05.581  PAC4  joan carles llompart i seguí

Si VAR_A=x i VAR_B=y llavors OUTB_1=z
Si VAR_C=x' i VAR_D=y' llavors OUTB_2=z'
Si OUTB_1=z i OUTB_2=z' llavors OUT=z''

$L_{VAR\_A}$={baix(B), mig(M), alt(A)}
$L_{VAR\_B}$={baix(B), mig(M), alt(A)}
$L_{VAR\_C}$={baix(B), mig(M), alt(A)}
$L_{VAR\_D}$= {baix(B), mig(M), alt(A)}
$L_{OUTB\_1}$={baix(B), mig(M), alt(A)}
$L_{OUTB\_2}$={baix(B), mig(M), alt(A)}
$L_{OUT}$={molt baix(MB), baix(B), mig(M), alt(A), molt alt(MA)}

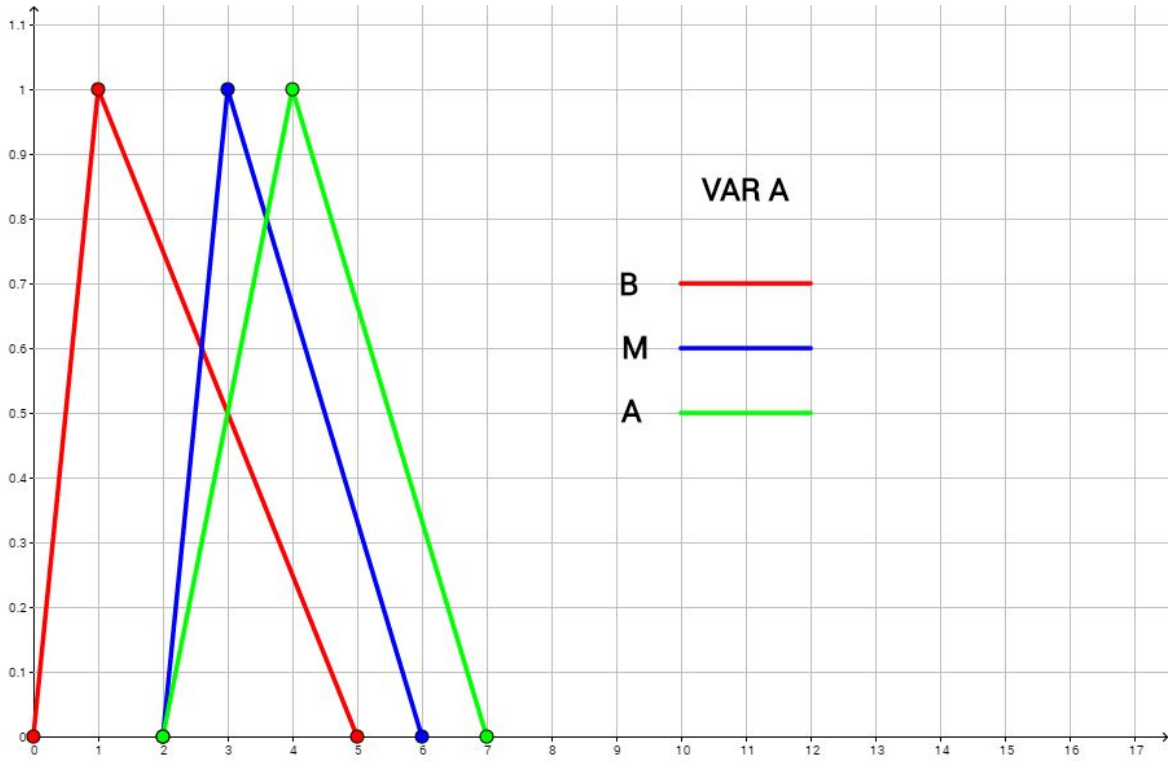
$U_{VAR\_A}$=[0,7]
$U_{VAR\_B}$=[0,9]
$U_{VAR\_C}$=[0,1]
$U_{VAR\_D}$=[0,1]
$U_{OUTB\_1}$=[0,5]
$U_{OUTB\_2}$=[5,10]
$U_{OUT}$ =[0,1]

VAR_A: B=(1,1,4), M=(3,1,3), A=(4,2,3)
VAR_B: B=(2,2,4), M=(4,2,2), A=(7,2,2)
VAR_C: B=(0.2,0.2,0.3), M=(0.4,0.2,0.3), A=(0.8,0.3,0.2)
VAR_D: B=(0.1,0.1,0.4), M=(0.5,0.1,0.5), A=(0.8,0.1,0.2)
OUTB_1: B=(1,1,4), M=(3,1,2), A=(4,1,1)
OUTB_2: B=(6,1,4), M=(8,1,2), A=(9,1,1)
OUT: MB=(0,0,0.25), B=(0.25,0.25,0.25), M=(0.5,0.25,0.25),
A=(0.75,0.25,0.25), MA=(1,0.25,0)

VAR_A:

$T_L$ VAR_A: B=(1,1,4), M=(3,1,3), A=(4,2,3)

$U_{VAR\_A}$=[0,7]




$\mu B(x)$= 0 si x=0
x/1 si 0<x<1
1-[(x-1)/(5-1)] si 1<=x<5
0 si x>=5

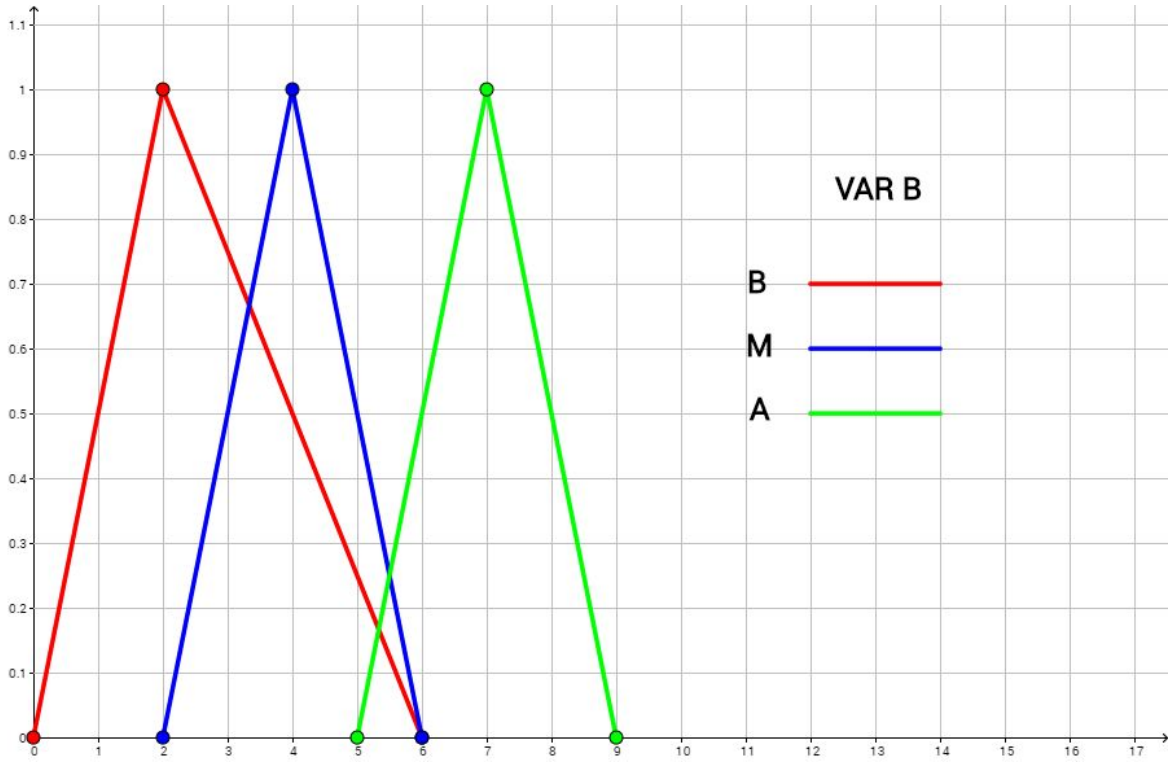
$\mu M(x)$= 0 si x<=2
(x-2)/(3-2) si 2<x<3
1-[(x-3)/(6-3)] si 3<=x<6
0 si x>=6

