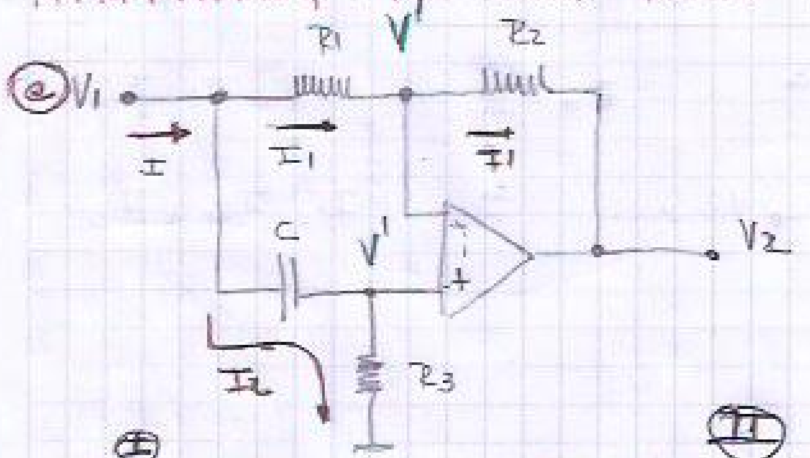
EJERCICIO #7 : UTILIZANDO MODELO IDEAL



$I = I_1 + I_2$

I

$$\left\{ \begin{aligned} I_1 &= \frac{V_1 - V'}{R_1} \\ I_1 &= \frac{V' - V_2}{R_2} \end{aligned} \right.$$

;

II

$$\begin{aligned} I_2 &= (V_1 - V')\, sC \\ I_2 &= \frac{V'}{R_3} \end{aligned}$$

;

I $\quad \dfrac{V_1 - V'}{R_1} = \dfrac{V' - V_2}{R_2} \rightarrow \dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} = V'\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$$\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)\left(\frac{R_1 R_2}{R_1 + R_2}\right) = V' \rightarrow \boxed{V_1\left(\frac{R_2}{R_1 + R_2}\right) + V_2\left(\frac{R_1}{R_1 + R_2}\right) = V'}$$

II $\quad (V_1 - V')\, sC = \dfrac{V'}{R_3} \rightarrow V_1\, sC = V'\left(sC + \dfrac{1}{R_3}\right) \Rightarrow V_1\, sC = V'\left(\dfrac{sCR_3 + 1}{R_3}\right)$

$$\boxed{V_1\left(\frac{sC R_3}{sCR_3 + 1}\right) = V'}$$

III $\quad V_1\left(\dfrac{R_2}{R_1 + R_2}\right) + V_2\left(\dfrac{R_1}{R_1 + R_2}\right) = V_1\left(\dfrac{sC R_3}{sCR_3 + 1}\right)$

$$V_2\left(\frac{R_1}{R_1 + R_2}\right) = V_1\left(\frac{sC R_3}{sCR_3 + 1} - \frac{R_2}{R_1 + R_2}\right)$$

$$V_2 \frac{R_1}{\cancel{R_1 + R_2}} = V_1 \frac{sCR_3(R_1 + R_2) - R_2(sCR_3 + 1)}{(sCR_3 + 1)\cancel{(R_1 + R_2)}}$$

$R_2 = R_3$

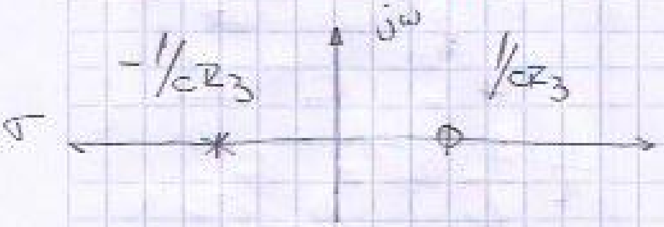
$$\frac{V_2}{V_1} = \frac{sCR_3R_1 + \cancel{sCR_3R_2} - \cancel{sCR_3R_2} - R_2}{sCR_3R_1 + R_1}$$

$$\boxed{\frac{V_2}{V_1} = \frac{sCR_3R_1 - R_2}{sCR_3R_1 + R_1}} \Rightarrow \frac{\left(s - \dfrac{R_2}{CR_3R_1}\right)}{\left(s + \dfrac{1}{CR_3}\right)}$$

