Continuous Random Variables

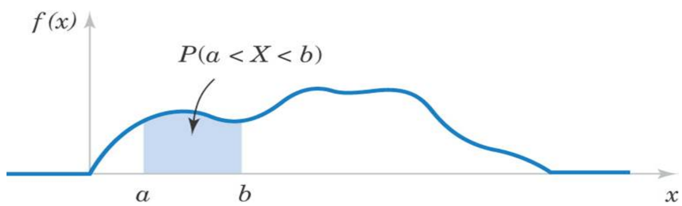
A ________________________________ has an interval or collection of intervals as its support
Ex:

- Y=maximum daily temperature (interval $[-40°F, 130°F]$).
- Y=lifetime (in years) of electronic equipment $0 < Y < \infty$
- Y=weight loss (or gain) after a 6 month period $-\infty < Y < \infty$.

For Discrete RVs we had the probability distribution, $P(y) = P(Y = y)$.

For Continuous RVs we can't assign probability to every y in the support. We now call the probability distribution by

- Probability a randomly chosen value will lie between any 2 given values is represented in terms of the area between the two values under the probability distribution.



A function $f(y)$ is a probability distribution if and only if

1. $f(y)$ is a ______________________________________, i.e. $f(y) \geq 0$ for all y
2. The area under $f(y)$ is 1, i.e.

Similar to the discrete case where $P(y) \geq 0$ and $\sum_y P(y) = 1$

We can find probabilities using integrals:

Example: Probabilities from a continuous probability distribution
Let Y be a random variable with probability distribution:

$$f(y) = \begin{cases} (1/2)y & 0 < y < 2 \\ 0 & \text{else} \end{cases}$$

1. Graph the probability distribution.

2. Find $P(1 \leq Y \leq 2)$ and $P(1 \leq Y < 10)$.

Expectations

Definition of Expectation
For a RV Y with probability distribution $f(y)$, the **expected value** of Y or mean of Y is defined as

$\mu_Y = E(Y)$ is then a ____________________________________ of all possible values of Y, with weighting function $f(y)$.

In general, the expectation of a function of Y, $g(Y)$, can be evaluated as

Variance of a Continuous RV
Definition of Variance is still the same as the discrete case:

Example: Let Y be a random variable with probability distribution:

$$f(y) = \begin{cases} 3y^2 & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Perhaps Y models the proportion of gas in a tank at a randomly selected time. Calculate μ_Y, σ_Y^2, and σ_Y.

(More practice will be provided in the problem session problems, of course these will be posted online if you can't make it!)

Named Distributions
How do we use continuous RVs?

- As before, for a particular experiment we assume a distribution and find characteristics of interest (probabilities, means, variances, etc)
- As with discrete RVs, many scenarios lead to similar distributions (such as the binomial for discrete RVs)
- The most important continuous distribution is the normal distribution. We will also discuss the t-distribution, χ^2 distribution and F-distribution later in the course.

Normal Distribution

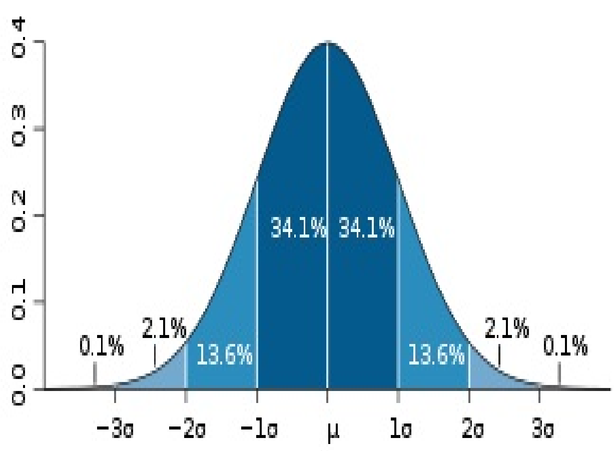
The Normal Distribution $Y \sim N(\mu, \sigma^2)$

A RV Y has a **normal distribution** with mean μ and variance σ^2 if the probability distribution of Y is

where $-\infty < y < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$.

