MacroEconomics - HW1

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1

1.1

LM-curve is (at equilibrium $L = M = \overline{M}$)

$$i = \frac{k}{h}Y - \frac{1}{h}\overline{M} \tag{1}$$

IS-curve is ($Y = C + I + G + NX$ and solving for i)

$$i = -\frac{1 - c + m_1}{b}Y + \frac{\overline{I} + G + x_1 Y^{\text{world}}}{b} + \frac{x_2 + m_2}{b}R \tag{2}$$

at the equilibrium the interest rate is the same $i_{IS} = i_{LM}$, thus solving for Y gives

$$Y = \frac{1}{kb + h(1 - c + m_1)}\left(b\overline{M} + h\left(\overline{I} + G + x_1 Y^{\text{world}} + (x_2 + m_2)R\right)\right) \tag{3}$$

1.2

1.2.1

An increase in money supply will raise income (more money available) and decrease i (people no longer have a high interest rate), please see fig 1

1.2.2

An increase in R see fig 2 gives rise to a better interest rate and a higher income. Indeed have more buying power so they can afford more (Y goes up) and thus consume more i goes up.

1.3

The curve will become steeper. Which means that effects after a shift in the LM curve will be stronger on the interest rate and less important on the income Y. See fig 3

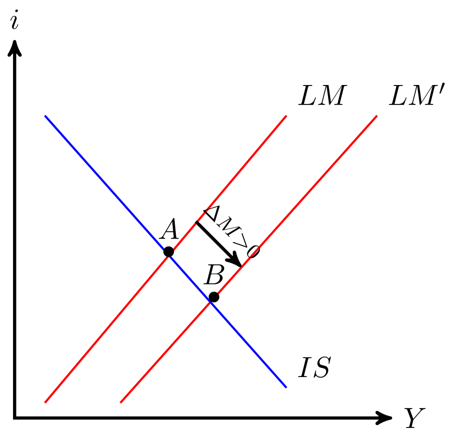


Figure 1: shift caused by an increase in Money supply

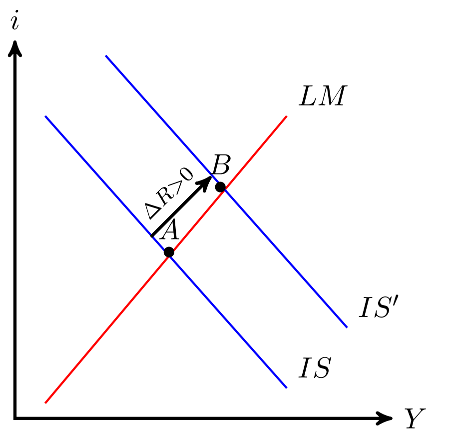


Figure 2: shift caused by an increase in R

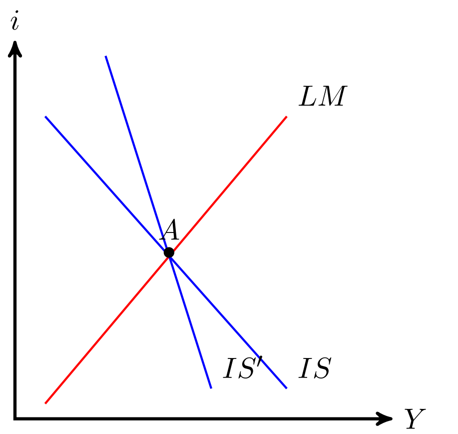


Figure 3: Illustration of a decrease in c

1.4

In this model, Y, i, R are endogenous variables. If we know set $i = \bar{i}$, i becomes an endogenous variable and we only have two endogenous variables left (Y and R) and two equations. This means that there are no degrees of freedom left and the equilibrium is determined.

Mathematically, it is

$$Y^* = \frac{h}{k}\bar{i} + \overline{M} \tag{4}$$

for the equilibrium income and

$$R^* = \frac{1}{x_2 + m_2}\left[b\bar{i} + (1 - c + m_1)Y^* - (\bar{I} + g + x_1 Y^{\text{world}})\right] \tag{5}$$

for the exchange rate.

This can be represented on a graph like on fig 4 (note that the LM curve is not dependent on R thus it is simply a vertical line).

