Week 9 - Friday

Friday, March 26, 2021 2:16 PM

Just a little bit more on PDEs...

- Midterm Next (friday) week, Monday is review (please complete quiz (HW8 survey))
- ~2 weeks on fourier transforms, random algs, HPC? (quiz) (11+5+5)
- ~2 weeks on project topics (handout coming after midterm)
- last class on presentations (?)

No final, No class (May 11.)

- PDEs in coordinate systems with singularities

eg. $\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right)$ + Angular terms

| In spherical coords.

$= \frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r}$

↑ how to handle when $r \to 0$?

L'Hopital's Rule: $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$

$$\lim_{r\to 0} \frac{2}{r}\frac{\partial f}{\partial r} = 2\frac{\partial^2 f}{\partial r^2}$$

- Why do diffusive, viscous terms $\sim \nabla^2 f$? (↑ why ∇^2) $\quad F \propto \frac{\Delta f}{\Delta V}$

$$\partial_x^2 f = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2}$$

$$= \frac{3}{\Delta x^2}\left[-f(x) + \frac{f(x+\Delta x) + f(x) + f(x-\Delta x)}{3}\right]$$

$-f(x)$: fn. value

$\frac{f(x+\Delta x) + f(x) + f(x-\Delta x)}{3}$: ~ local average around region

fn. value ~ local average around region

~ difference b/w f and surroundings.

- Can use Implicit schemes for solving $\partial_t f = A(f, t, x, \partial_x f, \partial_x^2 f, \ldots)$, $f(t,x)$
  - eg. Backward Euler method: $\frac{f(t+\Delta t) - f(t)}{\Delta t} = A\big|_{t+\Delta t}$
  - Explicit forward Euler method: $\frac{f(t+\Delta t) - f(t)}{\Delta t} = A\big|_{t}$
- Crank-Nicolson: (Very stable method) $\frac{f(t+\Delta t) - f(t)}{\Delta t} = \frac{1}{2}\left[A\big|_{t+\Delta t} + A\big|_{t} \right]$
  - eg. Diffusion Equation

$$\partial_t f = \partial_x^2 f = A$$

how does ε_f behave? $f = f_{true} + \varepsilon_f$

$$\varepsilon_f = E(t)\, e^{ikx}$$

$$|E(t)| \left| \frac{1 - 4\frac{\Delta t}{\Delta x^2} \sin^2\left(\frac{k\Delta x}{2}\right)}{1 + \frac{4\Delta t}{\Delta x^2} \sin^2\left(\frac{k\Delta x}{2}\right)} \right| \leq |E(t+\Delta t)|$$

want ≤ 1, and it always is!

Absolutely Stable.