Safety-Critical Control for Systems with Impulsive Actuators and Dwell-Time Constraints

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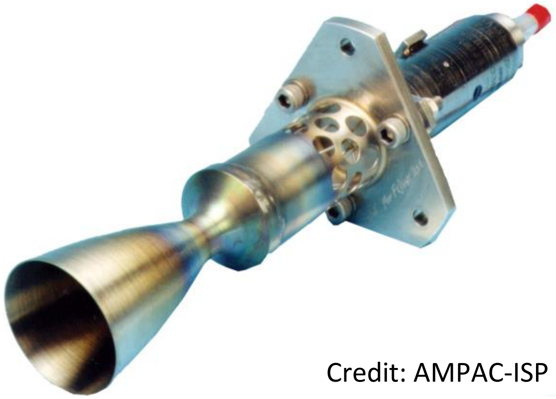
Objective

- State constrained control for satellites
- Problem setup
  - One controlled satellite needs to navigate in the vicinity of other objects in space
- Control Barrier Functions (CBFs) are a promising solution
- CBFs usually run in continuous time
- Most satellites possess <u>impulsive actuators</u>
- After an impulse, the satellite must wait a specified <u>dwell time</u> before the actuator can be used again
- Also relevant to chemical/population systems

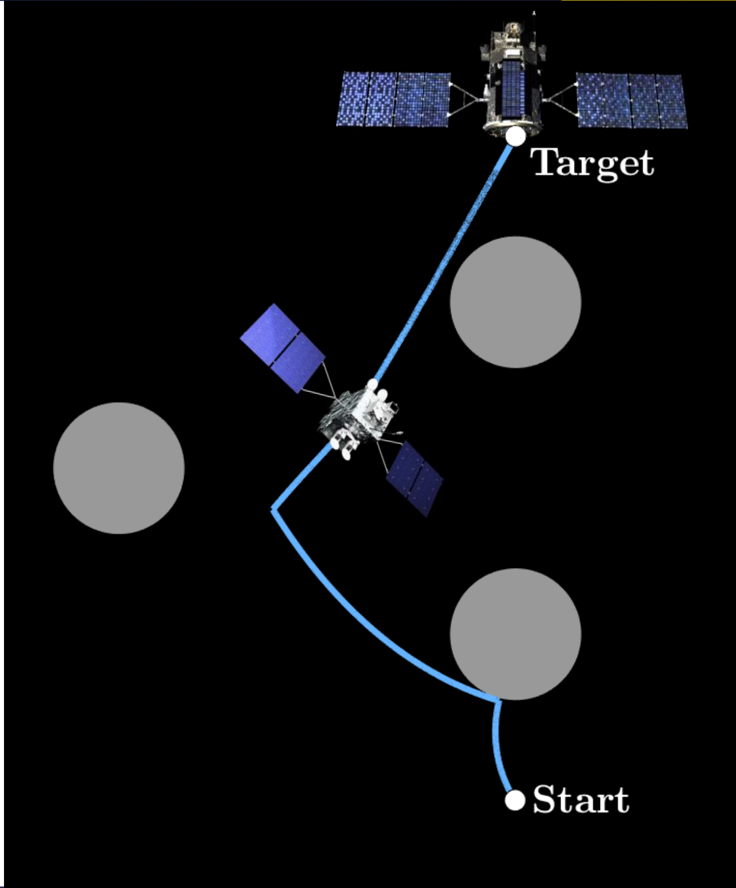
Credit: ESA



Credit: AMPAC-ISP

Objective

- Safety
  - The controlled spacecraft avoids all other objects in its vicinity
  - Equivalently, the spacecraft stays within a specified safe set
- Convergence
  - The controlled spacecraft reaches a target state
- Both subject to the actuator dynamics
- Real time instead of trajectory planning



Introduction - Control Barrier Functions

- CBFs are a tool for set invariance

[1] Chai and Sanfelice, "Forward invariance of sets for hybrid dynamical systems (part ii)," TAC, 2021.