$\mu A(x)$= 0 si x<=2
(x-2)/(4-2) si 2<x<4
1-[(x-4)/(7-4)] si 4<=x<7
0 si x=7

VAR_B:

$T_L$ VAR_B: B=(2,2,4), M=(4,2,2), A=(7,2,2)

$U_{VAR\_B}$=[0,9]




$\mu B(x)$= 0 si x=0
x/2 si 0<x<2
1-[(x-2)/(6-2)] si 2<=x<6
0 si x>=6

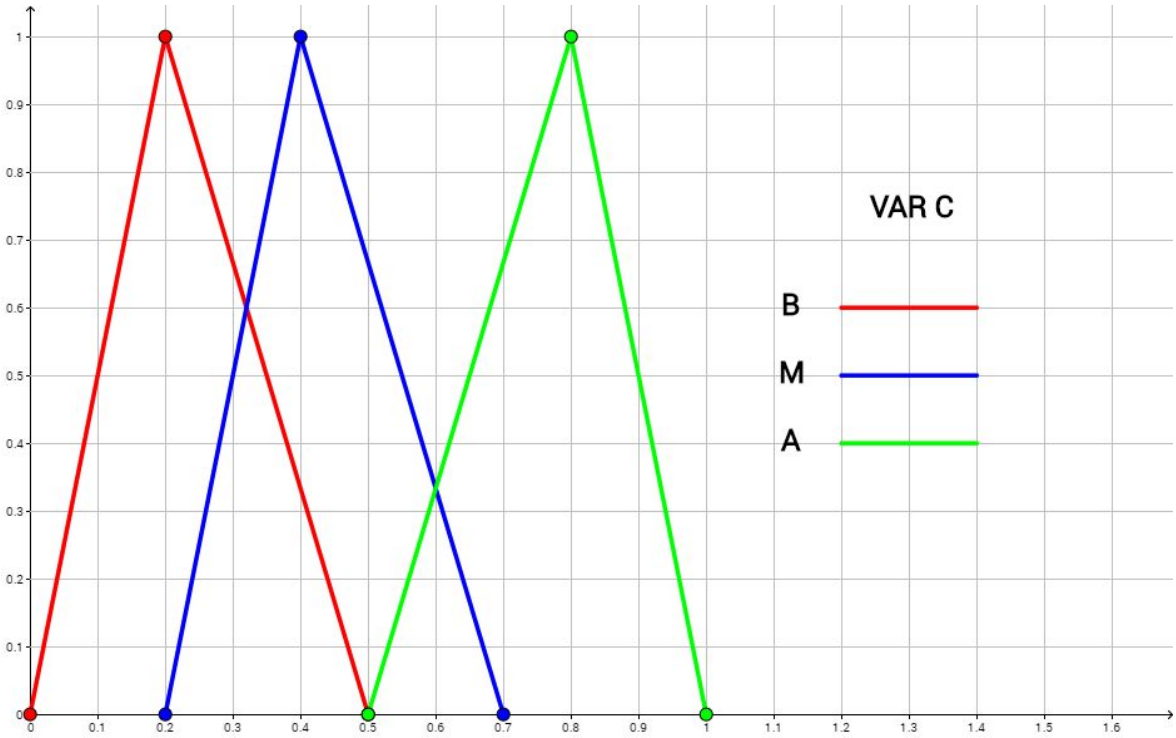
$\mu M(x)$= 0 si x<=2
(x-2)/(4-2) si 2<x<4
1-[(x-4)/(6-4)] si 4<=x<6
0 si x>=6

$\mu A(x)$= 0 si x<=5
(x-5)/(7-5) si 5<x<7
1-[(x-7)/(9-7)] si 7<=x<9
0 si x=9

VAR_C:

$T_L$ VAR_C: B=(0.2,0.2,0.3), M=(0.4,0.2,0.3), A=(0.8,0.3,0.2)

$U_{VAR\_C}$=[0,1]




$\mu B(x)$= 0 si x=0

x/0.2 si 0<x<0.2

1-[(x-0.2)/(0.5-0.2)] si 0.2<=x<0.5

0 si x>=0.5

$\mu M(x)$= 0 si x<=0.2

(x-0.2)/(0.4-0.2) si 0.2<x<0.4

1-[(x-0.4)/(0.7-0.4)] si 0.4<=x<0.7

0 si x>=0.7

$\mu A(x)$= 0 si x<=0.5

(x-0.5)/(0.8-0.5) si 0.5<x<0.8

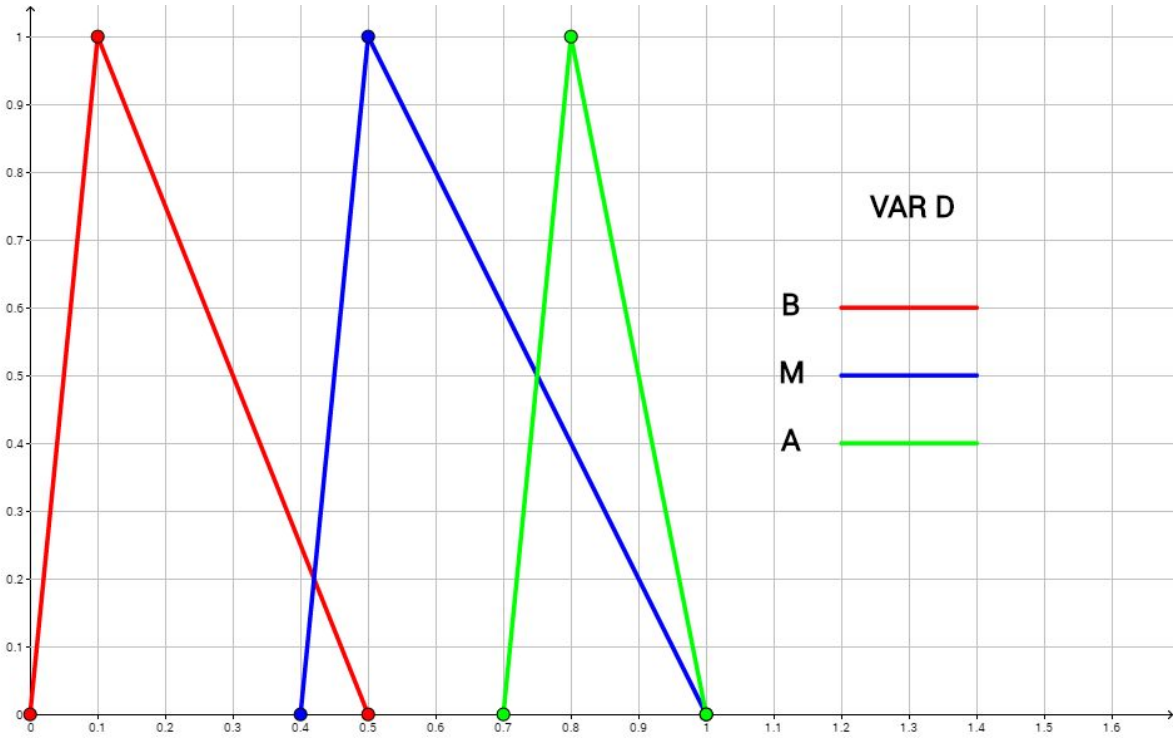
1-[(x-0.8)/(1-0.8)] si 0.8<=x<1

0 si x=1

VAR_D:

$T_L$ VAR_D: B=(0.1,0.1,0.4), M=(0.5,0.1,0.5), A=(0.8,0.1,0.2)

$U_{VAR\_D}$=[0,1]




$$\mu B(x) = \begin{cases} 0 & \text{si } x=0 \\ x/0.1 & \text{si } 0<=x<0.1 \\ 1-[(x-0.1)/(0.5-0.1)] & \text{si } 0.1<=x<0.5 \\ 0 & \text{si } x>=0.5 \end{cases}$$

