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Problem 1:

a) The perimeter up to N squares will be

$$P_N = \sum_{i=1}^{N} 4 \left(\frac{1}{\sqrt{2}}\right)^{i-1} = 8\frac{1-2^{-N/2}}{2-\sqrt{2}}. \tag{1}$$

In the limit where $N \to \infty$, this expression becomes

$$\lim_{N\to\infty} P_N = 8 + 4\sqrt{2}\,. \tag{2}$$

b) The area up to N squares will be

$$A_N = \sum_{i=1}^{N} 2^{1-i} = 2\left(1 - 2^{-N}\right)\,. \tag{3}$$

In the limit where $N \to \infty$, this expression becomes

$$\lim_{N\to\infty} A_N = 2\,. \tag{4}$$

c) Perimeter:

$$N > 6 - 2\log_2\left(\epsilon(2 - \sqrt(2)\right)\,. \tag{5}$$

Area:

$$N > -\log_2(\epsilon/2). \tag{6}$$

d) For $\epsilon = 10^{-7}$, $N \gtrsim 55$ terms for the perimeter and $N \gtrsim 25$ for the area.

Performing the sums explicitly, I find $|P_{55} - P| \simeq 7\times10^{-8}$ and $|A_{25} - A| \simeq 6\times10^{-8}$.

For $\epsilon = 10^{-15}$, I find $N \gtrsim 108$. When performing the sum in this case, I find $|P_{108} - P| \simeq 4 \times 10^{-15}$. Adding additional terms makes no improvement! Notice that the terms in the sum are now $\mathcal{O}(10^{-16})$, which is at the limit of numerical roundoff error when compared to the overall sum value of ~ 13.