Data and Modeling

What makes something a statistical model?

a math model that takes into randomness in the data

- ex: Linear Regression

random error

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

response

explanatory variable (predictor)

What is the difference between prediction and inference?

modeling a 'new' value

- either predicting a new response or classifying a new observation

Statements or claims about relationships in the population using sample data

Data

- When modeling, what should our data look like?

2-D table

Variables

observations

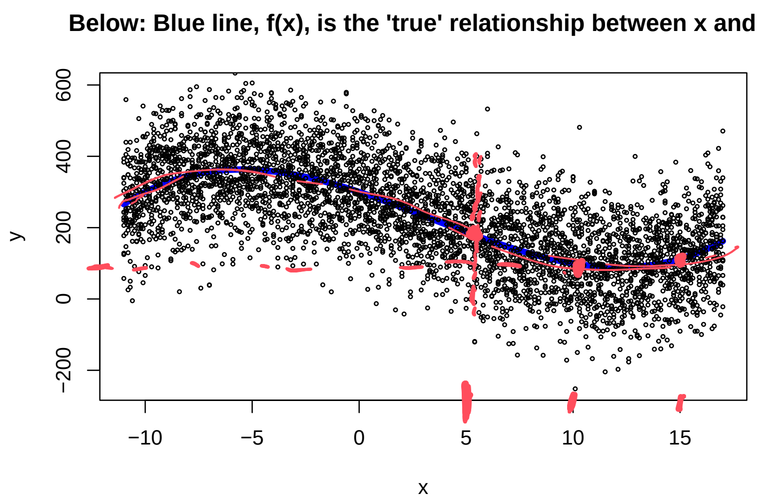
	A	B	C	D
1				
2				
3				
4				
⋮				
n				

Relating Explanatory Variables to a Response Variable

Consider the response Y as a random variable. We'll consider the x values fixed (for any explanatory variable). Our interest is in learning about the relationship between Y and x.

Y is random, so we don't have a **deterministic** relationship...

random - observed differently each time we see it



What should we try to relate/model?

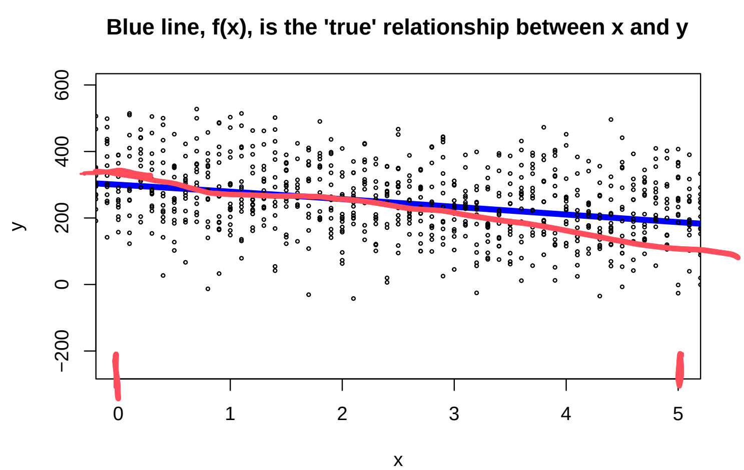
given

$$\boxed{f(x)} = E(Y|x)$$

$$E(Y|x=10) = 100$$

Approximating $f(x)$

Although the true relationship is most certainly nonlinear, we may be ok approximating the relationship linearly. For example, consider the same plot as above but between 0 and 5 only:



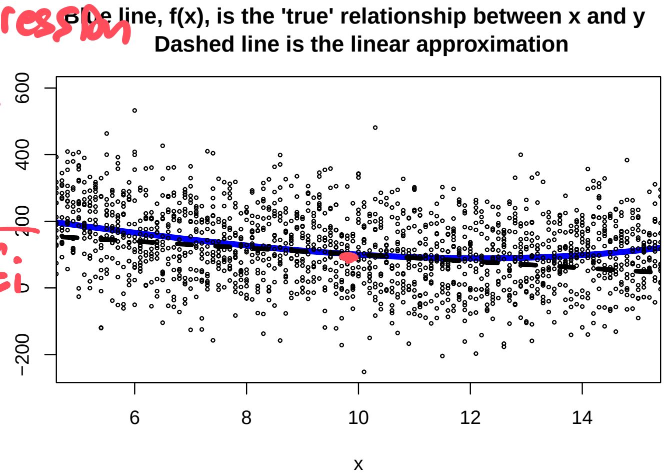
That's pretty linear. Consider plot between 5 and 15:

Quadratic Linear Regression

L in the predictor

$Y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i$

$Y_i = \boxed{\beta_0 e^{-\beta_1 x_i}}$



Line still does a reasonable job and is often used as a basic approximation.

Exploratory Data Analysis (EDA)

What are our first steps with data?

get to know your data, make sure it is what you think it should be, inform model choices

Common steps to EDA

1. Read in your data
2. Understand how your data is stored
3. Understand missing data values (NA)
4. Clean up data
5. Summarize the data (min, max, mean, frequency/count, scatter plot, bar graph, histogram)
6. Transformations? Repeat 5

Data Intro

This dataset contains information about used motorcycles and their cost.

From the information page: This data can be used for a lot of purposes such as price prediction to exemplify the use of linear regression in Machine Learning. The columns in the given dataset are as follows:

- name
- selling price response
- year
- seller type
- owner
- km driven
- ex showroom price

The data are available to download from this URL:
https://www4.stat.ncsu.edu/~online/datasets/bikeDetails.csv

Read in Data and Explore!

```
library(tidyverse)
bikeData <- read_csv("https://www4.stat.ncsu.edu/~online/datasets/bikeDetails.csv")
select(bikeData, selling_price, year, km_driven, ex_showroom_price, name, everything())
```

```
## # A tibble: 1,061 x 7
##    selling_price  year km_driven ex_showroom_price name        seller_type owner
##            <dbl> <dbl>     <dbl>             <dbl> <chr>       <chr>       <chr>
##  1        175000  2019       350                NA Royal Enfi~ Individual  1st ~
##  2         45000  2017      5650                NA Honda Dio   Individual  1st ~
##  3        150000  2018     12000            148114 Royal Enfi~ Individual  1st ~
##  4         65000  2015     23000             89643 Yamaha Faz~ Individual  1st ~
##  5         20000  2011     21000                NA Yamaha SZ ~ Individual  2nd ~
##  6         18000  2010     60000             53857 Honda CB T~ Individual  1st ~
##  7         78500  2018     17000             87719 Honda CB H~ Individual  1st ~
##  8        180000  2008     39000                NA Royal Enfi~ Individual  2nd ~
##  9         30000  2010     32000                NA Hero Honda~ Individual  1st ~
## 10         50000  2016     42000             60122 Bajaj Disc~ Individual  1st ~
## # ... with 1,051 more rows
```

Our 'response' variable here is the `selling_price` and we could use the variable `year`, `km_driven`, or `ex_showroom_price` as the explanatory variable. Let's make some plots and summaries to explore.

Linear Regression

Recap: Our goal is to predict a value of Y while including an explanatory variable x. We are assuming we have a sample of (x_i, y_i) pairs, $i = 1, ..., n$.