Continuous Time → **Hybrid**

$$\dot{x} = f(t, x, u)$$

$$\begin{cases} \dot{x} = f(t, x, u) & (t, x) \in \mathcal{C} \\ x^+ = g(t, x, u) & (t, x) \in \mathcal{D} \end{cases}$$

$$\mathcal{S}_{\text{safe}}(t) = \{x \in \mathcal{X} \mid \kappa(t, x) \leq 0\}$$

$$\mathcal{S}_{\text{cbf}}(t) = \{x \in \mathcal{X} \mid h(t, x) \leq 0\} \subseteq \mathcal{S}_{\text{safe}}(t)$$

$$\dot{h}(t, x, u) \leq \alpha(-h(t, x))$$

$$\text{where } \alpha \in \mathcal{K}$$

$$\begin{cases} \dot{h}(t, x, u) \leq \alpha(-h(t, x)) & (t, x) \in \mathcal{C} \\ h(t, g(t, x, u)) \leq 0 & (t, x) \in \mathcal{D} \end{cases}$$

is sufficient to render state trajectories always inside $\mathcal{S}_{\text{cbf}} \subseteq \mathcal{S}_{\text{safe}}$

- Assume that we already know how to pick $h : \mathcal{T} \times \mathcal{X} \to \mathbb{R}$

Introduction - Control Barrier Functions

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<u>Continuous Time</u>

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$$\dot{h}(t, x, u) \leq \alpha(-h(t, x))$$

where $\alpha \in \mathcal{K}$

<u>Sampled-Data</u>

$$\begin{cases} \dot{x} = f(t, x, u) \\ u(t) = \underbrace{u(t_k)}_{u_k}, \forall t \in [t_k, t_k + T) \end{cases}$$

$$h(t_k, x_k) + \dot{h}(t_k, x_k, u_k)T + \epsilon \leq 0$$

where ϵ is a bound on higher order terms

[2] Breeden, Garg, and Panagou, "Control barrier functions in sampled-data systems," LCSS, 2022.
[3] Shaw Cortez et al., "Control barrier functions for mechanical systems: Theory and application to robotic grasping," TCST, 2019.
[4] Yang, Belta, and Tron, "Self-triggered control for safety critical systems using control barrier functions," ACC, 2019.

Impulsive System Definition

- General hybrid system:

$$\begin{cases} \dot{x} = f(t, x, u) & (t, x) \in \mathcal{C} \\ x^+ = g(t, x, u) & (t, x) \in \mathcal{D} \end{cases}$$

- **Rule 0:** control is applied only via impulses and impulses only occur due to control

$$\begin{cases} \dot{x} = f(t, x) & (t, x) \notin \mathcal{D} \\ x^+ = g(t, x, u) & (t, x) \in \mathcal{D} \end{cases}$$

- **Rule 1:** the controller is sampled with fixed period and an impulse can only be applied at the samples

$$\begin{cases} \dot{x} = f(t, x) & t \notin \mathcal{D}_0 \\ x^+ = g(t, x, u) & t \in \mathcal{D}_0 \end{cases}$$

$$\mathcal{D}_0 = \{t \in \mathcal{T} \mid t = t_0 + k\Delta t, k \in \mathcal{Z}_{\geq 0}\}$$

- **Rule 2:** the controller can only apply an impulse a minimum dwell time after the last impulse

$$\mathcal{D} = \mathcal{D}_0 \times \{\sigma \in \mathbb{R}_{\geq 0} \mid \sigma \geq \Delta T\}$$

