Vanderbilt University
Political Science Department
Fall 2024

Stats 1
(PSCI 8356)
Professor Jim Bisbee

PROBLEM SET 3: Due Friday, September 13th by 11:59PM.

A reminder: you may work with others in the class on this problem set, and you are in fact encouraged to do so. However, the work you hand in must be your own. Handwritten work is acceptable, but word-processed work (e.g., using LaTeX or `RMarkdown`*) is preferred.*

1. A single unfair die is tossed once. Let Y be the number facing up. Find the expected value and variance of Y given the following probabilities for each face: $P(s_1) = 0.2$, $P(s_2) = 0.1$, $P(s_3) = 0.15$, $P(s_4) = 0.15$, $P(s_5) = 0.2$, $P(s_6) = 0.2$.

2. Consider a discrete random variable X with a mean μ and variance σ^2. Now define a new random variable $Y = X + 11$.

   (a) Is the mean of Y greater than, less than, or equal to the mean of X? Show your work using Theorems 3.3 and 3.5 from WMS.

   (b) Building on the answer to the preceding question, is the variance of Y greater than, less than, or equal to the variance of X?

   (c) Now consider a new random variable $Y = 2X$. Redo the analyses in 3.a and 3.b to determine if the mean and variable of Y is different from X.

   (d) Again, re-run the same analyses for a new random variable $Y = X/10$.

   (e) Finally, calculate the mean and variance of a new random variable $Y = aX + b$, where a and b are *constants*.

3. Djokovic and Medvedev play a series of games until one player wins three games. We assume that the games are played independently and that the probability that Djokovic wins any game is p. Compute the probability that the series lasts exactly five games. [Hint: Use what you know about the random variable, Y , the number of games that Djokovic wins among the first four games.]