MicroEconomics

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1 Introduction

Two Basic Postulates

- Rational Choices
- Equilibrium

Pareto Efficiency allows no "wasted warfare"

1.1 Comptetitive Equilibrium

p_e

1.2 Monopoly

Pareto inefficient $\Leftarrow$ not all appartments are rent

1.3 Rent control

Pareto inefficient

2 Budget Set

2.1 Budget Constraints

bundle $(x_1, ..., x_n)$ affordable at price $\mathbf{p} = (p_1, ..., p_n)$ when

$$\mathbf{p} \cdot \mathbf{x} = p_1 x_1 + ... + p_n x_n \leq m \tag{1}$$

m is the consumer's (disposable) income

2.1.1 Budget Constraint

$$\{(\mathbf{x}) | \mathbf{x} \geq 0 \text{ and } \mathbf{p} \cdot \mathbf{x} = m\} \tag{2}$$

2.1.2 Budget Set

$$B(\mathbf{p}, m) = \{(\mathbf{x}) | \mathbf{x} \geq 0 \text{ and } \mathbf{p} \cdot \mathbf{x} \leq m\} \tag{3}$$

2.2 Uniform *Ad Valorem* Sales Taxes

$$(1+t)\mathbf{p} \cdot \mathbf{x} \leq m \tag{4}$$

tax levied at rate t.

3 Preferences and Utility Functions

3.1 Preference Relations

$$x \succeq y \Leftrightarrow U(x) \geq U(y) \tag{5}$$

3.2 Non-unicity of utility function

Two utility functions represent the same preference relation if and only if there exists an increasing function ϕ such that

$$U = \phi(V) \tag{6}$$

3.3 Marginal Rate of Substitution

$$MRS = -\frac{dx_2}{dx_1} \tag{7}$$

rate at which consumer is willing to exchange commodity 2 for commodity 1.

Utility, however remains constant $du = 0$, taking the total derivative

$$\begin{aligned} du &= 0 \\ &= \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 \end{aligned} \tag{8}$$

Leads to

$$MRS = -\frac{dx_2}{dx_1} = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} \tag{9}$$

3.4 Cobb-Doublas

La fonction d'utilité de Cobb-Doublas est la suivante

$$U(x, y) = x^a y^b \tag{10}$$

Par transformation monotone, on a

$$V(x, y) = a \ln(x) + b \ln(y) \tag{11}$$

The Marginal utility is

$$\frac{\partial U}{\partial x_1} = a x^{a-1} y^b \tag{12}$$

$$\frac{\partial U}{\partial x_2} = b x^a y^{b-1} \tag{13}$$

Hence, the marginal rate of substitution is

$$MRS = \frac{a}{b} \frac{y}{x} \tag{14}$$

Note that the MRS is always the same for any utility function (U or V) satisfying the same preference relations.

4 Consumer Behavior

4.1 Consumer Problem

$$\max U(\mathbf{x}) \tag{15}$$

s.t.

$$\mathbf{p} \cdot \mathbf{x} \leq m \tag{16}$$

4.2 Kuhn-Tucker theorem

Consider the maximization problem

$$\max f(\mathbf{x}) \tag{17}$$

subject to

$$g_i(\mathbf{x}) \leq 0; i = 1..k \tag{18}$$

Note that $f, g_i : \mathbb{R}^n \to \mathbb{R}$.

Then for a solution $\mathbf{x}^*$ to the maximization problem there exists Kuhn-Tucker multipliers $\lambda_1, ..., \lambda_n \in \mathbb{R}$ such that

$$\nabla f(\mathbf{x}^*) = \sum_{i=1}^{k} \lambda_i \nabla g_i(\mathbf{x}^*) \tag{19}$$

$$\lambda_i \geq 0 \forall i \tag{20}$$

$$\lambda = 0 \text{ if } g_i(\mathbf{x}^*) < 0 \tag{21}$$

4.3 Perfect Substitutes Case

$$MRS = -1 \tag{22}$$

4.4 Corner Solutions

Corner solutions, as their name implies, are solutions that are located in a corner, mathematically

