Regression

Part 3

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Agenda

1. Recap of Movie Analysis
2. Multiple Regression
3. Categorical Predictors

Recap of Movie Analysis

```
require(tidyverse)

mv <-
read_rds('https://github.com/jbisbee1/ISP_Data_Science_2024/raw/main/da
```

- Theory: the more a movie costs, the more it should make
    - If not, Hollywood would go out of business!
- X: budget
- Y: gross

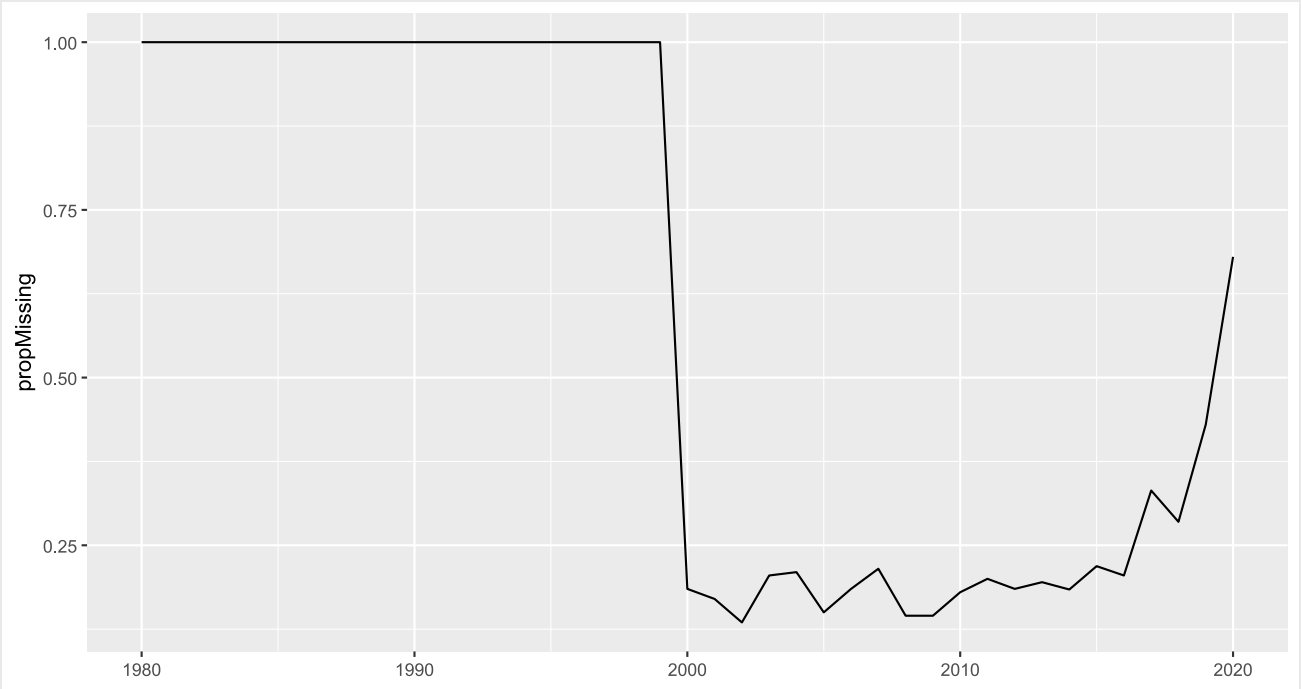
Step 1: Look

```
summary(mv %>% select(gross,budget))
```

```
##      gross                budget
##  Min.   :7.140e+02   Min.   :     5172
##  1st Qu.:1.121e+07   1st Qu.: 16865322
##  Median :5.178e+07   Median : 37212044
##  Mean   :1.402e+08   Mean   : 57420173
##  3rd Qu.:1.562e+08   3rd Qu.: 77844746
##  Max.   :3.553e+09   Max.   :387367903
##  NA's   :3668        NA's   :4482
```

Step 1: Look

```
mv %>%
  mutate(missing = ifelse(is.na(gross) | is.na(budget),1,0)) %>%
  group_by(year) %>%
  summarise(propMissing = mean(missing)) %>%
  ggplot(aes(x = year,y = propMissing)) +
  geom_line()
```



