Strategic and Cooperative Thinking

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1 Static Games of Complete Information

Complete Information Each player's payoff function is common knowledge among all players

1.1 Notation

- 1. Set of players: $I = \{1, ..., N\}$
- 2. S_i denotes set of strategies for player i
- 3. $s_i \in S_i$ denotes an arbitrary strategy of player i
- 4. $s = (s_1, ..., s_n)$ denotes a strategy profile
- 5. $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_n)$: strategy profile exept for player i
- 6. $u_i(s_1,...,s_n)$ is payoff to player i given strategy profile $(s_1,...,s_n)$

1.2 Normal form representation

- Strategy set: $S_1, ..., S_n$
- payoff functions, $u_1, ..., u_n$

denoted by

$$G_N = \{S_1, ..., S_N; u_1, ..., u_n\}$$
(1)

1.3 Classical Games

- 1.3.1 Prisoner's dilemma
- 1.3.2 "Chicken" or "Hawk-Dove" game
- 1.3.3 Battle of the sexes

1.4 Rationality and Common Knowledge

Rationality implies that every player maximizes his utility function

Common Knowledge if all the players know the game (mutual knowledge) and that all the players know that all the players know the game.

1.5 Dominance

when a strategy is better than another.

Strictly dominated if

$$u_i(s_i', s_{-i}) < u_i(s_i'', s_{-i}) \tag{2}$$

then strategy s_i^\prime is strictly dominated by $s_i^{\prime\prime}.$

Weakly dominated

$$u_i(s_i', s_{-i}) \le u_i(s_i'', s_{-i}) \tag{3}$$

1.6 Best Response

strategy s_i is a best response if

$$u_i(s_i, s_{-i}) < u_i(s_i', s_{-i})$$
 (4)

for every $s_i' \in S_i$

1.7 Nash Equilibrium

Strategies $(s_1^*,...,s_n^*)$ are a Nash equilibrium if for each player i

$$u_i(s_i^*, s_{-i}^*) < u_i(s_i, s_{-i}^*) \tag{5}$$