Name
Genome541 — Spring 2017 **Homework 4** Due: 4/27/2017

Problem 1

Review Gibbs Sampling

Suppose we have a model $y_i \sim Pois(\phi_i \lambda)$, with $\phi_i \sim Ga(\nu/2, \nu/2) \& \lambda \sim Ga(a, b)$. Derive the full conditional posterior distribution of each ϕ_i and of λ. Simulate data with $\nu = 0.5, \lambda = 5, n = 100$. Implement the Gibbs sampler and show a trace plot of samples of λ. Show an MCMC estimate of the posterior mean and 95% credible interval on the plot λ.

Problem 2

Sampling truncated distribution

Suppose we have $\theta \sim Beta(c, d)$, which has support on [0,1], we can modify the density of $f(\theta)$ to have support on a sub-interval $[a, b] \subset [0, 1]$. In other words, we start with pdf for an untruncated distribution such as $\theta \sim Beta(c, d)$ and we want to sample $\theta \sim Beta_{[a,b]}(c, d)$. Derive the form of the posterior density of θ updating the prior with a binomial (y, n, θ) likelihood. Compare posterior summaries including the mean, median and 95% credible interval with the posterior under an unconstrained beta prior, i.e., with support on [0,1]. In comparing posteriors, use $y = 8, , n = 10, a = 0.6, b = 0.9, c = 8, d = 2$

Problem 3

Bayesian Hypothesis Testing

In the fair coin example (Lecture 4 slides 12 to 17), derive the general form of Bayes Factor under prior distribution of Beta(a, b) (recall that in the lecture notes, we looked at an example under Beta(1, 1) prior). Based on the genereal form under Beta(a, b) prior, comment on what will happen if we let a and b goes infinitely close to 0 (in that case, the prior sample size a+b also goes to 0). Do we encounter Lindley's paradox? (if you are not familiar with how beta functions behave (B(a,b) on slide 15) as you change a and b, experiment in R with beta() function).