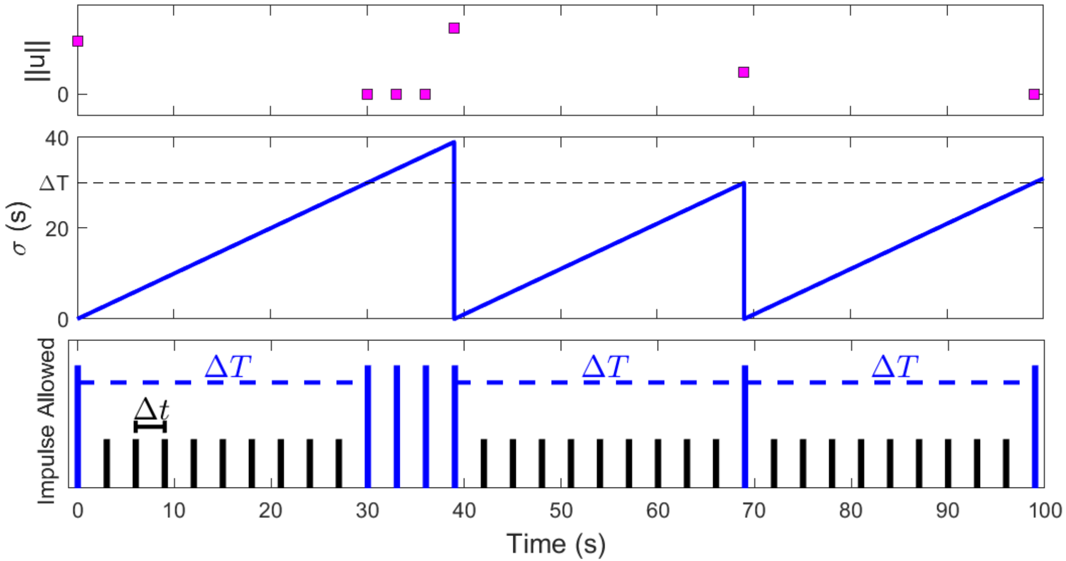
$$\begin{cases} \begin{cases} \dot{x} = f(t, x) \\ \dot{\sigma} = 1 \end{cases} & (t, \sigma) \notin \mathcal{D} \\ \begin{cases} x^+ = g(t, x, u) \\ \sigma^+ = \sigma \text{ if } u = 0 \\ \sigma^+ = 0 \text{ if } u \neq 0 \end{cases} & (t, \sigma) \in \mathcal{D} \end{cases}$$

Impulsive System Definition

$$\mathcal{D}_0 = \{t \in \mathcal{T} \mid t = t_0 + k\Delta t, k \in \mathcal{Z}_{\geq 0}\}$$

$$\mathcal{D} = \mathcal{D}_0 \times \{\sigma \in \mathbb{R}_{\geq 0} \mid \sigma \geq \Delta T\}$$

- $\mathcal{D}$ is the set of "impulse opportunities"



$$\begin{cases} \begin{cases} \dot{x} = f(t,x) \\ \dot{\sigma} = 1 \end{cases} & (t,\sigma) \notin \mathcal{D} \\ \begin{cases} x^+ = g(t,x,u) \\ \sigma^+ = \sigma \text{ if } u = 0 \\ \sigma^+ = 0 \text{ if } u \neq 0 \end{cases} & (t,\sigma) \in \mathcal{D} \end{cases}$$

Method: Impulsive Timed Control Barrier Functions

- Let p denote the flow of the uncontrolled system

$$p(\tau, t, x) = y(\tau) \text{ where } \dot{y}(s) = f(s, y(s)), \; y(t) = x$$

- Let ψ_h be an upper bound on h over a horizon

$$\psi_h(\tau, t, x) \geq h(s, p(s, t, x)), \; \forall s \in [t, \tau]$$

- For example,

$$\psi_h(\tau, t, x) = h(t, x) + \dot{h}(t, x)(\tau - t) + \epsilon$$

where ϵ is a bound on higher order terms

This was $\dot{h}(t_k, x_k, u_k)$ in the sampled-data example

Method: Impulsive Timed Control Barrier Functions

- Control Barrier Functions (CBFs) are a tool for set invariance

Continuous Time

$$\dot{x} = f(t, x, u)$$

$$\mathcal{S}_{\text{safe}}(t) = \{x \in \mathcal{X} \mid \kappa(t, x) \leq 0\}$$

$$\mathcal{S}_{\text{cbf}}(t) = \{x \in \mathcal{X} \mid h(t, x) \leq 0\} \subseteq \mathcal{S}_{\text{safe}}(t)$$

$$\dot{h}(t, x, u) \leq \alpha(-h(t, x))$$

where $\alpha \in \mathcal{K}$

Impulsive

$$\begin{cases} \begin{cases} \dot{x} = f(t, x) \\ \dot{\sigma} = 1 \end{cases} & (t, \sigma) \notin \mathcal{D} \\ \begin{cases} x^+ = g(t, x, u) \\ \sigma^+ = \sigma \text{ if } u = 0 \\ \sigma^+ = 0 \text{ if } u \neq 0 \end{cases} & (t, \sigma) \in \mathcal{D} \end{cases}$$

