MacroEconomics - HWII

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1 The FE curve, capital immobility and monetary policy

1.1

If $\kappa \to 0$, the FE curve becomes

$$i = i^{\text{world}} + \frac{E^e_{+1} - E}{E} \tag{1}$$

and if $i < i^{\text{world}}$ we have

$$i - i^{\text{world}} = \frac{E^e_{+1} - E}{E} < 0 \tag{2}$$

and thus in this case

$$i < i^{\text{world}} \Leftrightarrow E^e_{+1} < E \tag{3}$$

1.2

Capital perfectly immobile means $\kappa = 0$ and exchange rates flexible is $OR = 0$ and thus $BP = CA + CP = 0$.

Under these circumstances, we can rewrite the original FE curve (eq 1) setting $\kappa = 0$ which leads to

$$Y = \frac{x_1}{m_1} Y^{\text{world}} + \frac{m_2 + x_2}{m_1} R \tag{4}$$

and one immediately sees that it is independent from i, graphically it is a vertical line in the i-Y diagram.

1.3

The LM curve reads

$$i = \frac{k}{h} Y - \frac{1}{h} \overline{M} \tag{5}$$

so an increase in $\overline{M}$ will drive up the curve: higher i for a given Y.

At this point there is a disequilibrium between the three curves and the (Y_{FE}, i_{FE}) coordinates at which the LM curve crosses the FE are higher than the coordinates where the LM curve crosses the IS curve $(Y_{IS}, Y i_{is})$: $\begin{pmatrix} Y_{FE} \\ i_{FE} \end{pmatrix} > \begin{pmatrix} Y_{IS} \\ i_{IS} \end{pmatrix}$

The only way to get back to an equilibrium is for R to change.

If R increase the IS curve will increase by the associated multiplier and the FE curve also increases by its associated multiplier. However the multiplier of the FE curve is greater than the IS one (see section 1.4). If R increases, the equilibrium will never be reached because Y_{FE}, which is initially bigger than Y_{IS}, will always grow more than Y_{IS} and thus the only way to reach an equilibrium is to **decrease** R and thus the IS curve will decrease a little bit and the FE (which grows faster) will eventually catch-up.

To find the new value of i, we first find the equilibrium interest rate i in algebraic terms. We can do so in three steps

1. Substitute R in the IS curve from the FE curve, see eq 6
2. Substitute Y in the IS curve from the LM curve, see eq 7.
3. an rearrange eq 8

$$\begin{aligned}
i &= -\frac{1-c+m_1}{b}Y + \frac{x_2+m_2}{b}\left(\frac{m_1}{m_2+x_2}Y - \frac{x_1}{m_2+x_2}Y^{\text{world}}\right) + \frac{\overline{I}+G+x_1Y^{\text{world}}}{b} \\
&= -\frac{1-c}{b}Y + \frac{\overline{I}+G}{b} && (6) \\
i &= -\underbrace{\frac{1-c}{b}}_{\epsilon>0}\left(\frac{ih+\overline{M}}{k}\right) + \underbrace{\frac{\overline{I}+G}{b}}_{\gamma>0} && (7) \\
i &= \frac{1}{1+\epsilon\frac{h}{k}}\left(\gamma - \epsilon\frac{\overline{M}}{k}\right) && (8)
\end{aligned}$$

Since $\gamma, \epsilon, h, k > 0$ a decrease in $\overline{M}$ leads to a decrease in i.

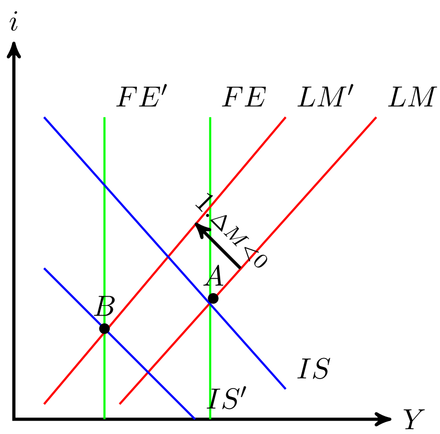

Figure 1: shift caused by a decrease in Money supply: IS and FE respond by shifting to the left ($\Delta R < 0$) ultimately leading to a lower Y and a lower i, see text for more explanations.

1.4

The FE-curve reads as in eq 4 and if we rewrite the IS-curve as $Y(i, R)$ we get

$$Y_{IS} = -\frac{b}{1-c+m_1}i + \frac{m_2+x_2}{1-c+m_1}R + \frac{1}{1-c+m_1}(\overline{I}+G+x_1Y^{\text{world}}) \tag{9}$$

If we now look at the partial derivative of those two curves (the derivative times the difference in Y is the amount by which the curves will be shifted; multiplier) we get

$$\frac{\partial Y_{IS}}{\partial R} = \frac{m_2 + x_2}{1 - c + m_1} \tag{10}$$

$$\frac{\partial Y_{FE}}{\partial R} = \frac{m_2 + x_2}{m_1} \tag{11}$$

since $c \in (0,1) \Rightarrow (1-c) \in (0,1)$ and $m_1 > 0$. We have $1 - c + m_1 > m_1$ and thus

$$\frac{\partial Y_{FE}}{\partial R} > \frac{\partial Y_{IS}}{\partial R} \tag{12}$$

which means that the shift in the FE curve is always going to be larger than the shift in the IS curve.

