Physics 427 – Introduction to Computational Physics
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Homework 7 – due March 18, 2021

Problem 1: We saw in class it was possible to derive a variety of finite-difference formulae for computing derivatives of arbitrary order.

a) On the midterm, we derived an expression for centered differencing that was accurate to third order, $\mathcal{O}(\Delta x^2)$. A higher-order expression, valid to order $\mathcal{O}(\Delta x^4)$, is given by

$$f'(x) = \frac{-f(x+2\Delta x) + 8f(x+\Delta x) - 8f(x-\Delta x) + f(x-2\Delta x)}{12\Delta x} + \mathcal{O}(\Delta x^4).$$

Verify this is correct by Taylor expanding the various f on the right-hand side of this expression and canceling terms.

b) We can similarly derive a formula accurate to $\mathcal{O}(\Delta x^2)$ for second derivatives. As above, verify that

$$f''(x) = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2} + \mathcal{O}(\Delta x^2).$$

Problem 2: In class, we looked at the PDE_Integration notebook for solving the advection equation. This notebook can be downloaded from Canvas.

a) Modify the notebook to solve the diffusion equation,

$$\partial_t f(x,t) = c\,\partial_x^2 f(x,t)\,.$$

Use the second-order finite-difference formula derived in problem 1b. You can use the same numerical coefficient c, Δx, and Δt in the notebook, although feel free to experiment with changing these. You should find that the initially Gaussian profile "diffuses", becoming a broader, shorter distribution.

b) It is very difficult to write down stable solutions for this system when c is negative. Verify that your solution will not provide a valid answer if you integrate for a sufficiently long time with a negative value of c. Can you think of a physical reason why this system is unstable?