Some quick wrangling

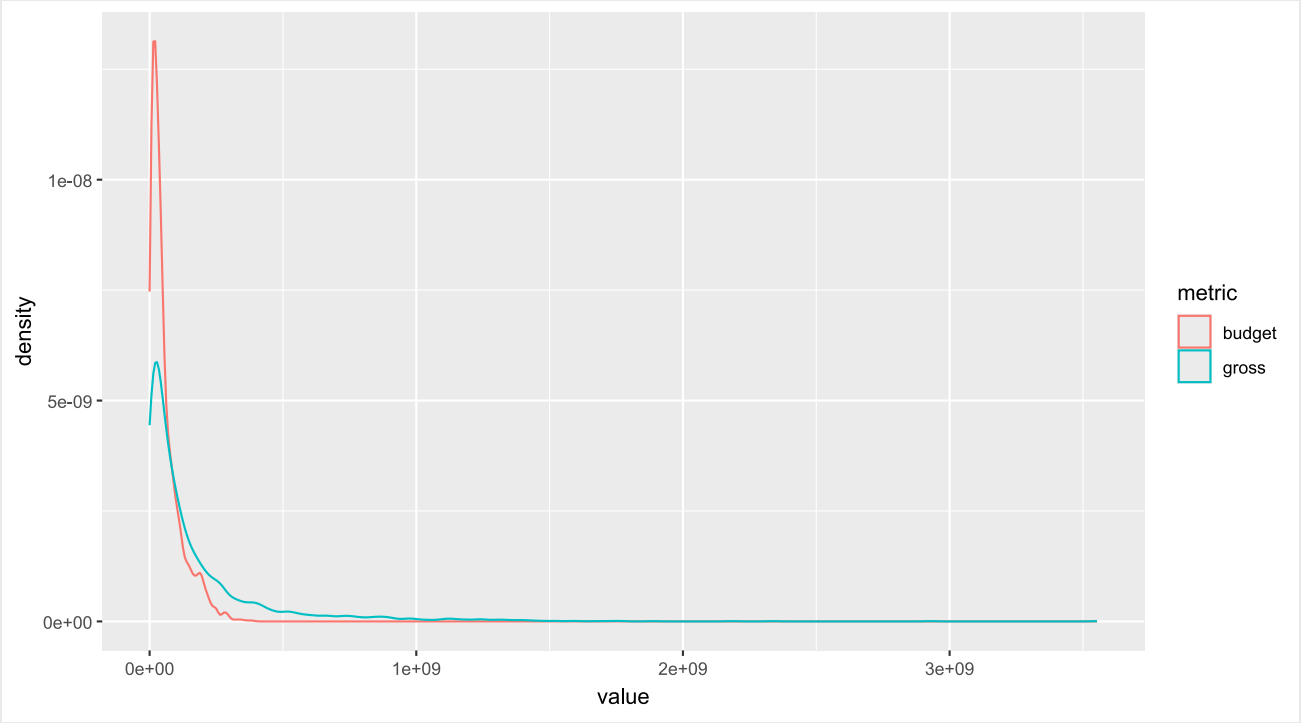
```
mv <- mv %>%
  drop_na(gross,budget)

mv %>%
  select(gross,budget) %>%
  glimpse()
```

```
## Rows: 3,179
## Columns: 2
## $ gross  <dbl> 73677478, 53278578, 723586629, 11490339, 62…
## $ budget <dbl> 93289619, 10883789, 160147179, 6996721, 139…
```

Step 2: Univariate Viz

```
mv %>%
  select(title,gross,budget) %>%
  gather(metric,value,-title) %>%
  ggplot(aes(x = value,color = metric)) +
  geom_density()
```



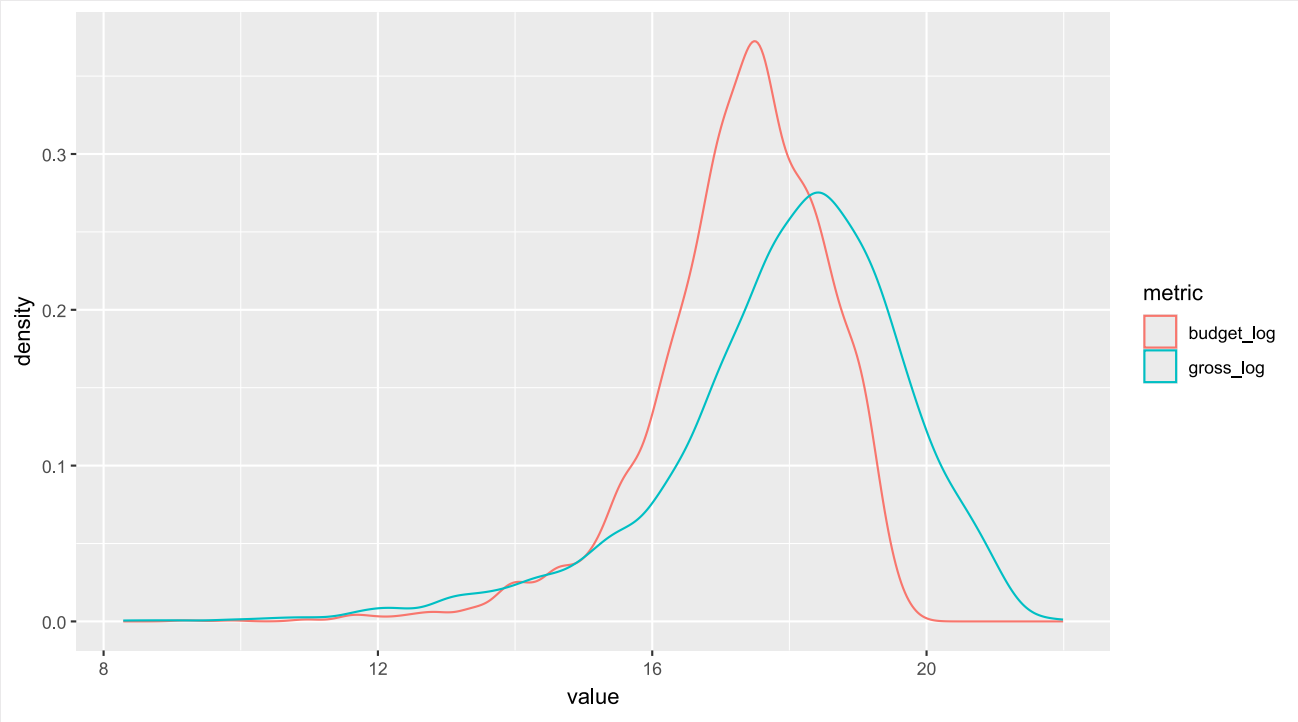
More Wrangling?