The Simple Linear Regression (SLR) model can be used:

$$Y_i = \beta_0 + \beta_1 x_i + E_i$$

where

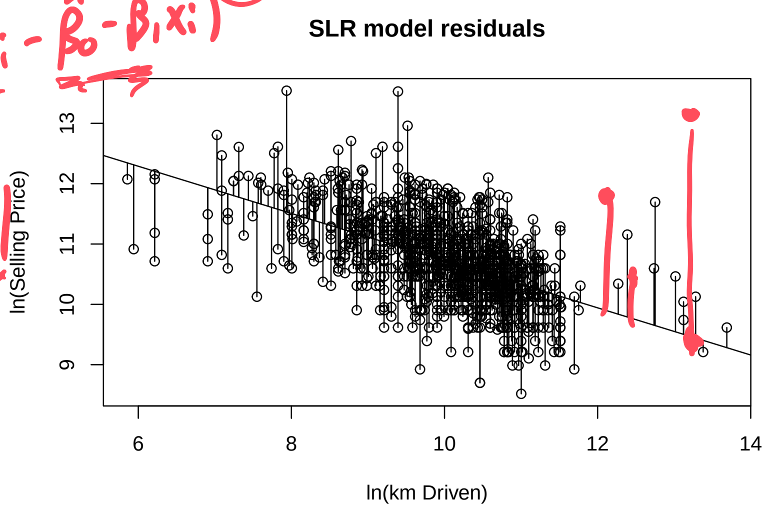
- y_i is our response for the i^{th} observation
- x_i is the value of our explanatory variable for the i^{th} observation
- β_0 is the y intercept
- β_1 is the slope
- $E_i \overset{iid}{\sim} N(0, \sigma^2)$

What is important to know from all that??

We **fit** this model to data. That is, find the **best** estimators of β_0 and β_1 (and σ^2) given the data. How to fit the line?

$\min SSE = \min \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$

$\min \sum_{i=1}^{n} |y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i|$



Fitting the line

The (fitted) linear regression model uses $\hat{f}(x) = \hat{\beta}_0 + \hat{\beta}_1 x$. Calculus allows us to find the 'least squares' estimators, $\hat{\beta}_0$ and $\hat{\beta}_1$ in a nice closed-form!

Making Inference

What hypothesis are we interested in and why?

How can we form a confidence interval for the quantity of interest?

Checking assumptions

How can we check our assumptions on the errors?

Fitting a Linear Regression Model in R

We can fit the model with the `lm()` function. Provide a `formula`

$$response \sim explanatory_variable_equation (\text{intercept fit by default})$$

```
library(tidyverse)
bikeData <- read_csv("https://www4.stat.ncsu.edu/~online/datasets/bikeDetails.csv")
bikeData <- bikeData %>% mutate(log_selling_price = log(selling_price),
                               log_km_driven = log(km_driven))

fit <- lm(log_selling_price ~ log_km_driven, data = bikeData)
```

Determine the fitted model by looking at the `coefficients` element.

```
fit$coefficients
```

```
##   (Intercept) log_km_driven
##    14.6355683    -0.3910865
```

Look at the hypothesis test of interest with `summary()`

```
summary(fit)
```

```
##
## Call:
## lm(formula = log_selling_price ~ log_km_driven, data = bikeData)
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9271 -0.3822 -0.0337  0.3794  2.5656
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)   14.63557    0.18455   79.31   <2e-16 ***
## log_km_driven -0.39109    0.01837  -21.29   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5953 on 1059 degrees of freedom
## Multiple R-squared:  0.2997, Adjusted R-squared:  0.299
## F-statistic: 453.2 on 1 and 1059 DF,  p-value: < 2.2e-16
```

What here is important and why?

