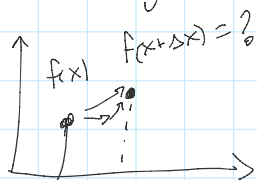


More ODE solving: $f'(x) = A(f(x), x)$

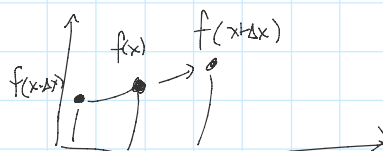
Runge-Kutta



given only $f(x)$, what is $f(x+\Delta x)$?

can we take different Δx steps + combine?

Multi-point Methods



$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x} = A$$

$$f(x+\Delta x) = f(x) + \Delta x A$$

Higher-order eg:

$$f'(x) \approx \frac{3f(x+\Delta x) - 4f(x) + f(x-\Delta x)}{2\Delta x} = A$$

$$f(x+\Delta x) = \frac{1}{3} \left[4f(x) - f(x-\Delta x) + 2\Delta x A \right]$$

More common
(most other cases)

Less common = Noisy
Systems
(sometimes)

Boundary-Value problems:

$$\frac{d^2 f}{dx^2} = A(f, x) \Rightarrow \begin{cases} \frac{df}{dx} = g \\ \frac{dg}{dx} = A(f, x) \end{cases}$$

Suppose we know $f(0)$, $f(L)$



- Use matrix inversion to solve if linear
- "Shooting method" is another option
 - ↳ Know $f(0)$, guess $f'(0)$, or $g(0)$.
 - ↳ Refine guess based on $f(L)$ from solution and desired $f(L)$
 - ↳ Can be mapped to an optimization problem to find "correct" $g(0) = f'(0)$.