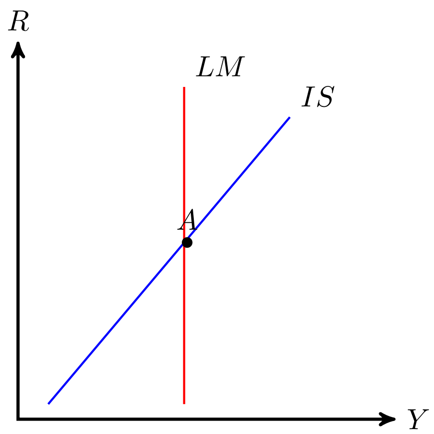


Figure 4: Equilibrium variables for R and Y

1.5

In the case where the central bank fixes the money supply ($M = \overline{M}$), the IS-curve fluctuates around the LM-curve but the difference in the IS-curve is damped by the slope of the LM-curve which can't move (no endogenous variable that is not i or Y). This results in an income (Y) that "fluctuates somewhat", see fig 5 top for graphical representation (taken from Gärtner, chapter 3).

On the other hand, when the interest rate is fixed, the LM-curve becomes

$$i = \frac{k}{h}Y - \frac{1}{h}M \tag{6}$$

where $M \neq \overline{M}$ and has become an endogenous variable. In this case the LM-curve, since it has an endogenous variable, can respond to the shift of the IS-curve. This results in an "income that fluctuates a lot", see fig 5 bottom.

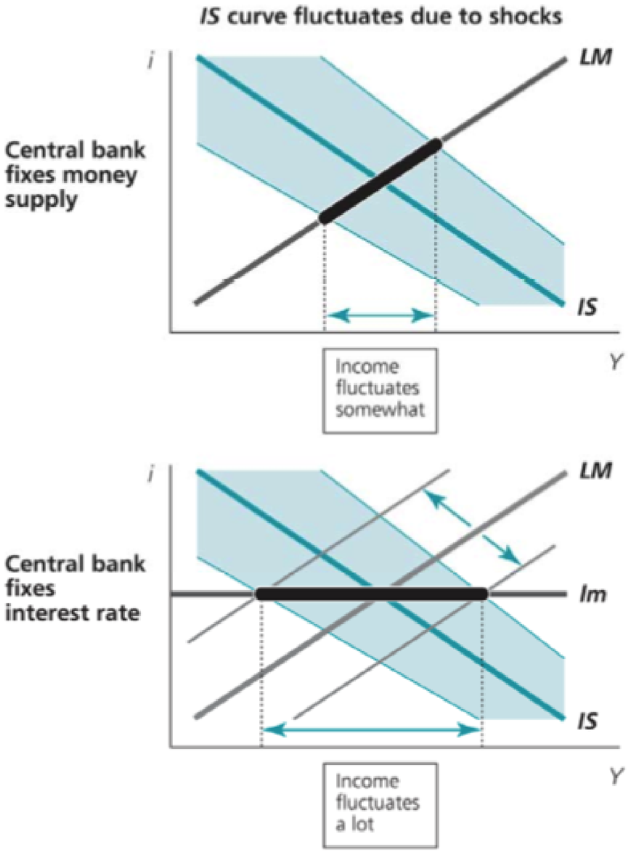


Figure 5: Effects of policy instruments on the income (from Gaertner)

1.5.1 Formally

For the fixed money supply, we have (rewriting in a simpler manner)

$$\begin{aligned} i &= aY - b && (7) \\ i &= -cY + d_0 && (8) \end{aligned}$$

where $a, b, c, d > 0$, 13 is the LM-curve and 14 is the IS-curve.

At equilibrium, we have

$$Y^* = \frac{d_0 + b}{a + c} \tag{9}$$

If IS moves, the change is

$$\Delta i = d_1 - d_0 = \Delta d \tag{10}$$

Thus the new equilibrium becomes

$$Y^* = \frac{d_1 + b}{a + c} \tag{11}$$

and the change is

$$\Delta Y^* = \frac{\Delta d}{a + c} \tag{12}$$

Now, if the interest rate is fixed, we have

$$\begin{aligned} \bar{i} &= aY - b_0 && (13) \\ \bar{i} &= -cY + d_0 && (14) \end{aligned}$$

where $a, b, c, d > 0$, 13 is the LM-curve ,14 is the IS-curve and $b = \frac{1}{h}M$.

