

Fitting and Evaluating Simple Linear Regression Models

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Modeling Ideas

What is a (statistical) model?

- A mathematical representation of some phenomenon on which you've observed data
- Form of the model can vary greatly!

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Simple Linear Regression Model

- response = intercept + slope*predictor + Error

$$Y_i = \beta_0 + \beta_1 x_i + E_i$$

- May make assumptions about how errors are observed

Simple Linear Regression Model

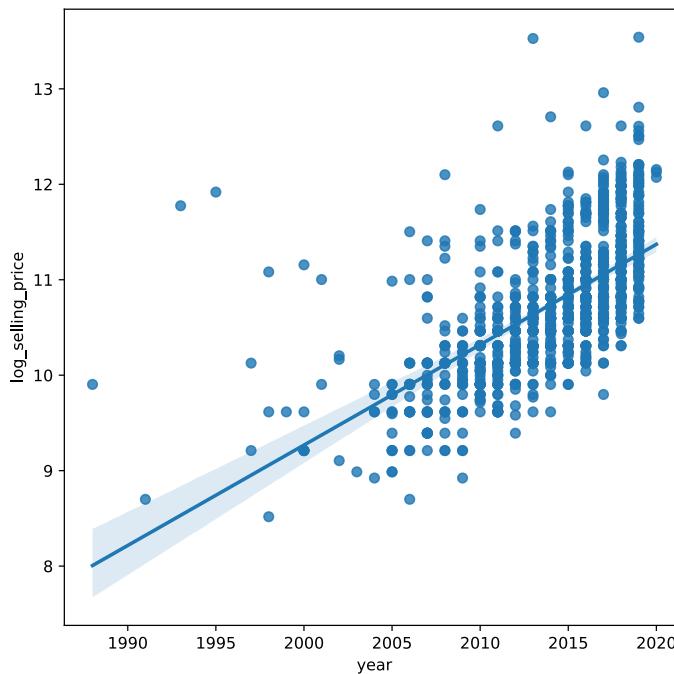
- First a visual

```
import pandas as pd
import numpy as np
import seaborn as sns
bike_data = pd.read_csv("https://www4.stat.ncsu.edu/~online/datasets/bikeDetails.csv")
bike_data['log_selling_price'] = np.log(bike_data['selling_price'])
bike_data['log_km_driven'] = np.log(bike_data['km_driven'])
```

Simple Linear Regression Model

- First a visual

```
sns.regplot(x = bike_data["year"], y = bike_data["log_selling_price"])
```



Statistical Learning

Statistical learning - Inference, prediction/classification, and pattern finding

- Supervised learning - a variable (or variables) represents an **output** or **response** of interest
 - May model response and
 - Make **inference** on the model parameters
 - **predict** a value or **classify** an observation

Goal: Understand what it means to be a good predictive model

Simple Linear Regression Model

Basic model for relating a numeric predictor to a numeric response

response = intercept + slope*predictor + Error

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Simple Linear Regression Model

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$$\text{response} = \text{intercept} + \text{slope} * \text{predictor} + \text{Error}$$

