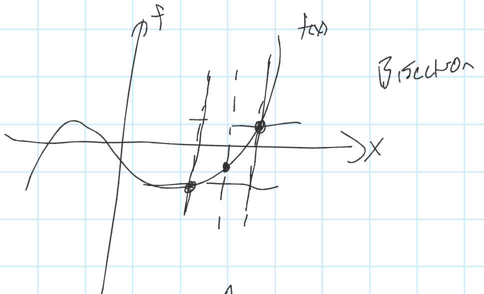
Week 5 - Monday

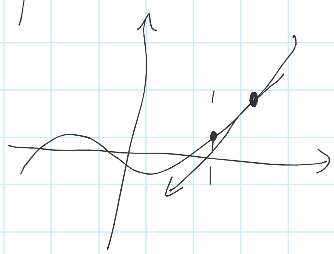
Monday, February 22, 2021 2:03 PM

Optimization		Interpolation	Linear Systems	fitting
Root finding	min/max	few methods connecting dots	solving $A\vec{x} = \vec{b}$	minimizing χ^2

Review!

Overview of Root finding



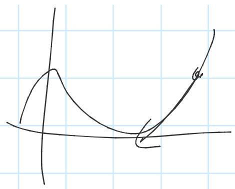


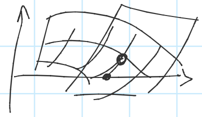
Method	Idea	Convergence
Bisection	bracket a function and divide interval to get a better bracket	guaranteed to converge (as long as continuous)
Linear Extrapolation Newton-Raphson → Secant method →	follow tangent line to zero or appx. tangent line	Not guaranteed to converge
Quadratic extrapolation (Halley's method) ↔ or Finite-difference →	write down 2nd-order polynomial, find zero-crossing using f, f', f'' approximate	Not guaranteed to converge
Brent's Method	Combine bisection with "inverse" quadratic extrapolation	guaranteed to converge
Dekker's method	Combine bisection w/ secant method	guaranteed to converge

Root-finding in more than 1 dimension

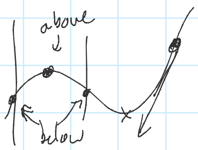
- Newton-Raphson could still work, $f' \to \nabla f$
- Bracketing still works
- Works for >1 function at a time

• works for 1 function at a time



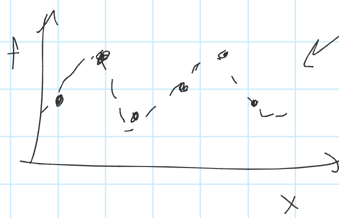


(1-d) Min/max Problems:



- Can still bracket functions to find max/mins
- Can still find tangent lines, head in up/down direction towards max/min
  Problem is finding an appropriate step size
- Can still find a parabola(-oid)
  look @ f, f', f'', find min/max
  (gives step size for linear method)

Interpolation:



How to connect the dots?

- Write down polynomial going through all points
- Piecewise polynomial "
  (and matching polynomial values, derivatives @ points)
- Rational function Interpolation

Linear Systems: $A\vec{x} = \vec{b}$

QM ex: $A \sim H > \nabla^2$

also: EM, Gravity, Wave equation, fluid dynamics
(different) (Navier-Stokes)

How can we solve this?

- Gauss-Jordan Elimination
- L-U decompositions
  (slightly more efficient way of)

$(\Delta)(\nabla)$
$A = LU$
$\hookrightarrow A^{-1}$ easily

- L-U decompositions
  (slightly more efficient way of)

  $A = LU$

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  easily back-sub, forward-sub

- Eigendecompositions – compute A^{-1}

  $A = PDP^{-1} \rightarrow PD^{-1}P^{-1}$

- SVD to handle over/under-determined systems

  Over: least-squares/"best" fit to system

  Under: Told us about null space

- Special routines for banded matrices $A = \begin{pmatrix} \ddots & \ddots & 0 \\ \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots \end{pmatrix}$

also routines for "sparse" matrices mostly/many-zeros

$$A = \begin{pmatrix} \ddots & 0 & \ddots \\ 0 & \ddots & 0 \\ \ddots & 0 & \ddots \end{pmatrix}$$

Least-Squares fitting : minimize χ^2

- In general, we want to perform a min/max problem
- Specific case of linear comb. of functions

Linear linear	generally
Can make analytic progress	SVD, Pseudo-inverse solutions

Curve-fit is usually OK.