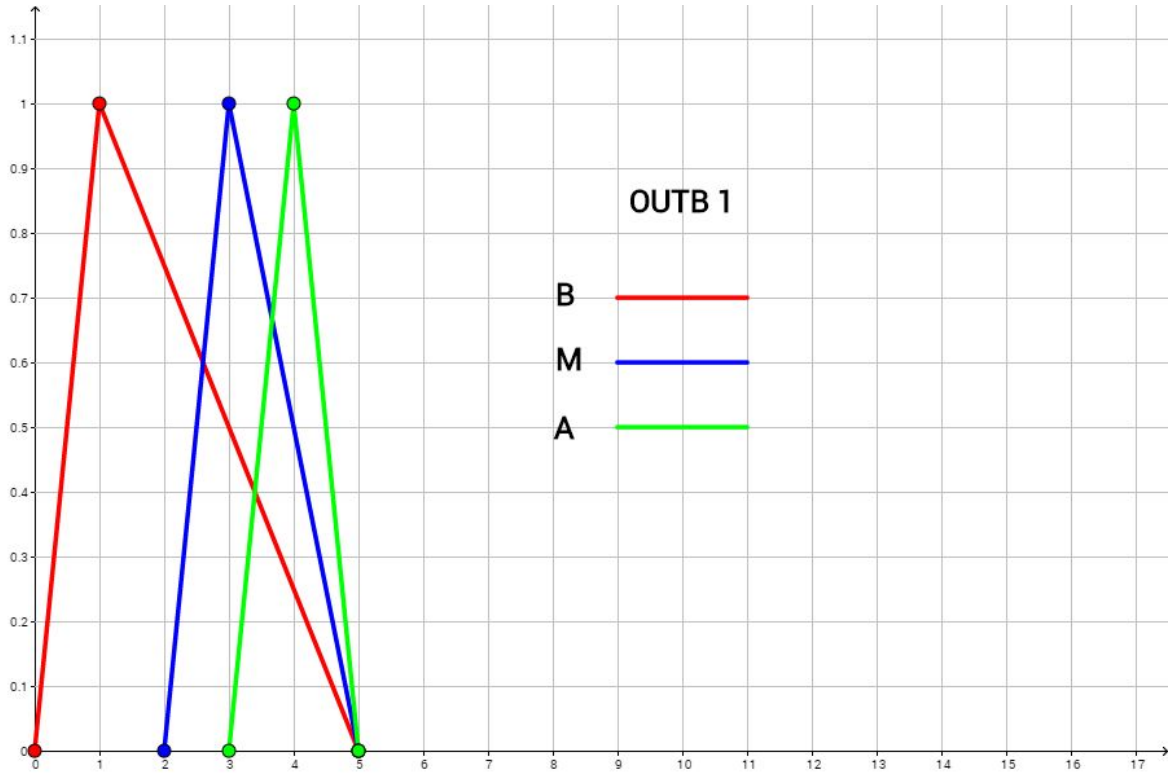
$$\mu M(x) = \begin{cases} 0 & \text{si } x<=0.4 \\ (x-0.4)/(0.5-0.4) & \text{si } 0.4<x<0.5 \\ 1-[(x-0.5)/(1-0.5)] & \text{si } 0.5<=x<1 \\ 0 & \text{si } x=1 \end{cases}$$

$$\mu A(x) = \begin{cases} 0 & \text{si } x<=0.7 \\ (x-0.7)/(0.8-0.7) & \text{si } 0.7<=x<0.8 \\ 1-[(x-0.8)/(1-0.8)] & \text{si } 0.8<=x<1 \\ 0 & \text{si } x=1 \end{cases}$$

OUTB_1:

$T_L$ OUTB_1: B=(1,1,4), M=(3,1,2), A=(4,1,1)

$U_{OUTB\_1}$=[0,5]




$\mu B(x)$= 0 si x=0
x/1 si 0<=x<1
1-[(x-1)/(5-1)] si 1<=x<5
0 si x=5

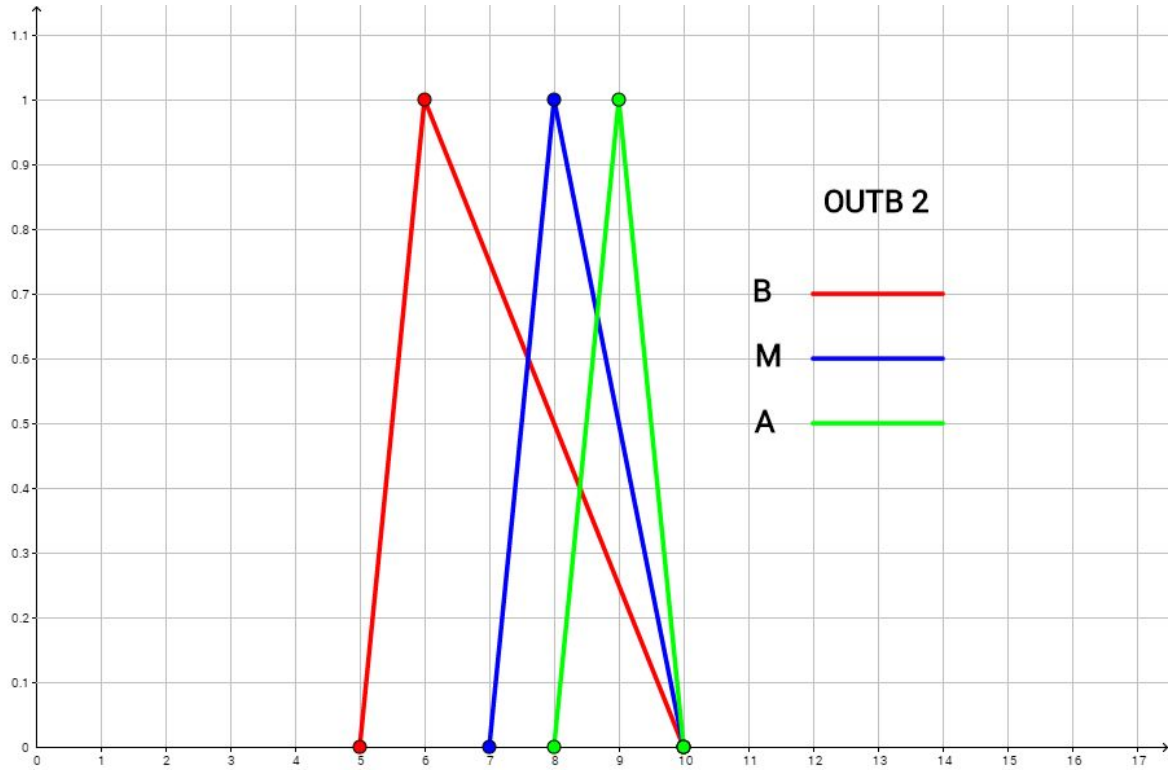
$\mu M(x)$= 0 si x<=2
(x-2)/(3-2) si 2<x<3
1-[(x-3)/(5-3)] si 3<=x<5
0 si x=5

$\mu A(x)$= 0 si x<=3
(x-3)/(4-3) si 3<x<4
1-[(x-4)/(5-4)] si 4<=x<5
0 si x=5

OUTB_2:

$T_L$ OUTB_2: B=(6,1,4), M=(8,1,2), A=(9,1,1)

$U_{OUTB\_2}$=[5,10]