$$\begin{cases} \psi_h(t + \Delta t, t, x) \leq 0 & u(t, \sigma, x) = 0 \\ \psi_h(t + \Delta T, t, g(t, x, u)) \leq 0 & \\ & u(t, \sigma, x) \neq 0 \end{cases}$$

$$\forall x \in \mathcal{S}_{\text{cbf}}(t), \forall (t, \sigma) \in \mathcal{D}$$

Applying ITCBFs

- Suppose $x = \begin{bmatrix} r \\ \dot{r} \end{bmatrix}$ and $\mathcal{S}_{\text{safe}}$ is the set of states outside an obstacle

$$\kappa(t, x) = \rho - \|r - r_0(t)\|$$

- Choose

$$h(t, x) = \kappa(t, x) + \gamma \dot{\kappa}(t, x)$$

where $\dot{\kappa}(t, x) = -\frac{(r - r_0)^{\mathrm{T}}(\dot{r} - \dot{r}_0(t))}{\|r - r_0(t)\|}$ is affine in $\dot{r}$

upper bound on $\ddot{\kappa}(t, x)$

- One possible choice of upper bound is

$$\psi_h(t + \delta, t, x) = \max\{h(t, x),\ \kappa(t, x) + (\gamma + \delta)\dot{\kappa}(t, x) + (\tfrac{1}{2}\delta^2 + \gamma\delta)\ddot{\kappa}_{\max}\}$$

- See paper for details on related Lyapunov functions V for convergence

[5] Nguyen and Sreenath, “Exponential control barrier functions for enforcing high relative-degree safety-critical constraints,” ACC, 2016
[6] Breeden and Panagou, “Autonomous spacecraft attitude reorientation using control barrier functions,” AIAA JGCD, 2023

Applying ITCBFs

- System

$$\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{-\mu x_1}{(x_1^2+x_2^2)^{3/2}} \\ \frac{-\mu x_2}{(x_1^2+x_2^2)^{3/2}} \end{bmatrix}, \qquad x^+ = \begin{bmatrix} x_1 \\ x_2 \\ x_3 + u_1 \\ x_4 + u_2 \end{bmatrix}, \qquad \Delta t = 3 \text{ s}$$

5 Obstacles
= 5 ITCBFs

- Control law is zero if zero is a safe choice

$$u = \begin{cases} 0 & \psi_v(t+\Delta t, t, x) \leq \gamma_1 V(t,x) \text{ and } \psi_{h_i}(t+\Delta t, t, x) \leq 0, \ \forall i \in \{1,2,3,4,5\} \\ u^* & \text{else} \end{cases}$$