Now, if the IS-curve moves, the change is the same as before (see eq 10) but the LM curve will compensate by the same amount through a change in the money supply described here by b; this is so because $\bar{i}$ can't change. Thus we have

$$\Delta i = -(d_1 - d_0) = -\Delta d \tag{15}$$

Taking in account the latter, the variation in income rewrites

$$\Delta Y^* = \frac{2\Delta d}{a + c} \tag{16}$$

By looking at eq 12 and eq 16, one immediately notices that the change is always bigger when the central bank fixes interest rate, and this is also in accordance with figure 5.

2

2.1

The equilibrium is not reached: the FE, LM and IS-curves do not cross at one single point. What happens economically is that the interest rate (i) is too high compared to the i^{world}, consequently the rest of the world has an incentive to invest in the domestic country which will eventually bring down the equilibrium towards i^{world}, either by decreasing R (IS-curve) or by increasing $\overline{M}$ (depends on policy).

2.2

For this sub-exercise we assume the FE-curve to be $i = i^{\text{world}}$

2.2.1 flexible exchange rate

Under flexible exchange rate, the endogenous variables of the model are R, i, Y and policy makers can only influence R directly. Since only the IS-curve is dependent on this variable, R needs to decrease so that the whole IS-curve goes down and eventually reaches the equilibrium. See fig 6

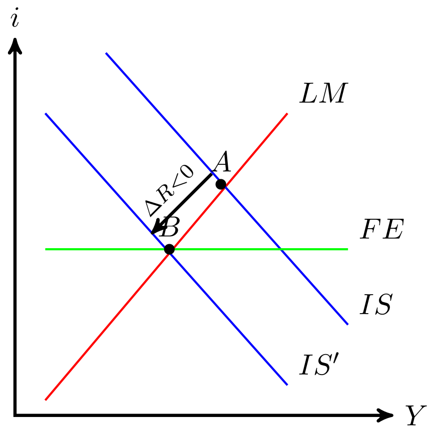


Figure 6: IS curve decrease and reaches equilibrium

2.2.2 fixed exchange rate

Under fixed exchange rate, the endogenous variables are i, Y, M. Once again i, Y can't be controlled directly, thus the LM-curve (since it is the only one dependent on M) goes down ($\Delta M > 0$), see fig 7.

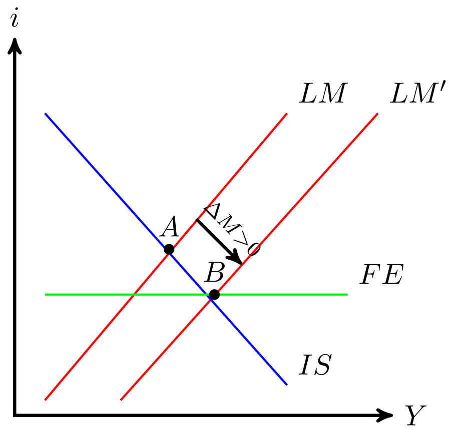


Figure 7: Increasing M shifts the FE-curve down eventually reaching equilibrium (B)

3

Large Economy A global economy model means that $NX = 0$ and that the interest rate i is the world's interest rate ($i = i^{\text{world}}$). Thus the FE-curve is irrelevant in this context and we reduce our analysis only to the IS and LM curve that writes, in this case, as follows

$$\begin{aligned} i_{LM} &= aY - b && (17)\\ i_{IS} &= -cY + d + eG && (18)\end{aligned}$$

where the constants (R assumed constant) and unused variables for this problem are hidden in $a, b, c, d, e > 0$.

Small economy Since the small economy relies on the large economy (the world's interest rate is determined by the large economy), we need to take in account the FE-curve ($i = i^{\text{world}}$ because of perfectly mobile capital).

