The parameter estimates and the variance-covariance matrix are very useful for making inference about our intercept and partial slope parameters (done very similary to SLR). Let's use the above to find the following

1. What is the estimate for β_2? What is the interpretation?
2. What is the standard error of $\hat{\beta}_2$?
3. Conduct a test to determine if $\beta_2 = 0$ plausible (technically, after accounting for the linear association between extractable aluminum and adsorption index). Hint: $t(0.025, 10) = 2.228$
4. Estimate the mean adsorption index among the population of ALL soil with extractable aluminum = 100 and extractable iron = 150. Report a standard error for this estimate and a 95% confidence interval and a 95% prediction interval.

Answers:

1. $\hat{\beta}_2 = 0.1127$, which represents the estimated change in adsorption for a one unit increase in extractable iron while holding the amount of extractable aluminum constant.
2. $\sqrt{0.00088} = 0.0297$ (square root of (3,3) element of $\widehat{\mathbf{\Sigma}}$)
3. $H_0 : \beta_2 = 0 \quad vs \quad H_A : \beta_2 \neq 0$, T-statistic: $t = (\hat{\beta}_2 - 0)/SE(\hat{\beta}_2) = 0.1127/0.0297 = 3.795$
   Since our obsered test statistic is greater than 2.228, we reject H_0 in favor of H_A, that is, at the 5% significance level, extractable iron has a significant linear association with adsorption (even after accounting for extractable aluminum).
4. Unknown population mean: $\theta = \beta_0 + \beta_1(100) + \beta_1(150)$
   Estimate : $\hat{\theta} = (1, 100, 150) * \hat{\beta} = 44.454$
   To find the standard error, find the variance and take the square root:

   $$Var((1, 100, 150) * \hat{\boldsymbol{\beta}}) = (1, 100, 150)Var(\hat{\boldsymbol{\beta}})(1, 100, 150)'$$

   estimated as

   $$= (1, 100, 150)\widehat{\mathbf{\Sigma}}(1, 100, 150)' = 19.832$$

   Which implies $SE(\hat{\theta}) = \sqrt{19.832} = 4.453$. Thus, we are 95% confident that the true mean adsorption index among the population of ALL soil with extractable aluminum = 100 and extractable iron = 150 is between

   $$(44.454 - 2.228(4.453), 44.454 - 2.228(4.453)) = (34.533, 54.375)$$

   To find the variance of a future value we need to find

   $$Var((1, 100, 150)*\hat{\boldsymbol{\beta}}+E_{new}) = (1, 100, 150)Var(\hat{\boldsymbol{\beta}})(1, 100, 150)'+Var(E_{new})$$

since we have independence of observations. We can use $MS(E)$ as an estimate of the error variance. The estimated variance of a future value is then $19.832 + 19.17897 = 39.01097$ and our SE is the square root $= 6.2459$. Therefore, we are 95% confident that a future absorption index for soil with extractable aluminum $= 100$ and extractable iron $= 150$ is between

$$(44.454 - 2.228(6.2459), 44.454 - 2.228(6.2459)) = (30.538, 58.370)$$