$\mu B(x)=$ 0 si x=5
(x-5/(6-5) si 5<x<6
1-[(x-6)/(10-6)] si 6<=x<10
0 si x=10

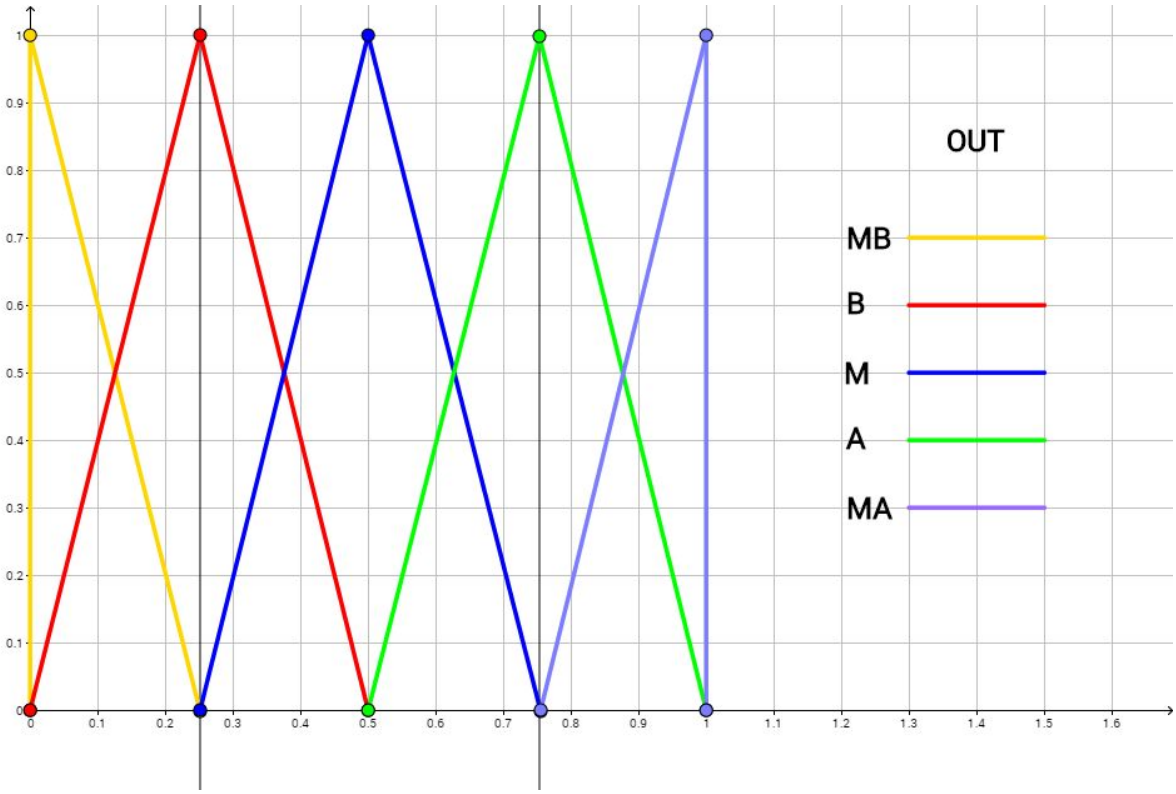
$\mu M(x)=$ 0 si x<=7
(x-7)/(8-7) si 7<x<8
1-[(x-8)/(10-8)] si 8<=x<10
0 si x=10

$\mu A(x)=$ 0 si x<=8
(x-8)/(9-8) si 8<x<9
1-[(x-9)/(10-9)] si 9<=x<10
0 si x=10

OUT:

$T_L$ OUT: MB=(0,0,0.25), B=(0.25,0.25,0.25), M=(0.5,0.25,0.25),
A=(0.75,0.25,0.25), MA=(1,0.25,0)

$U_{OUT}$ =[0,1]




$\mu MB(x)$= 1-(x/0.25) si 0<=x<0.25
0 si x>=0.25

$\mu B(x)$= 0 si x=0
x/0.25 si 0<x<=0.25
1- [(x-0.25)/(0.5-0.25)] si 0.25<x<0.5
0 si x>=0.5

$\mu M(x)$= 0 si x<=0.25
(x-0.25)/(0.50-0.25) si 0.25<x<0.5
1-[(x-0.5)/(0.75-0.5)] si 0.5=x<0.75
0 si x>=0.75

$\mu A(x)$= 0 si x<=0.5
(x-0.5)/(0.75-0.5) si 0.5<x<0.75
1-[(x-0.75)/(1-0.75)] si 0.75<=x<1
0 si x=1

$\mu MA(x)$= 0 si x<=0.75
(x-0.75)/(1-0.75) si 0.75<=x<=1

VAR_A=5:

μB(x)= 1-[(x-1)/(5-1)] si 1<=x<=5

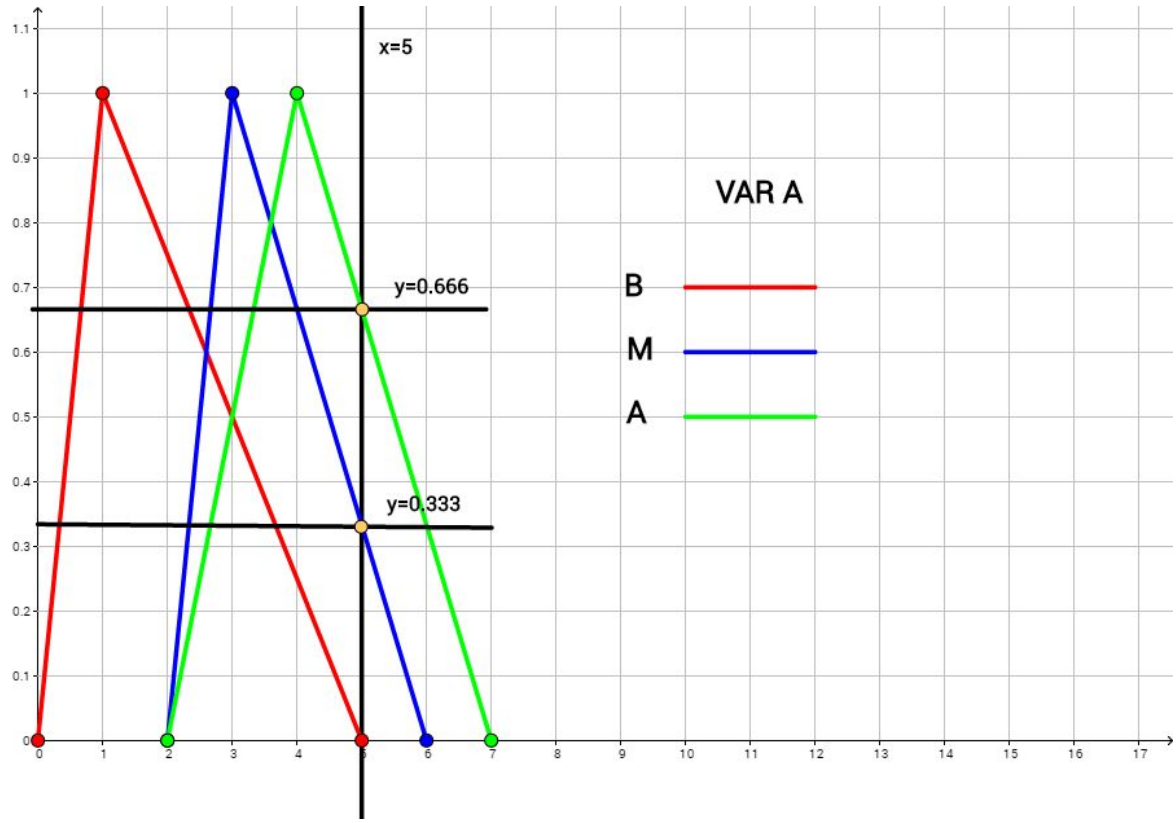
μB(5)= 1-[(5-1)/(5-1)] = 1-(4/4) = 0

μM(x)= 1-[(x-3)/(6-3)] si 3<=x<=6

μM(5)= 1-[(5-3)/(6-3)] = 1-(2/3) = 0.333

μA(5)= 1-[(x-4)/(7-4)] si 4<=x<=7

μA(5)= 1-[(5-4)/(7-4)] = 1-(1/3) = 0.666




VAR_A=5 activa M a 0.333 i A a 0.666.