$$Y_i = \beta_0 + \beta_1 x_i + E_i$$

Consider a data set on motorcycle sale prices

```
import pandas as pd
bike_data = pd.read_csv("https://www4.stat.ncsu.edu/~online/datasets/bikeDetails.csv")
print(bike_data.columns)

## Index(['name', 'selling_price', 'year', 'seller_type', 'owner', 'km_driven',
##        'ex_showroom_price'],
##       dtype='object')

bike_data.head()

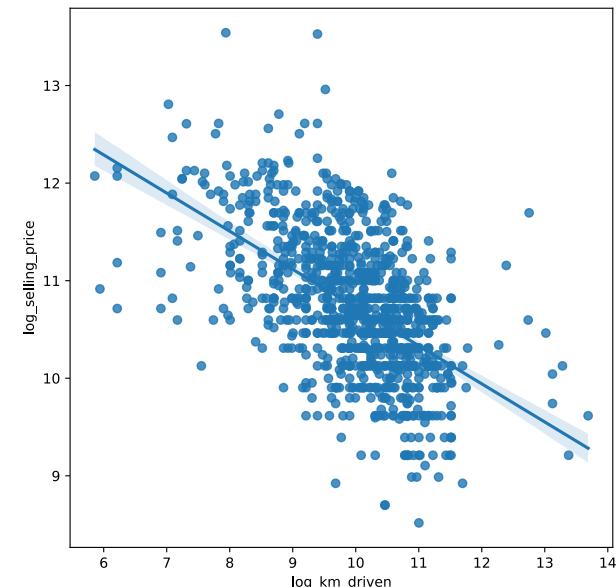
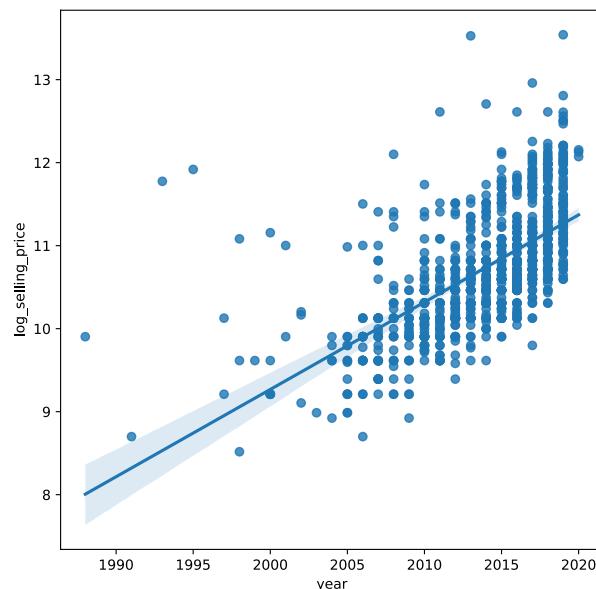
##                                     name    ...  ex_showroom_price
## 0      Royal Enfield Classic 350  ...          NaN
## 1                  Honda Dio  ...          NaN
## 2  Royal Enfield Classic Gunmetal Grey  ...  148114.0
## 3    Yamaha Fazer FI V 2.0 [2016-2018]  ...  89643.0
## 4      Yamaha SZ [2013-2014]  ...          NaN
##
## [5 rows x 7 columns]
```

Find a 'Best' Fitting Line

- We define some criteria to **fit** (or train) the model

Model 1: $\log_{\text{e}}\text{selling_price} = \text{intercept} + \text{slope} * \text{year} + \text{Error}$

Model 2: $\log_{\text{e}}\text{selling_price} = \text{intercept} + \text{slope} * \log_{\text{e}}\text{km_driven} + \text{Error}$



Training a Model

- We define some criteria to **fit** (or train) the model
- **Loss function** - Criteria used to fit or train a model
 - For a given **numeric** response value, y_i and prediction, \hat{y}_i

$$y_i - \hat{y}_i, (y_i - \hat{y}_i)^2, |y_i - \hat{y}_i|$$

Training a Model

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- **Loss function** - Criteria used to fit or train a model
 - For a given **numeric** response value, y_i and prediction, \hat{y}_i
$$y_i - \hat{y}_i, (y_i - \hat{y}_i)^2, |y_i - \hat{y}_i|$$
- We try to optimize the loss over all the observations used for training

$$\sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$\sum_{i=1}^n |y_i - \hat{y}_i|$$

Find a 'Best' Fitting Line

- In SLR, we often use squared error loss (least squares regression)
- Nice solutions for our estimates exist!

$$\hat{\beta}_0 = \bar{y} - \bar{x}\hat{\beta}_1$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

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```
y = bike_data['log_selling_price']
x = bike_data['log_km_driven']
b1hat = sum((x-x.mean())*(y-y.mean()))/sum((x-x.mean())**2)
b0hat = y.mean()-x.mean()*b1hat
print(round(b0hat, 4), round(b1hat, 4))

## 14.6356 -0.3911
```

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- These give us the values to use with \hat{y} !

Simple Linear Regression Model in Python

- Can use `linear_model` from `sklearn` module to fit the model
- Note the requirements on the shape of `X` and the shape of `y` to pass to the `.fit()` method

```
print(bike_data['log_km_driven'].shape)
## (1061,)

print(bike_data['log_km_driven'].values.reshape(-1,1).shape)
## (1061, 1)
```

Simple Linear Regression Model in Python

- Can use `linear_model` from `sklearn` module to fit the model

```
from sklearn import linear_model
reg = linear_model.LinearRegression() #Create a reg object
reg.fit(bike_data['log_km_driven'].values.reshape(-1,1), bike_data['log_selling_price'])

print(reg.intercept_, reg.coef_)