$$x_i = \tfrac{m}{p_1} \tag{23}$$

$$x_j = 0 \quad i \neq j \tag{24}$$

4.5 Maximization problem with Cobb-Doublas

We wanna solve

$$\max U = a \ln(x_1) + b \ln(x_2) \tag{25}$$

s.t. $g = p_1 x + p_2 x_2 = m$

We use Kuhn-Tucker

$$\nabla U = \sum \lambda_i \nabla g_i \tag{26}$$

$$\begin{pmatrix} \frac{a}{x_1^*} \\ \frac{b}{x_2^*} \end{pmatrix} = \lambda_1 \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \tag{27}$$

Hence

$$\begin{aligned}
x_1^* &= \tfrac{c}{d}\tfrac{p_2}{p_1}x_2 && (28)\\
&= \tfrac{c}{c+d}\tfrac{m}{p_1} && (29)\\
x_2^* &= \tfrac{d}{c+d}\tfrac{m}{p_2} && (30)
\end{aligned}$$

5 Dual Approaches to Consumer Behavior

Two ways of modeling rational consumer behavior

- utility maximization
- expenditure maximization

5.1 Utility Maximization

$$\max U(\mathbf{x}) \quad \text{s.t.} \quad \mathbf{p}\cdot\mathbf{x} = m \tag{31}$$

Marshallian demand function

$$(\mathbf{p}, m) \to \mathbf{x}(\mathbf{p}, m) \tag{32}$$

is called the **Marshallian demand function**

Indirect Utility function

$$v(\mathbf{p}, m) = U(\mathbf{x}(\mathbf{p}, m)) \tag{33}$$

5.2 Expenditure Minimization

$$\min \mathbf{p}\cdot\mathbf{h} \quad \text{s.t.} \quad U(\mathbf{h}) \geq u \tag{34}$$

Hicksian demand function (or compensated demand)

$$(\mathbf{p}, u) \to \mathbf{h}(\mathbf{p}, u) \tag{35}$$

Expenditure function

$$(\mathbf{p}, u) \to e(\mathbf{p}, u) = \underbrace{\mathbf{p}\cdot\mathbf{h}(\mathbf{p}, u)}_{m} \tag{36}$$

5.2.1 Relation between Hicksian and Marshallian demand function

$$\mathbf{h}(\mathbf{p}, u) = \mathbf{x}(\mathbf{p}, e(\mathbf{p}, u)) \tag{37}$$

5.3 Duality

The following is true

- Utility maximization $\Leftrightarrow$ Expenditure minimization
- Utility minimization $\Leftrightarrow$ Expenditure maximization

5.4 Properties of Marshallian demand function

- Homogeneity: $\mathbf{x}(\lambda\mathbf{p}, \lambda m) = \lambda\mathbf{x}(\mathbf{p}, m)$
- Additivity: $\mathbf{p} \cdot \mathbf{x} = m$
- Symmetry: see Slutsky
- $\mathbf{x}(\mathbf{p}, m)$ is
  - increasing in m for **normal goods**
  - decreasing in m for **inferior goods**
  - increasing in $\mathbf{p}$ for **ordinary goods**
  - decreasing in $\mathbf{p}$ for **Giffen goods**

5.5 Properties of Hicksian demand function

- Homogeneity: $\mathbf{h}(\lambda\mathbf{p}, u) = \lambda\mathbf{h}(\mathbf{p}, u)$
- $U(\mathbf{h}(\mathbf{p}, u)) = u$
- Symmetry: see Slutsky
- Monotonicity: $\mathbf{h}(\mathbf{p}, u)$ is decreasing in $\mathbf{p}$