VAR_B=3:

μB(x)= 1-[(x-2)/(6-2)] si 2<=x<=6

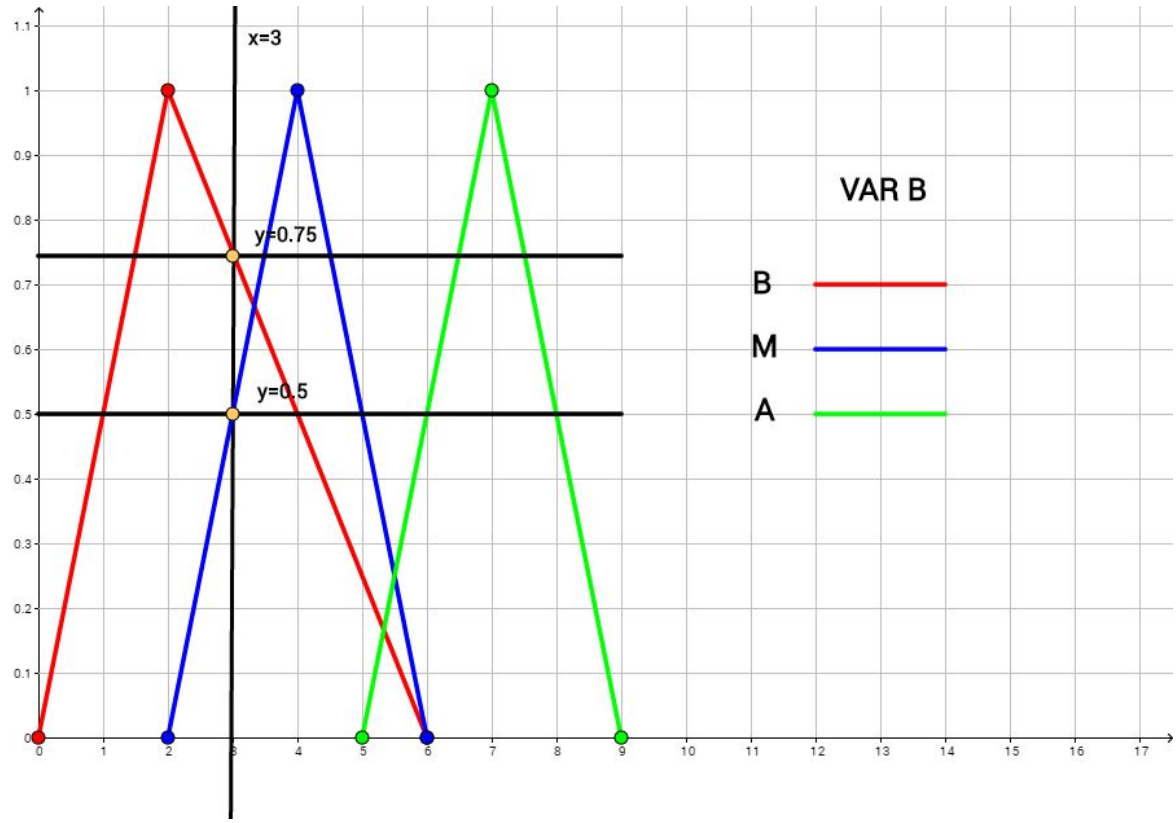
μB(3)= 1-[(3-2)/(6-2)] = 1-(1/4) = 0.75

μM(x)= (x-2)/(4-2) si 2<=x<4

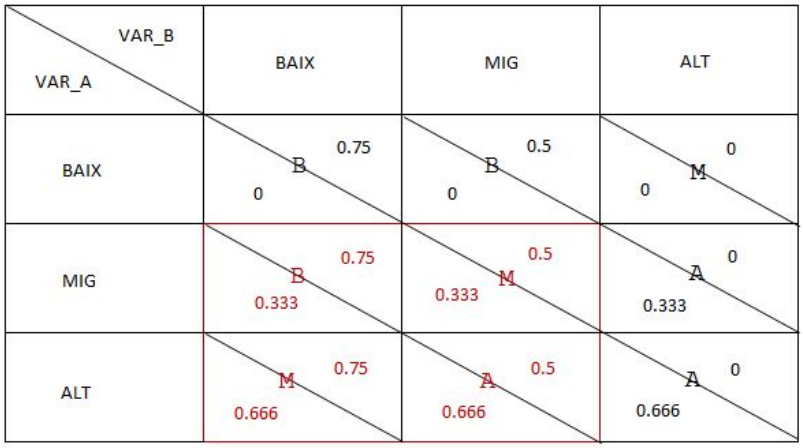
μM(3)= (3-2)/(4-2) = 1/2 = 0.5

μA(x)= 0 si x<5

μM(3)= 0




VAR_B=3 activa B a 0.75 i M a 0.5.




Regles OUTB_1:

| REGLA | VAR_A(5) | VAR_B(3) | OUT_B1 |
|---|---|---|---|
| 1 | B | B(0.75) | B |
| 2 | B | M(0.5) | B |
| 3 | B | A | M |
| 4 | M(0.333) | B(0.75) | B(0.333) |
| 5 | M(0.333) | M(0.5) | M(0.333) |
| 6 | M(0.333) | A | A |
| 7 | A(0.666) | B(0.75) | M(0.666) |
| 8 | A(0.666) | M(0.5) | A(0.5) |
| 9 | A(0.666) | A | A |

| REGLA | VAR_A | VAR_B | satisfacció-antecedent | consegüent-obtingut |
|---|---|---|---|---|
| 4 | 0.333 | 0.75 | min(0.333,0.75)= 0.333 | B(0.333) |
| 5 | 0.333 | 0.5 | min(0.333,0.5)= 0.333 | M(0.333) |
| 7 | 0.666 | 0.75 | min(0.666,0.75)= 0.666 | M(0.666) |
| 8 | 0.666 | 0.5 | min(0.666,0.5)= 0.5 | A(0.5) |

OUTB_1

| | |
|---|---|
| 0.333 | B |
| 0.333 | M |
| 0.666 | M |
| 0.5 | A |

VAR_C=0.3:

μB(x)= 1-[(x-0.2)/(0.5-0.2)] si 0.2<=x<=0.5

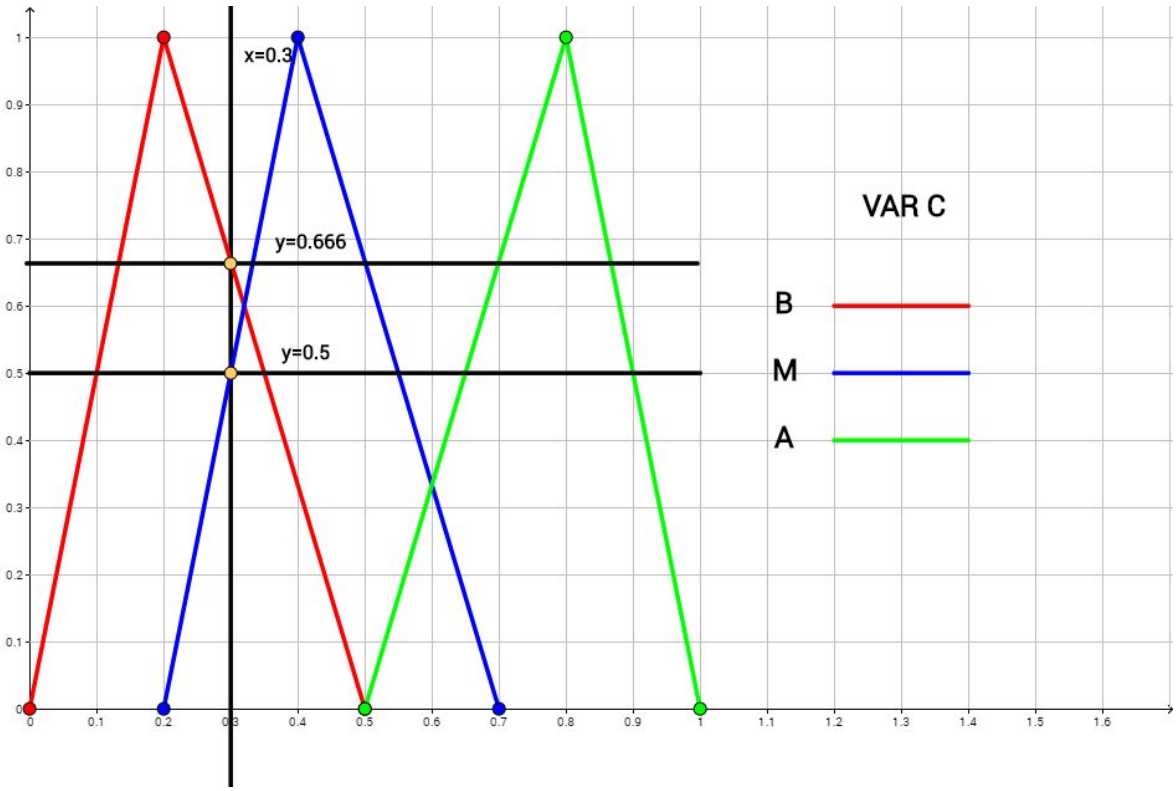
μB(0.3)= 1-[(0.3-0.2)/(0.5-0.2)] = 1-(0.1/0.3) = 0.666

