetype = 'dashed')

ggplotly(p)
```





As we can see, our model does poorly, as indicated by the curved `geom_smooth()` line. Substantively, this means that we are dramatically **underestimating** lower budget movies and, to a lesser extend, **underestimating** big budget movies. In both cases, our errors are positive.

Recall also that the perfect model is a uniform cloud of data. Meanwhile, our result has much larger mistakes for mid-budget movies compared to big budget movies. In the worst cases, we dramatically **underestimate** Paranormal Activity (the outlier in the top left of the plot), and **overestimate** Ginger Snaps (the outlier in the bottom center of the plot).

Based on these analyses, we would say that our model could be improved.

Root Mean Squared Error

It is always essential to look at the errors visually, with both univariate and multivariate plots. However, we also might want to save a single summary statistic that captures the overall performance, without having to rely on these somewhat subjective visualizations.

To do so, we will use a very standard method, Root Mean Squared Error, or RMSE. To calculate the RMSE, we just follow the recipe that is in its name. We square the errors (**S**), take the average (**M**), then take the square root of the result (**R**). The name RMSE is exactly what RMSE is– neat, huh?

$$RMSE(\hat{Y}) = \sqrt{1/n \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}$$

We can do this manually, and I think it's instructive to do so at least once for learning. This might seem like a scary equation with a lot of greek letters, but let's break it down into its separate pieces:

1. Let's start with the errors $Y_i - \hat{Y}_i$. Y_i are the true values and $\hat{Y}_i$ are the predictions. To calculate their difference, we just use a `mutate()` command and subtract the predicted values from the true values.
   `mvEval1 <- mvEval1 %>% mutate(diff = log_gross - pred_gross)` .
2. Now we square this: `mvEval1 <- mvEval1 %>% mutate(sqDiff = diff^2)` .
3. And now we take it's average (note that $\frac{1}{n}\sum_{i=1}^{n} X_i$ is just the way we write average…we literally sum up every value $\sum_{i=1}^{2} X_i$ and then divide this by n, which is the same as multiplying by $\frac{1}{n}$).
   `msEval1 <- mvEval1 %>% summarise(mse = mean(sqDiff))` .

4. Finally, we just take the square root! `sqrt(msEval1$mse)` and we're done!

```
mvEval1 %>%
  mutate(errors = log_gross - pred_gross) %>% # calculate the errors (E)
  mutate(sqErrors = errors^2) %>%             # square the errors (S)
  summarise(mse = mean(sqErrors)) %>%         # take the mean of the errors (M)
  summarise(rmse = sqrt(mse))                 # take the square root the mean (R)
```

```
## # A tibble: 1 × 1
##    rmse
##   <dbl>
## 1  1.28
```

That is a lot of work to calculate this! We can make it easier by calculating the errors (aka "residuals") directly with the `resid()` function.

```
e <- resid(m1)
se <- e^2
mse <- mean(se)
rmse <- sqrt(mse)
rmse
```

```
## [1] 1.280835
```

Calculating rmse from the data we used to fit the line is actually not correct, because it doesn't help us learn about out of sample data. To solve this problem, we need another concept: training and testing.

Training and Testing

The essence of prediction is discovering the extent to which our models can predict outcomes for data that *does not come from our sample*. Many times this process is temporal. We fit a model to data from one time period, then take predictors from a subsequent time period to come up with a prediction in the future. For instance, we might use data on team performance to predict the likely winners and losers for upcoming soccer games.

This process does not have to be temporal. We can also have data that is out of sample because it hadn't yet been collected when our first data was collected, or we can also have data that is out of sample because we designated it as out of sample.

The data that is used to generate our predictions is known as *training* data. The idea is that this is the data used to train our model, to let it know what the relationship is between our predictors and our outcome. So far, we have only worked with training data.

That data that is used to validate our predictions is known as *testing* data. With testing data, we take our trained model and see how good it is at predicting outcomes using out of sample data.

One very simple approach to this would be to cut our data. We could then train our model on half the data, then test it on the other portion. This would tell us whether our measure of model fit (e.g. rmse) is similar or different when we apply our model to out of sample data. That's what we're going to do in this lesson. We'll split the data randomly in two, with one part used to train our models and the other part used to test the model against out of sample data.

Training a model on one dataset, and then **testing** it on another dataset, is a method known as **cross validation**, because we **validate** (i.e., evaluate) our model on data it hasn't seen yet. We want to repeat this process multiple times, using different random divisions of the data into **training** and **test** datasets.

You will notice that this approach is very similar to the bootstrapping we have already done.

Note: the `set.seed` command ensures that your random split should be the same as my random split.

Training and Testing Data

The core idea is to separate the data into two random subsets: a training set (where you will estimate a model) and a testing set (where you will test the model).

