1 Power-Law

PDF

$$f_X(x) = Cx^{-\alpha} \tag{1}$$

defined for $\Gamma \in (x_{min}, \infty)$ and $\alpha < 1$

Normalizing $\int_\Gamma f_X(x)dx = 1$ which gives

$$C = \frac{\alpha - 1}{x_{min}^{1-\alpha}} \tag{2}$$

and thus

$$f_X(x) = \frac{1-\alpha}{x_{min}} \left(\frac{x}{x_{min}} \right)^{-\alpha}. \tag{3}$$

CDF

$$F_X(x) = 1 - \left(\frac{x}{x_{min}} \right)^{1-\alpha} \tag{4}$$

Expected value

$$\mathbb{E}(X) = \int_\Gamma x f_X(x)dx = \begin{cases} \frac{\alpha-1}{\alpha-2} x_{min}, & \alpha > 2 \\ \infty, & \text{else} \end{cases} \tag{5}$$

MLE best estimator $\hat{x}_{min}$ is the smallest x_i in the data (pretty obvious ..) and

$$\hat{\alpha} = 1 + n \left(\sum_{i=1}^{n} \ln \frac{x_i}{x_{min}} \right)^{-1} \tag{6}$$

2 M-files

misc general power_law.m