Week 3 - Friday

Thursday, February 11, 2021 5:29 PM

A bit on SVD - Singular value decomposition

Applications: { Least-Squares fitting
Principle component analysis
Statistical modeling, methods

Last time: • Solving $\underline{A\vec{x} = \vec{b}}$ { GJ-elimination
LU-decomposition
Matrix inversion
<u>Tridiagonal/banded</u> systems ← (Lab)

$A\vec{x} = \vec{0}$ ⇒ null space ← spanned by null vectors
vs.
row space ← spanned by eigenvectors

• Looked @ eigendecompositions

Col's are eigenvectors

$$A = PDP^{-1}$$

diagonal are eigenvalues

can only invert A if not singular (no 0 eigvals.)

$$A^{-1} = (PDP^{-1})^{-1} = PD^{-1}P^{-1}$$

SVD very analogous to eigenval decomp.

If complex, T → conj. T

(or "S")

$$A = U\Sigma V^T$$

Not need to be □!

$$UU^T = I = U^TU$$
$$VV^T = I = V^TV$$

A is MxN

$$\begin{array}{c} \quad N\to \\ M\downarrow \begin{pmatrix} & \\ A & \\ & \end{pmatrix} \end{array} = \begin{array}{c} \quad N\to \\ M\downarrow \begin{pmatrix} & \\ U & \\ & \end{pmatrix} \end{array} \begin{array}{c} \quad N\to \\ N\downarrow \begin{pmatrix} w_1 & & \\ & w_2 & \\ & & \ddots \end{pmatrix} \end{array} \begin{array}{c} \quad N\to \\ N\downarrow \begin{pmatrix} & \\ V^T & \\ & \end{pmatrix} \end{array}$$

$$\left(A \right) = \left(U \right) \begin{pmatrix} w_1 & & \\ & w_2 & \\ & & \ddots \end{pmatrix} \left(V^T \right)$$

$$\underset{M \times N}{\left(A \right)} = \underset{N \times N}{\left(U \right)} \underset{N \times N}{\left(\Sigma \right)} \underset{N \times N}{\left(V^T \right)}$$

$$A A^T = U \Sigma V^T V \Sigma U^T = U \Sigma^2 U^T \Rightarrow$$ U diagonalizes $A A^T$, Σ^2 contains eigenval's.

$$A^T A = V \Sigma U^T U \Sigma V^T = V \Sigma^2 V^T \Rightarrow$$ V diagonalizes $A^T A$

V has eigenvectors of $A^T A$

$V^T V = I \Rightarrow$ needs vectors that span null space

$$V \sim \begin{pmatrix} \left.\begin{matrix} v_1 \\ v_2 \end{matrix}\right\} \text{eigenvectors} \\ \vdots \\ \left.\begin{matrix} n_1 \\ n_2 \end{matrix}\right\} \text{null vectors} \\ \vdots \end{pmatrix}$$

Pseudo-Inverses: ($+ \equiv$ pseudo-inv.)

replace $\frac{1}{w}$ with zero if $w = 0$

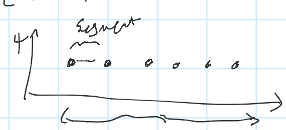
$$\left.\begin{matrix} A^+ A A^+ = A^+ \\ A A^+ A = A \end{matrix}\right\} A^+ = V \cdot \begin{pmatrix} \frac{1}{w_1} & & \\ & \frac{1}{w_2} & \\ & & \ddots \end{pmatrix} U^T$$

$\Rightarrow$ look into least-sq, Principle components

Lab: $H\psi = E\psi$

$\psi(x) = (\psi(x_1), \psi(x_2), \dots)$

BC's $\rightarrow H(L, N, V)$ $\leftarrow$ vector



$$\left(H \right) \left(\psi \right)$$

$[0, \quad L] \rightarrow$ Solving for

N segments

$N+1$ points

h is distance between points

$h = L/N$

h is distance points

$$\boxed{h = L/N}$$