87

ST 563 601 – SPRING 2025 – POST

Exam #1

Student's Name: Nick Zehnle

Date of Exam: Thursday, February 6, 2025 - Friday, February 7, 2025
Time Limit: 75 minutes
Allowed Materials: None (closed book & closed notes)

Student – NC State University Pack Pledge

I, Nick Zehnle have neither given nor received unauthorized aid on this exam or assignment. I have read the instructions and acknowledge that this is the correct exam.

STUDENT'S PRINTED NAME

Nick Zehnle

STUDENT SIGNATURE

2/7/25

DATE

Exam must be turned in by: 12:14

EXAM END TIME

STUDENT'S INITIAL AGREEMENT

NOTE: Failure to turn in exam on time may result in penalties at the instructor's discretion.

Exam 1

Please write your answers below each question. You should not have access nor use any materials during this exam.

A reminder that, by taking this exam, you are required to uphold the NC State honor pledge:

"I have neither given nor received unauthorized aid on this test or assignment."

1. In the statistical learning paradigm, we discussed three major goals: statistical inference, predictive modeling, and pattern finding.

   Give a brief real world example for each of these goals. Specify a possible model or method we discussed in class that would help answer the question from each real world example.

- Statistical Inference (4 pts)

Linear Regression ; interpretable such that you can infer relationships between predictors and response variable.

Ex: education on salary

ok

(OLS also predicts)

- Predictive Modeling (4 pts)

parametric

KNN ; non-parametric such that it loses out on interpretability but can fit a training set and produce estimates (predictions) on a test set

ok

Ex: tree height based on tree width

- Pattern Finding (4 pts)

Unsupervised, i.e. no response variable

Ex: patterns in accounting data across firms

model -2

-2

2. Consider having models characterized by flexibility with the scale going from not very flexible to very flexible.

   a. What type of relationship between flexibilty and squared bias would we expect? Why? (4 pts)

   More flexibility ⇒ less squared bias.
   Better suited to fit more complex relationships.

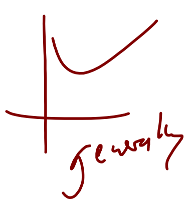
   b. What type of relationship between flexibilty and variance would we expect? Why? (4 pts)

   More flexibility ⇒ more variance
   Overfitting noise.

   c. What type of relationship between flexibilty and training error would we expect? Why? (4 pts)

   More flexibility ⇒ less training error
   Can fit more complex relationships and/or overfit such that training error is very small

   d. What type of relationship between flexibilty and test error would we expect? Why? (4 pts)



   Unknown. A flexible model may overfit and not generalize well and a non-flexible model may underfit (variance-bias trade-off)

   ok

   -1

3. What is a tuning parameter or hyperparameter? How does this differ from a ‘regular’ parameter in a parametric model? (4 pts)

A regular parameter is specified in a model while a tuning parameter is found through means such as CV or bootstrapping in the training set.

ok

-1

5. In the multiple linear regression setting, we discussed a number of model selection methods. State four model selection methods that can be used in the $p > n$ situation. (4 pts)

- ~~best subset~~
- forward stepwise
- ~~backward stepwise~~
- LASSO (can set coefficients to 0)

−2

6. State true or false (no need to explain). (3 pts each)
   a. Ordinary least squares performs variable selection. F
   b. Ordinary least squares performs shrinkage of coefficient estimates. F
   c. Best subset selection performs variable selection. T
   d. Best subset selection performs shrinkage of coefficient estimates. F
   e. Ridge Regression performs variable selection. F
   f. Ridge Regression performs shrinkage of coefficient estimates. T
   g. LASSO performs variable selection. T
   h. LASSO performs shrinkage of coefficient estimates. T

−2

7. Suppose we have a large data set where we want to perform a regression task. We want to determine the best overall model between a LASSO model and a kNN regression model. We want to use a train test split and compare the best kNN and LASSO model on the test set. We wish to determine the appropriate tuning parameters on the training set only using cross-validation. Fully outline the process for splitting the data, tuning, comparing, and fitting a final overall best model. (10 pts)

(1) split data into training/test sets (70/30 or 80/20 for example)

(2) tune λ and K
↳ use cross-validation (e.g. 5-fold CV)
↳ this creates 5 equal subsets of training set where 4 are selected to train on and 1 to test on
↳ get 5 test MSEs for each λ, k and take average along each λ, K how?
↳ select λ, K with smallest CV MSE