5.6 Roy's Identity

For any good i we have

$$x_i(\mathbf{p}, m) = -\frac{\frac{\partial v(\mathbf{p},m)}{\partial p_i}}{\frac{\partial v(\mathbf{p},m)}{\partial m}} \tag{38}$$

$$h_i(\mathbf{p}, u) = \frac{\partial e(\mathbf{p}, u)}{\partial p_i} \tag{39}$$

5.7 Slutsky matrix

$$S = (S_{ij}) = \left(\frac{\partial h_i}{\partial p_j}\right) \tag{40}$$

Using Roy's identity, one can write

$$(S_{ij}) = \left(\frac{\partial^2 e(\mathbf{p}, u)}{\partial p_i \partial p_j}\right) \tag{41}$$

$$S \cdot \mathbf{p} = \sum_j S_{ij} p_j = \sum_j \frac{\partial h_i}{\partial p_j} p_j = 0 \tag{42}$$

5.8 Slutsky Relations

$$
\begin{aligned}
S_{ij} &= \frac{\partial h_i}{\partial p_j} && (43)\\
&= \frac{\partial x_i}{\partial p_j} + \frac{\partial e}{\partial p_j}\frac{\partial x_i}{\partial e} && (44)\\
&= \frac{\partial x_i}{\partial p_j} + h_j \frac{\partial x_i}{\partial e} && (45)\\
&= \frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial m} && (46)
\end{aligned}
$$

and therefore we get the Slutsky relations

$$\frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial m} = \frac{\partial x_j}{\partial p_i} + x_i \frac{\partial x_j}{\partial m} \tag{47}$$

5.8.1 Interpretation

$$\frac{\partial x_i}{\partial p_j} + x_j \frac{\partial x_i}{\partial m} = S_{ij} = \frac{\partial h_i}{\partial p_j} \tag{48}$$

thus

$$\frac{\partial x_i}{\partial p_j} = \underbrace{\frac{\partial h_i}{\partial p_j}}_{\text{substitution effect}} + \underbrace{-x_j \frac{\partial x_i}{\partial m}}_{\text{income effect}} \tag{49}$$

6 Labor Supply and Intemporal Choice

6.1 Present Values

$$c_1 + c_2 \frac{1}{1+r} = y_1 + (1+r)y_2 \tag{50}$$

where c_i is consumption at time t, y_i amount spent and r real interest rate $(1+r) = \frac{1+R}{1+i}$.
Consumer at time 1 can either save or borrow for/on period 2.

- if r increase, consumer **saves** more
- if r decrease, consumer **borrows** more

6.2 The N-period case

$$\max U(c_1, c_2, ..., c_N) \tag{51}$$

$$\text{s.t.} \tag{52}$$

$$\sum \frac{1}{(1+r)^{i-1}} c_i = \sum \frac{1}{(1+r)^{i-1}} y_i \tag{53}$$

7 Choice under Uncertainty

if we set:

- c_a: car accident
- c_{na} no car accident

π_a and π_{na} are the associated probabilities, in case of accident L$ are lost.

$$c_{na} = \quad m \tag{54}$$
$$c_a = \quad m - L \tag{55}$$

after buying premium γK

$$c_{na} = \quad m - \gamma K \tag{56}$$
$$c_a = \quad m - L - \gamma K + K = m - L + (1-\gamma)K \tag{57}$$

thus $K = \frac{1}{1-\gamma}(c_a - m + L)$ and

$$c_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} c_a \tag{58}$$

7.1 competitive insurance (fair)

expected economic profit is zero

$$\gamma K - \pi_a K - (1-\pi_a)0 = (\gamma - \pi_a)K = 0 \tag{59}$$
$$\gamma = \pi_a \tag{60}$$

The rational choice must satisfy

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a}{1-\pi_a} = \frac{\pi_a}{\underbrace{\pi_{na}}_{1-\pi_a}} \frac{MU(c_a)}{MU(c_{na})} \tag{61}$$

and thus

$$MU(c_a) = MU(c_{na}) \tag{62}$$

8 Market Demand

The consumer i's ordinary demand function for commodity j is

$$x_j^{*i}(\mathbf{p}, m) \tag{63}$$

8.1 Aggregate Demand

When all consumers are price takers, the market demand for commodity j is

$$X_j(\mathbf{p}, \mathbf{m}) = \sum_{i=1}^{n} x_j^{*i}(\mathbf{p}, m^i) \tag{64}$$