The IS and LM curves writes as for the global economy with exception that they have to equal $i^{world} \neq i$

Solution If there is an expansionary tax policy, this means that G is increased, and thus the IS curve of the global economy goes up leading to a higher income and a higher interest rate. This new interest rate is also the new i^{world} which means that the FE curve of the small economy ($i = i^{\text{world}}$) has to shift up to adapt to this new interest rate. This in turns obliges either the IS or the LM curve to shift respectively to the right or to the left. However, R is kept constant so the only endogenous variable left is the money supply M that has to decrease to permit the equilibrium to be reached. Since only the LM curve shift, and shift to the right, the small economy will suffer from a lower income with a higher interest rate. The situation is depicted in figures 8a and 8b.

4

4.1

We have

$$\begin{aligned} \Delta L &= -sL + fU + (1 - e_u)eN - (1 - q_u)qN && (19)\\ \Delta U &= sL - fU + e_u eN - q_u qN && (20)\end{aligned}$$

the equilibtrium condition reads $\Delta U = 0$ and thus

$$fU^* = sL + e_u eN - (1 - q_u)qN \tag{21}$$

dividing by N gives

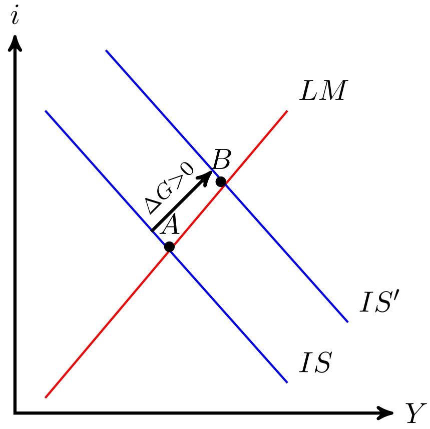
$$fu^* = s\frac{L}{N} + e_u e - q_u q \tag{22}$$

substituting $\frac{E}{N}$ by $(1 - u^*)$ leads to

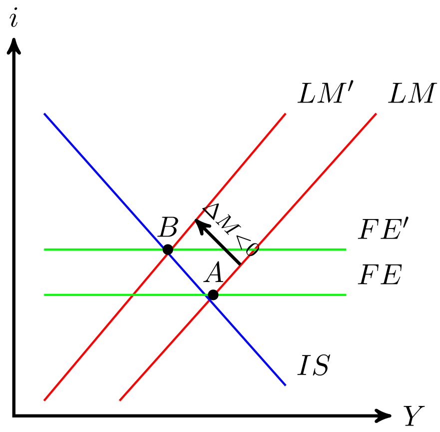
$$fu^* = s(1 - u^*) + e_u e - q_u q \tag{23}$$

and thus

$$u^* = \frac{1}{s + f}(s + e_u e - q_u q) \tag{24}$$



(a) Global/large economy



(b) Small economy

Figure 8: Illustration of the problem 3

4.2

$$\frac{\partial u^*}{\partial f} = -\frac{1}{(s+f)^2}(s + e_u e - q_u q) \tag{25}$$
$$= -\frac{1}{s+f}u^* \tag{26}$$

Thus as long as $u^* > 0$, u^* will decrease when f increase. Intuitively, this seems reasonable: if the finding rate increase, logically the unemployment should decrease.

4.3

$$\frac{\partial^2 u^*}{\partial f \partial s} = \frac{s - f + 2(e_u e - q_u q)}{(s+f)^3} \tag{27}$$

Since $s > f - 2(e_u e - q_u q)$ the expression is strictly positive (we assume $s, f > 0$). This means that an increase in s will make $\frac{\partial u^*}{\partial f}$ bigger and thus an increase in the separation rate gives a higher unemployment rate for the same f in other words s compensates with f.

5

To keep the exchange rate constant ($R = E\frac{P^{\text{world}}}{P}$) the domestic price level (P) must adjust to the world price level (P^{world}). In this case, P increase and thus the growth rate increase going from μ_{LO} to μ_{HI} this is reflected by the upward shift of EAD (EAD_{old} to EAD_{new} on fig 9 and EAD_{LO} to EAD_{hi} on fig 10).