and otherwise is computed as the quadratic program

$$u^* = \underset{u \in \mathbb{R}^2}{\arg\min}\, u^{\mathrm{T}} u$$

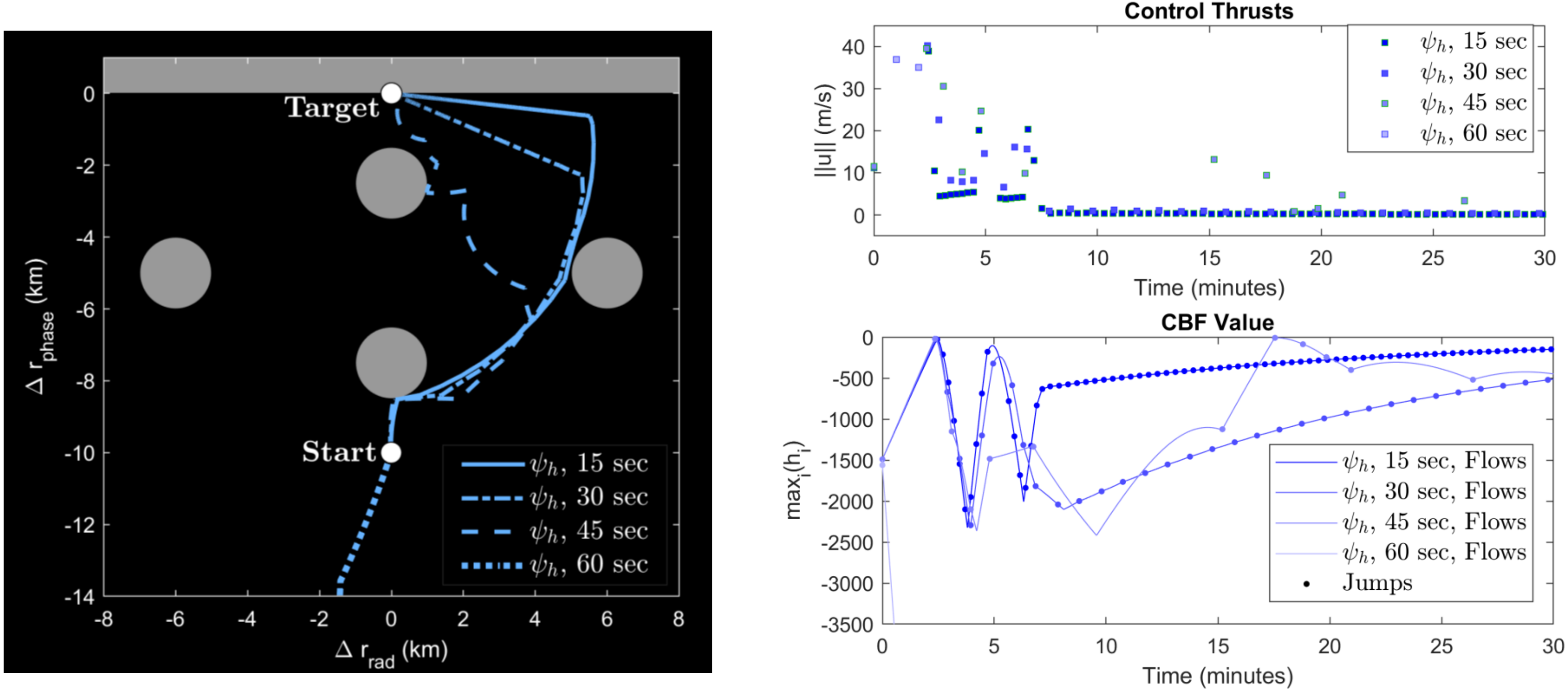
$$\text{s.t. } \psi_{h_i}(t+\Delta T, t, g(t,x,u)) \leq 0, \ \forall i \in \{1,2,3,4,5\}$$

$$(\text{and other conditions on } V)$$

This condition
is affine in u

Simulation Part I

- Results converted to Hill's Frame for visualization



Simulation Part I

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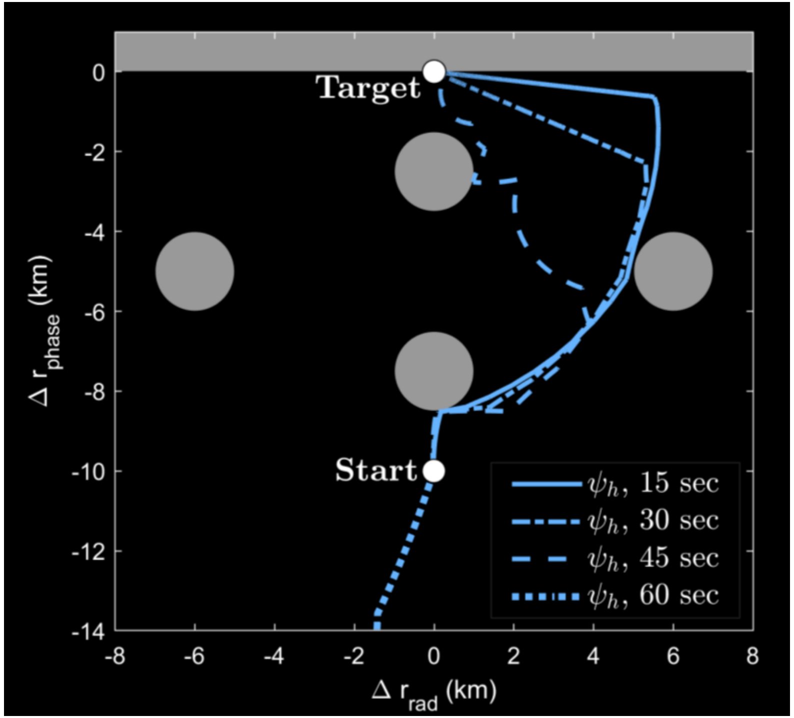