$$\boxed{\frac{V_2}{V_1} = \frac{s - \dfrac{1}{CR_3}}{s + \dfrac{1}{CR_3}}}$$

HOJA Nº
FECHA

- **Diagrama de polos y ceros**



- **Modulo y fase**

$$T(j\omega) = \frac{j\omega - \frac{1}{R_3C}}{j\omega + \frac{1}{R_3C}} \rightarrow |T(j\omega)| = \frac{\sqrt{\omega^2 + \frac{1}{R_3^2C^2}}}{\sqrt{\omega^2 + \frac{1}{R_3^2C^2}}}$$

$$\boxed{|T(j\omega)| = 1}$$

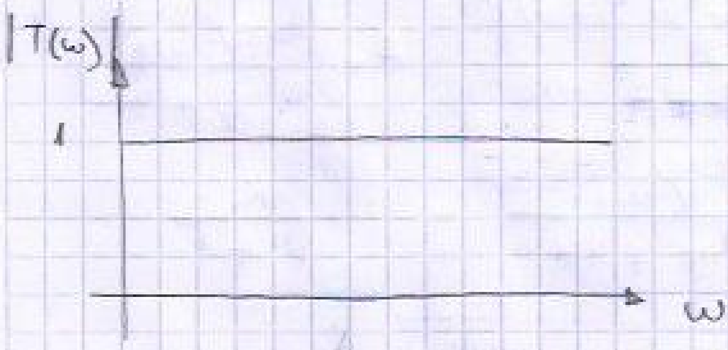
$$\varphi(j\omega) = \operatorname{arctg}\left(-\frac{\omega}{1/R_3C}\right) - \operatorname{arctg}\left(\frac{\omega}{1/R_3C}\right)$$

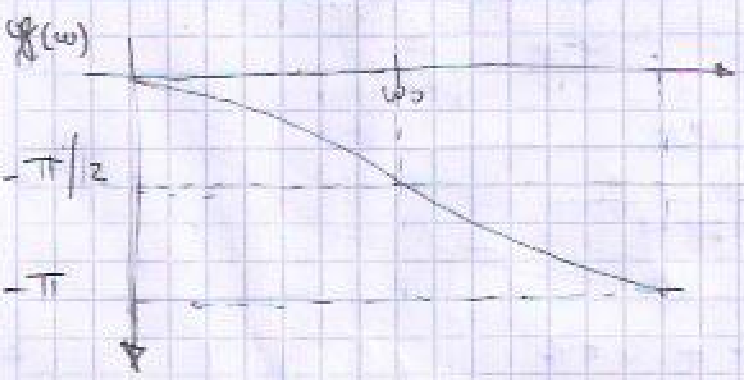
$$\boxed{\varphi(j\omega) = -2\operatorname{arctg}(\omega R_3 C)}$$

$\omega \to 0 \Rightarrow \varphi = 0$

$\omega \to \infty \Rightarrow \varphi = -\pi$

$\omega \to \omega_0 \Rightarrow \varphi = -\pi/2$





NOTA

UTILIZANDO MODELO REAL

$(V^+) - (V^-) = \frac{V_2}{A}$ (A)

(b) y (c) en (A)

(*)

$$\begin{cases} (V^- - V_1) G_1 + (V^- - V_2) G_2 = 0 \\ (V^+ - V_1) \$C + (V^+) G_3 = 0 \end{cases}$$

$$\begin{cases} V^- (G_1 + G_2) - V_1 G_1 - V_2 G_2 = 0 \\ V^+ (\$C + G_3) - V_1 \$C = 0 \end{cases}$$

(b) $V^- = \frac{V_1 G_1 - V_2 G_2}{G_1 + G_2}$; (c) $\frac{V_1 \$C}{\$C + G_3} = V^+$

$V_1 (G_1 + \$C) - \frac{V_1 G_1 G_1 - V_2 G_1 G_2}{G_1 + G_2} - \frac{V_1 \$^2 C^2}{\$C + G_3} = 0$

(*) $\frac{V_1 \$C}{\$C + G_3} - \frac{V_1 G_1}{G_1 + G_2} + \frac{V_2 G_2}{G_1 + G_2} = \frac{V_2}{A}$