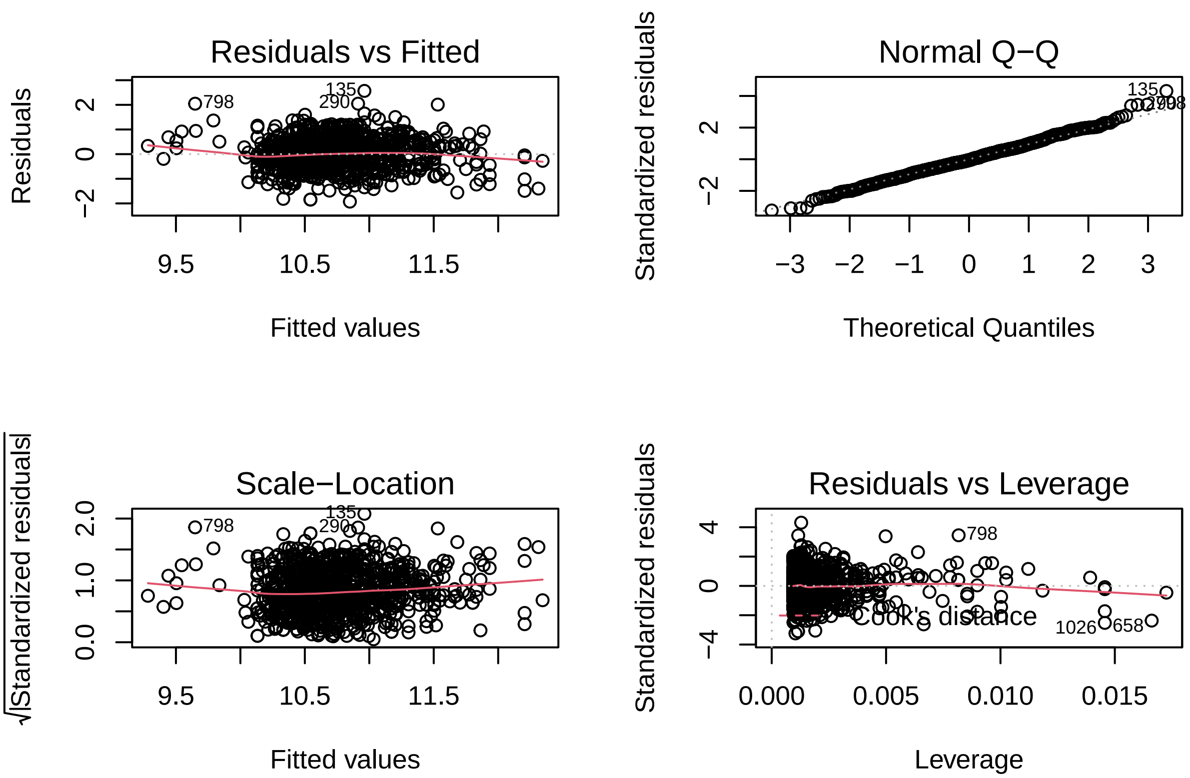
Find a confidence interval with `confint()`

```
confint(fit)
```

```
##                    2.5 %     97.5 %
## (Intercept)   14.2734501 14.9976864
## log_km_driven -0.4271342 -0.3550389
```

Check conditions! `plot()` on the model fit will work.

```
par(mfrow = c(2,2))
plot(fit)
```



Logistic Regression Model

Used when you have a **binary** response variable

- Using SLR is not appropriate!

Example:

- Consider data about water potability

```
library(tidyverse)
water <- read_csv("water_potability.csv")
water
```

```
## # A tibble: 3,276 x 10
##       ph Hardness Solids Chloramines Sulfate Conductivity Organic_carbon
##    <dbl>    <dbl>  <dbl>       <dbl>   <dbl>        <dbl>          <dbl>
##  1 NA        205. 20791.        7.30    369.         564.           10.4
##  2  3.72     129. 18630.        6.64     NA          593.           15.2
##  3  8.10     224. 19910.        9.28     NA          419.           16.9
##  4  8.32     214. 22018.        8.06    357.         363.           18.4
##  5  9.09     181. 17979.        6.55    310.         398.           11.6
##  6  5.58     188. 28749.        7.54    327.         280.            8.40
##  7 10.2      248. 28750.        7.51    394.         284.           13.8
##  8  8.64     203. 13672.        4.56    303.         475.           12.4
##  9 NA        119. 14286.        7.80    269.         389.           12.7
## 10 11.2      227. 25485.        9.08    404.         564.           17.9
## # ... with 3,266 more rows, and 3 more variables: Trihalomethanes <dbl>,
## #   Turbidity <dbl>, Potability <dbl>
```