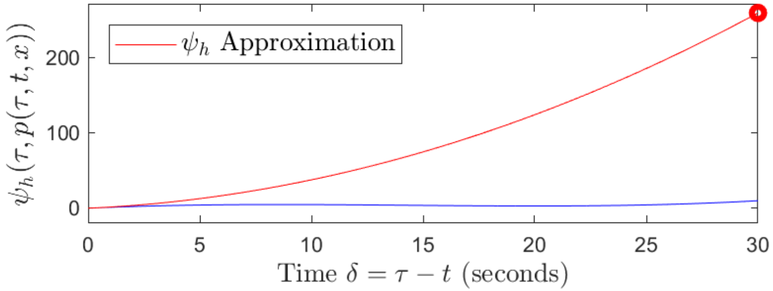
- The strategy works up to 45 seconds of dwell time
- At 60 seconds, the controller is so conservative that it turns away from the target
- All trajectories used a lot of fuel (about 13x what a path planner achieved under the same constraints and time of convergence)

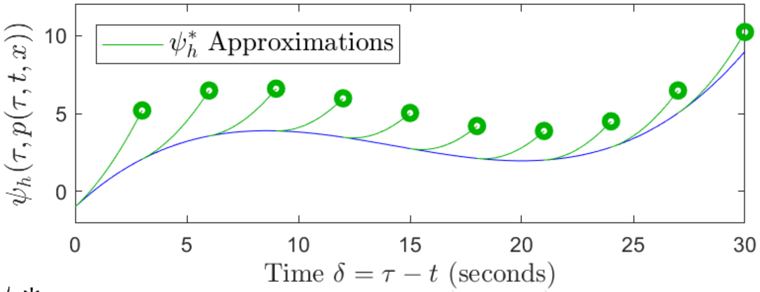
Extending ITCBFs

- The conservatism of ψ_h is quadratic in ΔT
- Replace ψ_h with n_ψ predictions to reduce conservatism

$$\psi_h^*(\tau,t,x) = \begin{bmatrix} \psi_h(\tau_1,t,x) \\ \vdots \\ \psi_h(\tau_j,\tau_{j-1},p(\tau_{j-1},t,x)) \\ \vdots \\ \psi_h(\tau_{n_\psi},\tau_{n_\psi-1},p(\tau_{n_\psi-1},t,x)) \end{bmatrix}$$