μM(x)= (x-0.2)/(0.4-0.2) si 0.2<=x<0.4

μM(0.3)= (0.3-0.2)/(0.4-0.2) = 0.1/0.2 = 0.5

μA(x)= 0 si x<0.5

μA(0.3)= 0




VAR_C=0.3 ativa B a 0.666 i M a 0.5.

VAR_D=0.8:

μB(x)= 0 si x>0.5

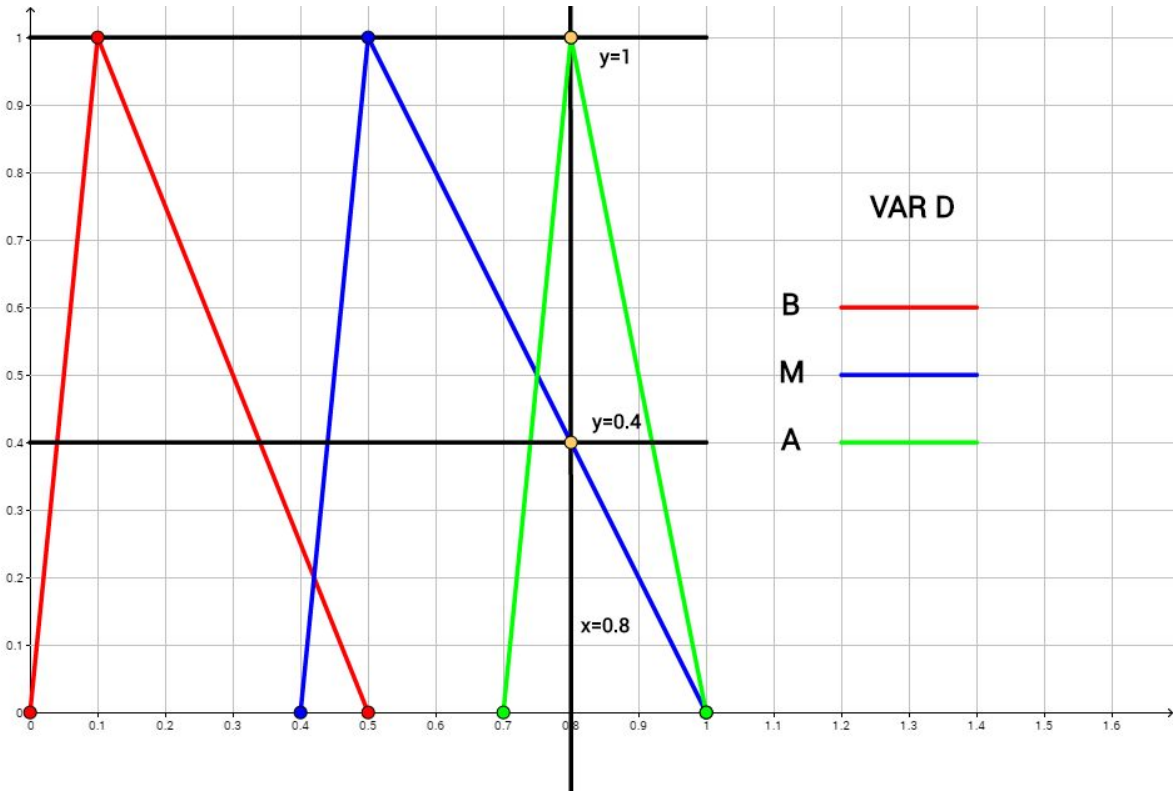
μB(0.8)= 0

μM(x)= 1-[(x-0.5)/(1-0.5)] si 0.5<=x<=1

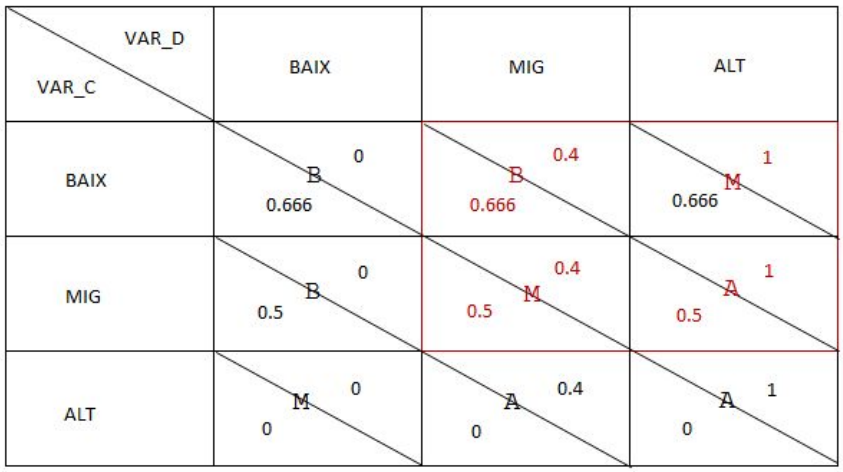
μM(0.8)= 1-[(0.8-0.5)/(1-0.5)] = 1-(0.3/0.5) = 0.4

μA(x)= 1-[(x-0.8)/(1-0.8)] si 0.8<=x<=1

μA(0.8)= 1-[(0.8-0.8)/(1-0.8)] = 1-(0/0.2) = 1




VAR_D=0.8 activa M a 0.4 i A a 1.




Regles OUTB_2:

| REGLA | VAR_C(0.3) | VAR_D(0.8) | OUT_B2 |
|---|---|---|---|
| 1 | B(0.666) | B | B |
| 2 | B(0.666) | M(0.4) | B(0.4) |
| 3 | B(0.666) | A(1) | M(0.666) |
| 4 | M(0.5) | B | B |
| 5 | M(0.5) | M(0.4) | M(0.4) |
| 6 | M(0.5) | A(1) | A(0.5) |
| 7 | A | B | M |
| 8 | A | M(0.4) | A |
| 9 | A | A(1) | A |

| REGLA | VAR_C | VAR_D | satisfacció-antecedent | consegüent-obtingut |
|---|---|---|---|---|
| 2 | 0.666 | 0.4 | min(0.666,0.4)= 0.4 | B(0.4) |
| 3 | 0.666 | 1 | min(0.666,0.1)= 0.666 | M(0.666) |
| 5 | 0.5 | 0.4 | min(0.5,0.4)= 0.4 | M(0.4) |
| 6 | 0.5 | 1 | min(0.5,0.1)= 0.5 | A(0.5) |

OUTB_2

| | |
|---|---|
| 0.4 | B |
| 0.666 | M |
| 0.4 | M |
| 0.5 | A |

S’aplica la t-conorma max per obtenir els termes lingüístics que s’activaran amb els corresponents nivells:

