$$\begin{split} P\left(-t_{n-1,\alpha/2} < \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} < t_{n-1,\alpha/2}\right) &= 1 - \alpha \\ P\left(-t_{n-1,\alpha/2}S_D/\sqrt{n} < \bar{D} - \mu_D < t_{n-1,\alpha/2}S_D/\sqrt{n}\right) &= 1 - \alpha \\ P\left(-\bar{D} - t_{n-1,\alpha/2}S_D/\sqrt{n} < -\mu_D < -\bar{D} + t_{n-1,\alpha/2}S_D/\sqrt{n}\right) &= 1 - \alpha \\ P\left(\bar{D} + t_{n-1,\alpha/2}S_D/\sqrt{n} > \mu_D > \bar{D} - t_{n-1,\alpha/2}S_D/\sqrt{n}\right) &= 1 - \alpha \\ P\left(\bar{D} - t_{n-1,\alpha/2}S_D/\sqrt{n} > \mu_D > \bar{D} - t_{n-1,\alpha/2}S_D/\sqrt{n}\right) &= 1 - \alpha \\ P\left(\bar{D} - t_{n-1,\alpha/2}S_D/\sqrt{n} < \mu_D < \bar{D} + t_{n-1,\alpha/2}S_D/\sqrt{n}\right) &= 1 - \alpha \\ t_{\nu,\alpha} \quad t_{\nu,\alpha/2} \quad -t_{\nu,\alpha/2} \\ & \quad t_{\nu,\alpha} \quad t_{\nu,\alpha/2} \quad -t_{\nu,\alpha/2} \\ & \quad \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} \sim t_{n-1} \\ & \quad \frac{\hat{p} - p}{\sqrt{\frac{p(1-\bar{p})}{p}}} \sim N(0,1) \\ & \quad -3.52 \pm 1.968(13.60)/\sqrt{300} = (-5.06, -1.97) \\ & \quad \bar{d} \pm t_{n-1,\alpha/2}S_D/\sqrt{n} \\ & \quad \bar{T} - \mu \\ & \quad \bar{S}/\sqrt{n} \sim T_{n-1} \\ & \quad \bar{Y} - \mu \\ & \quad \bar{S}/\sqrt{n} \sim T_{n-1} \\ & \quad \bar{Y} - \mu \\ & \quad \bar{S}/\sqrt{n} \sim T_{n-1} \\ & \quad \bar{Y} - \mu \\ & \quad \bar{T} - 1 \\ & \quad \bar{$$

$$\begin{split} E\left(\bar{D}\right) &= E\left(\frac{1}{n}\sum_{i=1}^{n}(Y_{1i}-Y_{2i})\right) = \frac{1}{n}\sum_{i=1}^{n}\left(E\left(Y_{1i}\right)-E\left(Y_{2i}\right)\right) = \frac{1}{n}\sum_{i=1}^{n}(\mu_{1}-\mu_{2}) = \mu_{D} \\ E\left(\bar{Y}_{1}\right) &= E\left(\frac{1}{n}\sum_{i=1}^{n}Y_{1i}\right) = \frac{1}{n}\sum_{i=1}^{n}E(Y_{1i}) = \frac{1}{n}\sum_{i=1}^{n}\mu_{1} = \mu_{1} \\ Var\left(\bar{Y}_{1}\right) &= Var\left(\frac{1}{n}\sum_{i=1}^{n}Y_{1i}\right) = \frac{1}{n^{2}}Var\left(\sum_{i=1}^{n}Y_{1i}\right) \\ &= \frac{1}{n^{2}}\left(\sum_{i=1}^{n}Var(Y_{1i}) + \sum_{i \neq j}Cov(Y_{1i},Y_{1j})\right) = \frac{1}{n^{2}}\sum_{i=1}^{n}\sigma_{1}^{2} = \sigma_{1}^{2}/n \\ Var(aX) &= a^{2}Var(X) \\ Var(X-Y) &= Var(X) + Var(Y) - 2Cov(X,Y) \\ Var\left(\bar{Y}_{1}-\bar{Y}_{2}\right) &= Var\left(\bar{Y}_{1}\right) + Var\left(\bar{Y}_{2}\right) \\ Var\left(\bar{D}\right) &= Var\left(\frac{1}{n}\sum_{i=1}^{n}D_{i}\right) + \frac{1}{n^{2}}Var\left(\sum_{i=1}^{n}D_{i}\right) = \frac{1}{n^{2}}\sum_{i=1}^{n}Var(D_{i}) \\ &= \frac{1}{n}Var(D_{1}) = \frac{1}{n}Var(Y_{11}-Y_{21}) = \frac{1}{n}\left(Var(Y_{11}) + Var(Y_{21}) - 2Cov(Y_{11},Y_{21})\right) \\ &= \sigma_{1}^{2}/n + \sigma_{2}^{2}/n - 2Cov(Y_{11},Y_{21})/n = \sigma_{D}^{2}/n \\ Var\left(\bar{D}\right) &= \sigma_{1}^{2}/n + \sigma_{2}^{2}/n - 2Cov(Y_{11},Y_{21}) \\ &= E\left(\bar{Y}_{1}-\bar{Y}_{2}\right) = \mu_{D} \\ E\left(\bar{Y}_{1}-\bar{Y}_{2}\right) &= \mu_{D} \\ Z &= \frac{\bar{Y}-\mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0,1) \\ \bar{Y} \sim N(\mu_{D},\sigma_{D}^{2}/n) \\ \bar{D} \sim N(\mu_{D},\sigma_{D}^{2}/n) \\ \bar{D} \sim N(\mu_{D},\sigma_{D}^{2}/n) \\ T &= \frac{\bar{D}-\mu_{D}}{S_{D}/\sqrt{n}} \sim t_{n-1} \end{split}$$