$V_1 \left[\frac{\$C}{\$C + G_3} - \frac{G_1}{G_1 + G_2} \right] = V_2 \left(\frac{1}{A} - \frac{G_2}{G_1 + G_2} \right)$

$\frac{\$C(G_1 + G_2) - (\$C + G_3) G_1}{(\$C + G_3) \cancel{(G_1 + G_2)}} = \frac{V_2}{V_1} \quad \frac{G_1 + G_2 - G_2 A}{\cancel{(G_1 + G_2)} \cdot A}$

$\frac{\$C G_1 + \$C G_2 - \$C G_1 - G_3 G_1}{\$C + G_3} \cdot \frac{A}{G_1 + G_2(1 - A)} = \frac{V_2}{V_1}$

$\frac{\$C G_2 - G_3 G_1}{\$C + G_3}$

$\frac{\cancel{C} G_2 \left(\$ - \frac{G_3 G_1}{C G_2} \right)}{\cancel{C} \left(\$ + \frac{G_3}{C} \right)} \cdot \frac{A}{\underbrace{G_1 + G_2 (1 + A)}_{\cancel{G_2} \left[\frac{G_1}{G_2} + (1 + A) \right]}} = \frac{V_2}{V_1}$

$$\frac{\left(s - \frac{G_3 G_1}{C G_2}\right)}{\left(s + \frac{G_3}{C}\right)} \cdot \frac{A}{\frac{G_1}{G_2} + 1 + A} = \frac{V_2}{V_1}$$

con $R_2 = R_1 \rightarrow G_2 = G_1$

$$\boxed{\frac{\left(s - \frac{G_3}{C}\right)}{\left(s + \frac{G_3}{C}\right)} \cdot \frac{A}{2 + A} = \frac{V_2}{V_1}}$$

• Polos y ceros

Polo en $s = -G_3/C$

Cero en $s = G_3/C$

• Módulo y Fase

$$|T(j\omega)| = \left|\frac{A}{2+A}\right| \frac{\sqrt{\omega^2 + (G_3/C)^2}}{\sqrt{\omega^2 + (G_3/C)^2}} = \left|\frac{A}{2+A}\right|$$

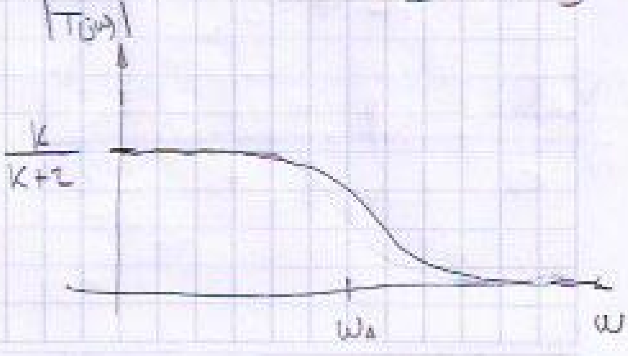
$$|T(j\omega)|\Big|_{A = \frac{K\omega_A}{j\omega + \omega_A}} = \left| \frac{\frac{K\omega_A}{j\omega+\omega_A}}{2 + \frac{K\omega_A}{j\omega + \omega_A}} \right| = \left| \frac{\frac{K\omega_A}{\cancel{j\omega+\omega_A}}}{\frac{j2\omega + 2\omega_A + K\omega_A}{\cancel{j\omega + \omega_A}}} \right|$$

$$\boxed{|T(j\omega)| = \left| \frac{K\omega_A}{\omega_A(K+2) + j2\omega} \right| = \frac{K\omega_A}{\sqrt{\omega_A^2 (K+2)^2 + 4\omega^2}}}$$

$$|T(j\omega)|\Big|_{\omega=0} = \frac{K\cancel{\omega_A}}{\cancel{\omega_A}(K+2)} = \frac{K}{K+2}$$

$$|T(j\omega)|\Big|_{\omega=\omega_A} = \frac{K\omega_A}{\sqrt{\omega_A^2 (K+2)^2 + 4\omega_A^2}} = \frac{K\omega_A}{\sqrt{\omega_A^2 \cdot \left[(K+2)^2 + 4\right]}}$$