If all consumers are identical then

$$X_j(\mathbf{p}, M) = n \cdot x_j^{*}(\mathbf{p}, m) \tag{65}$$

where $M = nm$ and $x^{*i} = x^* \ \forall i$

8.2 Elasticity

$$\epsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y} = \frac{\partial x}{\partial y}\frac{y}{x} = \left|\frac{\partial \ln(x)}{\partial \ln(y)}\right| \tag{66}$$

8.3 Revenue and Own-Price Elasticity of demand

The seller's revenue is

$$R(p) = p \cdot X^*(p) \tag{67}$$

Variation around price is

$$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp} = X^*(p)(1+\epsilon) \tag{68}$$

- $\epsilon \in (-1, 0)$, price rise cause revenue to rise
- $\epsilon = -1$, price rise has no effect on revenue
- $\epsilon < -1$ price rise causes revenue to fall

8.4 Marginal Revenue

$$MR(q) = \frac{dR(q)}{dq} = p(q) \cdot \left(1 + \frac{1}{\epsilon}\right) \tag{69}$$

9 The Producer

9.1 Technology

A technology is a process by which inputs are converted to an output.

Input bundle

$$\mathbf{x} = (x_1, x_2, ..., x_n) \tag{70}$$

9.2 Production Functions

States the **maximum** amount of output possible from the input bundle

$$y = f(x_1, ..., x_n) \tag{71}$$

where y is the level of output.

Technology sets A **production plan** is an input bundle and an output level:$(\mathbf{x}, y)$. A production plan is feasible if

$$y \leq f(\mathbf{x}) \tag{72}$$

The collection of all feasible production plan is called technology set.
The technology set is

$$T = \{(\mathbf{x}, y) | y \leq f(\mathbf{x}) \text{ and } \mathbf{x} \geq 0\} \tag{73}$$

9.2.1 Example of technologies

- Cobb-Douglas Production function

$$y = A\prod_{i=1}^{n} x_i^{a_i} \tag{74}$$

- Fixed proportions (perfect complements)

$$y = \min(a_1 x_1, ..., a_n x_n) \tag{75}$$

- Perfect substitutes

$$y = \sum_{i=1}^{n} a_i x_i \tag{76}$$

9.3 Marginal Product

$$MP_i = \frac{\partial y}{\partial x_i} \tag{77}$$

9.4 Return-to-scale

$$f(k\mathbf{x}) = kf(\mathbf{x}) \tag{78}$$

- Diminishing return-to-scale

$$f(k\mathbf{x}) < kf(\mathbf{x}) \tag{79}$$

- Increasing return-to-scale

$$f(k\mathbf{x}) > kf(\mathbf{x}) \tag{80}$$

9.5 Technical rate of substitution

$$\frac{dx_2}{dx_1} = \frac{\partial y/\partial x_1}{\partial y/\partial x_2} \tag{81}$$

9.6 Well-behaved technologies

is **monotonic** and **convex**

Monotonic more of any input generates more output

10 Cost Minimization

10.1 Cost minimization problem

$$\min w_1 x_1 + w_2 x_2 \tag{82}$$

subject to $f(x_1, x_2) \geq y$

Iso-cost lines

$$x_1 x_1 + w_2 x_2 = C \tag{83}$$

Isoquant

$$f(x_1, x_2) = C \tag{84}$$

Firm's conditional demand for input 1

$$x_1^*(w_1, w_2, y) \tag{85}$$

Firm's total cost function

$$c(w_1, w_2, y) \tag{86}$$

Average total cost

$$AC = \frac{c(w_1, w_2, y)}{y} \tag{87}$$

10.2 Short-run and long-run

The short-run total cost minimization problem, is the long-term problem with the additional constraint:

$$x_i = x_i' = C \tag{88}$$

(one input can't be changed)