OUTB_1:
B = 0.333
M = max(0.333, 0.666) = 0.666
A = 0.5

OUTB_2:
B = 0.4
M = max(0.666, 0.4) = 0.666
A = 0.5

| OUTB_1 | | OUTB_2 | |
|---|---|---|---|
| 0.333 | B | 0.4 | B |
| 0.666 | M | 0.666 | M |
| 0.5 | A | 0.5 | A |

Aquests es traslladen directament al bloc de regles d’OUT, a l’hora que s’aplica la t-norma min per obtenir OUT:

| REGLA | OUTB_1 | OUTB_2 | OUT |
|---|---|---|---|
| 1 | B(0.333) | B(0.4) | MB(0.333) |
| 2 | B(0.333) | M(0.666) | B(0.333) |
| 3 | B(0.333) | A(0.5) | M(0.333) |
| 4 | M(0.666) | B(0.4) | B(0.4) |
| 5 | M(0.666) | M(0.666) | M(0.666) |
| 6 | M(0.666) | A(0.5) | A(0.5) |
| 7 | A(0.5) | B(0.4) | M(0.4) |
| 8 | A(0.5) | M(0.666) | A(0.5) |
| 9 | A(0.5) | A(0.5) | MA(0.5) |

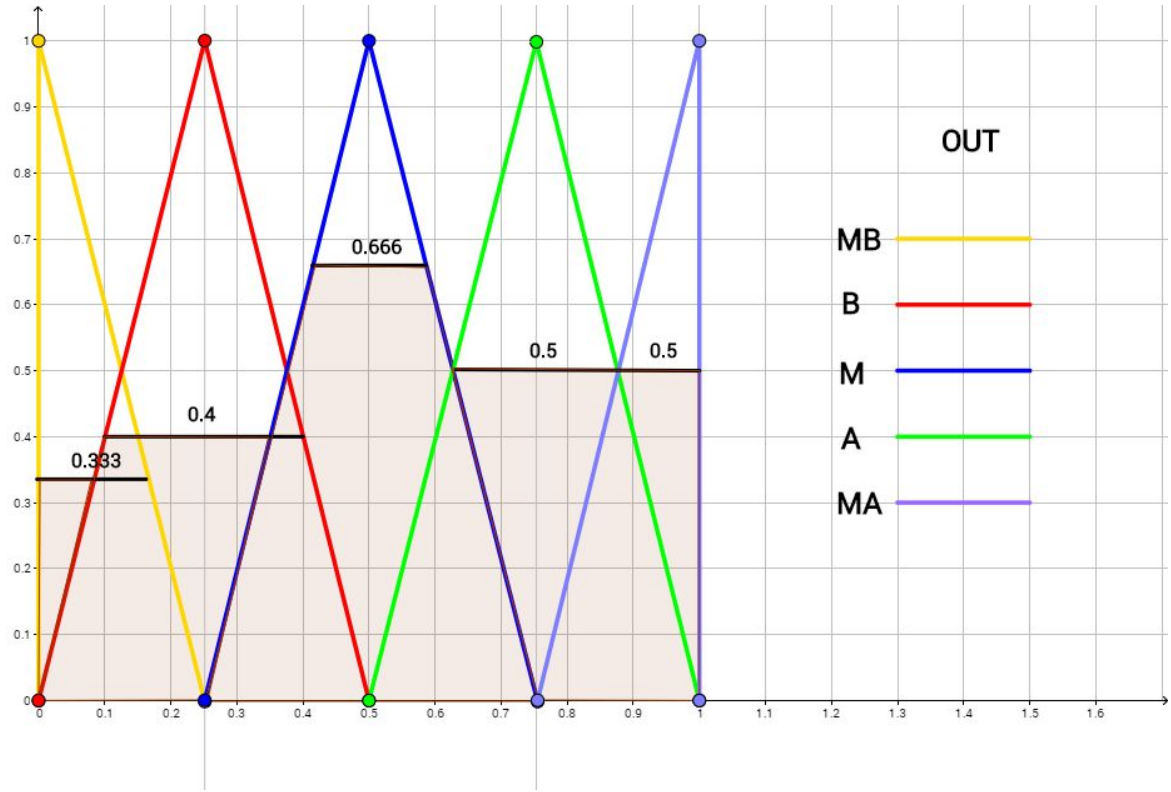
S'aplica ara t-conorma max als següents:

OUT:

MB = 0.333
B = max(0.333, 0.4) = 0.4
M = max(0.333, 0.666, 0.4) = 0.666
A = max(0.5, 0.5) = 0.5
MA = 0.5

| OUT | |
|---|---|
| 0.333 | MB |
| 0.4 | B |
| 0.666 | M |
| 0.5 | A |
| 0.5 | MA |

Ara ja es pot representar la gràfica d'activació d'OUT:




$\mu MB(x) = 1-(x/0.25)$ si $0<=x<0.25$
$0$ si $x>=0.25$

$\mu B(x) = 0$ si $x=0$
$x/0.25$ si $0<x<=0.25$
$1-[(x-0.25)/(0.5-0.25)]$ si $0.25<x<0.5$
$0$ si $x>=0.5$

$\mu M(x) = 0$ si $x<=0.25$
$(x-0.25)/(0.50-0.25)$ si $0.25<x<0.5$
$1-[(x-0.5)/(0.75-0.5)]$ si $0.5=x<0.75$
$0$ si $x>=0.75$

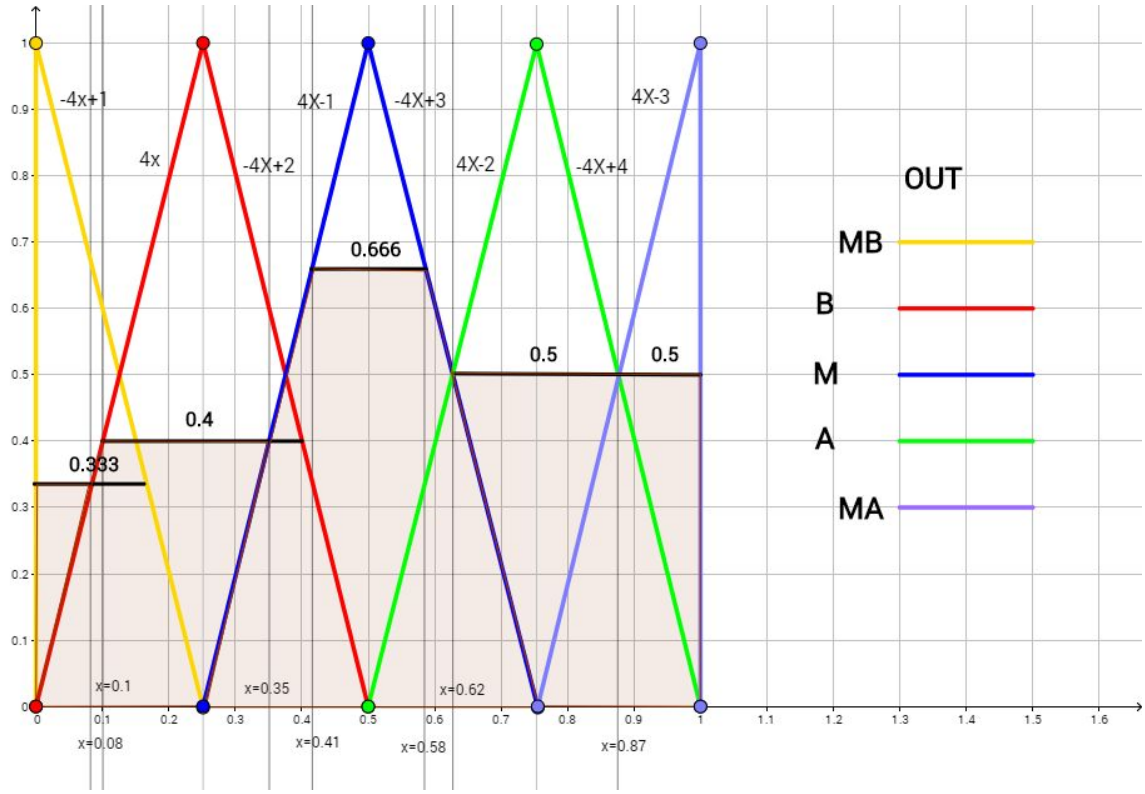
$\mu A(x) = 0$ si $x<=0.5$
$(x-0.5)/(0.75-0.5)$ si $0.5<x<0.75$
$1-[(x-0.75)/(1-0.75)]$ si $0.75<=x<1$
$0$ si $x=1$