- Univariate visualization higlights significant **skew** in both measures
    - Most movies don't cost a lot and don't make a lot
    - But there are a few blockbusters that pull the density way out
- Let's **wrangle** two new variables that take the log of these skewed measures
    - Logging transforms skewed measures to more "normal" measures

```
mv <- mv %>%
  mutate(gross_log = log(gross),
         budget_log = log(budget))
```

Step 2: Univariate Viz

```
mv %>%
  select(title,gross_log,budget_log) %>%
  gather(metric,value,-title) %>%
  ggplot(aes(x = value,color = metric)) +
  geom_density()
```



Step 3: Multivariate Viz

```
pClean <- mv %>%
  ggplot(aes(x = budget,y = gross)) +
  geom_point() +
  scale_x_log10(labels = scales::dollar) +
  scale_y_log10(labels = scales::dollar) +
  labs(title = "Movie Costs and Returns",
       x = "Costs (logged budget)",
       y = "Returns (logged gross)")
```

Step 3: Multivariate Viz

```
pClean + geom_smooth(method = 'lm')
```

```
## `geom_smooth()` using formula = 'y ~ x'
```

Step 4: Regression!

```
require(broom) # Make the output nicer looking
```

```
## Loading required package: broom
```

```
m <- lm(gross_log ~ budget_log,data = mv)
tidy(m)
```

```
## # A tibble: 2 × 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)    1.26     0.310       4.07 0.0000473
## 2 budget_log     0.964    0.0179     54.0  0
```

Step 5.1: Univariate Viz of Errors

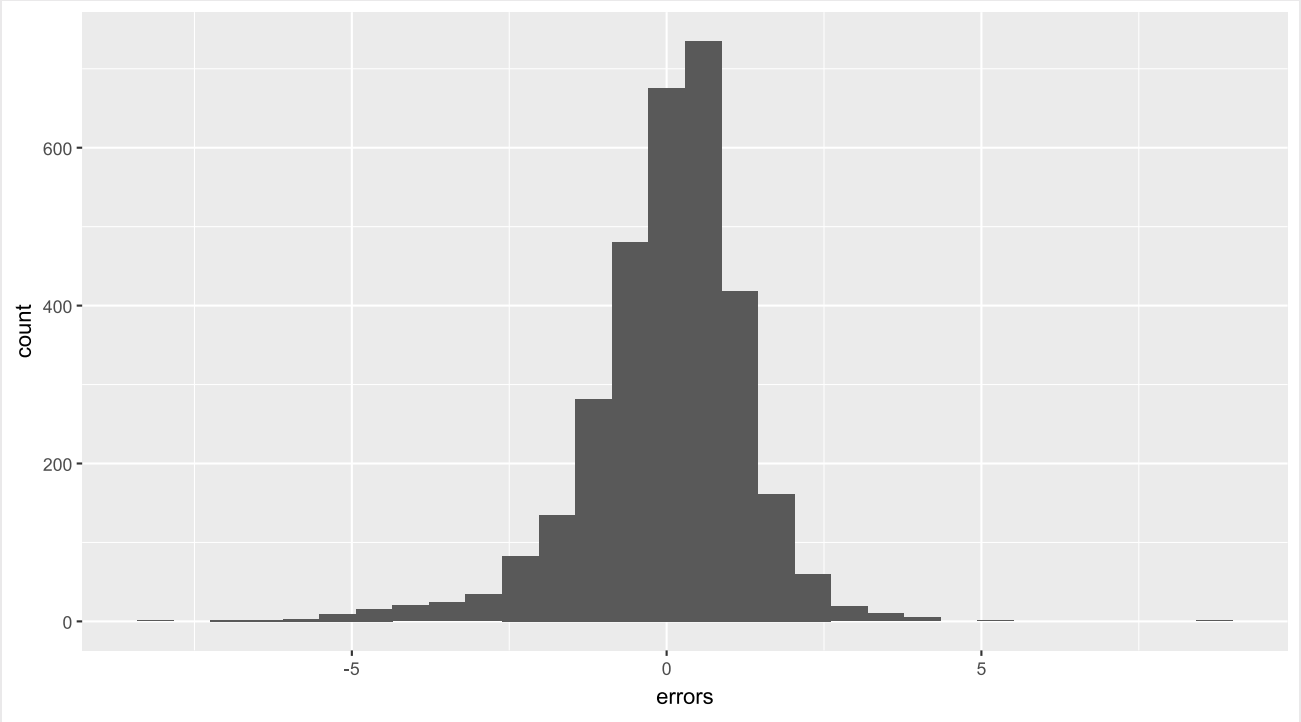
- Errors $\varepsilon = Y - \hat{Y}$
  - In `R`, can also get them via `resid()` function

```
mv %>%
  mutate(errors_manual = gross_log - predict(m),
         errors_resid = resid(m))
```

```
## # A tibble: 3,179 × 24
##    title   rating genre  year released score  votes director
##    <chr>   <chr>  <chr> <dbl> <chr>    <dbl>  <dbl> <chr>
##  1 Almost… R      Adve…  2000 Septemb…   7.9 2.6 e5 Cameron…
##  2 Americ… R      Come…  2000 April 1…   7.6 5.14e5 Mary Ha…
##  3 Gladia… R      Acti…  2000 May 5, …   8.5 1.4 e6 Ridley …
##  4 Requie… Unrat… Drama  2000 Decembe…   8.3 7.86e5 Darren …
##  5 Memento R      Myst…  2000 May 25,…   8.4 1.20e6 Christo…
##  6 Cast A… PG-13  Adve…  2000 Decembe…   7.8 5.42e5 Robert …
##  7 Scary … R      Come…  2000 July 7,…   6.2 2.38e5 Keenen …
##  8 The Pe… PG-13  Acti…  2000 June 30…   6.4 1.6 e5 Wolfgan…
##  9 Coyote… PG-13  Come…  2000 August …   5.7 1.08e5 David M…
## 10 X-Men   PG-13  Acti…  2000 July 14…   7.4 5.82e5 Bryan S…
## # ℹ 3,169 more rows
```