where $\tau_j = t + \frac{j}{n_\psi}(\tau - t)$

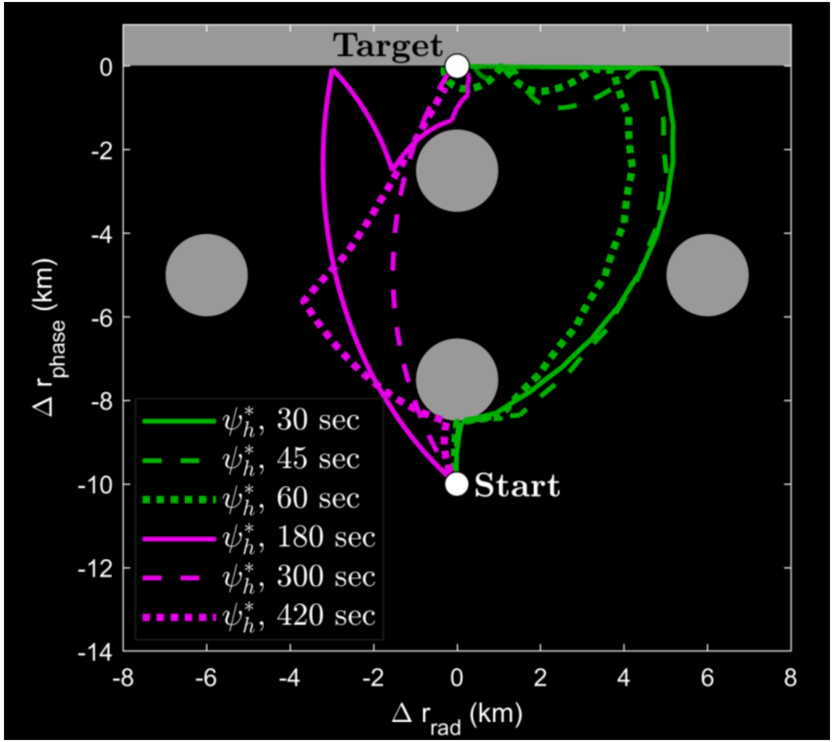


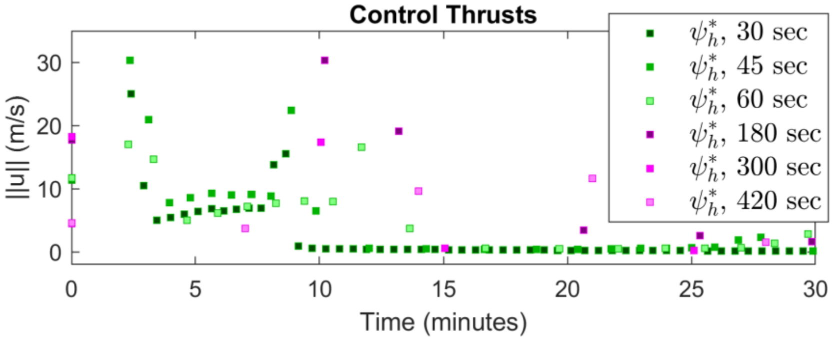


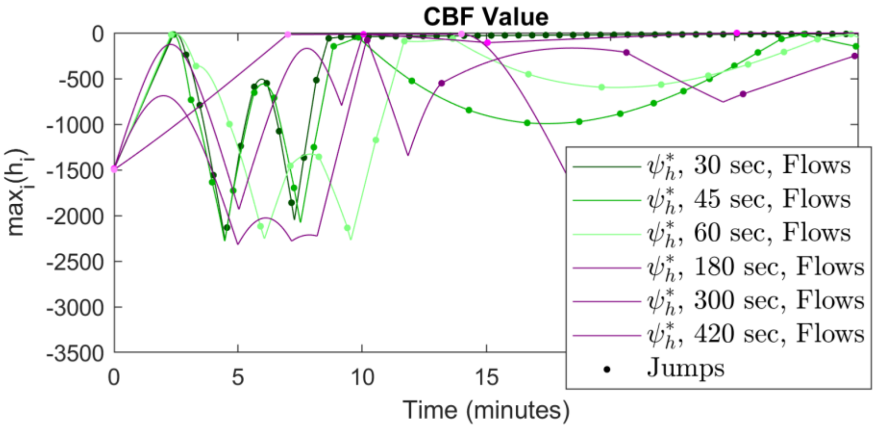
- Replace ψ_h in the control law with the vector ψ_h^*
  - Analogous to MPC with a control horizon of 1 and a long state horizon
  - The controller is now a nonlinear program instead of quadratic program
    - Assume that this is acceptable because of the long time between actuations

