`PHSX815_Project2`:
The Frequency of Meteor Impacts

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1 Abstract

The Solar System is full of detritus left over from its initial formation processes. As planetesimals and protoplanets accrete material from their surrounding disk, they continue to collide with each other. These collisions lead to the formation of new, larger objects, but also throw off smaller material ejected above the new bodies' escape velocities. Now, these planetary pencil shavings litter the solar system, continuing to occasionally impact each other and the major planets. Since the frequency of these objects heavily depends on their mass, the impact frequency of these objects and the Earth also depends on their mass, leading to extremely frequent small object impacts and very few large object impacts. From a dataset of simulated "observed" meteorite impacts, we use Monte Carlo methods to identify the rate parameter of the observed meteorite impacts.

2 Introduction

Solar system solid bodies range in sizes from microscopic dust grains to thousands of kilometers in diameter. For convenience, we deem every observable body outside of the eight major planets to be a minor planet, and many of these minor planets are also deemed "asteroids" for their starlike appearance in optical observations. Every so often, an asteroid (or other small body) impacts the Earth, and if this produces a visible trail in the atmosphere, these are called "meteors". Some meteors are large enough that they reach the ground whole, or in pieces, and these recovered objects are called "meteorites".

The main parameter affecting the survivability of a meteor is its size. Under a few tens of meters, these tend to burn up in the Earth's atmosphere. Over a few tens of meters and under a hundred meters, these may reach the Earth's surface after ablating their outer material away, or explode into fragments that may themselves either burn up or reach the ground. Over a few hundred meters and these will pose significant risk of the bulk of the asteroid and its fragments causing significant damage on impact.

In this project we examine whether given some observed population of meteor impacts over a period of time, we can estimate the average size of the incident meteors.

3 Meteor Impact Rates and Poisson Processes

Poisson processes consist of independent, non-simultaneous events occurring at some constant average rate. Although this is clearly not accurate for certain meteorite falls (as the periodic meteor showers show), we may approximate the total meteorite flux on the Earth as a Poisson process given that we can't a priori predict meteor impacts outside of the few periodic meteor showers. We also don't know a priori the size of the meteors impacting the Earth, although we may expect that the size and the abundance of these meteors are related, with the impact rate depending on the size of the meteors.

For a general Poisson process, we have:

$$P(x|\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

and

$$\sum_{x=0}^{\infty} P(x|\lambda) = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = 1$$

If we know that the impact rate of meteors depends on their sizes, then by observing many meteor impacts over a period of time we can estimate the sizes of the meteors that struck the Earth. From studies like [1] the observed impact rate of asteroids under a few hundred meters in diameter is described by, what else, a power law in meteor diameter:

$$log(N) = c_0 - d_0 Log(D)$$

where D is the diameter in meters, $c_0 = 1.568 \pm 0.03$ and $d_0 = 2.70 \pm 0.08$.

This paper looks like a pretty comprehensive reference, so we may assume that it gives a good estimate of the true diameter-impact rate relationship. However, we would like to observe meteors on our own, and these are not guaranteed to fit this relationship exactly.

4 Code

Unfortunately, converting the power law to a distribution I could have sampled from was more difficult than expected. In this case, we use Markov Chain Monte Carlo methods to draw observed meteors from a Poisson distribution, where the rate parameters have been drawn from a gamma distribution, and the gamma distribution parameters have Gaussian priors with some arbitrary means and sigmas. The process was ported from Dr. Rogan's example C++ MCMC/Gibbs sampling code.

As we can't use this to simulate the global population of meteors, we instead can treat our observations as if we were observing some related meteor event, like a shower. Here our two models are stand-ins for two different meteor showers with their own characteristic rates. Model 0 assumes that the shower has a perfectly consistent distribution from which to draw meteor rates, and model 1 allows the gamma distribution parameters to vary, changing with each step along the Markov chain. Our challenge now is to determine, given some observed meteor impact rate, which shower we are observing. For both models, the starting parameters are $\alpha = 2.7$ and $\beta = 1.57$. (Somewhat of a misuse of the original distribution's parameters, but this is an arbitrary choice anyway). The Gaussian priors on α and β both have $\sigma = 1$.

5 Analysis

Figure 1 shows the probability distributions of the number of meteors observed per session. These distributions are skewed to the right, showing that for most of the observations we made, we saw very few meteors (and often none) within that specific time period. Figure 2 shows the log-likelihood

