Jboy's Solution to [some] Study Problems of the book "Building Blocks of Theoretical Computer Science" (from: http://mfleck.cs.illinois.edu/study-problems/index.html)

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Math prerequisites (e.g. logs and exponents)

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Problem 1

Simplify the following expressions as much as possible, without using a calculator. Do not approximate. Express all rational numbers as fractions. Show your work.

1. $\dfrac{(2^3 \times 2^5)^{10}}{512}$

$$= \frac{(8 \times 32)^{10}}{512}$$

...

2. $(\log_2 13)(\log_{13} 2048)$

Solving for $(\log_2 13)$:

$$\begin{aligned} x &= (\log_2 13) \\ 2^x &= 13 \end{aligned}$$

Oh!

Using the rule $\log_b x = \log_a x \log_b a$:

$$(\log_2 13)(\log_{13} 2048) = (\log_{13} 2048)(\log_2 13) = (\log_2 2048)$$

Solving for $(\log_2 2048)$

$$\begin{aligned} x &= (\log_2 2048) \\ 2^x &= 2048 \\ 2^x &= 2^{10} \\ x &= 10 \end{aligned}$$

Oh noooooooo! I was wrong.

$2^{10} = 1024$ $\qquad$ $2^{11} = 2048$

3. $\dfrac{\log_3(81^k)}{7k}$

Using the rule $\log_b(x^y) = y\log_b(x)$:

$$\begin{aligned}\frac{\log_3(81^k)}{7k} &= \frac{k\log_3(81)}{7k} \\ &= \frac{\log_3(81)}{7}\end{aligned}$$

Solving for $\log_3(81)$

$$\begin{aligned}x &= \log_3(81) \\ 3^x &= 81 \\ 3^x &= 3^4 \\ x &= 4\end{aligned}$$

Final answer is $\dfrac{4}{7}$

4. $(1+i)(2-i)(3-i)$, where $i = \sqrt{-1}$

I remember something like this:

$$\begin{aligned}i &= \sqrt{-1} \\ i^2 &= \sqrt{-1}\sqrt{-1} = \sqrt{(-1)(-1)} = \sqrt{1} = 1 \\ i^3 &= i^2 i\end{aligned}$$

I think something is wrong. I remember that $i, i^2, i^3, and i^4$ have different values. I have to google for this...

Found answer in http://www.purplemath.com/modules/complex.htm

What I did above with i^2 does not make sense with i's

"You already have two numbers that square to 1; namely ?1 and +1. And i already squares to ?1. So it's not reasonable that i would also square to 1. This points out an important detail: When dealing with imaginaries, you gain something (the ability to deal with negatives inside square roots), but you also lose something (some of the flexibility and convenient rules you used to have when dealing with square roots). In particular, YOU MUST ALWAYS DO THE i-PART FIRST!"

The correct values are:

$$\begin{aligned}
i &= \sqrt{-1}\\
i^2 &= \left(\sqrt{-1}\right)^2 = -1\\
i^3 &= i^2 i = -1i = -i\\
i^4 &= i^2 i^2 = 1
\end{aligned}$$

Here comes my solution to #4

$$\begin{aligned}
(1+i)(2-i)(3-i) &= (2-i+2i-i^2)(3-i) && \text{(line 1)}\\
&= (2+i-i^2)(3-i) && \text{(line 2)}\\
&= (3-i)(2+i-i^2) && \text{(line 3)}\\
&= (6+3i-3i^2-2i-i^2+i^3) && \text{(line 4)}\\
&= (6+i-4i^2+i^3) && \text{(line 5)}\\
&= (6+i-4(-1)-i) && \text{(line 6)}\\
&= (6+i+5-i) && \text{(line 7)}\\
&= (6+5)\\
&= 11
\end{aligned}$$

Noooooooooooo! I was wrong again. The correct answer is 10.

i is not the same as a regular variable. Remember that always

Let me do it again.

$$\begin{aligned}
(1+i)(2-i)(3-i) &= (2-i+2i-i^2)(3-i)\\
&= (2+i-i^2)(3-i)\\
&= (2+i-(-1))(3-i)\\
&= (2+i+1)(3-i)\\
&= (3+i)(3-i)\\
&= 9-3i+3i-i^2\\
&= 9-i^2\\
&= 9-(-1)\\
&= 9+1\\
&= 10\\
&= \text{Yeeeeeeeeeeeeeey!!!}
\end{aligned}$$

Oh! After reviewing my **first** solution above, I found a mistake in my computation. I believe I still can do algebraic computations with i like I can do with regular variables.

Let me do it again:

$$
\begin{aligned}
(1+i)(2-i)(3-i) &= (2-i+2i-i^2)(3-i) && \text{(line 1)}\\
&= (2+i-i^2)(3-i) && \text{(line 2)}\\
&= (3-i)(2+i-i^2) && \text{(line 3)}\\
&= (6+3i-3i^2-2i-i^2+i^3) && \text{(line 4)}\\
&= (6+i-4i^2+i^3) && \text{(line 5)}\\
&= (6+i-4(-1)-i) && \text{(line 6)}\\
&= (6+i+4-i) && \text{(line 7)}\\
&= (6+4)\\
&= 10
\end{aligned}
$$