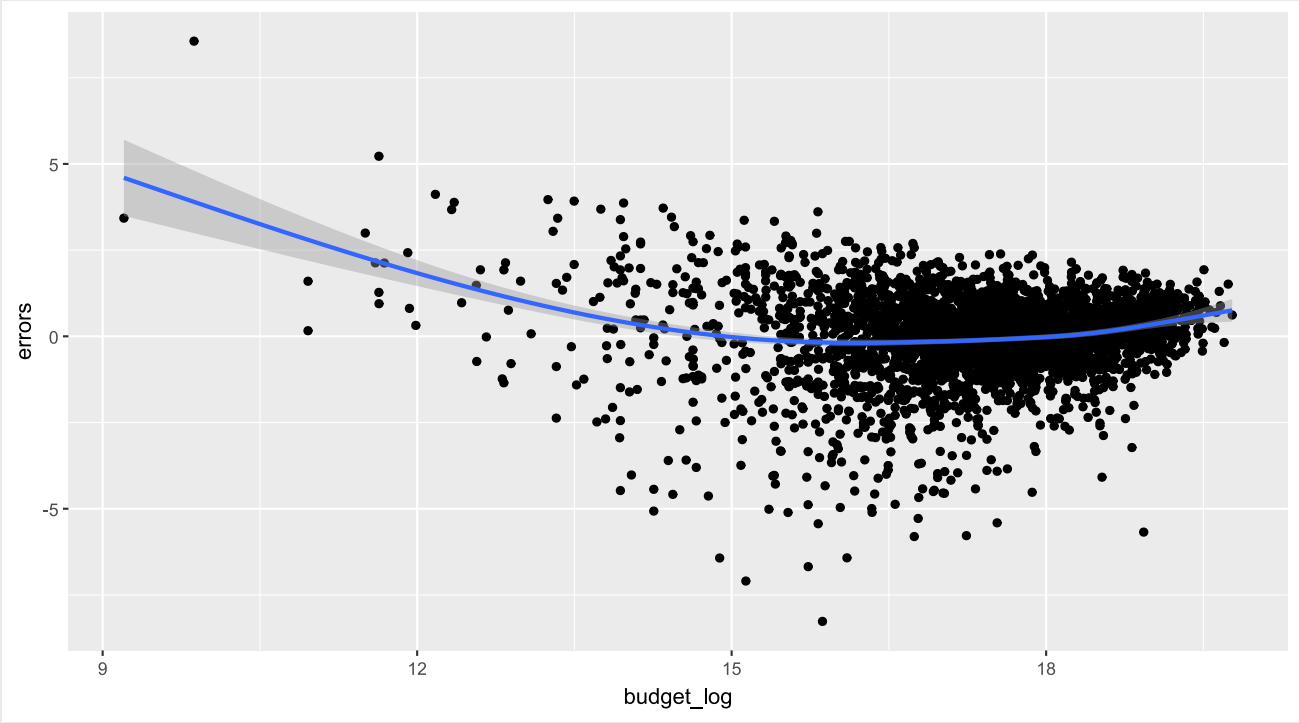
Step 5.1: Univariate Viz of Errors

```
mv %>%
  ggplot(aes(x = errors)) +
  geom_histogram()
```



Step 5.2: Multivariate Viz of Errors

```
mv %>%
  ggplot(aes(x = budget_log,y = errors)) +
  geom_point() +
  geom_smooth()
```



Step 5.3: Cross Validated RMSE

```
set.seed(123)
rmseBudget <- NULL
for(i in 1:100) {
  train <- mv %>%
    sample_n(size = round(nrow(mv)*.5),replace = F)
  test <- mv %>% anti_join(train)

  mTrain <- lm(gross_log ~ budget_log,train)

  test$preds <- predict(mTrain,newdata = test)

  rmse <- sqrt(mean((test$gross_log - test$preds)^2,na.rm=T))
  rmseBudget <- c(rmseBudget,rmse)
}

mean(rmseBudget)
```

```
## [1] 1.279899
```

Thinking like a scientist

- Our previous model predicted `gross` as a function of `budget`
- Theoretically, is this sensible?
    1. Bigger budgets → famous actors → mass appeal → more tickets
    2. Bigger budgets → advertising money → mass appeal → more tickets
- But what if the movie is just...not good?