2 Economic growth and capital markets

2.1

According to the Solow model, at equilibrium we have

$$\Delta K^* = sY - \delta K^* = sF(K^*, L) - \delta K^* = 0 \tag{13}$$

Setting $F = \sqrt{KL}$ and solving for K^* gives

$$K^* = \left(\frac{s}{\delta}\right)^2 L \tag{14}$$

Since country B does not save ($s = 0$), its steady capital is $K_B^* = 0$ and $Y_B^* = 0$ whereas country A has $s = .25$ and $K_A^* = 625$ and $Y_A^* = 250$. Thus the consumptions per capita are

$$c_A = \tfrac{Y_A(1-s_a)}{L} = \frac{250(1-.25)}{100} = 1.875 \tag{15}$$

$$c_B = \tfrac{Y_B(1-s_B)}{L} = \frac{0(1-0)}{100} = 0 \tag{16}$$

2.2

Since $s_B = 0$, all investments will have to be done by country A. At equilibrium both saving rates are the same thus $K = K_B = K_B$.

We can write for the world (note the $\frac{1}{2}$ in front of Y_B, this is because we take only the capital income in account)

$$s_a(Y_A(K_A^*) + \frac{1}{2}Y_B(K_B^*)) = \delta(K_A^* + K_B^*) \tag{17}$$

$$\frac{3}{2}s_a\sqrt{K^*L} = 2\delta K^* \tag{18}$$

$$\begin{aligned} K^* &= \left(\frac{3}{4}\frac{s_a}{\delta}\right)^2 L \\ &= \left(\frac{3}{4}\frac{.25}{.1}\right)^2 100 = 351.56 \end{aligned} \tag{19}$$

Hence $Y_A = Y_B = 10\sqrt{351.56} = 187.50$.

2.3

GNP is the sum of the GDP plus the capital income from abroad minus the capital income generated at home by foreign capital.

$$\begin{aligned} GNP_A &= GDP_A + \frac{1}{2}GDP_B && (20)\\ &= 187.5 + \frac{1}{2}187.5 = 281.25 && (21)\\ GNP_B &= GDP_B - \frac{1}{2}GDP_A && (22)\\ &= 187.5 - \frac{1}{2}187.5 = 93.75 && (23)\end{aligned}$$

If we define the consumption level, c, as the portion of the GDP that makes consumption (ie $c = C/Y$) so that $c = 1 - s$, we have

$$\begin{aligned} c_A &= 1 - s_A = 75\% && (24)\\ c_B &= 1 - s_B = 100\% && (25)\end{aligned}$$

2.4

We see that the sum of the GDPs when there is no trade ($250 + 0 = 250$) is smaller as the GDP sum when there is trade ($187.5 + 187.5 = 375$), so the world aggregate consumption is to be maximized, one has to favor a global capital market.

When there is a global capital market K_A decreased and since the labour income is defined as half the GDP of A that is in turn increasing with K. The higher is K the higher is the labour income, so one does not want to favor global capital market in this case.

2.5

As argued above, for wealthy countries (in this problem, country A), globalization decrease the capital and consequently the labor income whereas GNP increases.

3 Disinflation and oil price shocks

3.1

We have $Y = Y^*$ and thus substituting this into the SAS curve gives

$$\pi = \underbrace{\pi_{-1}}_{10} + \underbrace{Y - Y^*}_{0} + 10 \ln 0.5 = 3.069\% \qquad (26)$$

3.2

If income is to remain at $Y = 100$, then the DAD curves becomes ($Y = Y_{-1}$)

$$\pi = \mu \qquad (27)$$

Moreover note that the price of oil has fallen **permanently** to .5 and thus

$$\pi_{\mathrm{OIL},i} = \ln P_{\mathrm{OIL}}, i - \ln P_{\mathrm{OIL},i-1} = 0 \qquad (28)$$

for $i > 1$.

Which leads to the following SAS curve (for $i > 1$)

$$\pi_i = \pi_{i-1} \tag{29}$$

For every period the DAD curve adjusts to the SAS curve which gives

- Period 1:

$$\pi_1 = \quad 3.069\% \quad DAD \tag{30}$$
$$\mu_1 = \quad \pi_1 = 3.069\% \quad SAS \tag{31}$$

- Period 2:

$$\pi_2 = \quad \pi_1 = 3.069\% \quad DAD \tag{32}$$
$$\mu_2 = \quad \pi_2 = 3.069\% \quad SAS \tag{33}$$

- Period 3:

$$\pi_3 = \quad \pi_2 = 3.069\% \quad DAD \tag{34}$$
$$\mu_3 = \quad \pi_3 = 3.069\% \quad SAS \tag{35}$$

- ...

