Week 10 - Monday

Monday, March 29, 2021 2:03 PM

Review for midterm #2

Friday – Midterm: both computing + written

Covering: PDEs, ODEs, Integration

PDE: equations: $\partial_t f^A(t, x, y, z, \ldots) = A\left(f^A, \partial_x f^A, \partial_y f^A, \partial_x^2 f^A, \ldots, t, x, y, z, \ldots\right)$

↖ more than 1 time-deriv: $\partial_t^2 f = A \Rightarrow \begin{cases} \partial_t f = g \\ \partial_t g = A \end{cases}$

ODE: $\partial_t f(t) = A(f, t)$ (no $\partial_x f$, etc..., only 1-dim.)

Integrals: $\partial_t f(t) = A(t)$

Integrals: eg. quad function is generally "best"

- other routines may be more convenient, or problem-specific, but quad usually works.
- includes routines for handling ∞'s

eg. Romberg integration: sample function at many interval sizes

- Combine answers to improve accuracy, eliminate error,
- ~ Extrapolate to $\Delta t \to 0$ limit

ODEs: solve_ivp – good, generic function, for IVPs (initial value problems)

solve_bvp – similar, but for boundary value problems

eg. $\partial_x^2 f = a(1-f^2) f' - f$

$\partial_x f = g$ ←

eg. $\partial_x^2 f = a(1-f^2)f' - f$

↳ $\begin{cases} \partial_x f = g \\ \partial_x g = a(1-f^2)g - f \end{cases}$ ↩

- Stability of ODE integration methods: $|E(t+\Delta t)| \leq |E(t)|$
- Looked at higher-order methods:
  - RK2 – midpoint method
  - RK4 – 4th-order classic method ← solve_ivp default
  - Implicit methods
    - Forward, backward Euler
    - Crank-Nicolson
    - ⋮

PDEs: needed to discretize in $t, x, y, z, \ldots$ dimensions. → $\partial_t f = A(f, \partial_x f, \ldots, t, x, \ldots)$

Classify PDEs:
- Elliptic: No time-dependence ~ static systems ⇒ BVPs
- Parabolic Eqs: ~ single time-deriv. advective, diffusive behavior ↘ IVP
- Hyperbolic: ~ two time derivatives ~ wave-like behavior