- Summarize water potability

```
table(water$Potability)
```

```
##
##    0    1
## 1998 1278
```

```
water %>%
  group_by(Potability) %>%
  select(Hardness, Chloramines) %>%
  summary()
```

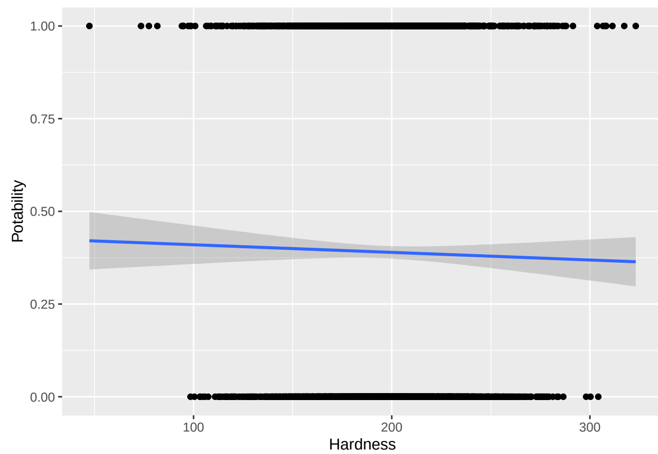
```
## Adding missing grouping variables: `Potability`
```

```
##    Potability        Hardness       Chloramines
##  Min.   :0.0000   Min.   : 47.43   Min.   : 0.352
##  1st Qu.:0.0000   1st Qu.:176.85   1st Qu.: 6.127
##  Median :0.0000   Median :196.97   Median : 7.130
##  Mean   :0.3901   Mean   :196.37   Mean   : 7.122
##  3rd Qu.:1.0000   3rd Qu.:216.67   3rd Qu.: 8.115
##  Max.   :1.0000   Max.   :323.12   Max.   :13.127
```

Why is linear regression not appropriate?

```
fit <- lm(Potability ~ Hardness, data = water)
ggplot(water, aes(x = Hardness, y = Potability)) +
  geom_point() +
  geom_smooth(method = "lm")
```

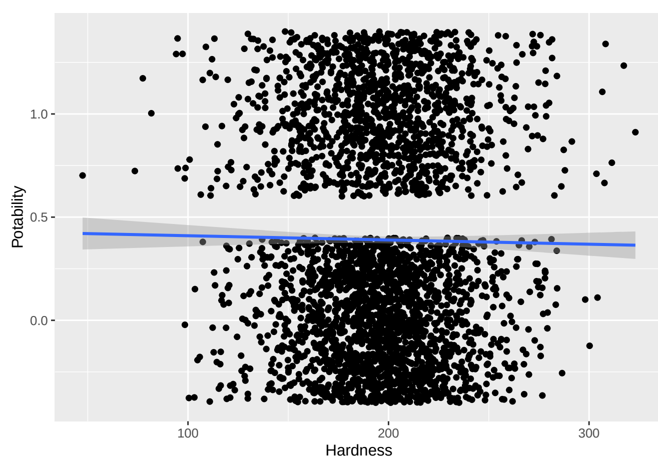
```
## `geom_smooth()` using formula 'y ~ x'
```