(3) fit models with selected λ, k on entire training set

(4) predict on test set and evaluate better test MSE

(could use other metrics such as RMSE, R^2, MAE, ...)

final model? −1

−1

8. Consider the Ridge Regression procedure for fitting a multiple linear regression model. With this model we minimize the following criterion (recall $\lambda \geq 0$):

$$\sum_i \left(Y_i - \beta_0 - X_{i1}\beta_1 - \cdots - X_{ip}\beta_p\right)^2 + \lambda \sum_{j=1}^{p} \beta_j^2$$

a. What are the benefits of fitting a Ridge Regression model as compared to an ordinary least squares model? (4 pts)

By applying a constraint on coefficients, it reduces variability but may introduce bias (i.e. model becomes less flexible). Useful when OLS is plagued by high variance.

b. What happens to our coefficient estimates for a 'large' value of the tuning parameter? What happens for a tuning parameter value near 0? (4 pts)

$\lambda \to 0 \Rightarrow$ RR $\to$ OLS

$\lambda >> 0 \Rightarrow$ coefficients get nearer to 0 but never equal 0 (differs from LASSO in this respect)

0

$X_1 = \begin{cases} 1 & \text{married} \\ 0 & \text{o.w.} \end{cases}$

$X_2 = \begin{cases} 1 & \text{never married} \\ 0 & \text{o.w.} \end{cases}$

9. Suppose we fit a multiple linear regression model to data about how much people earn. Our response variable is the `wage` (in 1000's of dollars) and our predictors are `marital_status` (`married`, `never_married`, or `divorced`), and `age`.

   We fit a linear and quadratic term for `age` and include an interaction between `marital_status` and `age` and an interaction between `marital_status` and `age` squared in the model. Output for the model is given below.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	25.293	38.116	0.664	0.507
X_1 – marital_statusmarried	-19.780	40.405	-0.490	0.624
X_2 – marital_statusnever_married	-31.760	40.992	-0.775	0.439
age – X_3	2.846	1.611	1.767	0.077
I(age^2)	-0.024	0.017	-1.470	0.142
marital_statusmarried:age	2.024	1.716	1.179	0.238
marital_statusnever_married:age	2.230	1.820	1.225	0.221
marital_statusmarried:I(age^2)	-0.025	0.018	-1.412	0.158
marital_statusnever_married:I(age^2)	-0.032	0.020	-1.607	0.108

   a. Write down the fitted equation for $\hat{y}$. Define any indicator variables as needed. (4 pts)

label these

−1

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3 + \hat{\beta}_4 X_3^2 + \hat{\beta}_5 X_1 X_3 + \hat{\beta}_6 X_2 X_3 + \hat{\beta}_7 X_1 X_3^2 + \hat{\beta}_8 X_2 X_3^2$$

   b. One column of the output represents the t-value or t-statistic. What is the usefulness of this t-value? (2 pts)

To assess whether to reject the null of no relationship (i.e $\hat{\beta}_i = 0$)

−1

c. Write down the form of a predicted value for somone that is `married` and has an age of 30. No need to simplify. (2 pts)

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 + 30\hat{\beta}_3 + 900\hat{\beta}_4 + 30\hat{\beta}_5 + 900\hat{\beta}_7$$

ok

d. Write down the form of a predicted value for somone that is `divorced` and has an age of 30. No need to simplify. (2 pts)

$$\hat{Y} = \hat{\beta}_0 + 30\hat{\beta}_3 + 900\hat{\beta}_4$$

ok

f. Conceptually, what does including an interaction between `marital_status` and age and an interaction between `marital_status` and age squared do to our model as compared to a model without those interactions (that still includes a main effect for `marital_status` and a linear and quadratic term for age)? (3 pts)

Produce other slopes $\hat{\beta}_4$, $\hat{\beta}_5$ for age and age^2 dependent on marital status

−2 separate quadratics for each marital status

g. The F-statistic for the global model test is 46.26 on 8 numerator and 2991 denominator degrees of freedom. The p-value for the test is very close to zero.

ANOVA F-test

i. Write down the null and alternative hypotheses for this global test. (3 pts)

Null: model 2 doesn't capture more variability of the response than model1

Alt: reject null

−2

ii. We see a significant global test but none of the coefficient tests are significant. What do you think could be causing this issue? (3 pts)

Adding more predictors intrinsically reduces variability

−2

h. What type of plot might we look at to investigate the homogenous error variance (i.e. the assumption of equal error variance)? (3 pts)

Resid vs. Fitted ✓

← hetero-scedasticity

−6