Week 13 - Monday

Monday, April 19, 2021 2:23 PM

HW: 1) Complete in-class Notebook
2) look @ sol'n's & correct FFT from last week
3) Work on projects

Probability & Randomness

1) Random # generators
2) PDF transforms
3) Markov Chains
4) Monte-Carlo Algs. (Ising Model)

RNGs.

"True" RNGs

e.g.. specialized hardware,
- RF Noise/static
- Atmospheric Noise
- Lava lamps
- ⋮

Pseudo-RNGs

- Recurrence Relation to get another random # from a previous #
- Mersenne Twister ~ 6000 digits
  ~ 10^{6000} #'s

PDF Transforms:

given some PDF $p_x(x)$, $\overset{\text{prob. } P}{P(x_a \le x \le x_b)} = \int_{x_a}^{x_b} p_x(x)\, dx$

Want to know $P_y(y)$, or $x \longleftrightarrow y$

$\Updownarrow$ given $y(x)$

$$\int_{y(x_a)}^{y(x_b)} dy \underbrace{\frac{dx}{dy} P_x(x(y))}_{= P_y(y)}$$

e.g. have $P_x(x) = \{ 1 \text{ if } 0 \le x < 1$

e.g. have $P_X(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$

want $P_y(y) = \begin{cases} y & \text{if } 0 \le y < \sqrt{2} \\ 0 & \text{otherwise} \end{cases}$ ← Normalization

$y\,dy = 1\,dx$ where $p \neq 0$

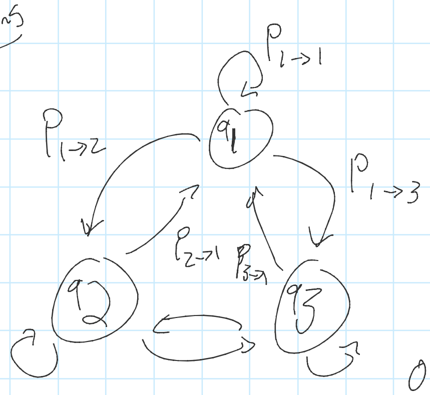
$\frac{1}{2}y^2 = x + c$ Want $y = \sqrt{2}$ @ $x = 1$

$y = 0$ @ $x = 0$

$\Rightarrow c = 0$

$x = y^2/2, \quad y = \sqrt{2x}$

Markov Chains



State vector $\vec{q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$ ← Prob. of being in state 1, 2, 3

"one step": $\vec{q}\,' = \underline{\overleftrightarrow{P}}\,\vec{q}$

$$\begin{pmatrix} q_1' \\ q_2' \\ q_3' \end{pmatrix} = \begin{pmatrix} P_{1\to1} & P_{2\to1} & P_{3\to1} \\ P_{1\to2} & P_{2\to2} & P_{3\to2} \\ P_{1\to3} & P_{2\to3} & P_{3\to3} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

Col's should sum to 1

2 steps: $q' = P^2 q$

N steps: $q' = P^N q$

Can take $N \to \infty$, $P = V D V^{-1}$

$P^N = V D^N V^{-1}$

eigen vals = 1 states are "left"

"Stationary" States