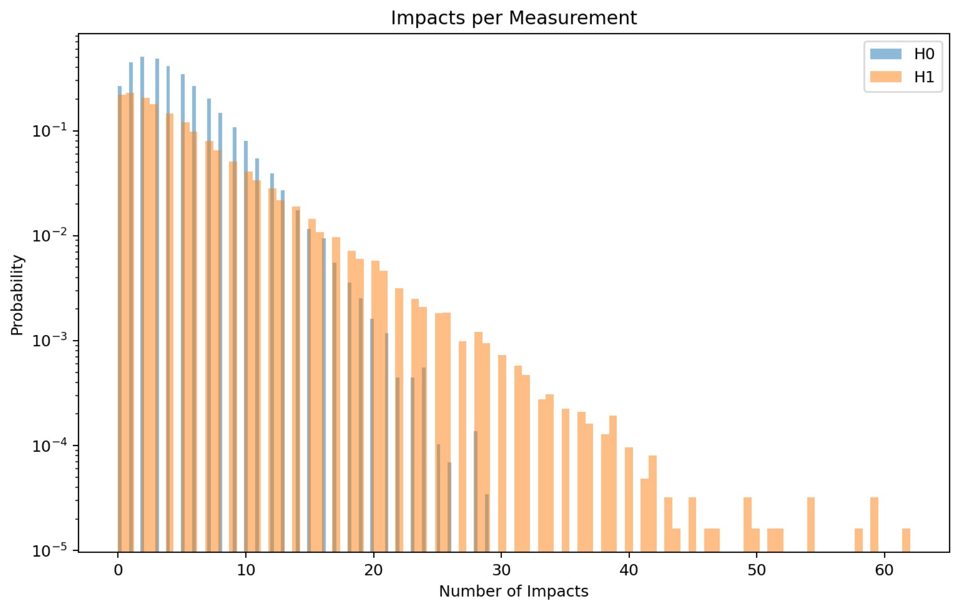

Figure 1: Probability distributions for observed meteors per measurement.

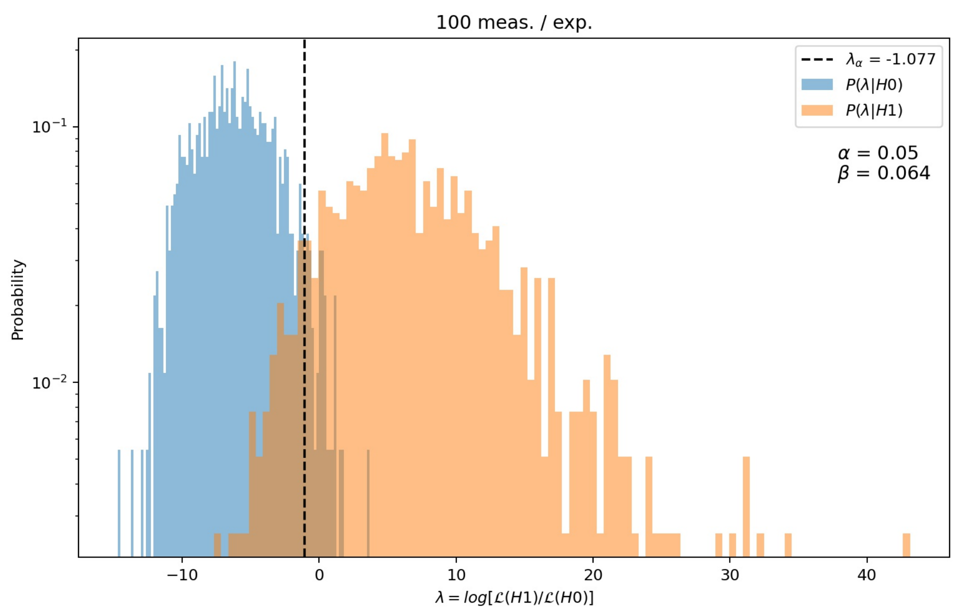

Figure 2: Log-likelihood plots for the two models.

ratio plotted for our many observations and the hypothesis testing parameters. In performing our likelihood ratio test, we chose $\alpha = 0.05$, finding that there is a 5% chance of a false positive result (i.e. observing a shower actually consistent with model 1 and assuming it to be consistent with model 0) when measuring a log-likelihood ratio greater than -1.077. We found β to be 0.064, so our false negative (i.e. observing a shower actually consistent with model 0 and assuming it to be consistent with model 1) rate is 6.4% when measuring a log-likelihood ratio less than -1.077, and the power of the test is 0.936.

6 Conclusions

By observing a meteor shower, we can measure the impact rates of the meteors in the shower and compute likelihoods for that rate consistent with each model. For example, we may go outside one night and see 10 meteors: this would give us a log-likelihood ratio of 0.08. This is greater than our test statistic -1.077, and therefore we reject the null hypothesis with a confidence level of 95%. Since we know we were observing a shower with characteristics of model 1, we can find the mean incident meteors from our model 1 data, which is 4.748. The standard deviation of our model 1 data is 4.87. Assuming this is an accurate value for one night's meteors over an entire year (a bad assumption but let's go with it), we find $\log(4.748 * 365) = 1.57 - 2.7\log(D)$, and after incorporating our $\pm 1\sigma$ constraints, $D \approx 0.113 \pm 0.087$ meters. Given what we know about meteors in our real-world solar system, this is shockingly large! These meteors would be extremely bright and persistent, but as we drew our samples from non-physical distributions, it's not too surprising.

References

[1] P. Brown, R. E. Spalding, D. O. ReVelle, E. Tagliaferri, and S. P. Worden, *The flux of small near-Earth objects colliding with the Earth*, **420** no. 6913, (Nov., 2002) 294–296.