$\mu MA(x) = 0$ si $x<=0.75$
$(x-0.75)/(1-0.75)$ si $0.75<=x<=1$

1-(x/0.25)=-4x+1 → pendent negativa d'MB
x/0.25=4x → pendent positiva de B
1-[(x-0.25)/(0.5-0.25)]=1-[(x-0.25)/0.25]=
=1-4(x-0.25)=-4x+2 → pendent negativa de B
(x-0.25)/(0.50-0.25)=(x-0.25)/0.25=4(x-0.25)=
=4x-1 → pendent positiva d'M
1-[(x-0.5)/(0.75-0.5)]=1-[(x-0.5)/0.25]=
=1-4(x-0.5)=-4x+3 → pendent negativa d'M
(x-0.5)/(0.75-0.5)=(x-0.5)/0.25=4(x-0.5)=
=4x-2 → pendent positiva d'A
1-[(x-0.75)/(1-0.75)]=1-[(x-0.75)/0.25]=
=1-4(x-0.75)=-4x+4 → pendent negativa d'A
(x-0.75)/(1-0.75)=(x-0.75)/0.25=4(x-0.75)=
=4x-3 → pendent positiva d'MA

Càlcul dels punts de tall:
a) Punt de tall de recta 0.333 amb pendent positiva de B:
0.333=4x → x = 0.08
b) Punt de tall de recta 0.4 amb pendent positiva d'M:
0.4=4x-1 → x = 0.35
c) Punt de tall de recta 0.666 amb pendent positiva d'M:
0.666=4x-1 → x = 0.41
d) Punt de tall de recta 0.666 amb pendent negativa d'M:
0.666=-4x+3 → x = 0.58
e) Punt de tall de recta 0.5 amb pendent positiva d'A:
0.5=4x-2 → x = 0.62
f) Punt de tall de recta 0.5 amb pendent negativa d'A i punt de tall de recta 0.5 amb pendent positiva d'MA (es comproven els tres punts, encara que la gràfica dóna a entendre que tots tres punts són el mateix):
0.5=4x-3 → x = 0.87
0.5=-4x+4 → x = 0.87
(nota: no s'ha calculat el punt de tall de recta 0'4 amb pendent positiva de B perquè es veu clarament que 0.1x4=0.4)

Gràfica amb tots els punts d'interès marcats:




Ara ja es pot passar a crear la funció de pertinença:

$$\mu OUT(x) = \begin{cases} 0.333 & si \quad 0 \le x < 0.08 \\ 4x & si \quad 0.08 \le x < 0.1 \\ 0.4 & si \quad 0.1 \le x < 0.35 \\ 4x-1 & si \quad 0.35 \le x < 0.41 \\ 0.666 & si \quad 0.41 \le x < 0.58 \\ -4x+3 & si \quad 0.58 \le x < 0.62 \\ 0.5 & si \quad 0.62 \le x \le 1 \end{cases}$$

Càlcul del valor nítid: s'empren integrals segons el llibre de text per a funcions contínues:

$$\frac{\int \mu(x) x dx}{\int \mu(x) dx}$$

En aquest cas d'estudi s'empraran integrals definides perquè la funció de pertinença està definida en parts:

$$\int_0^{0.08} 0.333x dx = \left[\frac{0.333x^2}{2}\right]_0^{0.08} = \frac{0.333 \cdot 0.08^2}{2} - \frac{0}{2} = \frac{0.02}{2} = 0.001$$

$$\int_0^{0.08} 0.333 dx = [0.333x]_0^{0.08} = 0.333 \cdot 0.08 - 0 = 0.02$$

$$\int_{0.08}^{0.1} 4xxdx = \int_{0.08}^{0.1} 4x^2dx = \left[\tfrac{4x^3}{3}\right]_{0.08}^{1} = \tfrac{4}{3} - \tfrac{4\cdot 0.08^3}{3} = 1.33 - 0.0006 = 1.33$$

$$\int_{0.08}^{0.1} 4xdx = \left[\tfrac{4x^2}{2}\right]_{0.08}^{1} = \tfrac{4}{2} - \tfrac{4\cdot 0.08^2}{2} = 0.5 - 0.01 = 0.49$$

$$\int_{0.1}^{0.35} 0.4xdx = \left[\tfrac{0.4x^2}{2}\right]_{0.1}^{0.35} = \tfrac{0.4\cdot 0.35^2}{2} - \tfrac{0.4\cdot 0.1^2}{2} = 0.02 - 0.002 = 0.01$$

$$\int_{0.1}^{0.35} 0.4dx = [0.4x]_{0.1}^{0.35} = 0.4\cdot 0.35 - 0.4\cdot 0.1 = 0.14 - 0.04 = 0.1$$

$$\int_{0.35}^{0.41} (4x-1)xdx = \int_{0.35}^{0.41} (4x^2-x)dx = \left[\tfrac{4x^3}{3} - \tfrac{x^2}{2}\right]_{0.35}^{0.41} = \tfrac{4\cdot 0.41^3}{3} - \tfrac{.41^2}{2} - \tfrac{4\cdot 0.35^3}{3} + \tfrac{.35^2}{2} =$$
$$= 0.09 - 0.08 - 0.05 + 0.06 = 0.02$$

$$\int_{0.35}^{0.41} (4x-1)dx = \left[\tfrac{4x^2}{2} - x\right]_{0.35}^{0.41} = \tfrac{4\cdot 0.41^2}{2} - 0.41 - \tfrac{4\cdot 0.35^2}{2} + 0.35 = 0.82 - 0.41 - 0.24 + 0.35 = 0.52$$

$$\int_{0.41}^{0.58} 0.666xdx = \left[\tfrac{0.666x^2}{2}\right]_{0.41}^{0.58} = \tfrac{0.666\cdot 0.58^2}{2} - \tfrac{0.666\cdot 0.41^2}{2} = 0.11 - 0.05 = 0.06$$

$$\int_{0.41}^{0.58} 0.666dx = [0.666x]_{0.41}^{0.58} = 0.666\cdot 0.58 - 0.666\cdot 0.41 = 0.38 - 0.27 = 0.11$$

$$\int_{0.58}^{0.62} (4x-2)xdx = \int_{0.58}^{0.62} (4x^2-2x)dx = \left[\tfrac{4x^3}{3} - \tfrac{2x^2}{2}\right]_{0.58}^{0.62} = \tfrac{4\cdot 0.62^3}{3} - \tfrac{.2\cdot 0.62^2}{2} - \tfrac{4\cdot 0.58^3}{3} + \tfrac{.2\cdot 0.58^2}{2} =$$
$$= 0.31 - 0.38 - 0.26 + 0.33 = 0$$

(nota: la superfície d'aquest interval dóna 0, cosa que no és normal, però com es tracten intervals tan petits pot ser que hi hagi decimals per enmig que s'hagin obviat).

$$\int_{0.58}^{0.62} (4x-2)dx = \left[\tfrac{4x^2}{2} - 2x\right]_{0.58}^{0.62} = \tfrac{4\cdot 0.62^2}{2} - 2\cdot 0.62 - \tfrac{4\cdot 0.58^2}{2} + 2\cdot 0.58 = 0.76 - 1.24 - 0.67 + 1.16 = 1.35$$

$$\int_{0.62}^{1} 0.5xdx = \left[\tfrac{0.5x^2}{2}\right]_{0.62}^{1} = \tfrac{0.5}{2} - \tfrac{0.5\cdot 0.62^2}{2} = 0.25 - 0.15 = 0.1$$

$$\int_{0.62}^{1} 0.5dx = [0.5x]_{0.62}^{1} = 0.5 - 0.5\cdot 0.61 = 0.2$$

Així,

$$\frac{\int_0^1 \mu OUT(x)xdx}{\int_0^1 \mu OUT(x)dx} = \frac{0.01+1.33+0.01+0.02+0.06+0+0.1}{0.02+0.49+0.1+0.52+0.11+1.35+0.2} = \frac{1.53}{2.75} = 0.54$$