Week 2 - Monday

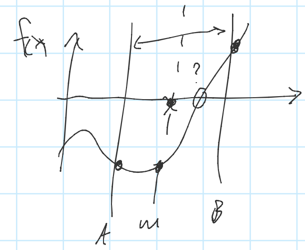
Monday, February 1, 2021 12:47 PM

- By Eye!
- Bisection
- Newton's Method
  ↓
  Secant Method

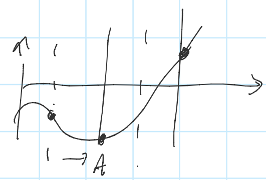
- Halley's Method
- Dekker's Method
- Brent's Method

Bisection

- guaranteed to converge
- Pick up factor of ~2 in root accuracy

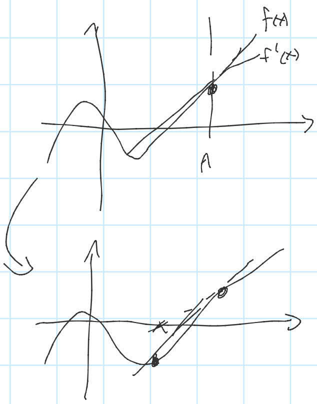


If $A<0$, $B>0$ | is $m<0$ $A\to m$; $m>0$ $B\to m$



Repeat until $f(x)\simeq 0$ within some tolerance

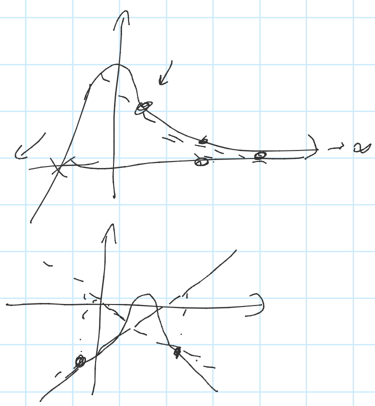
Newton's Method



$$\cancel{Y} - f(A) = f'(A)(\underline{x}-A)$$

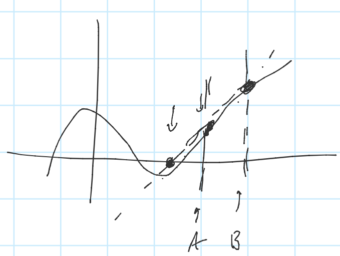
$$x = A - \frac{f(A)}{f'(A)} = A_{new}$$

- Not guaranteed to converge!
- "Quadratic" Convergence



Secant Method

replace $f'(A) \to \dfrac{f(A)-f(B)}{A-B}$

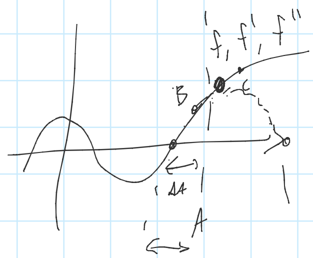


- Same issues as ↑
- But... don't need to know $f'(A)$

(Householder's Method)

Halley's Method

- Converges cubically
- Not guaranteed to converge (less stable than Newton's)



$$f(A+\Delta A) \simeq \underbrace{f(A) + \Delta A \overset{\uparrow \frac{f(B)-f(C)}{B-C}}{f'(A)} + \frac{\Delta A^2}{2} \overset{\nearrow \frac{\frac{f(B)-f(A)}{B-A} - \frac{f(A)-f(C)}{A-C}}{\frac{1}{2}(B-2A-C)}}{f''(A)}} = 0$$

$\uparrow$ $f=0$

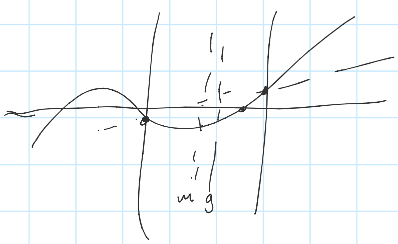
Expand root for small f

$$\Delta A = \frac{-f'}{f''}\left(1 \pm \sqrt{1 - \underbrace{\frac{2ff''}{(f')^2}}_{\sim \epsilon}}\right) \qquad (1+\epsilon)^{1/2} \simeq 1 + \frac{1}{2}\epsilon + \frac{\epsilon^2}{8} + \dots$$

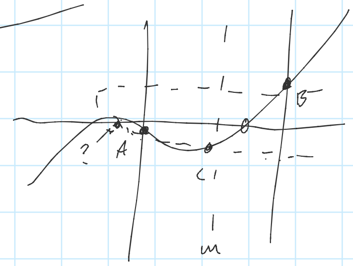
$$\Delta A \simeq -\frac{f}{f'} - \frac{f^2 f''}{2(f')^3} + O(\Delta A^3)$$

Dekker's Method

Combine Secant + Bisection { Use bisection as a backup to guarantee convergence.



Brent's Method



- Use inverse quadratic zero when inside Bracket
- Otherwise use Bisection

(plus caveats in order to improve convergence.

Minimization

- Bracketing

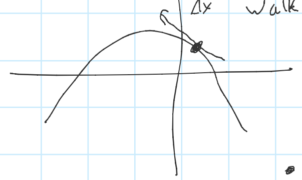
→ Bisection still works

→ Can fit parabola, use maximum of parabola as guess for max.

- guaranteed to converge

- "Gradient descent" ~ Newton's Method

- Δx "walk" along the function in the



"walk" along the function in the direction of the gradient.

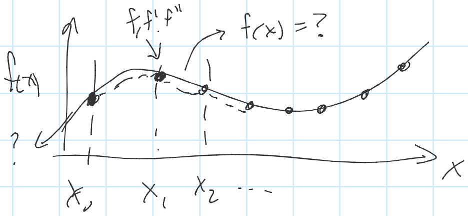
- Not guaranteed to converge

↳ talk more on Friday.

Interpolation

Know f at some x_i

what is f between?



① Lagrange Polynomial

② "Spline" (cubic)

① Lagrange poly:
$$f(x) = \frac{(x-x_1)(x-x_2)\cdots}{(x_0-x_1)(x_0-x_2)\cdots} f_0 + \frac{(x-x_0)(x-x_2)\cdots}{(x_1-x_0)(x_1-x_2)\cdots} f_1 + \cdots$$

- Unstable Alg. for ~20+ pts in function

② Cubic Spline:
- Cubic function
- agree with f at x_i
- 1^{st}, 2^{nd} derivs agree at x_i
- 2^{nd} derivative $=0$ at ends (not unique)