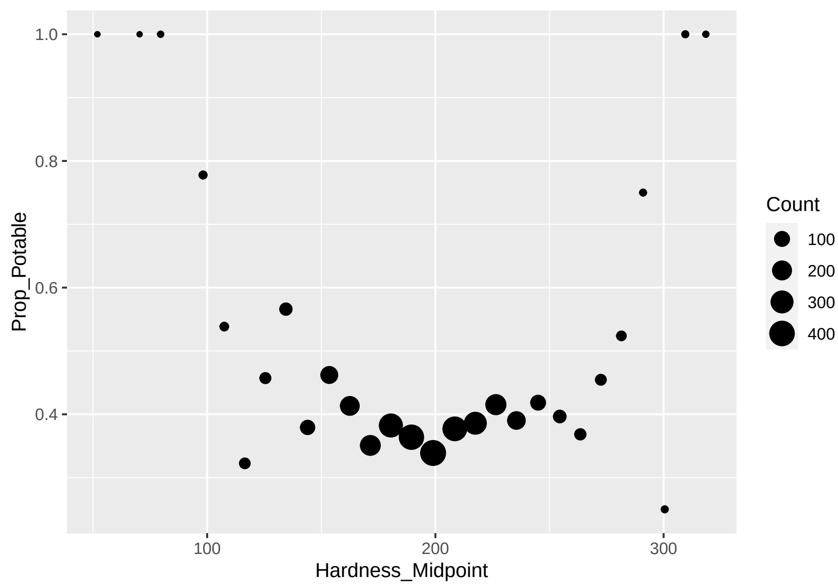
Better view…

```
fit <- lm(Potability ~ Hardness, data = water)
ggplot(water, aes(x = Hardness, y = Potability)) +
  geom_jitter() +
  geom_smooth(method = "lm")
```

```
## `geom_smooth()` using formula 'y ~ x'
```



A better view of the data is to visualize the proportions of successes as a function of hardness.

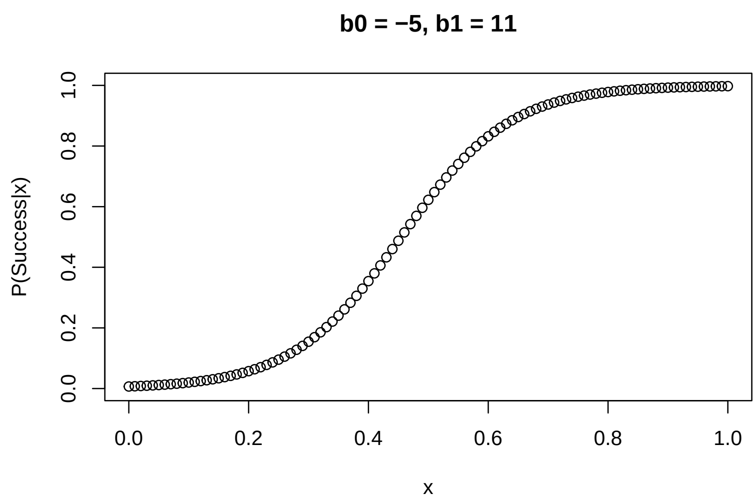


- In SLR, we modeled the average of the response as a linear function. What does the average of the responses represent here? Why does using a linear function not make sense?

- Basic Logistic Regression models success probability using the *logistic function*

$$P(success|x) = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}$$

- This function never goes below 0 and never above 1 - works great for many applications!
- The logistic regression model doesn't have a closed form solution (maximum likelihood often used to fit parameters)



- Back-solving shows the *logit* or *log-odds* of success is linear in the parameters

$$log\left(\frac{P(success|x)}{1-P(success|x)}\right) = \beta_0 + \beta_1 x$$

- Coefficient interpretation changes greatly from linear regression model!
- β_1 represents a change in the log-odds of success

Hypotheses of Interest

What do you think would indicate that x is related to the probability of success here?

Fitting a Logistic Regression Model in R

Fit in R using `glm()` with `family = binomial` and a formula just like `lm()`.

```
fit <- glm(Potability ~ Hardness, data = water, family = "binomial")
```

Get coefficients by looking at `coefficients` element:

```
fit$coefficients
```

```
##   (Intercept)      Hardness
## -0.2774792831 -0.0008629619
```

Get hypothesis test via `summary()`:

```
summary(fit)
```

```
##
## Call:
## glm(formula = Potability ~ Hardness, family = "binomial", data = water)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.0279  -0.9963  -0.9853   1.3678   1.4209
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -0.277479   0.216758  -1.280    0.200
## Hardness    -0.000863   0.001090  -0.792    0.428
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 4382.0  on 3275  degrees of freedom
## Residual deviance: 4381.3  on 3274  degrees of freedom
## AIC: 4385.3
##
## Number of Fisher Scoring iterations: 4
```

