$$\mu_1 \quad \mu_2 \quad \mu_D$$

$$\bar{Y}_1 - \bar{Y}_2$$

$$E\left(\bar{Y}_1 - \bar{Y}_2\right) = E\left(\bar{Y}_1\right) - E\left(\bar{Y}_2\right) = \mu_1 - \mu_2 = \mu_D$$

$$Var\left(\bar{Y}_1 - \bar{Y}_2\right) = \sigma_1^2/n + \sigma_2^2/n$$

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i = \frac{1}{n}\sum_{i=1}^{n}(Y_{1i} - Y_{2i})$$

$$E\left(\bar{D}\right) = E\left(\frac{1}{n}\sum_{i=1}^{n}(Y_{1i} - Y_{2i})\right) = \frac{1}{n}\sum_{i=1}^{n}\left(E\left(Y_{1i}\right) - E\left(Y_{2i}\right)\right) = \frac{1}{n}\sum_{i=1}^{n}(\mu_1 - \mu_2) = \mu_D$$

$$E\left(\bar{Y}_1\right) = E\left(\frac{1}{n}\sum_{i=1}^{n} Y_{1i}\right) = \frac{1}{n}\sum_{i=1}^{n} E(Y_{1i}) = \frac{1}{n}\sum_{i=1}^{n}\mu_1 = \mu_1$$

$$Var\left(\bar{Y}_1\right) = Var\left(\frac{1}{n}\sum_{i=1}^{n} Y_{1i}\right) = \frac{1}{n^2} Var\left(\sum_{i=1}^{n} Y_{1i}\right)$$

$$= \frac{1}{n^2}\left(\sum_{i=1}^{n} Var(Y_{1i}) + \sum_{i \neq j} Cov(Y_{1i}, Y_{1j})\right) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma_1^2 = \sigma_1^2/n$$

$$Var(aX) = a^2 Var(X)$$

$$Var(X - Y) = Var(X) + Var(Y) - 2Cov(X, Y)$$

$$Var\left(\bar{Y}_1 - \bar{Y}_2\right) = Var\left(\bar{Y}_1\right) + Var\left(\bar{Y}_2\right)$$

$$Var\left(\bar{D}\right) = Var\left(\frac{1}{n}\sum_{i=1}^{n} D_i\right) + \frac{1}{n^2} Var\left(\sum_{i=1}^{n} D_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} Var(D_i)$$

$$= \frac{1}{n} Var(D_1) = \frac{1}{n} Var(Y_{11} - Y_{21}) = \frac{1}{n}\left(Var(Y_{11}) + Var(Y_{21}) - 2Cov(Y_{11}, Y_{21})\right)$$

$$= \sigma_1^2/n + \sigma_2^2/n - 2Cov(Y_{11}, Y_{21})$$

$$Var\left(\bar{D}\right) = \sigma_1^2/n + \sigma_2^2/n - 2Cov(Y_{11}, Y_{21})$$

$$E\left(\bar{D}\right) = \mu_D$$

$$E\left(\bar{Y}_1 - \bar{Y}_2\right) = \mu_D$$

$$Z = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

$$\bar{Y} \sim N(\mu, \sigma^2/n)$$

$$\sigma_D^2$$

$$\bar{D} \sim N(\mu_D, \sigma_D^2/n)$$