Physics 427 – Introduction to Computational Physics
James B. Mertens
Homework 8 Solutions

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Problem 1:

a) The finite-difference version of this scheme also determines the behavior of the error,

$$\epsilon_f(t+\Delta t, x) = \epsilon_f(t,x) + \Delta t \frac{\epsilon_f(t, x+\Delta x) - 2\epsilon_f(t,x) + \epsilon_f(t, x-\Delta x)}{\Delta x^2}\,.$$

b) Assuming the error takes the form

$$\epsilon_f(t,x) = E(t)e^{ikx}\,,$$

we end up with the equation

$$E(t+\Delta t)e^{ikx} = E(t)e^{ikx} + \frac{\Delta t}{\Delta x^2}E(t)e^{ikx}\left[e^{ik\Delta x} - 2 + e^{-ik\Delta x}\right]\,.$$

which reduces to

$$E(t+\Delta t) = E(t)\left[1 - \frac{\Delta t}{\Delta x^2}4\sin^2(k\Delta x/2)\right]\,.$$

c) We would like for

$$\left|E(t)\left[1 - \frac{\Delta t}{\Delta x^2}4\sin^2(k\Delta x/2)\right]\right| \leq |E(t)|\,,$$

or

$$\left|1 - \frac{\Delta t}{\Delta x^2}4\sin^2(k\Delta x/2)\right| \leq 1\,.$$

Note that $\sin^2$ is always positive, between 1 and 0, and Δx and Δt will be positive assuming usual finite differencing integration. We would therefore like

$$\frac{\Delta t}{\Delta x^2} \leq \frac{1}{2}$$

for stability: we can therefore carefully choose a Δx and Δt to ensure stability of this method.