```
set.seed(123)

# First I'm going to add two columns of logged gross and logged budget to the dataset
mv <- mv %>%
  mutate(log_gross = log(gross),
         log_budget = log(budget))

# Get a list of row numbers for the training set
inds <- sample(1:nrow(mv),size = round(nrow(mv)/2),replace = F) # NB: we set replace = F
for cross validation

# Now use these indices to create two data frames, the first that includes them, and the
second that doesn't include them
train <- mv %>%
  slice(inds)

test <- mv %>%
  slice(-inds)
```

Now we estimate our model on the training set!

```
mTrain <- lm(log_gross ~ log_budget,train)
```

Instead of calculating the RMSE directly on this model though, we want to *use* this model to *predict* outcomes on the test dataset. To do this, we will use the `predict()` command again, but tell it to use a new dataset. To save a few steps, I'm going to add it directly to the `test` dataset.

```
test$predGross <- predict(mTrain,newdata = test)
```

Now we can calculate the RMSE here!

```
e <- test$predGross - test$log_gross
se <- e^2
mse <- mean(se,na.rm=T)
rmse <- sqrt(mse)
```

Great! Now let's put it in a loop, just like with bootstrapping!

```
cvRes <- NULL

for(i in 1:100) {
  # Get a list of row numbers for the training set
  inds <- sample(1:nrow(mv),size = round(nrow(mv)/2),replace = F) # NB: we set replace =
F for cross validation

  # Now use these indices to create two data frames, the first that includes them, and t
he second that doesn't include them
  train <- mv %>%
    slice(inds)

  test <- mv %>%
    slice(-inds)

  mTrain <- lm(log_gross ~ log_budget,train)

  test$predGross <- predict(mTrain,newdata = test)

  e <- test$predGross - test$log_gross
  se <- e^2
  mse <- mean(se,na.rm=T)
  rmse <- sqrt(mse)

  cvRes <- c(cvRes,rmse)
}
```

And now we have a vector of crossvalidated measures of model fit! Take the average to see how well we're doing!

```
mean(cvRes,na.rm=T)
```

```
## [1] 1.286128
```

```
summary(cvRes)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   1.235   1.270   1.290   1.286   1.303   1.346
```

Evaluate Predictions in the Testing Data

Is this good? Who knows! RMSE is very context dependent. One way to think about it for us is that the model predicts that a 10 million dollar investment will generate about 18 million: sounds good, right?

```
summary(m1)
```

```
##
## Call:
## lm(formula = log_gross ~ log_budget, data = mv %>% mutate(log_gross = log(gross),
##     log_budget = log(budget)))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.2672 -0.6354  0.1648  0.7899  8.5599
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.26107    0.30953   4.074 4.73e-05 ***
## log_budget   0.96386    0.01786  53.971  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.281 on 3177 degrees of freedom
##   (12 observations deleted due to missingness)
## Multiple R-squared:  0.4783, Adjusted R-squared:  0.4782
## F-statistic:  2913 on 1 and 3177 DF,  p-value: < 2.2e-16
```

```
quick_est<-exp(1.26+ (.96* log(1e7)))
dollar(quick_est)
```

```
## [1] "$18,501,675"
```

Except the RMSE says that gross could be between 66 million (yay) and 5.1 million. Right now, that's our prediction– a 10 million dollar investment, base on this model, will make somewhere between 5.1 million and 66 million. Your investors are probably not thrilled and are thinking about just putting the money in T Bills.

```
quick_upper_bound<-exp(1.26+ .96*log(1e7) +1.28)

dollar(quick_upper_bound)
```

```
## [1] "$66,543,859"
```

```
quick_lower_bound<-exp(1.26+ .96* log(1e7) -1.28)

dollar(quick_lower_bound)
```

```
## [1] "$5,144,156"
```