Week 2 - Friday

Friday, February 5, 2021 2:11 PM

- Multi-dimensional Newton's Method
- Higher-Order methods

- Rational Function Interpolation
- Multi-dimensional Interpolation

Multi-d Newton's

$(f_1, f_2, \ldots)$ $\quad (x, y, z, \ldots)$ $\quad x_i = (x_0, x_1, x_2, \ldots)$ $\quad \# f_A \neq \# x_i$

$$f_A(x_i + \Delta x_i) \simeq f_A(x_i) + \sum_i \Delta x_i \frac{\partial}{\partial x_i} f_A(x_i) + O(\Delta x_i^2)$$

Newton-Raphson

$$\vec{f}(\vec{x} + \vec{\Delta x}) \simeq \vec{f}(\vec{x}) + (\vec{\Delta x} \cdot \vec{\nabla})\vec{f}(\vec{x}) + \ldots$$

$$-f_A(x_i) = \sum_i \frac{\partial f_A}{\partial x_i} \Delta x_i$$

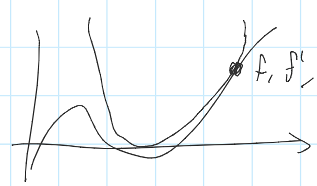
$$\text{Jacobian Matrix} = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y}, & \ldots \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \ldots \\ \vdots & \vdots & \end{pmatrix} \equiv J_{Ai}$$

$$\Delta x_i = -J_{Ai}^{-1} f_A$$

☆ also works for minimum finding
↳ finding a good step size

Newton's Method
- Use information about 2nd derivative to find a zero

1-d picture (Halley's method)



$f, f', f'' \Rightarrow$ determine parabola
lets you find a min/max or zero.

Multi-dimension picture | finding a paraboloid

Rational function Interpolation

$$P(x) = \frac{A_0 + A_1 x + A_2 x^2 + \ldots}{B_0 + B_1 x + B_2 x^2 + \ldots}$$

$$\lim_{x \to \infty} P(x) = \frac{A_2}{B_2}$$

$$\lim_{x \to \infty} P(x) = \frac{A_2}{B_2}$$

- Can choose asymptotic behavior
  - ↳ Useful for extrapolation

- Padé Approximation

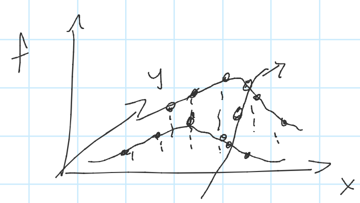
  Choose $P(x)$ s.t. Taylor expansion matches a function

- Pro/Con: has poles in denominator
  - ↳ Barycentric Interpolation (fix coeff's s.t. no poles)

See Numerical Recipes.

Multi-dimensional Interpolation

Know function at regularly spaced grid points

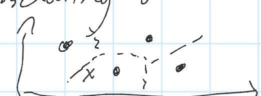


Can successively apply 1-d interpolation

Details: N.R.

Irregular Grid

- Tesselating data (see NR)



- Radial Basis function

$$f(x) \simeq \sum w_i \, \phi(\vec{x} - \vec{x}_i)$$

Weights

Basis functions

See N.R.