5.1 Adaptive expectation

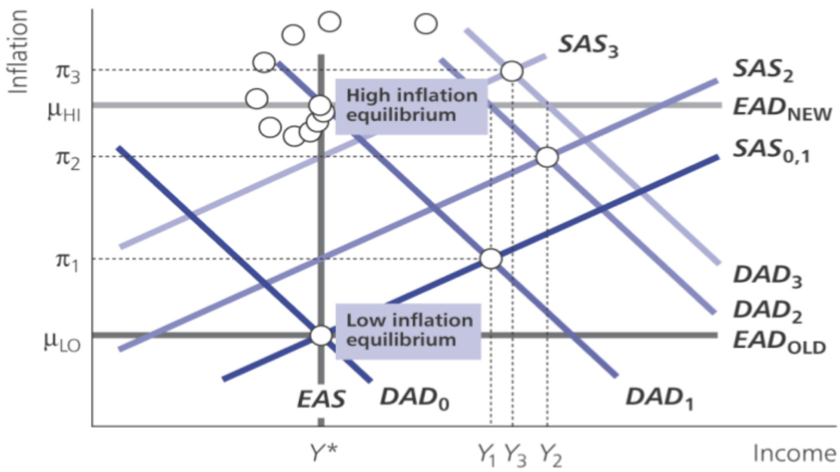


Figure 9: Effect of an increase of μ with people having an adaptive expectation (from Gaertner)

With adaptive expectations people don't know the model that is behind and behave based on what happened in the past.

At first the DAD will move up to cross the new the EAD curve. This give rise to a new income Y_1 and π_1. In the next cycle, the labor force noticed that the inflation is higher and thus negotiate higher wages which utlimately lead to a new DAD curve that cross the EAS curve at π_1 but at the same time DAD moves also to the point where $EAD = Y_1$. This keeps going on like this until it reaches the new equilibrium (on the long run) (Y^*, μ_{HI}). The effect is graphically depicted on fig 9.

5.2 Rational expectation

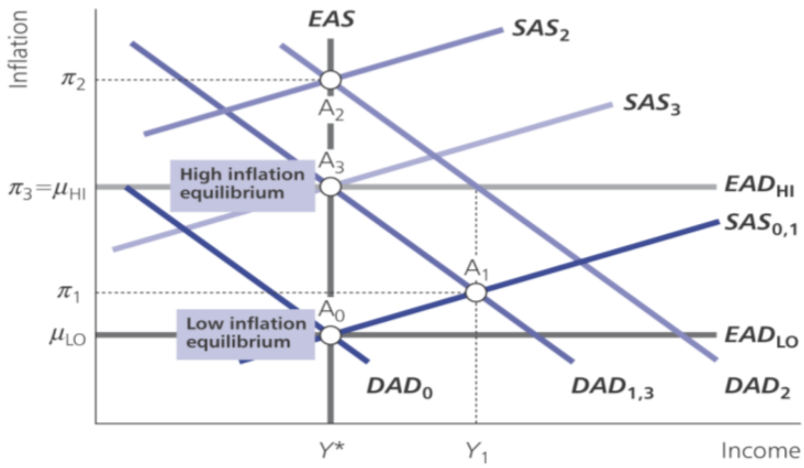


Figure 10: Effect of an increase of μ with people having an rational expectation (from Gaertner)

Here, people understand the model and will behave accordingly.

In the first period, DAD moves up from μ_{LO} to μ_{HI}, the SAS does not move as change in the money supply can not be foreseen. Thus, the output is $Y_1 > Y^*$ and the inflation rate is also larger $\pi_1 > \mu_{LO}$ but still not as high as the new one $(\pi_1 < \mu_{HI})$.

In the second phase, as people know the model they can anticipate. DAD_2 has to cross the EAD at Y_1. If people want the output to come back to Y^*, DAD_2 and SAS_2 have to cross at the EAS. This leads to an output $Y_2 = Y^*$ but an inflation rate (π_2) that is this time higher than μ_{HI}.

In the last period SAS_3 adjusts via Y^* and the EAD curve. SAS_3 is corrected by the same amount. Hence the equilibrium (where EAD and EAS cross, (Y^*, μ_{HI})) is reached.

The effect is graphically depicted on fig 10.

Note that for the two cases, the equilibrium is the same on the long-run. But the adaptive expectation mechanism takes longer to reach the equilibrium and causes more fluctuations whereas in the rational expectation mechanism it takes only 3 steps to reach the equilibrium.