Physics 427 – Introduction to Computational Physics
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Homework 6 Solutions

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Problem 1: The second-order multi-point method

a) We should arrive at a formula similar to

$$f(x+\Delta x) = f(x-\Delta x) + 2\Delta x A(f(x), x)\,.$$

b) Given $f = f^{\text{true}} + \epsilon$, we have

$$f^{\text{true}}(x+\Delta x) + \epsilon(x+\Delta x) = f^{\text{true}}(x-\Delta x) + \epsilon(x-\Delta x) + 2\Delta x A(f^{\text{true}}(x) + \epsilon(x), x)\,.$$

Assuming the true solution satisfies the result from part a, and expanding A, we have

$$\epsilon(x+\Delta x) = \epsilon(x-\Delta x) + 2\Delta x \epsilon(x) \left.\frac{\partial A}{\partial f}\right|_x\,.$$

c) We would like the following inequality to hold true for stability,

$$|\epsilon(x+\Delta x)| \leq |\epsilon(x)|\,.$$

Substituting in the result from part (b), letting $\partial A/\partial f = \lambda$, and simplifying, the condition becomes

$$\left|\frac{\epsilon(x-\Delta x)}{\epsilon(x)} + 2\Delta x\lambda\right| \leq 1\,.$$

Whether the inequality holds will thus depend on the precise values of the error at previous steps along with λ. There is, unfortunately, no value of Δx which will *guarantee* the inequality holds, so stability is not guaranteed.