Mod 4 Hw

4.1.1

a.) True

b.) False

c.) True

d.) False

4.1.3

a.) True

b.) True

c.) False

d.) True

4.1.6

a.) False

b.) True

c.) True

d.) False

4.2.1

a.) True

b.) True

c.) False

d.) False

4.2.2

a.) $P(\{a\}) = \{\emptyset, \{a\}\}$

b.) $P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

4.3.1

a.) $A \cup B = \{-12, -5, -3, 0, 1, 4, 6, 17\}$

b.) $A \cap B = \{1, 4\}$

c.) $A \cap C = \{-3, 1, 17\}$

d.) $A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

4.3.3

a.) $\{1\}$

b.) $\bigcup_{i=2}^{5} A_i = \{1, 2, 4, 3, 9, 16, 5, 25\}$

c.) $\bigcap_{i=1}^{100} B_i = \{x \in \mathbb{R} : -1 \leq x \leq 1/100\}$

d.) $\bigcup_{i=1}^{100} B_i = \{x \in \mathbb{R} : -100 \leq x \leq 1\}$

4.4.1

a.) $A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{3, 4, 5, 7\}$

$C = \{1, 2, 5, 6, 7\}$

$A - (B - C) = \{1, 2, 5, 6, 7\} = K$

$(A - B) - C = \{5, 7\} = t$

$K \neq t$

4.4.4

a.) False

b.) True

c.) True

d.) False $\Rightarrow \{-1, 0\}$

4.5.1

a.) Compliment law

b.) Absorbtion law

c.) De Morgan's laws

d) Double complement law

4.5.2

(a.) $(\bar{A} \cap C) \cup (A \cap C) = C$

$\equiv (\bar{A} \cup A) \cap C$ distributave law

$= U \cap C$ complement

$\equiv C$ identity

(b.) $(B \cup A) \cap (\bar{B} \cup A)$

$= (B \cap \bar{B}) \cup A$ distributave

$= \emptyset \cup A$ Complement

$= A$ identity

4.5.2

c.

$$\overline{A \cap \bar{B}}$$

~~$= \bar{A} A$~~

$= \bar{A} \cup B$ de Morgan's laws

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d. $\bar{A} \cap (A \cup B) = \bar{A} \cap B$

$= (\bar{A} \cap A) \cup (\bar{A} \cap B)$ distributative

$= \emptyset \cup (\bar{A} \cap B)$ complemant

$=$ ~~$\bar{A} \cup$~~

$= \bar{A} \cap B$ Identity

4.6.1

a.) $A \times B \times C$

$= \{tall, foam, Non-fat\}$

b.) $B \times A \times C$

$= \{foam, tall, Non-fat\}$

c.) $\{(a,b) : a \in B \text{ and } b \in C\}$

4.6.3

a.) False

b.) True

c.) True

d.) True

4.6.4

a.) $A^2 = \{++, --, +-, -+\}$

b.) $A^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

4.6.8

(a.) $(x,y) \in A \times (B \cup C) \leftrightarrow (x \in A) \wedge (y \in (B \cup C)$

Cartesian

$\leftrightarrow (x \in A) \wedge ((y \in B) \vee (y \in C))$ union

$\leftrightarrow ((x \in A) \wedge (y \in B)) \vee ((x \in A) \wedge (y \in C))$ distributive

$\leftrightarrow ((x,y) \in A \times B) \vee ((x,y) \in A \times C)$ Cartesian

$\leftrightarrow ((x,y) \in (A \times B) \cup (A \times C)$ union

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(b.) $(x,y) \in A \times (B \cap C) \leftrightarrow (x \in A) \wedge (y \in (B \cap C)$

$\leftrightarrow (x \in A) \wedge ((y \in B) \wedge (y \in C))$ intersection

$\leftrightarrow (x \in A) \wedge (y \in B) \wedge (y \in C)$ associative

$\leftrightarrow (x \in A) \wedge (y \in B) \wedge (x \in A) \wedge (y \in C))$

~~$\leftrightarrow ((x \in A)$~~

$\leftrightarrow ((x,y) \in (A \times B)) \wedge ((x,y) \in (A \times C))$ Cartesian

$\leftrightarrow ((x,y) \in ((A \times B) \cap (A \times C))$ intersection

4.7.1

a.) $A = \{1, 2, 6\}$ $D = \{1, 2, 3, 4, 5, 6\}$

$B = \{2, 3, 4\}$

$C = \{5\}$

$A \cap B = \{2\} \neq \phi$

$\{A, B, C\}$ does not form a partition of D

b.) $B = \{2, 3, 4\}$

$C = \{5\}$

No, $\nexists B \cup C = \{2, 3, 4, 5\} \neq D$

c) $B = \{2, 3, 4\}$ $E = \{2, 3, 4, 5\}$

$C = \{5\}$

Yes

4.7.2

a.) $(0,1)^3 =$

$\{(0,0,0),(0,0,1),(0,1,0),(0,1,1),(1,0,0),(1,0,1),(1,1,0),(1,1,1)\}$

i) $D \cup E \cup F$ is a partition

1.) $D \cup E \cup F = (0,1)^3$

2.) $D \cap E = D \cap F = E \cap F = \phi$

3.) $E, F, D \neq \phi$

ii) $C \cup E$

1.) $C \cup E = (0,1)^3$

2.) $C \cap E = \phi$

3.) $C \neq \phi, E \neq \phi$