Get confidence interval for β_1 with:

```
confint(fit)
```

```
## Waiting for profiling to be done...
```

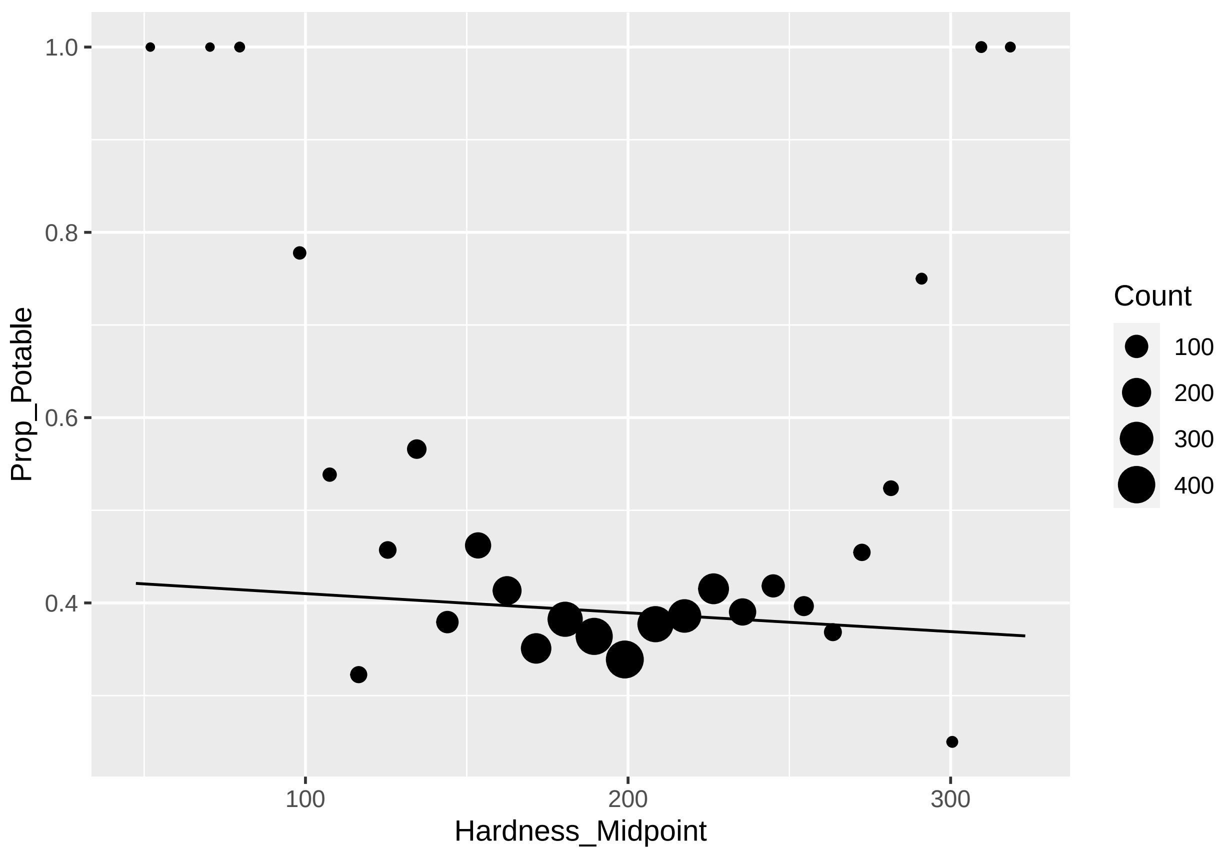
```
##                    2.5 %      97.5 %
## (Intercept) -0.702803063 0.147169863
## Hardness    -0.003000628 0.001272738
```

