Growth Rates

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1 Definitions of growth rates

1.1 Absolute growth rate

Discrete

$$g_t = \frac{\Delta x}{\Delta t} \tag{1}$$

Continuous

$$g_t = \frac{dx}{dt} \tag{2}$$

1.2 Relative growth rate

$$R = \frac{x_t}{x_{t-1}} = 1 + \frac{g_1}{x_{t-1}} \tag{3}$$

Note: $\Delta t = 1$

1.3 log growth rate

This growth rate is the most commonly used in the economic field, most probably because it represents a percentage growth.

Discrete

$$r = \log R = \log\left(\frac{x_t}{x_{t-1}}\right) \approx \frac{x_t - x_{t-1}}{x_t} \tag{4}$$

Continuous

$$r = \frac{\mathrm{d}\log x}{\mathrm{d}t} = \frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\dot{x}}{x} \tag{5}$$

Note that the following is always true (see section *operator algebra* for more informations)

$$\frac{\mathrm{d}y}{y} = \mathrm{d}\ln(y) \tag{6}$$

2 Continuously compounded rates

Usually rates are compounded discretely and have the following form:

$$\left(1+\frac{r}{k}\right)^{(n\cdot k)} \tag{7}$$

Where r is the annual rate, n the number of years and k the frequency (of payment for instance). If the frequency goes towards infinity, we observe the following

$$\lim_{k\to\infty}\left(1+\frac{r}{k}\right)^{(n\cdot k)} = e^{rn} \tag{8}$$

3 Rule of the 70

How many years does it take for an amount to double with rate r?

One can simply answer this question by using the following formula

$$n = \frac{70}{100}r \tag{9}$$

3.1 Explanation

The real formula is

$$(1+r)^n = 2 \tag{10}$$

which solving for n gives

$$n = \frac{\log(2)}{\log(1+r)} \tag{11}$$

As Taylor's expansion of $\ln(x)$ around 1 is

$$\ln(x) = \sum_{k=1}^{n}(-1)^{(k+1)}\frac{(x-1)^k}{k} \tag{12}$$

and since $0 < r < \frac{1}{10}$ we can keep only the first term, which is $\ln(x) = (x-1)$ and thus $\ln(1+r) = r$, which leads to

$$n \approx \frac{\ln(2)}{r} = \frac{.693...}{r} \tag{13}$$