CS 371 HW 1

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2/8/19

1. Chapter 1, Problem 2.
   True. This must be the case, because w would never reject m, and m would always propose to w first. If w was already matched with another man, then she would dump that man to be with m.

2. Chapter 1, Problem 4.

Data: Preferences for hospitals and medical students
Result: A stable matching of hospitals and medical students
Initialize all students and hospitals to unpaired
while *some hospital position is unfilled* **do**
  Choose one such hospital h.
  Hospital h offers the next most desirable student s a job, a student that they haven't offered yet.
  if *s has no job* **then**
    s accepts job from hospital h.
    Hospital h has 1 less job to offer.
  else
    if *student s prefers current job with Hospital h'* **then**
      s declines job from h and remains with h'.
    else
      (s prefers h to h')
      s leaves hospital h' for hospital h.
      Hospital h' has one more job available, hospital h has one less job available.
    end
  end
end
All job-fillings are finalized.

If there are n hospitals and m students, the upper bound for the run time of this algorithm is $O(mn)$. This algorithm is capable of filling every hospital's positions because every hospital offers positions to students as long as it has positions available. Students have the agency to reject offers

from hospitals if they favor the position of another hospital they've already accepted.
There are two types of instabilities:

(a) The first instability is one in which there is a student s assigned to a hospital h, a student s that is not assigned to a hospital, and hospital h prefers s' to s. Using this algorithm, this instability will not occur because the hospital will always offer the next most desirable student a job. Because of this, h will have offered s' before it offered s, given that s' has no job, s' would have accepted the job from h, and this instability will not have occurred.

(b) The second instability is one in which there are students s and s', and hospitals h and h'. This instability occurs when student s is matched with hospital h and prefers hospital h', and student s' is matched with hospital h' and prefers hospital h. Using this algorithm, this instability cannot occur, because hospital h would've offered student s a job before offering student s' and student s would've accepted the offer.

3. Chapter 1, Problem 8.
In the preference matrix above, the matching would be: M1-W1 (Her

M1	W1	W2	W3
M2	W2	W1	W3
M3	W1	W2	W3
W1	M2	M1	M3
W2	M1	M2	M3
W3	M1	M2	M3

second choice), M2-W2, M3-W3. If W1 decided to lie in order to improve her chances to match with M2 (rather than ending up with M1), she would change her preference to: 2,3,1. If we change W1's preferences to this, the preference matrix becomes:

M1	W1	W2	W3
M2	W2	W1	W3
M3	W1	W2	W3
W1	M2	M3	M1
W2	M1	M2	M3
W3	M1	M2	M3

When the matrix table changes to this, the matches become: M1-W2,M2-W1 (Her top choice), M3-W3.

This works, because by switching her preferences from 2,1,3, to 2,3,1, W1 is able to dump M1 for M3, once she dumps M1, she is able to accept a proposal from M2.