The constants ________________________ are the 'parameters' of the distribution.

We write $Y \sim N(\mu, \sigma^2)$.
Most Famous Bell-Shaped Curve



Properties of the $N(\mu, \sigma^2)$ **RV**

- ____

- ____

- ____

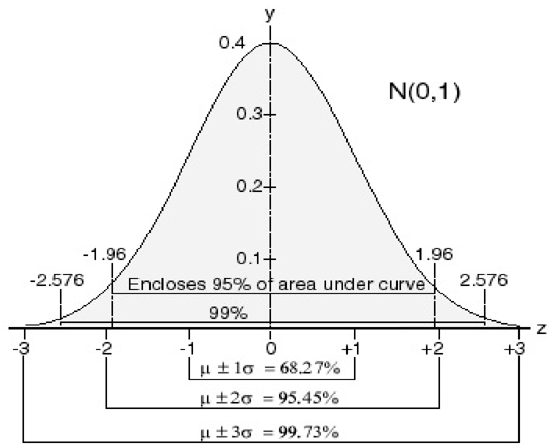
Expectation and Variance of $N(\mu, \sigma^2)$**.**:

$$E(Y) = \_\_\_\_\_\_\_\_\_\_, \quad Var(Y) = \_\_\_\_\_\_\_\_\_\_\_.$$

Note: these are the parameters of the distribution - (i.e. the distribution is ________________________ by its mean and variance)

$Z \sim N(0,1)$ (i.e. a normal distribution with $\mu = 0$ and $\sigma^2 = 1$) is said to follow a

The standard normal is centered at zero and its probabilities are concentrated between (-3,+3).



Standardization of Normal Random Variables Theorem

If $Y \sim N(\mu, \sigma^2)$, then ____________________________ follows the std normal distribution:

Suppose that $Y \sim N(\mu, \sigma^2)$. By standardizing Y, we have

Likewise, if $Z \sim N(0,1)$ __________________________________

CDF of $Z \sim N(0,1)$: $\Phi(z)$
The CDF (cumulative distribution function) of the standard normal random variable Z is by definition:

No closed form, has to be calculated through ________________________________ .

Tables or calculators often used.

We will use SAS to find probabilities (for homework, on test you will just need to find the answer in terms of a standard normal distribution). See SAS file on web.

Bottle Example
A company that manufactures and bottles apple juice uses a machine that automatically fills 16-ounce bottles. There is some variation, however, in the amounts of liquid dispensed into the bottles. The amount dispensed has been observed to be approximately normally distributed with mean 16 ounces and standard deviation 1 ounce.
What is the probability a randomly selected bottle will:

- have more than 17.5 ounces?

- have between 15.2 and 16 ounces?

- have less than 15 ounces?

(See example 4.15, 4.16, 4.17 in the book on page 174 for more practice.)

Percentiles of the Normal Distribution
The $(100p)$**th percentile** of Y (also called the **pth quantile** of Y) is the value y that solves $P(Y \leq y) = p$.

Suppose $Z \sim N(0, 1)$. Find (see SAS file)

1. the 97.5th percentile of Z: $z =$

2. the 2.5th percentile of Z: $z =$

Suppose $Y \sim N(100, 9)$. Then find

1. the 97.5th percentile of Y: $y =$

2. the 2.5th percentile of Y: $y =$

(See examples 4.18 and 4.19 on page 177/178 for more practice.)

SAT/ACT Example

The mathematics portion of the SAT and ACT exams produce scores that are approximately normally distributed. The SAT scores have averaged 480 with a S.D. of 100. The ACT scores average 18 with a S.D. of 6.

1. An engineering school sets 550 as the minimum SAT math score for students. What percentage of students will score below 550 typically?

2. What score should the engineering school set as a comparable standard on the ACT?