$$\begin{cases} y'' = p(x)y' + q(x)y + r(x), \quad x \in [a,b] \\ y(a) = \alpha \\ y(b) = \beta \end{cases} \xrightarrow{\text{SOL}} \boxed{y(x) = y_1(x) + \frac{\beta - y_1(b)}{y_2(b)} y_2(x)}$$

donde $y_1(x)$ es solución de

$$\text{PVI 1: } \begin{cases} y_1'' = p(x)y_1' + q(x)y_1 + r(x) \\ y_1(a) = \alpha \\ y_1'(a) = 0 \end{cases}$$

donde $y_2(x)$ es solución de

$$\text{PVI 2: } \begin{cases} y_2'' = p(x)y_2' + q(x)y_2 \\ y_2(a) = 0 \\ y_2'(a) = 1 \end{cases}$$

Demostración

$$y = y_1 + \frac{\beta - y_1(b)}{y_2(b)} y_2 \rightarrow \begin{cases} y' = y_1' + \frac{\beta - y_1(b)}{y_2(b)} y_2' \\ y'' = y_1'' + \frac{\beta - y_1(b)}{y_2(b)} y_2'' \end{cases}$$

Sustituimos en y'' los PVIs:

$$y'' = \left(p(x)y_1' + q(x)y_1 + r(x)\right) + \frac{\beta - y_1(b)}{y_2(b)} \left(p(x)y_2' + q(x)y_2\right)$$

$$= p(x) \underbrace{\left(y_1' + \frac{\beta - y_1(b)}{y_2(b)} y_2'\right)}_{y'} + q(x) \underbrace{\left(y_1 + \frac{\beta - y_1(b)}{y_2(b)} y_2\right)}_{y} + r(x)$$

$$= p(x)y' + q(x)y + r(x) \rightarrow y(x) \text{ cumple la ED}$$

CF:

- $y(a) = y_1(a) + \frac{\beta - y_1(b)}{y_2(b)} y_2(a) \overset{y_2(a)=0}{=} y_1(a) = \alpha$

- $y(b) = y_1(b) + \frac{\beta - y_1(b)}{\cancel{y_2(b)}} \cancel{y_2(b)} =$

$$= y_1(b) + \beta - y_1(b) = \beta$$

$\rightarrow$ $y(x)$ satisface el Problema de Frontera