

$$\frac{\bar{Y}_1 - \bar{Y}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim T_{df}$$

$$\frac{\bar{Y}_1 - \bar{Y}_2 - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim T_{n_1+n_2-2}$$

$$P - value = P(T_{13} \geq 0.8488 | \mu_1 = \mu_2) = 0.2057$$

$$H_0 : \mu_1 - \mu_2 = 0 \iff H_0 : \mu_1 = \mu_2$$

$$H_A : \mu_1 - \mu_2 > 0 \iff H_A : \mu_1 > \mu_2$$

$$t_{obs} = \frac{(1.679 - 1.520) - 0}{\sqrt{\frac{(8-1)0.3743^2 + (7-1)0.3469^2}{8+7-2} \left(\frac{1}{8} + \frac{1}{7} \right)}} = 0.8488$$

$$t_{obs} = \frac{(1.679 - 1.520) - 0}{\sqrt{0.3743^2/8 + 0.3469^2/7}} = 0.8535$$

$$\frac{\bar{V}_1 - \bar{V}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim T_{df}$$

$$\frac{\bar{V}_1 - \bar{V}_2 - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \sim T_{n_1+n_2-2}$$

$$SE(\bar{V}_1 - \bar{V}_2) = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} = \sqrt{S_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$SE(\bar{V}_1 - \bar{V}_2) = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$SD(\bar{V}_1 - \bar{V}_2) = \sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}$$

$$\frac{(\bar{V}_1 - \bar{V}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \sim N(0, 1)$$

$$\bar{V}_1 - \bar{V}_2 \sim N(\mu_1 - \mu_2, \sigma_1^2/n_1 + \sigma_2^2/n_2)$$

$$\bar{V}_1 \sim N(\mu_1, \sigma_1^2/n_1) \quad \bar{V}_2 \sim N(\mu_2, \sigma_2^2/n_2)$$

$$V_{11} = \log_{10}(Y_{11}), V_{12} = \log_{10}(Y_{12}), \dots, V_{1n_1} = \log_{10}(Y_{1n_1})$$

$$V_{21} = \log_{10}(Y_{21}), V_{22} = \log_{10}(Y_{22}), \dots, V_{2n_2} = \log_{10}(Y_{2n_2})$$

$$Var(\bar{Y}_1 - \bar{Y}_2) = \sigma_1^2/n_1 + \sigma_2^2/n_2$$

$$Y_{11}, Y_{12}, \dots, Y_{1n_1}$$

$$Y_{21}, Y_{22}, \dots, Y_{2n_2}$$

$$P\left(-t_{n-1,\alpha/2} < \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} < t_{n-1,\alpha/2}\right) = 1 - \alpha$$

$$P\left(-t_{n-1,\alpha/2}S_D/\sqrt{n} < \bar{D} - \mu_D < t_{n-1,\alpha/2}S_D/\sqrt{n}\right) = 1 - \alpha$$

$$P\left(-\bar{D} - t_{n-1,\alpha/2}S_D/\sqrt{n} < -\mu_D < -\bar{D} + t_{n-1,\alpha/2}S_D/\sqrt{n}\right) = 1 - \alpha$$

$$P\left(\bar{D} + t_{n-1,\alpha/2}S_D/\sqrt{n} > \mu_D > \bar{D} - t_{n-1,\alpha/2}S_D/\sqrt{n}\right) = 1 - \alpha$$

$$P\left(\bar{D} - t_{n-1,\alpha/2}S_D/\sqrt{n} < \mu_D < \bar{D} + t_{n-1,\alpha/2}S_D/\sqrt{n}\right) = 1 - \alpha$$

