Astr 423, Spring 2019

Homework 7: equations of state and the temperature-density plane

1 Different formulas for pressure

Put together the formulas for:

1. Radiation pressure
2. Ideal gas pressure
3. Non-relativistic electron degeneracy pressure
4. Extremely relativistic electron degeneracy pressure

Prepare a diagram showing the $\log T$, $\log \rho$ (g cm^{-3}) plane: $\log T$ increases vertically and log density increases horizontally. The limits for your diagram are: $\log T$ from 3 to 12, log density from -9 to +9.

2 Areas of preponderance in the $\log T$ - $\log \rho$ plane

Find the equation of a boundary separating the region where radiation pressure dominates from the region where gas pressure (take $P_g = nkT$) dominates. Plot the boundary you have found in the $\log T$ - $\log \rho$ diagram.

Find the boundary separating the nondegenerate region from the degenerate non-relativistic region (neglect radiation pressure for this estimate), and add it to your diagram.

Find the boundary separating the nondegenerate region from the degenerate extremely relativistic region (neglect radiation pressure for this estimate), and add it to your diagram.

Find the position of a vertical line equally distant from the non-relativistic and extremely relativistic degenerate regions, and add it to the diagram. This line should intersect the boundary between non-degenerate and degenerate regions near the point where the boundary's slope changes.

Your diagram should show 4 regions in the $\log T$ - $\log \rho$ plane, one region for each dominant source of pressure. Label each region accordingly. Indicate the position of $(T,\ \rho)$ for the core of the Sun in the plot. Take $T = 15$ million K, $\rho = 150$ g cm^{-3}. What equation of state is most adequate for solar conditions at the core?

Note: to calculate the equations of the different boundaries, you need to know the mean molecular weight μ and the mean molecular weight per electron μ_{e}. Assume that $\mu = 0.85$ and $\mu_{\mathrm{e}} = 1.5$.