# Week 1 - Monday

Sunday, January 24, 2021 11:10 PM

- Perturbation Theory
- Iterative
- Halley's Method

$\sqrt{q} = x$

$q = x^2$

(P.T.) $X = x_{(0)} + \epsilon x_{(1)} + \epsilon^2 x_{(2)} + \dots$

$q = q + \epsilon = x_{(0)}^2 + 2\epsilon x_{(1)} x_{(0)} + \epsilon^2 \left( x_{(1)}^2 + 2 x_{(0)} x_{(2)} \right) + \dots$

$\epsilon = -1$

$q = x_{(0)}^2 \Rightarrow x_{(0)} = 3$ (0th)

$\epsilon \cdot 1 = 2 x_{(1)} x_{(0)} \Rightarrow x_{(1)} = 1/6$ (1st)

$0 = x_{(1)}^2 + 2 x_{(0)} x_{(2)} \Rightarrow x_{(2)} = \frac{-1}{6^3}$ (2nd)

| $x^2 = q$ |
|---|
| $(3 - 1/6)^2 = 8.029$ |
| $(3 - \frac{1}{6} - \frac{1}{6^3})^2 = 8.0016$ |

- Iterative

$(x + \epsilon)^2 = q = x^2 + 2x\epsilon + O(\epsilon^2)$

$\epsilon = \frac{q - x^2}{2x}$

0th $x = 3$, $x^2 = 9$

1st $x = 3 - \frac{1}{6}$, $x^2 = 8.028$

2nd $x = 3 - \frac{1}{6} - \frac{1}{204}$, $x^2 = 8.00002$ ✓

- Halley's Method | $f(x) = 0 = x^2 - q$

$f' = 2x$

$f'' = 2$

$x_{new} = x_{old} - \frac{f}{f'} - \frac{f'' f^2}{2(f')^2}$

$q$

9

0th $x=3$, $x^2=9$

1st $x=3+\dots$, $x^2=9.0015$

2nd $x=3+\dots+\dots$, $x^2=9+7e^{-12}$