PHY982: Nuclear Dynamics

Homework 1 — Deadline: 26 Mar 2015

Scattering states and Phase shifts

Please read Chapters 3.1 and 4.2 before starting.

Consider a simple two particle description of the halo nucleus ^{11}Be as a single neutron and an inert ^{10}Be core. The effective nuclear interaction between the neutron and ^{10}Be is often taken to be a Woods Saxon form:

$$V(R) = \frac{V_0}{1 + \exp(\frac{R - R_{ws}}{a_{ws}})} \tag{1}$$

where the radius is $R_{ws} = 1.2A^{1/3}$ fm, A being the mass of the core, and the diffuseness $a_{ws} = 0.65$ fm. The depth of the interaction V_0 is fitted to the well known binding energy of this system $S_n = 0.5$ MeV, considering that the valence neutron is in a $2s_{1/2}$ orbital. After solving the radial equation with the following constants ($hc = 197.32705$ MeV.fm and $2\mu/\hbar^2 = 0.0478450$) and taking integer masses, one verifies that the correct depth that reproduces the experimental binding energy is $V_0 = -61.1$ MeV. For other states, one usually needs to consider spin-orbit effects, but we will neglect these for now.

In this example, we will consider the scattering states of this system within the Schrödinger formalism.

1. Why is the valence neutron in a $2s_{1/2}$ orbital in the ^{11}Be ground state? Considering the shell structure, what excited states would you expect to find? How do your expectations compare to experiment?

2. Write down the radial scattering equations for arbitrary relative n-^{10}Be energy and relative angular momentum l. How can you solve these equations? Describe the numerical method (e.g. Runge-Kutta Method) and implement it with scattering boundary conditions.

3. Plot the radial behaviour of the scattering wavefunctions $u_l(r)$ for $l = 0, 1, 2$ at energies 0.1 MeV and 3.0 MeV, out to large enough radius so that one can recover the asymptotic behavior.

4. Study the energy dependence of the phase shifts $\delta_l(E)$ for $l = 0, 1, 2$ for energies from $[0.1, 4.0]$ MeV. Make sure you regularize the functions $\delta_l(E)$ to get rid of discontinuities. Present in graphical form and draw conclusions about the continuum structure of ^{11}Be.

5. Did you find any resonances in the system? For which partial waves? What are the corresponding energies and widths?

6. Prepare a report with your results and conclusions. It should not exceed 10 pages.