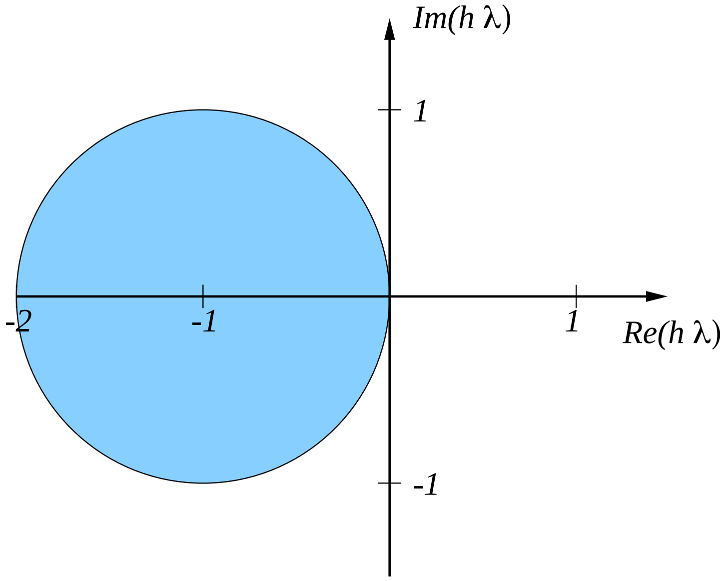

Figure 2.1.2: Region of absolute stability for Euler's method.

As shown in Figure 2.1.2, the region of absolute stability is a unit circle centered at $(-1, 0)$ in the complex $h\lambda$ plane.

Example 2.1.7. Let us solve the IVP

$$y' = y, \qquad y(0) = 1, \qquad 0 \leq t \leq 1$$

by Euler's method. The exact solution is, of course, $y(t) = e^t$. The interval $0 \leq t \leq 1$ is divided into $N = 2^k$, $k = 0, 1, \ldots,$ uniform subintervals of width $h = 1/N$. In order to enhance roundoff effects, arithmetic was done on a simulated computer where floating point numbers have 21-bit fractions. The computed solutions and errors at $T = 1$ are presented in Table 2.1.4 for k ranging from 0 to 16. A ˜ signifies results computed with 21-bit rounded arithmetic.

Roundoff errors are introduced at each step of the computation because of the imprecise addition, multiplication, and evaluation of $f(t, y)$. They can, and typically do, accumulate to limit accuracy. In the present case, approximately four digits of accuracy can be achieved with a step size of approximately 2^{-14}. Decreasing the step size further will not increase accuracy. In order to study this, let

- $y(t_n)$ be the solution of the IVP at $t = t_n$,