UNDERGRADUATE SEMINARS II: KNOTS AND DYNAMICS

JONATHAN BLOOM

1. Knots and Links (Delia)

We introduce knots and links.

Exercise 1: Show that the Figure 8 knot is equivalent to its mirror using Reidemeister moves to go between the standard diagrams.

2. Properties and Invariants (Adam)

Two classes of invariants.

3. Topology of Surfaces (Kari)

The Classification of Surfaces Euler characteristics and its relationship to genus, $\chi = 2 - 2g - b$.

4. Seifert Surfaces (Sasha)

Seifert's algorithm, genus of a Seifert surface, genus is additive (so no inverses)

Exercise: Use Seifert's algorithm to find a Seifert surface for...

5. The Seifert Matrix

Seifert graph, Seifert matrix, examples (2,p) torus knot, compute the determinant by induction.

6. Matrix Invariants

Congruent matrices, Sylvester's Theorem, signature, determinant, overview of proof of invariance, additivity/multiplicativity, (2,p) torus knot.

$(2, p)$ torus knot for $p \geq 3$ has signature $p - 1$ and determinant p. Therefore these torus knots (2,p) are prime for prime p. And you can use this to determine unknotting number bounds on unknotting number.

Alexander polynomial

7. Braids and the Braid Group

Braid group, Alexander's Theorem (10.1.2), braid index (Corollary 10.1.4), positive braids

8. The Lorenz Template

Introducing the Lorenz template and branch line, knots on the template. Trip number one implies unknotted. Lorenz links are unsplittable (Theorem 4.1)

9. Symbolic Dynamics I

The map in Proposition 2.4.1 is onto. All torus knots occur (Theorem 6.1).

10. Symbolic Dynamics II

The map in Proposition 2.4.1 is invertible. This proves Corollary 2.4.

11. Lorenz knots as positive braids

Proof of Theorem 5.1. Proof of Conjecture: Trip number = braid index, using Theorem 10.5.1 in Cromwell. Prop 10.5.2 and 10.5.3 on torus knots.

12. Genus bounds

Genus of the positive braid template (part of Theorem 5.2). Proof of Corollary 5.3. Corollary 5.4 and 5.5 imply many knots are not Lorenz.