## 14.6355682846293 [-0.39108654]
```

Simple Linear Regression Model

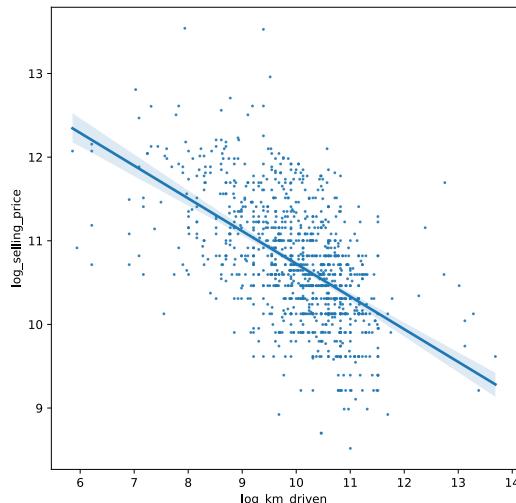
- Can use the line for prediction with the `.predict()` method!

```
print(reg.intercept_, reg.coef_)

## 14.6355682846293 [-0.39108654]

pred1 = reg.predict(np.array([[10], [12], [14]]))
pred1 #each of these represents a 'y-hat' for the given value of x

## array([10.72470291,  9.94252984,  9.16035677])
```



Recenter

Supervised Learning methods try to relate predictors to a response variable through a model

- Lots of common models
 - Regression models
 - Tree based methods
 - Naive Bayes
 - k Nearest Neighbors
- For a set of predictor values, each will produce some prediction we can call \hat{y}

Recentre

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Goal: Understand what it means to be a good predictive model. **How do we evaluate the model?**

Quantifying How Well the Model Predicts

We use a **loss** function to fit the model. We use a **metric** to evaluate the model!

- Often use the same loss function for fitting and as the metric
- For a given **numeric** response value, y_i and prediction, \hat{y}_i

$$(y_i - \hat{y}_i)^2, |y_i - \hat{y}_i|$$

- Incorporate all points via

$$\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2, \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|$$

Metric Function

- For a numeric response, we commonly use squared error loss as our metric to evaluate a prediction

$$L(y_i, \hat{y}_i) = (y_i - \hat{y}_i)^2$$

- Use Root Mean Square Error as a **metric** across all observations

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n L(y_i, \hat{y}_i)} = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2}$$

Commonly Used Metrics

For prediction (numeric response)

- Mean Squared Error (MSE) or Root Mean Squared Error (RMSE)
- Mean Absolute Error (MAE or MAD - deviation)

$$L(y_i, \hat{y}_i) = |y_i - \hat{y}_i|$$

- Huber Loss
 - Doesn't penalize large mistakes as much as MSE

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For classification (categorical response)

- Accuracy
- log-loss
- AUC
- F1 Score

Evaluating our SLR Model

- We could find our metric for our SLR model using the training data...
- Import our **MSE metric** from `sklearn.metrics`

```
import sklearn.metrics as metrics
pred = reg.predict(bike_data["log_km_driven"].values.reshape(-1,1))
print(np.sqrt(metrics.mean_squared_error(bike_data["log_selling_price"], pred)))
## 0.5947022682215317
print(metrics.mean_absolute_error(bike_data["log_selling_price"], pred))
## 0.46886132002881753
```

Useful for Comparison!

- Fit a competing model with `year` as the predictor

```
reg1 = linear_model.LinearRegression() #Create a reg object
reg1.fit(bike_data['year'].values.reshape(-1,1), bike_data['log_selling_price'])

## LinearRegression()
print(reg1.intercept_, reg1.coef_)

## -201.06317651252058 [0.10516552]
```

- Compare the performance on the training data...

```
pred1 = reg1.predict(bike_data["year"].values.reshape(-1,1))
print(np.sqrt(metrics.mean_squared_error(bike_data["log_selling_price"], pred)),
      np.sqrt(metrics.mean_squared_error(bike_data["log_selling_price"], pred1)))

## 0.5947022682215317 0.5482751462879227
```

Training vs Test Sets

Ideally we want our model to predict well for observations **it has yet to see!**

- For multiple linear regression models, our training MSE will always decrease as we add more variables to the model...
- We'll need an independent **test** set to predict on (more on this shortly!)

Recap

- SLR is one type of model for a continuous type response
- SLR Model is fit using some criteria (usually least squares, squared error loss)
- Must determine a method to judge the model's effectiveness (a metric)
 - Metric function measures *loss* for each prediction
 - Combined overall all observations
- To obtain a better understanding of the predictive power of a model, we need to apply our metric to prediction made on a different set of data than that used for training!