$$\hat{p}_1 = \frac{\text{\# with a degree that care a lot}}{\text{Total \# with a degree}} = \frac{181}{620}$$

$$\hat{p}_2 = \frac{\text{\# with no degree that care a lot}}{\text{Total \# with no degree}} = \frac{97}{406}$$

$$\hat{p}_1 - \hat{p}_2 = \frac{181}{620} - \frac{97}{406} = 0.053$$

$$SE(\hat{p}_1 - \hat{p}_2) \approx 0.0280$$

$$\hat{p}_1 - \hat{p}_2$$

$$P(\hat{p}_1 - \hat{p}_2 < 0)$$

$$\hat{p}$$

$$\hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$$

$$Y = \begin{cases} 0 & \text{person doesn't care} \\ 1 & \text{person cares} \end{cases}$$

$$Y \sim Ber(p)$$

$$p(y) = P(Y = y) = \begin{cases} 1 - p & \text{if } y = 0 \\ p & \text{if } y = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$p(y) = \begin{cases} 1 - p & \text{if } y = 0 \\ p & \text{if } y = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$Y_i = \begin{cases} 0 & \text{person i doesn't care} \\ 1 & \text{person i cares} \end{cases}$$

$$Y \sim Bin(5, p)$$

$$Y \sim Bin(n, p)$$

$$P(\text{1st F} \cap \text{2nd F} \cap \text{3rd F} \cap \text{4th F} \cap \text{5th F})$$

$$P(A \cap B) = P(A)P(B)$$

$$= P(\text{1st F})P(\text{2nd F})P(\text{3rd F})P(\text{4th F})P(\text{5th F})$$

$$= (1-p)(1-p)(1-p)(1-p)(1-p) = (1-p)^5$$

$$P(\text{1st S} \cap \text{2nd F} \cap \text{3rd F} \cap \text{4th F} \cap \text{5th F})$$

$$= P(\text{1st S})P(\text{2nd F})P(\text{3rd F})P(\text{4th F})P(\text{5th F})$$

$$= p(1-p)(1-p)(1-p)(1-p) = p(1-p)^4$$

$$P(\text{1st S} \cap \text{2nd S} \cap \text{3rd F} \cap \text{4th F} \cap \text{5th F})$$

$$= P(\text{1st S})P(\text{2nd S})P(\text{3rd F})P(\text{4th F})P(\text{5th F})$$

$$= (p)(p)(1-p)(1-p)(1-p) = p^2(1-p)^3$$

$$\binom{5}{2} = \frac{5!}{2!(5-2)!} = 10$$

$$\binom{n}{y} = \frac{n!}{y!(n-y)!}$$

$$Y \sim Bin(n, p)$$

$$p(y) = \begin{cases} \binom{n}{y} p^y (1-p)^{n-y} & y = 0, 1, ..., n \\ 0 & \text{otherwise} \end{cases}$$

$$Y \sim Bin(25, 0.1)$$

$$Y \sim Bin(25, 0.2)$$

$$Y \sim Bin(25, 0.4)$$

$$Y \sim Bin(100, 0.2)$$

$$P(Y \geq 9) = \sum_{y=9}^{25} \binom{25}{y} 0.2^y 0.8^{25-y} = 0.0468$$

$$P(Y > 9) = \sum_{y=10}^{25} \binom{25}{y} 0.2^y 0.8^{25-y} = 0.0173$$

$$P(\hat{p} < 0.2) = P(Y/25 < 0.2) = P(Y < 5)$$

$$= \sum_{y=0}^{4} \binom{25}{y} 0.2^y 0.8^{25-y} = 0.4207$$

$$\hat{p} = \frac{Y}{n}$$

$$Y \sim N(\mu, \sigma^2)$$

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(y-\mu)^2} \text{ for } -\infty < y < \infty$$

$$F(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(y-\mu)^2} dy$$

$$Z = \frac{Y-\mu}{\sigma} \sim N(0,1)$$

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}z^2} dz$$

$$\hat{p} \sim N\left(0.2, \frac{0.2 * 0.8}{100}\right) \sim N(0.2, 0.0016)$$

$$P(\hat{p} < 0.15) = P\left(\frac{\hat{p} - 0.2}{\sqrt{0.0016}} < \frac{0.15 - 0.2}{\sqrt{0.0016}}\right)$$