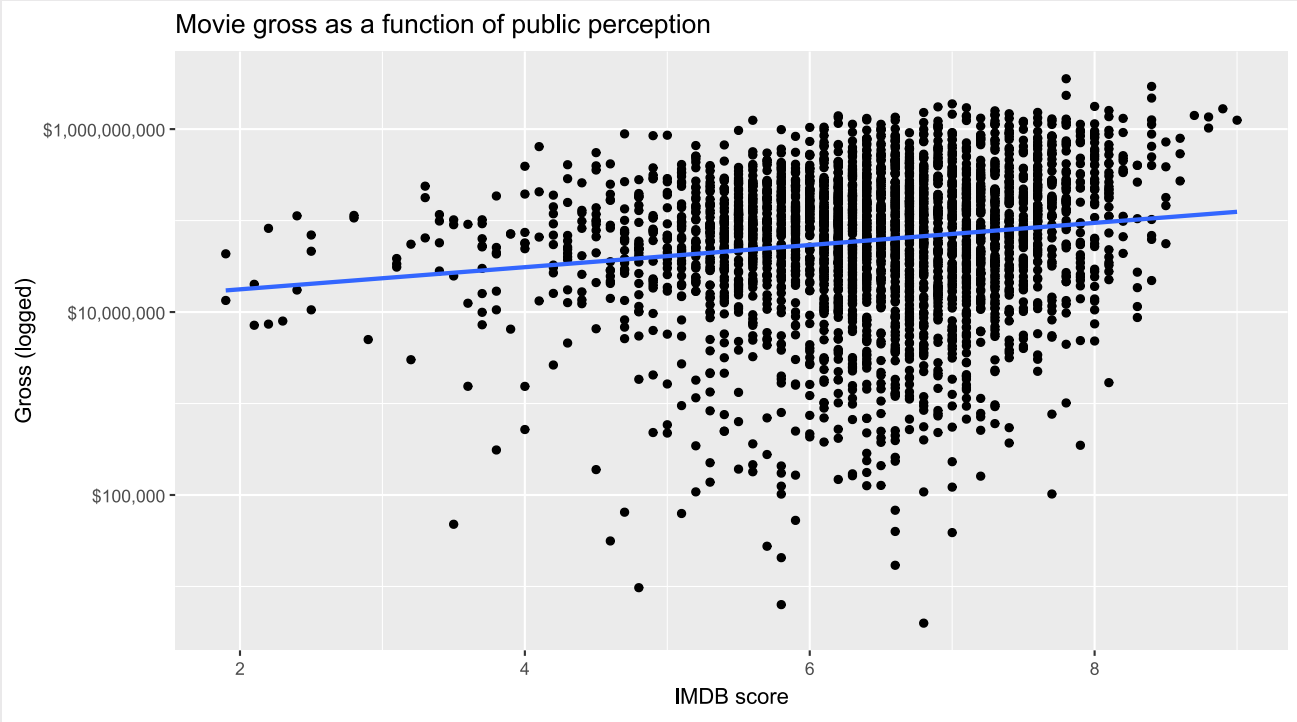
Alternative Theory

- Good movies make more money
  - Theory: good movies → recommendations → more tickets
- Predict gross with IMDB rating (`score`)

```
pIMDB <- mv %>%
  ggplot(aes(x = score,y = gross)) +
  geom_point() +
  labs(title = "Movie gross as a function of public perception",
       x = "IMDB score",
       y = "Gross (logged)") +
  scale_y_log10(label = scales::dollar) +
  geom_smooth(method = 'lm',se = F)
```

Alternative Model

```
pIMDB
```



Evaluating the Model

- Let's go straight to RMSE
    - We can have `R` calculate errors for us with `residuals()` command

```
m2 <- lm(gross_log ~ score,mv)
error <- residuals(m2)
(rmseScore <- sqrt(mean(error^2)))
```

```
## [1] 1.753146
```

- Even worse!

Multivariate Regression

- Recall that we can **model** our outcome with multiple **predictors**

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \cdots + \varepsilon$$

- How much better can we predict `gross` with **BOTH** `budget` and `score`?

```
m3 <- lm(gross_log ~ budget_log + score,mv)
error <- residuals(m3)
(rmseBudgScore <- sqrt(mean(error^2)))
```

```
## [1] 1.248817
```

Comparing Models

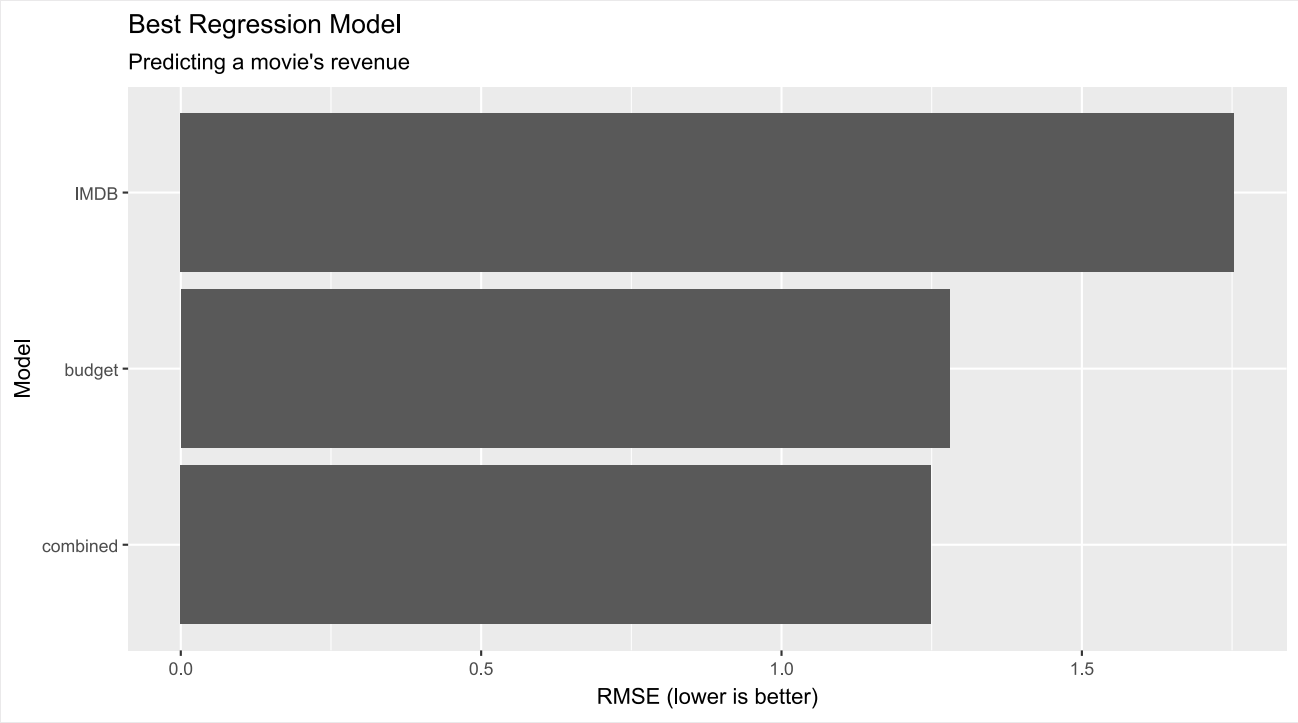
- Which model best predicts movie revenues?

```
p <- data.frame(budget = mean(rmseBudget),
           IMDB = rmseScore,
           combined = rmseBudgScore) %>%
  gather(model,rmse) %>%
  ggplot(aes(x = reorder(model,rmse),y = rmse)) +
  geom_bar(stat = 'identity') +
  labs(title = "Best Regression Model",
       subtitle = "Predicting a movie's revenue",
       y = "RMSE (lower is better)",
       x = "Model") +
  coord_flip()
```

