

1 Probabilité: concepts

- Sum of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (1)$$

- Multiplication (when independent)

$$P(A \cap B) = P(A)P(B) \quad (2)$$

- Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (3)$$

1.1 Fonctions de probabilités

- Fonction de densité (pdf)

$$f_X(x) \in [0, 1] \quad (4)$$

$$\int_{-\infty}^{\infty} f_X(x) dx = 1 \quad (5)$$

- Fonction de répartition (cdf)

$$F_X(x) = \int_{-\infty}^x f_X(y) dy \quad (6)$$

$$F'_X(x) = f_X(x) \quad (7)$$

$$F_X(-\infty) = 0 \quad (8)$$

$$F_X(+\infty) = 1 \quad (9)$$

$$\mathbb{P}(a \leq X \leq b) = \int_a^b f_X(x) dx = F(b) - F(a) \quad (10)$$

$$\mathbb{P}(X \leq a) = \int_{-\infty}^a f_X(x) dx = F(a) \quad (11)$$

1.2 Espérance $\mathbb{E}(\cdot) = \mu$.

First moment

- discrete

$$\mathbb{E}(X) = \sum x_i P(x_i) = \sum x_i f_X(x_i) \quad (12)$$

- continuous

$$\mathbb{E}(X) = \int_{\mathbb{R}} x f_X(x) dx \quad (13)$$

Second Moment

$$\mathbb{E}(X^2) = \int_{\mathbb{R}} x^2 f_X(x) dx \quad (14)$$

In general

$$\mathbb{E}(g(X)) = \int_{\mathbb{R}} g(x) f_X(x) dx \quad (15)$$

1.2.1 Properties

$$\mathbb{E}(X + a) = \mathbb{E}(X) + a \quad (16)$$

$$\mathbb{E}(cX) = c\mathbb{E}(X) \quad (17)$$

$$\mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y) \quad (18)$$

if X and Y are independent:

$$\mathbb{E}(XY) = \mathbb{E}(X)\mathbb{E}(Y) \quad (19)$$

1.3 Variance $Var(\cdot) = \sigma^2$

Centered $(X - \mu)$ second moment

$$Var(X) = \mathbb{E}[(X - \underbrace{\mu}_{\mathbb{E}(X)})^2] \quad (20)$$

$$= \mathbb{E}(X^2) - \mathbb{E}^2(X) \quad (21)$$

$$= \int_{\mathbb{R}} (x - \mu)^2 f_X(x) dx \quad (22)$$

1.3.1 Properties

$$Var(c) = 0 \quad (23)$$

$$Var(X + c) = Var(X) \quad (24)$$

$$Var(cX) = c^2 Var(X) \quad (25)$$

$$Var(X \pm Y) = Var(X) + Var(Y) \pm 2 Cov(X, Y) \quad (26)$$

where c is a constant