$$t \qquad n_t$$

$$Y_1 = \# \text{ of ionized bulbs that are marketable} \sim Bin(180, p_1)$$

$$Y_2 = \# \text{ of non-ionized bulbs that are marketable} \sim Bin(180, p_2)$$

$$Y \sim Bin(n,p) \qquad E(Y) = np \qquad Var(Y) = np(1-p)$$

$$\hat{p} = \frac{Y}{n} \qquad E(\hat{p}) = p \qquad Var(\hat{p}) = \frac{p(1-p)}{n}$$

$$\hat{p}_1 = \frac{Y_1}{n_1} \qquad E(\hat{p}_1) = p_1 \qquad Var(\hat{p}_1) = \frac{p_1(1-p_1)}{n_1}$$

$$\hat{p}_2 = \frac{Y_2}{n_2} \qquad E(\hat{p}_2) = p_2 \qquad Var(\hat{p}_2) = \frac{p_2(1-p_2)}{n_2}$$

$$\hat{p}_1 = \frac{Y_1}{n_1} = \text{Proportion of ionized bulbs that are marketable}$$

$$\hat{p}_2 = \frac{Y_2}{n_2} = \text{Proportion of non-ionized bulbs that are marketable}$$

$$Var(Y) = E\left((Y - E(Y))^2\right) = E(Y^2) - (E(Y))^2 = \sum_{y=0}^{n} y^2 \binom{n}{y} p^y (1-p)^{n-y} - (np)^2$$

$$f_{Y_1,Y_2}(y_1,y_2) = f_{Y_1}(y_1) f_{Y_2}(y_2) = f(y_1) f(y_2)$$

$$E(Y_1 Y_2) = \int\int y_1 y_2 f_{Y_1,Y_2}(y_1,y_2)\,dy_1 dy_2$$

$$= \int\int y_1 y_2 f_{Y_1}(y_1) f_{Y_2}(y_2)\,dy_1 dy_2 = \left(\int y_1 f_{Y_1}(y_1)\,dy_1\right)\left(\int y_2 f_{Y_2}(y_2)\,dy_2\right) = E(Y_1)E(Y_2)$$

$$E(\hat{p}_1 - \hat{p}_2) = E(\hat{p}_1) - E(\hat{p}_2) = p_1 - p_2$$

$$Var(\hat{p}_1 - \hat{p}_2) = Var(\hat{p}_1) + Var(\hat{p}_2) - 2Cov(\hat{p}_1, \hat{p}_2)$$

$$= \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}$$

$$Var(aY_1 + bY_2 + c) = a^2 Var(Y_1) + b^2 Var(Y_2) + 2ab Cov(Y_1, Y_2)$$

$$E(aY_1 + bY_2 + c) = aE(Y_1) + bE(Y_2) + c$$

$$Cov(Y_1,Y_2) = E\left[(Y_1 - E(Y_1))(Y_2 - E(Y_2))\right] = E(Y_1Y_2) - E(Y_1)E(Y_2)$$

$$Cov(Y_1,Y_1) = E\left[(Y_1 - E(Y_1))^2\right] = Var(Y_1)$$

$$E(Y_1 + Y_2 + Y_3 + Y_4 + Y_5) = p + p + p + p + p = 5p \qquad (np)$$

$$Var(Y_1 + Y_2 + Y_3 + Y_4 + Y_5) = p^2 + p^2 + p^2 + p^2 + p^2 = 5p^2 \qquad (np^2)$$

$$n_1\hat{p}_1 \geq 10, n_1(1-\hat{p}_1) \geq 10, n_2\hat{p}_2 \geq 10, n_2(1-\hat{p}_2) \geq 10$$

$$Cov(\hat{p}_1, \hat{p}_2) = E\left((\hat{p}_1 - p_1)(\hat{p}_2 - p_2)\right) = E(\hat{p}_1 - p_1)E(\hat{p}_2 - p_2) = 0$$

$$\hat{p}_1 = \text{Sample proportion of ionized bulbs that are marketable} = \frac{153}{180}$$

$$\hat{p}_2 = \text{Sample proportion of non-ionized bulbs that are marketable} = \frac{119}{180}$$

$$\hat{p}_1 - \hat{p}_2$$

$$\neq$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$\hat{p} = \frac{Y_1 + Y_2}{n_1 + n_2}$$

$$\hat{p}_1 - \hat{p}_2 = 34/180$$

$$z = \frac{34/180}{\sqrt{\frac{272}{360}\left(1 - \frac{272}{360}\right)\left(\frac{1}{180} + \frac{1}{180}\right)}} = 4.17$$

$$P(Z \geq 4.17 | \text{No Difference in } p_1 - p_2) \approx 0$$

$$P\left(\hat{p}_1 - \hat{p}_2 \geq 34/180 | p_1 - p_2 = 0\right)$$

$$= P(Z \geq 4.17 | p_1 - p_2 = 0) \approx 0$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} \sim N(0,1)$$

$$z = \frac{0.19 - 0.27}{\sqrt{\frac{46}{200}\left(1 - \frac{46}{200}\right)\left(\frac{1}{100} + \frac{1}{100}\right)}} = -1.34$$

$$P(Z \leq -1.34) = \int_{-\infty}^{-1.34} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz = 0.0901$$

$$E\left(\hat{p}_1 - \hat{p}_2\right) = E\left(\hat{p}_1\right) - E\left(\hat{p}_2\right)$$

$$Var(\hat{p}_1 - \hat{p}_2)$$

$$E(g(Y_1, Y_2)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(y_1, y_2) f_{Y_1, Y_2}(y_1, y_2) dy_1 dy_2$$

$$E(\hat{p}_1 - \hat{p}_2) = \int \int_{supports} (\hat{p}_1 - \hat{p}_2) f_{\hat{p}_1, \hat{p}_2}(\hat{p}_1, \hat{p}_2) d\hat{p}_1 d\hat{p}_2$$

$$= \int \int_{supports} \hat{p}_1 f_{\hat{p}_1}(\hat{p}_1) f_{\hat{p}_2}(\hat{p}_2) d\hat{p}_1 d\hat{p} - \int \int_{supports} \hat{p}_2 f_{\hat{p}_1}(\hat{p}_1) f_{\hat{p}_2}(\hat{p}_2) d\hat{p}_1 d\hat{p}$$