Phy 981 Assignment 2

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Exercise 3

(a) For $N = 3$, Φ^{AS} is

$$\Phi_\lambda^{\mathrm{AS}} = \frac{1}{\sqrt{3!}} \sum_p (-)^p \, \hat{P} \, \psi_{\alpha_1}(x_1) \, \psi_{\alpha_2}(x_2) \, \psi_{\alpha_3}(x_3)$$

$$\begin{aligned}\Phi_\lambda^{\mathrm{AS}} = \frac{1}{\sqrt{6}} [&\psi_{\alpha_1}(x_1) \, \psi_{\alpha_2}(x_2) \, \psi_{\alpha_3}(x_3) - \psi_{\alpha_1}(x_2) \, \psi_{\alpha_2}(x_1) \, \psi_{\alpha_3}(x_3) \\ &- \psi_{\alpha_1}(x_3) \, \psi_{\alpha_2}(x_2) \, \psi_{\alpha_3}(x_1) - \psi_{\alpha_1}(x_1) \, \psi_{\alpha_2}(x_3) \, \psi_{\alpha_3}(x_2) \\ &+ \psi_{\alpha_1}(x_3) \, \psi_{\alpha_2}(x_1) \, \psi_{\alpha_3}(x_2) + \psi_{\alpha_1}(x_2) \, \psi_{\alpha_2}(x_3) \, \psi_{\alpha_3}(x_1)]\end{aligned}$$

(b) The integral can be written out as

$$\int dx_1 \ldots dx_N \frac{1}{N!} \left[\sum_p (-)^p \, \hat{P} \prod_{i=1}^N \psi_{\alpha_i}(x_i) \right] \left[\sum_p (-)^p \, \hat{P} \prod_{i=1}^N \psi_{\alpha_i}(x_i) \right].$$

However, since the basis functions $\psi_{\alpha_i}(x_i)$ are orthogonal, the only terms that will survive the integration are those with corresponding permutations from each sum. In other words, the integral can be written as

$$\frac{1}{N!} \int dx_1 \ldots dx_N \sum_p (-)^{2p} \, \hat{P} \prod_{i=1}^N \psi_{\alpha_i}^2(x_i).$$

Now, using the normality of the basis functions, this can be rewritten as follows:

$$\frac{1}{N!} \sum_p \int dx_1 \ldots dx_N \, \hat{P} \prod_{i=1}^N \psi_{\alpha_i}^2(x_i)$$

$$\frac{1}{N!} \sum_p \prod_{i=1}^N \int dx_i \psi_{\alpha_i}^2(x_i)$$

$$\frac{1}{N!} \sum_p (1) = \frac{N!}{N!} = 1$$