$$|T(j\omega)|\Big|_{\omega=\omega_A} = \frac{K}{(K+2)^2 + 4}$$



$$|T(j\omega)|\Big|_{\omega=\infty} = \rightarrow 0$$

HOJA Nº 1

FECHA

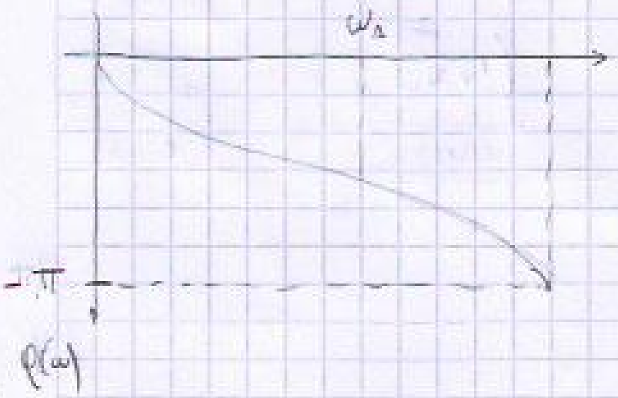
$$T(j\omega) = \frac{j\omega - \frac{G_3}{C}}{j\omega + \frac{G_3}{C}} \cdot \frac{\dfrac{K\omega_A}{j\omega + \omega_A}}{\dfrac{K\omega_A}{j\omega + \omega_A} + 2}$$

$$T(j\omega) = \frac{j\omega - G_3/C}{j\omega + G_3/C} \cdot \frac{K\omega_A}{K\omega_A + 2(j\omega + \omega_A)}$$

$$T(j\omega) = \frac{j\omega - G_3/C}{j\omega + G_3/C} \cdot \frac{1}{j\frac{2\omega}{K\omega_A} + \left(\frac{2\omega_A}{K\omega_A} + 1\right)}$$

$$\varphi(\omega) = -2\,\mathrm{arctg}\left(\omega R_3 C\right) - \mathrm{arctg}\left[\frac{2\omega}{K\omega_A\left(\frac{2}{K}+1\right)}\right]$$

$$\boxed{\varphi(j\omega) = -2\,\mathrm{arctg}\left(\omega R_3 C\right) - \mathrm{arctg}\left(\frac{2\omega}{\omega_A(2+K)}\right)}$$



$\omega = 0 \rightarrow \varphi(\omega) = 0$

$\omega = \infty \rightarrow \varphi(\omega) = -\pi$

NOTA

- **Normalización en frec. (Ideal)**

$$T(s) = \frac{s - \frac{1}{C R_3}}{s + \frac{1}{C R_3}}$$

$$s = s_N \cdot \omega_0$$

$$T(s_N \omega_0) = \frac{s_N \omega_0 - \frac{1}{C R_3}}{s_N \omega_0 + \frac{1}{C R_3}}$$

$$T(s_N \omega_0) = \frac{s_N - \frac{1}{C R_3 \omega_0}}{s_N + \frac{1}{C R_3 \omega_0}}$$

$$C R_3 \omega_0 = 1 \rightarrow \omega_0 = \frac{1}{C R_3}$$

$$\boxed{T(s_N) = \frac{s_N - 1}{s_N + 1}}$$

UNIDADES

$$i = C \frac{dv}{dt}$$

$$A = C \frac{V}{s}$$

$$\frac{1}{R} \left(\frac{A}{V}\right) s = C$$

$$R \cdot \frac{s}{R} = C \cdot R$$

$$[s] = CR$$

- **Normalización en frec (Real)**

$$T(s) = \frac{s - \frac{1}{C R_3}}{s + \frac{1}{C R_3}} \cdot \frac{A}{2 + A} \quad ; \quad A = \frac{K \omega_A}{s + \omega_A}$$

$$s = s_N \cdot \omega_A \rightarrow A = \frac{K \omega_A}{s_N \omega_A + \omega_A} = \frac{K}{s_N + 1}$$

$$T(s_N) = \frac{s_N - 1}{s_N + 1} \cdot \frac{\frac{K}{s_N + 1}}{2 + \frac{K}{s_N + 1}}$$

$$T(s_N) = \frac{s_N - 1}{s_N + 1} \cdot \frac{\frac{K}{s_N + 1}}{\frac{2 s_N + 2 + K}{s_N + 1}}$$

$$\boxed{T(s_N) = \frac{s_N - 1}{s_N + 1} \cdot \frac{K}{s_N \cdot 2 + 2 + K}}$$