- Period n:

$$\pi_n = \quad \pi_{n-1} = 3.069\% \quad DAD \tag{36}$$
$$\mu_n = \quad \pi_n = 3.069\% \quad SAS \tag{37}$$

3.3

Here we set $\pi_1 = 0$, and solve

$$0 = 10 + (Y_1 - 100) - 6.9 \tag{38}$$

and find $Y_1 = 96.6$.

To find μ we solve

$$0 = \mu - \underbrace{\frac{1}{2}(96.9 - 100)}_{-1.55} \tag{39}$$

For period $i > 1$,eq 28 applies and the DAD curve becomes ($\pi_i = 0$)

$$Y_i = Y^* = 100 \tag{40}$$

note that for $i > 2$, the DAD curve becomes ($Y_i = Y_{i-1}$)

$$\mu_i = 0 \tag{41}$$

Keeping this in mind we find

- Period 1:

$$Y_1 \quad = \quad 96.6 \tag{42}$$
$$\mu_1 \quad = \quad .5(Y_1 - Y_{-1}) = -1.55 \tag{43}$$

- Period 2:

$$\begin{aligned} Y_2 &= 100 && (44)\\ \mu_2 &= .5(Y_2 - Y_1) = 1.55 && (45) \end{aligned}$$

- Period $i > 2$:

$$\begin{aligned} Y_i &= 100 && (46)\\ \mu_i &= .5(Y_i - Y_{i-1}) = 0 && (47) \end{aligned}$$

3.4

We have (where $P_{OIL,i} = 1$ for $i \neq 1$ and $P_{OIL,1} = .5$)

$$\begin{aligned} \pi_{OIL,1} &= \ln P_{OIL,1} - \ln P_{OIL,0} &&= -.69 && (48)\\ \pi_{OIL,2} &= \ln P_{OIL,2} - \ln P_{OIL,1} &&= .69 && (49)\\ \pi_{OIL,i} &= \ln P_{OIL,i} - \ln P_{OIL,i-1} &&= 0 \quad i > 2 && (50) \end{aligned}$$

as in 3.2 $Y_i = Y^*$ for $i \in \mathbb{N}^+$, we have

- Period 1:

$$\begin{aligned} \pi_1 = \pi_0 + 10\pi_{OIL,1} = 10 - 6.9 = 3.1 && (51)\\ \mu_1 = \pi_1 && (52) \end{aligned}$$

- Period 2:

$$\begin{aligned} \pi_2 = \pi_1 + 10\pi_{OIL,2} = 3.1 + 6.9 = 10 && (53)\\ \mu_2 = \pi_2 && (54) \end{aligned}$$

- Period $i > 2$:

$$\begin{aligned} \pi_i = \pi_{i-1} + 10\pi_{OIL,i} = 10 + 0 = 10 && (55)\\ \mu_i = \pi_i && (56) \end{aligned}$$

3.5

the sacrifice ratio is defined as

$$\text{Sacrifice ratio} = \frac{\text{total income loss}}{\text{inflation reduction}} \tag{57}$$

if we want to compute this for period 1 to 3 the formumla becomes

$$SR = 100\frac{1}{Y^*}\frac{(Y^* - Y_1) + (Y^* - Y_2) + (Y^* - Y_3)}{\pi_0 - \pi 4} \tag{58}$$

For the big-leap we have

$$SR = \frac{100}{100}\frac{(100 - 96.9) + (100 - 100) + (100 - 100)}{10} = 0.31 \tag{59}$$

Before calculating the SR when price are reduced for one year only, we first have to calculate the different Y_i and μ_i.

Firstly note that eq 48 is still valid and hence we have

- Period 1:

$$
\begin{aligned}
Y_1 &= 96.9 && (60)\\
\mu_1 &= .5(Y_1 - Y_0) = -1.55 && (61)
\end{aligned}
$$

- Period 2:

$$
\begin{aligned}
Y_2 &= 100 - 6.9 = 93.1 && (62)\\
\mu_2 &= .5(Y_2 - Y_1) = -0.95 && (63)
\end{aligned}
$$

- Period 3:

$$
\begin{aligned}
Y_3 &= 100 + 0 = 100 && (64)\\
\mu_3 &= .5(Y_3 - Y_2) = 3.45 && (65)
\end{aligned}
$$

- Period $i > 3$:

$$
\begin{aligned}
Y_i &= 100 + 0 = 100 && (66)\\
\mu_i &= .5(Y_i - Y_{i-1}) = 0 && (67)
\end{aligned}
$$

Now using 58, we find when the oil price is only reduced for one year

$$SR = \frac{100}{100}\frac{(100 - 96.9) + (100 - 93.1) + (100 - 100)}{10} = 1. \quad (68)$$