Comparing Models

- Which model best predicts movie revenues?

p



Why RMSE?

- Want to understand how good / bad our model is
- Can use it to compare models

Why RMSE?

- Do we improve our model with `score`?

```
set.seed(123)
bsRes <- NULL
for(i in 1:100) {
  train <- mv %>%
    sample_n(size = round(nrow(mv)*.8),replace = F)
  test <- mv %>% anti_join(train)

  mB <- lm(gross_log ~ budget_log,train)
  mS <- lm(gross_log ~ score,train)
  mC <- lm(gross_log ~ budget_log + score,train)

  bsRes <- test %>%
    mutate(pB = predict(mB,newdata = test),
           pS = predict(mS,newdata = test),
           pC = predict(mC,newdata = test)) %>%
    summarise(Budget = sqrt(mean((gross_log - pB)^2,na.rm=T)),
              Score = sqrt(mean((gross_log - pS)^2,na.rm=T)),
              Combined = sqrt(mean((gross_log - pC)^2,na.rm=T))) %>%
    bind_rows(bsRes)
}
```

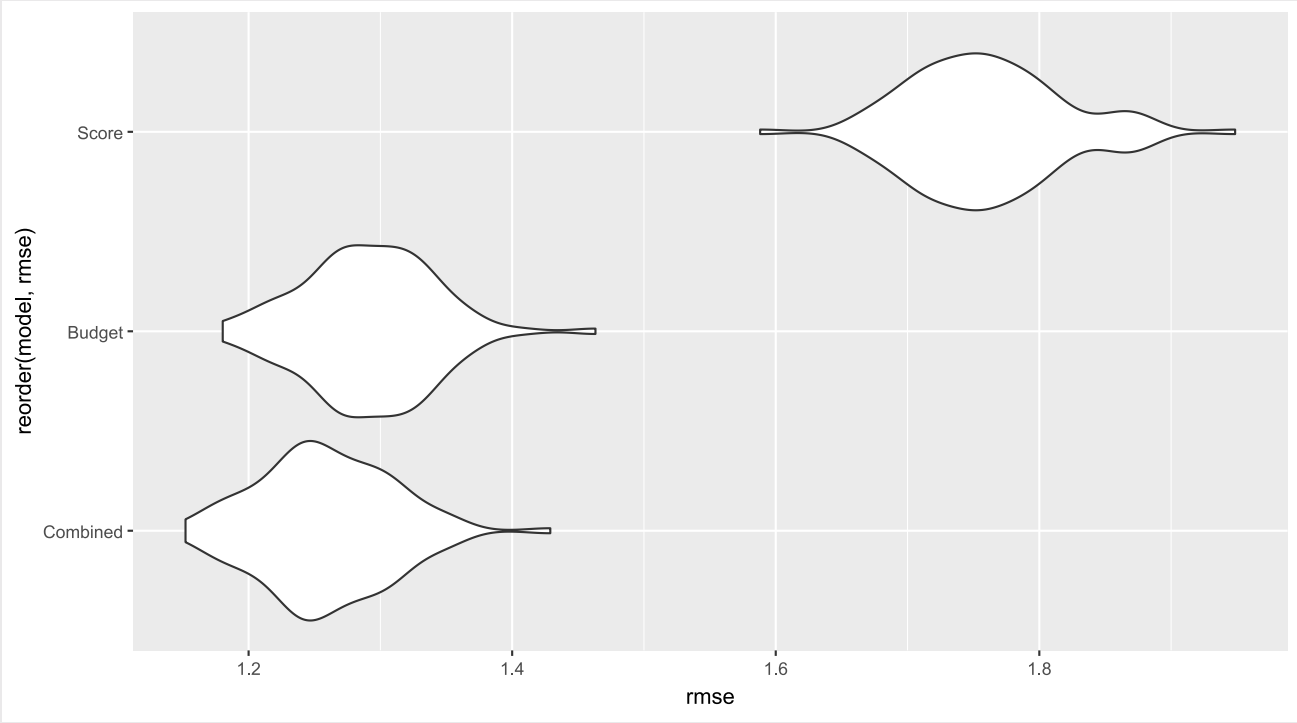
Why RMSE?

```
bsRes %>%
  summarise_all(mean,na.rm=T)
```

```
## # A tibble: 1 × 3
##   Budget Score Combined
##    <dbl> <dbl>    <dbl>
## 1   1.29  1.76     1.26
```

