

Jboy's Solution to [some] Exercises in The Art of Problem Solving
Volume 1

Jboy Flaga

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Chapter 1: Exponents and Logarithms

EXERCISE 1-1 Evaluate each of the following.

(August 5, 2016 about 12:10 AM)

i. $3^4 = 3 \times 27 = 81$

ii. $2^5 2^2 = 2^7 = 128$

iii. $5^{-3} 5^5 5^{-1} 5^1 = 5$

iv. $4^3/4 = 4^2 = 16$

v. $2^7/2^2 = 2^5 = 32$

vi. $(3^4 3^{-2})/(3^5 3^{-2}) = 3^2/3^3 = \frac{1}{3}$

vii. $2^5 3^2 2^{-3} = 2^2 3^2 = 4 \times 9 = 36$

viii. $5^2 3^{-1} 2^4 5^{-1} 2^{-2} = (5 \times 2^2)/3 = \frac{20}{3} = 6\frac{2}{3}$

EXERCISE 1-2 Try these.

i. $9^{3/2} = (9^{1/2})^3 = 3^3 = 9$

ii. $\left(\sqrt[3]{81}\right)^{3/2} = (81^{1/3})^{3/2} = 81^{(1/3)(3/2)} = 81^{1/2} = 9$

iii. $64^{-4/3} = \left(64^{1/3}\right)^{-4} = 4^{-4} = \frac{1}{4^4} = \frac{1}{64 \times 4}$

iv.

v.

vi.

EXERCISE 1-3 Find all real x in each of the following.

i. $x = (-2)^5$ $x = -32$

ii. $x = \sqrt[3]{-1/8}$ $x = (-1/8)^{1/3}$

$$x = \left(\frac{-1^{1/3}}{8^{1/3}} \right)$$

$$x = \left(\frac{-1^{1/3}}{8^{1/3}} \right)$$

$$x = -\frac{1}{2}$$

iii. $x^6 = 64$ $x^6 = 2^6$
 $x = 2$

iv. $x^3 = 64$ $x^3 = 4^3$
 $x = 4$

v.

vi. $x^{5/3} = 243$ $(x^{1/3})^5 = (27^{1/3})^5$
 $x = 27$

EXAMPLE 1-11

(August 5, 2016 11:37 PM)

Here, I will try to answer #3 of Example 1-11 and see if my answer is the same with that in the book

#3. Simplify $\sqrt[6]{6912}$

I will first create a table of some prime numbers with exponents:

$$\begin{aligned} 2^2 &= 4 \\ 2^3 &= 8 \\ 2^4 &= 16 \\ 2^5 &= 32 \\ 2^6 &= 64 \\ 2^7 &= 128 \end{aligned}$$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

Now I will do some trial and error to find the prime factorization of 6912

$$81 \times 128 = 10,368$$

$$81 \times 64 = 5,184$$

$$27 \times 128 = 3,456$$

Let's try to use 2^8

$$2^8 = 256$$

$$256 \times 27 = 6,912$$

$$2^8 \times 3^3 = 6,912$$

I found it!

$$\sqrt[6]{6912} = \sqrt[6]{2^8 3^3} = 2^{8/6} 3^{3/6} = 2^{4/3} 3^{1/2} = \left(2^{1/3}\right)^4 3^{1/2}$$

I'm stuck! I'm going to look for a hint.

Ahh! He did $2 \times 2^{1/3}$.

I think he went through this step:

$$\dots = 2^{4/3} 3^{1/2} = (\sqrt[3]{2^4}) 3^{1/2} = (\sqrt[3]{2^3 2^1}) 3^{1/2} = 2(\sqrt[3]{2}) 3^{1/2} = 2(\sqrt[3]{2})(\sqrt{3})$$

The answer from the book is $2\sqrt[6]{108}$

But I used <http://web2.0calc.com/> to verify if my answer is also correct.

It is correct!

$$\begin{aligned} \sqrt[6]{6912} &= 4.3644945438868856 \\ 2(\sqrt[3]{2})(\sqrt{3}) &= 4.3644945438868856 \\ 2\sqrt[6]{108} &= 4.3644945438868856 \end{aligned}$$

Yeeey!

EXERCISE 1-4 Find the following

I will not be doing Exercise 1-4 anymore. I can use a calculator to do this. Hahahaha.

i. $\sqrt{27} =$

...

vi. $\sqrt{\frac{56}{126}} =$