

$$t \quad n_t$$

$$Y_1 = \# \text{ of ionized bulbs that are marketable} \sim \text{Bin}(180, p_1)$$

$$Y_2 = \# \text{ of non-ionized bulbs that are marketable} \sim \text{Bin}(180, p_2)$$

$$Y \sim \text{Bin}(n, p) \quad E(Y) = np \quad \text{Var}(Y) = np(1 - p)$$

$$\hat{p} = \frac{Y}{n} \quad E(\hat{p}) = p \quad \text{Var}(\hat{p}) = \frac{p(1 - p)}{n}$$

$$\hat{p}_1 = \frac{Y_1}{n_1} \quad E(\hat{p}_1) = p_1 \quad \text{Var}(\hat{p}_1) = \frac{p_1(1 - p_1)}{n_1}$$

$$\hat{p}_2 = \frac{Y_2}{n_2} \quad E(\hat{p}_2) = p_2 \quad \text{Var}(\hat{p}_2) = \frac{p_2(1 - p_2)}{n_2}$$

$$\hat{p}_1 = \frac{Y_1}{n_1} = \text{Proportion of ionized bulbs that are marketable}$$

$$\hat{p}_2 = \frac{Y_2}{n_2} = \text{Proportion of non-ionized bulbs that are marketable}$$

$$\text{Var}(Y) = E((Y - E(Y))^2) = E(Y^2) - (E(Y))^2 = \sum_{y=0}^n y^2 \binom{n}{y} p^y (1 - p)^{n-y} - (np)^2$$

$$f_{Y_1, Y_2}(y_1, y_2) = f_{Y_1}(y_1) f_{Y_2}(y_2) = f(y_1) f(y_2)$$

$$E(Y_1 Y_2) = \int \int y_1 y_2 f_{Y_1, Y_2}(y_1, y_2) dy_1 dy_2$$

$$= \int \int y_1 y_2 f_{Y_1}(y_1) f_{Y_2}(y_2) dy_1 dy_2 = \left(\int y_1 f_{Y_1}(y_1) dy_1 \right) \left(\int y_2 f_{Y_2}(y_2) dy_2 \right) = E(Y_1) E(Y_2)$$

$$E(\hat{p}_1 - \hat{p}_2) = E(\hat{p}_1) - E(\hat{p}_2) = p_1 - p_2$$

$$\text{Var}(\hat{p}_1 - \hat{p}_2) = \text{Var}(\hat{p}_1) + \text{Var}(\hat{p}_2) - 2\text{Cov}(\hat{p}_1, \hat{p}_2)$$

$$= \frac{p_1(1 - p_1)}{n_1} + \frac{p_2(1 - p_2)}{n_2}$$

$$\text{Var}(aY_1 + bY_2 + c) = a^2 \text{Var}(Y_1) + b^2 \text{Var}(Y_2) + 2ab \text{Cov}(Y_1, Y_2)$$

$$E(aY_1 + bY_2 + c) = aE(Y_1) + bE(Y_2) + c$$

$$\text{Cov}(Y_1, Y_2) = E[(Y_1 - E(Y_1))(Y_2 - E(Y_2))] = E(Y_1 Y_2) - E(Y_1)E(Y_2)$$

$$\text{Cov}(Y_1, Y_1) = E[(Y_1 - E(Y_1))^2] = \text{Var}(Y_1)$$

$$E(Y_1 + Y_2 + Y_3 + Y_4 + Y_5) = p + p + p + p + p = 5p \quad (np)$$

$$\text{Var}(Y_1 + Y_2 + Y_3 + Y_4 + Y_5) = p^2 + p^2 + p^2 + p^2 + p^2 = 5p^2 \quad (np^2)$$

$$n_1 \hat{p}_1 \geq 10, n_1(1 - \hat{p}_1) \geq 10, n_2 \hat{p}_2 \geq 10, n_2(1 - \hat{p}_2) \geq 10$$

$$Cov(\hat{p}_1, \hat{p}_2) = E((\hat{p}_1 - p_1)(\hat{p}_2 - p_2)) = E(\hat{p}_1 - p_1)E(\hat{p}_2 - p_2) = 0$$

$$\hat{p}_1 = \text{Sample proportion of ionized bulbs that are marketable} = \frac{153}{180}$$

$$\hat{p}_2 = \text{Sample proportion of non-ionized bulbs that are marketable} = \frac{119}{180}$$

$$\hat{p}_1 - \hat{p}_2$$

$$\neq$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$$

$$\hat{p} = \frac{Y_1 + Y_2}{n_1 + n_2}$$

$$\hat{p}_1 - \hat{p}_2 = 34/180$$

$$z = \frac{34/180}{\sqrt{\frac{272}{360} \left(1 - \frac{272}{360}\right) \left(\frac{1}{180} + \frac{1}{180}\right)}} = 4.17$$

$$P(Z > 4.17 | \text{No Difference in } p_1 - p_2) \approx 0$$

$$P(\hat{p}_1 - \hat{p}_2 > 34/180 | p_1 - p_2 = 0)$$

$$= P(Z > 4.17 | p_1 - p_2 = 0) \approx 0$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} \sim N(0, 1)$$

$$z = \frac{0.19 - 0.27}{\sqrt{\frac{46}{200} \left(1 - \frac{46}{200}\right) \left(\frac{1}{100} + \frac{1}{100}\right)}} = -1.34$$

$$P(Z \leq -1.34) = \int_{-\infty}^{-1.34} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = 0.0901$$

$$E(\hat{p}_1 - \hat{p}_2) = E(\hat{p}_1) - E(\hat{p}_2)$$

$$Var(\hat{p}_1 - \hat{p}_2)$$

$$E(g(Y_1, Y_2)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(y_1, y_2) f_{Y_1, Y_2}(y_1, y_2) dy_1 dy_2$$

$$E(\hat{p}_1 - \hat{p}_2) = \int \int_{\text{supports}} (\hat{p}_1 - \hat{p}_2) f_{\hat{p}_1, \hat{p}_2}(\hat{p}_1, \hat{p}_2) d\hat{p}_1 d\hat{p}_2$$

$$= \int \int_{\text{supports}} \hat{p}_1 f_{\hat{p}_1}(\hat{p}_1) f_{\hat{p}_2}(\hat{p}_2) d\hat{p}_1 d\hat{p}_2 - \int \int_{\text{supports}} \hat{p}_2 f_{\hat{p}_1}(\hat{p}_1) f_{\hat{p}_2}(\hat{p}_2) d\hat{p}_1 d\hat{p}_2$$