$$= P(Z < -1.25) = \Phi(-1.25) = 0.1056$$

$$E(Y) = \sum_{support} yp(y)$$

$$E(Y) = \int_{-\infty}^{\infty} yf(y)dy$$

$$E(Y) = \sum_{y=0}^{n} y \binom{n}{y} p^y (1-p)^{n-y} = np$$

$$E(\hat{p}) = E\left(\frac{Y}{n}\right) = \sum_{y=0}^{n} \frac{y}{n} \binom{n}{y} p^y (1-p)^{n-y}$$

$$= \frac{1}{n} \sum_{y=0}^{n} \frac{y}{n} \binom{n}{y} p^y (1-p)^{n-y} = \frac{1}{n} np = p$$

$$E(a + bY) = a + bE(Y)$$

$$E(g(Y)) = \begin{cases} \sum_{support} g(y)f(y) & \text{if Y discrete} \\ \int_{-\infty}^{\infty} g(y)f(y)dy & \text{if Y continuous} \end{cases}$$

$$Var(Y) = E\left((Y - E(Y))^2\right) = E(Y^2) - (E(Y))^2$$

$$Var(Y) = E(Y^2) - (np)^2 = \sum_{y=0}^{n} y^2 \binom{n}{y} p^y (1-p)^{n-y} - (np)^2$$

$$= n(n-1)p^2 + np - (np)^2 = np(1-p)$$

$$Var(\hat{p}) = Var\left(\frac{Y}{n}\right) = E\left(\left(\frac{Y}{n} - \frac{np}{n}\right)^2\right) = \frac{1}{n^2} Var(Y) = \frac{p(1-p)}{n}$$

$$Var(a + bY) = b^2 Var(Y)$$

$$Y \sim Bin(n, p)$$

$$Y \sim N(np, np(1-p))$$

$$\hat{p} = \frac{Y}{n} \sim N\left(p, \frac{p(1-p)}{n}\right)$$

$$Y_1 \sim Bin(n_1, p_1), Y_2 \sim Bin(n_2, p_2)$$

$$\hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$$

$$n_1\hat{p}_1 \geq 10, n_1(1-\hat{p}_1) \geq 10, n_2\hat{p}_2 \geq 10, n_2(1-\hat{p}_2) \geq 10$$

$$\hat{p}_1 - \hat{p}_2 \pm 1.96\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$\hat{p}_1 - \hat{p}_2 = 181/620 - 97/406 = 0.053$$

$$SE(\hat{p}_1 - \hat{p_1}) \approx \sqrt{\frac{(181/620)(1-181/620)}{620} + \frac{(97/406)(1-97/406)}{406}} = 0.0280$$

$$\hat{p} \pm z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\hat{p}_1 - \hat{p}_2 \pm z^*\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$\text{Point Estimate} \pm \text{Margin of Error}$$

$$\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$$

$$z^* SE(\text{Point Estimator})$$

$$Y = \mu + \sigma Z \sim N(\mu, \sigma^2)$$

$$\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$$

$$\frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} \sim N(0,1)$$

$$P\left(-1.96 < \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} < 1.96\right) = 0.95$$

$$P\left(-1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < \hat{p} - p < 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 0.95$$

$$P\left(-\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < -p < -\hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 0.95$$

$$P\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 0.95$$

$$\Longleftrightarrow$$

$$\textbf{Point Estimate} \pm z^* SE(\textbf{Point Estimator})$$

$$\hat{p} = 276/480$$

$$SE(\hat{p}) \approx \sqrt{\frac{(276/480)(1 - 276/480)}{480}}$$

$$276/480 \pm 2.58 * \sqrt{\frac{(276/480)(1 - 276/480)}{480}}$$

$$n\hat{p} \geq 10 \qquad n(1 - \hat{p}) \geq 10$$

$$\hat{p}_1 - \hat{p}_2 = 175/290 - 101/190 = 0.072$$

$$SE(\hat{p}_1 - \hat{p}_2) \approx= 0.046$$

$$0.072 \pm 2.58 * 0.046$$