Visualizing

```
bsRes %>%
  gather(model,rmse) %>%
  ggplot(aes(x = rmse,y = reorder(model,rmse))) +
  geom_violin()
```



Categorical Data

- Thus far, only using continuous variables
- But we can do regression with categorical data too!
- The Bechdel Test: 3 questions of a movie

1. Does it have two women in it?
2. Who talk to each other?
3. About something other than a man?

```
mv %>%
  count(bechdel_score)
```

```
## # A tibble: 5 × 2
##   bechdel_score     n
##           <dbl> <int>
## 1             0   141
## 2             1   526
## 3             2   206
## 4             3  1185
## 5            NA  1121
```

Research Question

- Do movies that pass the Bechdel Test make more money?
    - Theory: Women are ~50% of the population. Movies that pass the test are more appealing to women.
    - Hypothesis: Movies that pass the test make more money.
- Wrangling: Let's turn the `bechdel_score` variable into a binary

```
mv <- mv %>%
  mutate(bechdel_bin = ifelse(bechdel_score == 3,"Pass","Fail"))
```

Regression

- We can add the binary factor to our regression

```
tidy(lm(gross_log ~ bechdel_bin,mv))
```

```
## # A tibble: 2 × 5
##   term            estimate std.error statistic p.value
##   <chr>              <dbl>     <dbl>     <dbl>   <dbl>
## 1 (Intercept)       18.3      0.0563    325.    0
## 2 bechdel_binPass   -0.160    0.0742     -2.15  0.0316
```

Regression

- Coefficient is negative
- What is the interpretation?
    - Movies that pass make less money...
    - ...than what?
    - Movies that fail the Bechdel Test
- Categorical variables are **always interpreted in relation to the hold-out category**!

Regression

- Movies that fail the test make more money!?
- **REMEMBER**: Correlation $\neq$ causation
    - What might explain this pattern?
    - Budgets in a sexist Hollywood!
    - Movies that fail the test get larger budgets
    - Budgets are positively associated with gross
- So we want to "control" for budget by adding it to our regression

```
mBechCtrl <- lm(gross_log ~ budget_log + bechdel_bin,mv)
```

Regression

```
tidy(mBechCtrl)
```

```
## # A tibble: 3 × 5
##   term            estimate std.error statistic   p.value
##   <chr>              <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)        2.12     0.353       6.01 2.25e-  9
## 2 budget_log         0.921    0.0199     46.2  3.57e-320
## 3 bechdel_binPass    0.188    0.0525      3.58 3.56e-  4
```

Regression

- Our hypothesis is supported!
- What about non-binary categorical variables?

```
mv %>%
  count(rating)
```

```
## # A tibble: 9 × 2
##   rating        n
##   <chr>     <int>
## 1 G            54
## 2 NC-17         6
## 3 Not Rated    34
## 4 PG          434
## 5 PG-13      1249
## 6 R          1388
## 7 TV-MA         2
## 8 Unrated       7
## 9 <NA>          5
```

Categorical

- Let's first remove rarely-occurring ratings

```
mvAnalysis <- mv %>%
  filter(!rating %in% c('Approved','TV-14','TV-MA','TV-PG','X'))
```

Categorical

```
tidy(lm(gross_log ~ rating,mvAnalysis))
```

```
## # A tibble: 7 × 5
##   term            estimate std.error statistic  p.value
##   <chr>              <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)       19.2       0.218     88.1  0
## 2 ratingNC-17       -2.45      0.688     -3.56 3.81e- 4
## 3 ratingNot Rated   -4.43      0.350    -12.7  7.62e-36
## 4 ratingPG          -0.391     0.231     -1.69 9.08e- 2
## 5 ratingPG-13       -0.763     0.222     -3.43 6.05e- 4
## 6 ratingR           -1.91      0.222     -8.62 1.06e-17
## 7 ratingUnrated     -4.66      0.643     -7.25 5.38e-13
```

Categorical

- Everything makes less money than the hold-out category!
    - "G"-rated movies are powered by children
- What if we wanted to compare to a different reference category?

```
mvAnalysis <- mvAnalysis %>%
  mutate(rating = factor(rating,
                         levels = c('R','PG-13','PG','G','Not
Rated')))

mRating2 <- lm(gross_log ~ rating,mvAnalysis)
```

Categorical

```
tidy(mRating2)
```

```
## # A tibble: 5 × 5
##   term            estimate std.error statistic  p.value
##   <chr>              <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)        17.3     0.0430    402.   0
## 2 ratingPG-13         1.15    0.0624     18.4  5.81e-72
## 3 ratingPG            1.52    0.0880     17.3  4.66e-64
## 4 ratingG             1.91    0.222       8.61 1.09e-17
## 5 ratingNot Rated    -2.52    0.278      -9.07 2.04e-19
```

