Week 7 - Friday

Friday, March 12, 2021 1:45 PM

Hw written problem –

$$f'(x) = A(f(x), x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

a) $f(x+\Delta x) = f(x-\Delta x) + 2\Delta x\, A(f(x), x)$

b) $f = f_{true} + \epsilon$, $A(f+\epsilon, x) \simeq A(f) + \epsilon \frac{\partial A}{\partial f}$

$\hookrightarrow \epsilon(x+\Delta x) = \epsilon(x-\Delta x) + 2\Delta x\, \epsilon(x) \frac{\partial A}{\partial f}$

c) for "stability", want $|\epsilon(x+\Delta x)| \leq |\epsilon(x)|$ $\Big|$ $\frac{\partial A}{\partial f} = \lambda$, $\lambda > 0$, $\lambda < 0$, $\lambda = i\omega$

$$|\epsilon(x-\Delta x) + 2\Delta x\, \epsilon(x)\lambda| \leq |\epsilon(x)|$$

$$\hookrightarrow \left| \frac{\epsilon(x-\Delta x)}{\epsilon(x)} + 2\Delta x \lambda \right| \leq 1$$

$$\hookrightarrow |\pm 1 + 2\Delta x \lambda| \leq 1$$

No way to <u>ensure</u> stability

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1) <u>Monte-Carlo Integration</u>

2) Finite-Element Methods (ODEs)

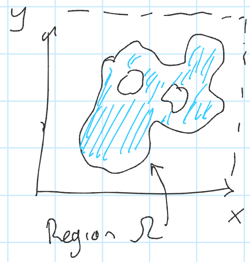
1) MC – want to know $I = \int_a^b f(x)\,dx$

$= \langle f \rangle (b-a)$

choose random points x_i, let $\langle f \rangle = \frac{1}{N}\sum_{x_i} f(x_i)$

<u>Useful for</u>

- High-dimensional Integrals
- Multi-dimensional problems w/ complicated boundaries



$f(x,y)$, what is $\int_\Omega f(x,y)\,dx\,dy$? $= \langle f \rangle \cdot V$

↳ want to know $\langle f \rangle$, but only in Ω

want to know V of region.

↳ schematically, • compute $\langle f \rangle$ by choosing random x_i

• Count fraction of points in Ω to determine the volume.

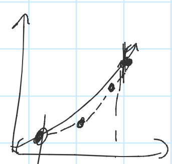
↳ Est. for uncertainty in integral is $\sim \sigma_f / \sqrt{N} \equiv \sigma_N$ ← st. dev. of f

$$I = V \cdot \langle f \rangle \pm V \cdot \sigma_N$$

2) Finite Element Methods — for ODEs

↳ Write some interpolating function, choose coefficients to best satisfy differential equation.

e.g. solve $f''(x) = 6x$ → know soln is x^3



⇒ useful for boundary value problems

Use b.c.'s → $f(0)=0$ $f(1)=1$

⇒ suppose we want $f^{guess}(x) = ax^2 + bx + c$

↳ b, c fixed by boundary conditions

$f''(x) - 6x = 0$, $2a - 6x = R$ = residual

want this ↑ to be as close to zero as possible

$$I = \int_0^1 R^2\,dx \leftarrow \text{make small}$$

$$\frac{\partial I}{\partial a} = 2\int_0^1 R \frac{\partial R}{\partial a}\,dx = 4\int_0^1 (2a - 6x)\,dx = 0$$

$$\underline{a = 3/2}$$

↳ $f_{guess}(x) = \dfrac{3x^2 - x}{2}$

Galerkin method

- linear, interpolating polynomial is common
- minimize weighted version of R in each interpolating segment

↳ Works in higher dimensions as well!

↳ Equivalent to the linear schrodinger system we saw earlier on.