Brief Article

The Author

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1 Markov Chain

A markov chain describes the evolution of a system state over time given that we know the change of state probability for each time and state. It is described by

$$\mathbf{b}_{n+1} = A\mathbf{b}_n \tag{1}$$

where $\mathbf{b}$ is a $n \times 1$ vector and A a $n \times n$ matrix; n denoting the number of state in the system. The element a_{ij} can be read as the probability to jump from state i to state j. Note that $\sum_{j=1}^{n} a_{ij} = 1 \; \forall i \in (1, .., n)$, $\sum_{i=1}^{n} b_i = 1$ and $a_{ij}, b_i \in (0, 1)$

If A remains constant over time $(A = A(n))$ the state of the system at time n is

$$\mathbf{b}_n = A^{n-k}\mathbf{b}_k = A^n \mathbf{b}_0 \tag{2}$$

Sometimes there exists a state where the system reaches an equilibrium and is described when

$$\mathbf{b} = A\mathbf{b} \tag{3}$$

1.1 Market evolution

The market shift from bull to bear and recession can be described (see wikipedia example and PUT IMAGE) using a Markov chain.

If state

1. is Bull market
2. is Bear Market
3. is recession

and we have

$$A = \begin{pmatrix} 0.9 & 0.15 & .25 \\ 0.075 & .8 & .25 \\ 0.025 & 0.05 & .5 \end{pmatrix} \tag{4}$$

(e.g. transition from bull to bear market is $a_{21} = 7.5\%$). One can show that eventually the market will tend to

$$\mathbf{b} = A\mathbf{b} = \begin{pmatrix} 62.5\% \\ 31.25\% \\ 6.25\% \end{pmatrix} \tag{5}$$