

泊松分布:  $P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$

$L(\lambda; x) = \prod P(x_i | \lambda) = \prod \left( \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \right)$

$l(\lambda; x) = \sum [x_i \log \lambda - \lambda - \log(x_i!)]$

$\frac{\partial l}{\partial \lambda} = \sum \left[ \frac{x_i}{\lambda} - 1 \right] = 0 \Rightarrow \frac{\sum x_i}{n} = \lambda$

即  $\hat{\lambda} = \frac{1}{n} \sum_{i=1}^n x_i = \bar{x}$

即  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$  是  $\lambda$  最大似然估计值

本题:  $\eta = \alpha + \beta_1 x_1 + \beta_2 x_2$

$L = \prod_{i=1}^{35} \frac{e^{-\hat{\eta}_i} \hat{\eta}_i^{y_i}}{y_i!} \Rightarrow l = -\sum_{i=1}^{35} \hat{\eta}_i + \sum_{i=1}^{35} y_i \log \hat{\eta}_i - \sum_{i=1}^{35} \log(y_i!)$

$= -\sum_{i=1}^{35} \lambda + \sum_{i=1}^{35} x_i \log \lambda - \sum_{i=1}^{35} \log(x_i!)$

$= -10e^{\hat{\alpha} + \hat{\beta}_1} - 5e^{\hat{\alpha} + \hat{\beta}_2} - 20e^{\hat{\alpha}} + 18\hat{\alpha} + 11\hat{\beta}_1 + 3\hat{\beta}_2 - 2.772589$

$\begin{matrix} 5 & 5 & 4 \\ 1+2+2+1+2+2+1+1+1+1+4 & = & 18 \end{matrix}$

$-4.112$

$-1.116$

注意: 10对应 class 1 中观测值数

5对应 class 2 中

20对应 class 3 中

18对应 所有观测值之和

11对应 class 1 中

3对应 class 2 中

$x_1$   
 $x_2$   
整体

$\begin{cases} \frac{\partial l}{\partial \alpha} = -10e^{\hat{\alpha} + \hat{\beta}_1} - 5e^{\hat{\alpha} + \hat{\beta}_2} - 20e^{\hat{\alpha}} + 18 = 0 \\ \frac{\partial l}{\partial \hat{\beta}_1} = -10e^{\hat{\alpha} + \hat{\beta}_1} + 11 = 0 \\ \frac{\partial l}{\partial \hat{\beta}_2} = -5e^{\hat{\alpha} + \hat{\beta}_2} + 3 = 0 \end{cases}$