$$t_{\nu,\alpha} \quad t_{\nu,0.1/2} \quad -t_{\nu,0.1/2}$$

$$t_{\nu,\alpha} \quad t_{\nu,\alpha/2} \quad -t_{\nu,\alpha/2}$$

$$\frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} \sim t_{n-1}$$

$$\frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} \sim N(0, 1)$$

$$-3.52 \pm 1.968(13.60)/\sqrt{300} = (-5.06, -1.97)$$

$$\bar{d} \pm t_{n-1,0.025} s_D/\sqrt{n}$$

$$\bar{D} \pm t_{n-1,\alpha/2} S_D/\sqrt{n}$$

$$\alpha$$

$$\frac{\bar{Y} - \mu}{S/\sqrt{n}} \sim T_{n-1}$$

$$\frac{\bar{Y} - \mu}{S/\sqrt{n}} \sim ?$$

$$Z = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

$$T \sim t_\nu \quad E(T) = \nu \quad Var(T) = 2\nu$$

$$\mu_1 \quad \mu_2 \quad \mu_D$$

$$\bar{Y}_1 - \bar{Y}_2$$

$$E\left(\bar{Y}_1 - \bar{Y}_2\right) = E\left(\bar{Y}_1\right) - E\left(\bar{Y}_2\right) = \mu_1 - \mu_2 = \mu_D$$

$$Var\left(\bar{Y}_1 - \bar{Y}_2\right) = \sigma_1^2/n + \sigma_2^2/n$$

$$\bar{D} = \frac{1}{n} \sum_{i=1}^n D_i = \frac{1}{n} \sum_{i=1}^n (Y_{1i} - Y_{2i})$$

$$E(\bar{D}) = E\left(\frac{1}{n} \sum_{i=1}^n (Y_{1i} - Y_{2i})\right) = \frac{1}{n} \sum_{i=1}^n (E(Y_{1i}) - E(Y_{2i})) = \frac{1}{n} \sum_{i=1}^n (\mu_1 - \mu_2) = \mu_D$$

$$E(\bar{Y}_1) = E\left(\frac{1}{n} \sum_{i=1}^n Y_{1i}\right) = \frac{1}{n} \sum_{i=1}^n E(Y_{1i}) = \frac{1}{n} \sum_{i=1}^n \mu_1 = \mu_1$$

$$\begin{aligned} Var(\bar{Y}_1) &= Var\left(\frac{1}{n} \sum_{i=1}^n Y_{1i}\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^n Y_{1i}\right) \\ &= \frac{1}{n^2} \left(\sum_{i=1}^n Var(Y_{1i}) + \sum_{i \neq j} Cov(Y_{1i}, Y_{1j}) \right) = \frac{1}{n^2} \sum_{i=1}^n \sigma_1^2 = \sigma_1^2/n \end{aligned}$$

$$Var(aX) = a^2 Var(X)$$

$$Var(X - Y) = Var(X) + Var(Y) - 2Cov(X, Y)$$

$$Var(\bar{Y}_1 - \bar{Y}_2) = Var(\bar{Y}_1) + Var(\bar{Y}_2)$$

$$\begin{aligned} Var(\bar{D}) &= Var\left(\frac{1}{n} \sum_{i=1}^n D_i\right) + \frac{1}{n^2} Var\left(\sum_{i=1}^n D_i\right) = \frac{1}{n^2} \sum_{i=1}^n Var(D_i) \\ &= \frac{1}{n} Var(D_1) = \frac{1}{n} Var(Y_{11} - Y_{21}) = \frac{1}{n} (Var(Y_{11}) + Var(Y_{21}) - 2Cov(Y_{11}, Y_{21})) \\ &= \sigma_1^2/n + \sigma_2^2/n - 2Cov(Y_{11}, Y_{21})/n = \sigma_D^2/n \\ Var(\bar{D}) &= \sigma_1^2/n + \sigma_2^2/n - 2Cov(Y_{11}, Y_{21}) \end{aligned}$$

$$E(\bar{D}) = \mu_D$$

$$E(\bar{Y}_1 - \bar{Y}_2) = \mu_D$$

$$Z = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

$$\bar{Y} \sim N(\mu, \sigma^2/n)$$

$$\sigma_D^2$$

$$\bar{D} \sim N(\mu_D, \sigma_D^2/n)$$

$$T = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} \sim t_{n-1}$$