11 Profit Maximization

11.1 Economic Profit

$$\Pi = \sum p_i x_i - w_i x_i \tag{89}$$

Present value of firm

$$PV = \sum \frac{1}{(1+r)^i} \Pi_i \tag{90}$$

11.2 Profit maximization problem

assuming one output, y

$$\max py - \sum w_i x_i \tag{91}$$

st $f(\mathbf{x}) \geq= y$, solving with the lagrangian gives

$$\frac{\partial f}{\partial x_i} = \frac{w_i}{p} \tag{92}$$

11.3 Hotelling's Lemma

$$y^*(p, w) = \frac{\partial \Pi}{\partial p}(p, w) \tag{93}$$

$$x_i^* = -\frac{\partial \Pi}{\partial w_i}(p, w) \tag{94}$$

similar as Roy's identity, for consumer's problem

11.4 Profit maximization and marginal cost

we have

$$py - C(y, w) \tag{95}$$

and thus

$$\frac{\partial C}{\partial y} = p \tag{96}$$

With constant return to scale, profit maximization gives $\Pi = 0$

11.5 Revealed profitability

The firm's technology set must lie under all the iso profit lines

12 Monopoly

12.1 Pure Monopoly

- Single seller
- alter market price by altering output level

Pareto ineficient

12.2 Profit Maximization

$$\Pi(y) = p(y)y - c(y) \tag{97}$$

where $p(y)$ is the inverse demand function.

$$\begin{aligned}
\max_y \Pi &\Leftrightarrow & (98)\\
\frac{d\Pi}{dy} &= 0 & (99)\\
&\Leftrightarrow & (100)\\
MR &= MC & (101)\\
p'y^* + p &= c' & (102)\\
p(y^*)(1 + \frac{dp}{dy}\frac{y^*}{p}) &= \frac{dc}{dy} & (103)\\
p(y^*)(1 + \frac{1}{\epsilon}) &= \frac{dc}{dy} & (104)
\end{aligned}$$

where ϵ is the price-owner elasticity.

12.2.1 Markup pricing

Markup is the difference between the price and the marginal cost, that make up the profit

$$p(y^*) - MC \tag{105}$$

12.3 Tax Levied on Monopolist

12.3.1 Profit Tax

Monopolist has now to solve

$$\max_y (1-t)\Pi(y) \Leftrightarrow \max_y \Pi(y) \tag{106}$$

thus, **neutral tax**.

12.3.2 Quantity Tax

$$MC' = MC + t \tag{107}$$

- higher price
- less quantity

called **distortionary tax**

12.4 Natural Monopoly

Arises when firm's technology has economies-of-scale high enough to supply whole market.

Can't set $MC = p$ because it implies $ATC > p \Rightarrow$ economic loss.

12.5 Price discrimnation

12.5.1 First Degree

Each output sold at different price.

Pareto Efficient.

12.5.2 Third degree

Supply different markets with different demands

13 Oligopoly

Oligopoly is an industry consisting of a few firms. Each firm's own price or output decisions affect its competitors profit.

13.1 Duopoly

$$\Pi_1(y_1|y_2) = p(y_1+y_2)y_1 - c_1(y_1) \tag{108}$$

Maximizing:

$$\frac{\partial \Pi_1}{\partial y_1} = p(y_1+y_2) + y_1\frac{\partial p(y_1+y_2)}{\partial y_1} - \frac{dc}{dy_1} = 0 \tag{109}$$

Solution is $y_1 = R(y_2)$, same story for Π_2 that leads $y_2 = R(y_1)$.

The solution $y_1^* = R(y_2^*)$ and $y_2^* = R(y_1^*)$ is called the **Cournot-Nash equilibrium** (quantity competition)

13.2 Bertrand Games

Price competitio, equilibrium is $p = p_1 = p_2 = MC$.