Simulation Part II

- Let $n_\psi = 10$

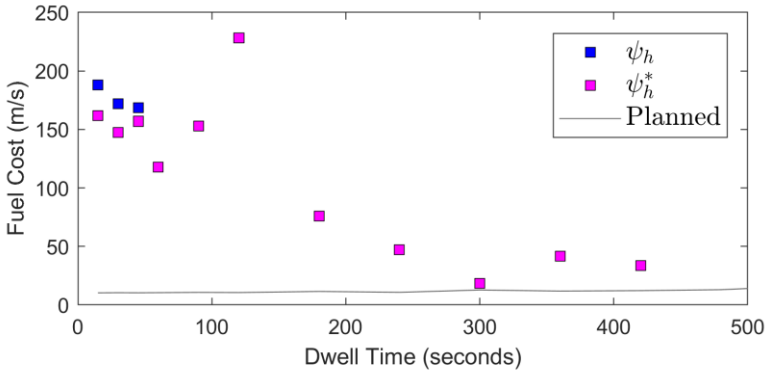






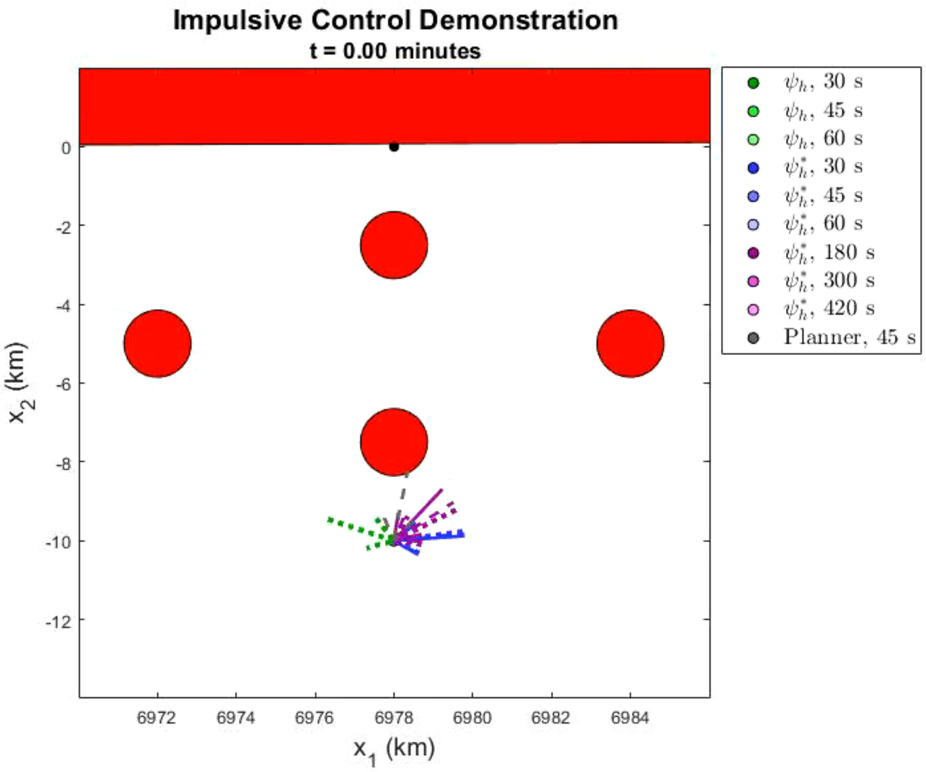
Simulation Part II

- The strategy works up to at least 420 s dwell time for the choice $n_\psi = 10$
- The strategy works for longer dwell times at $n_\psi = 10$ but convergence is slow



- Fuel consumption and time to convergence are nonlinear with dwell time
- The lowest fuel trajectory (of those simulated) was $\Delta T = 300$ sec, which took 1.4x as much fuel as the path planned trajectory

Animation



Impulsive Control Demonstration
t = 0.00 minutes
0
-2
-4
-6
-8
-10
-12
x_2 (km)
6972
6974
6976
6978
6980
6982
6984
x_1 (km)
ψ_h, 30 s
ψ_h, 45 s
ψ_h, 60 s
ψ_h^*, 30 s
ψ_h^*, 45 s
ψ_h^*, 60 s
ψ_h^*, 180 s
ψ_h^*, 300 s
ψ_h^*, 420 s
Planner, 45 s

Conclusions

- Extended CBFs to <u>impulsive</u> systems with minimum <u>dwell time</u> constraints
- Impulses modeled as hybrid system but invariance conditions more similar to the sampled-data CBF literature
- Larger dwell times generally reduce fuel consumption
  - Not a strict rule
  - Note that we did not set out to minimize fuel
- We derived similar conditions for Lyapunov convergence in the paper
- Future work
  - Consider <u>robustness</u> to disturbances that occur during and/or between impulses
  - Using this technique in conjunction with optimal control methods to minimize fuel

Thank You To Our Sponsors

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