Cross Validation

```
set.seed(123)
rmseRes_rating <- NULL
for(i in 1:100) {
  train <- mvAnalysis %>%
    group_by(rating) %>%
    sample_n(size = round(n()*.8),replace = F)
  test <- mvAnalysis %>% anti_join(train)

  m <- lm(gross_log ~ rating,train)
  rmseRes_rating <- test %>%
    mutate(preds = predict(m,newdata = test)) %>%
    summarise(rmse = sqrt(mean((gross_log - preds)^2,na.rm=T))) %>%
    bind_rows(rmseRes_rating)
}
rmseRes_rating %>%
  summarise(rmse = mean(rmse))
```

```
## # A tibble: 1 × 1
##    rmse
##   <dbl>
## 1  1.60
```

BREAK

Practice

- How to interpret a regression table:
    1. Check if any of the variables are logged
    2. Check if the X variable is categorical
    3. (Coming soon) Check if the Y variable is binary

Practice: 1, 2, 3 = "No"

- If 1, 2 & 3 are "No", then
  - α is the value of Y when X is zero
  - β is the amount Y increases when X increases by 1

Practice: 1, 2, 3 = "No"

- RQ: What is the relationship between head shots and eliminations?

```
fn <-
read_rds('https://github.com/jbisbee1/ISP_Data_Science_2024/raw/main/da
tidy(lm(eliminations ~ head_shots,fn))
```

```
## # A tibble: 2 × 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)    2.00     0.0768      26.0 2.31e-113
## 2 head_shots     0.109    0.0102      10.7 2.71e- 25
```

Practice: 1, 2, 3 = "No"

- **NB**: Pay attention to the units of X and Y!
- RQ: What is the relationship between accuracy and head shots?

```
tidy(lm(head_shots ~ accuracy,fn))
```

```
## # A tibble: 2 × 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)     2.61     0.390      6.68 3.94e-11
## 2 accuracy        8.52     1.32       6.43 1.99e-10
```

Practice: Logged = "Yes"

- Then need to remember THESE rules:

1. `log(Y) ~ X`: 1 unit change in $X \rightarrow (exp(\beta) - 1) * 100$ % change in Y
2. `Y ~ log(X)`: 1% increase in $X \rightarrow \beta/100$ unit change in Y
3. `log(Y) ~ log(X)`: 1% increase in $X \rightarrow \beta$ % change in Y

Practice: Logged = "Yes"

1. `log(Y) ~ X`: 1 unit change in $X \rightarrow (exp(\beta) - 1) * 100$ % change in Y

- RQ: What is the relationship between movie gross and IMDB score?

```
tidy(lm(gross_log ~ score,mv))
```

```
## # A tibble: 2 × 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   16.1      0.211      76.5  0
## 2 score          0.279    0.0325      8.58 1.49e-17
```

Practice: Logged = "Yes"

1. `log(Y) ~ X`: 1 unit change in $X \rightarrow (exp(\beta) - 1) * 100$ % change in Y
2. `Y ~ log(X)`: 1% increase in $X \rightarrow \beta/100$ unit change in Y

RQ: What is the relationship between IMDB score and budget?

```
tidy(lm(score ~ budget_log,mv))
```

```
## # A tibble: 2 × 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)   6.61      0.231     28.6   6.21e-160
## 2 budget_log   -0.0109    0.0133    -0.817 4.14e-  1
```

Practice: Logged = "Yes"

1. `log(Y) ~ X`: 1 unit change in $X \rightarrow (exp(\beta) - 1) * 100$ % change in Y
2. `Y ~ log(X)`: 1% increase in $X \rightarrow \beta/100$ unit change in Y
3. `log(Y) ~ log(X)`: 1% increase in $X \rightarrow \beta$ % change in Y

RQ: What is the relationship between gross and budget?

```
tidy(lm(gross_log ~ budget_log,mv))
```

```
## # A tibble: 2 × 5
##   term        estimate std.error statistic   p.value
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)    1.26     0.310       4.07 0.0000473
## 2 budget_log     0.964    0.0179     54.0  0
```

Practice: Categorical X = "Yes"

- RQ: What is the relationship between a movie's PG rating and it's IMBD rating?

```
tidy(lm(score ~ rating,mv))
```

```
## # A tibble: 8 × 5
##   term            estimate std.error statistic p.value
##   <chr>              <dbl>     <dbl>     <dbl>   <dbl>
## 1 (Intercept)       6.37       0.129    49.2    0
## 2 ratingNC-17       0.431      0.409     1.06   0.291
## 3 ratingNot Rated   0.481      0.208     2.31   0.0207
## 4 ratingPG         -0.0775     0.137    -0.565  0.572
## 5 ratingPG-13      -0.0540     0.132    -0.409  0.683
## 6 ratingR           0.168      0.132     1.27   0.204
## 7 ratingTV-MA       0.481      0.684     0.704  0.482
## 8 ratingUnrated     0.546      0.382     1.43   0.153
```

Practice: Categorical X = "Yes"

- RQ: What is the relationship between mental state and accuracy (Fortnite data)?

```
tidy(lm(accuracy ~ mental_state,fn))
```

```
## # A tibble: 2 × 5
##   term              estimate std.error statistic   p.value
##   <chr>                <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)         0.245    0.00640     38.3  4.19e-195
## 2 mental_statesober   0.0301   0.00889      3.38 7.47e-  4
```

Aside: scientific notation

- What is `7.47e-4`?
  - Decimal, then 3 zeros, then the number
  - `0.000747`
- What is `1.39e-1`?
  - `0.139`
- Rule: take the number, and move the decimal n places to the left, where n is the negative number following the `e`
- What about `7.47e+4`
- Rule: take the number, and move the decimal n places to the **right**, where n is the positive number following the `e`
  - 74,700

Practice: Categorical Y = "Yes"

- Next class!