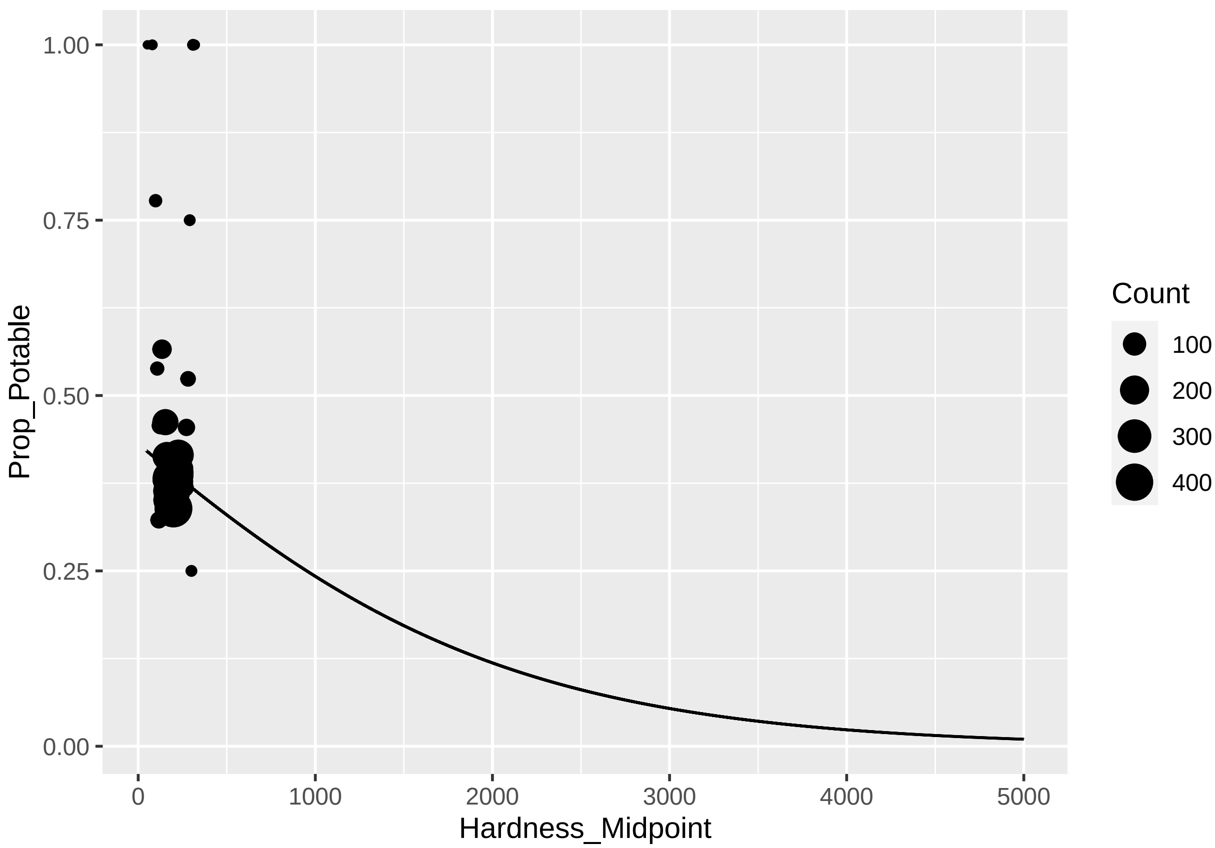
If we want a probability estimate back, use `predict()` with `type = 'link'`:

```r
predict(fit, newdata = data.frame(Hardness = c(200, 300)), type = "link", se.fit = TRUE)
```

```
## $fit
##          1          2
## -0.4500717 -0.5363679
##
## $se.fit
##          1          2
## 0.03606504 0.11869267
##
## $residual.scale
## [1] 1
```

Visualize the fit:





Is a logistic curve!