Lattice data in the JAM framework

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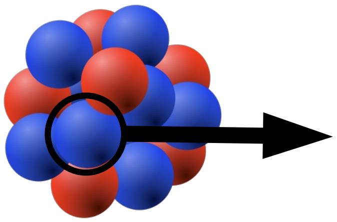
Big picture:
What's the structure of the nucleon?



PDFs

Can't be directly
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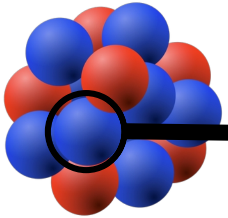

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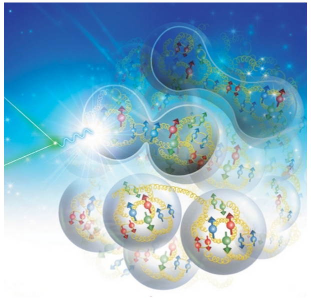

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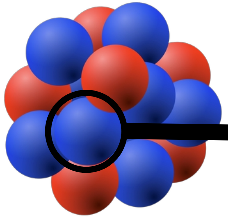

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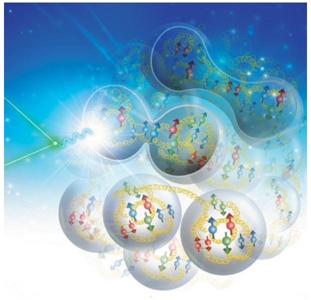

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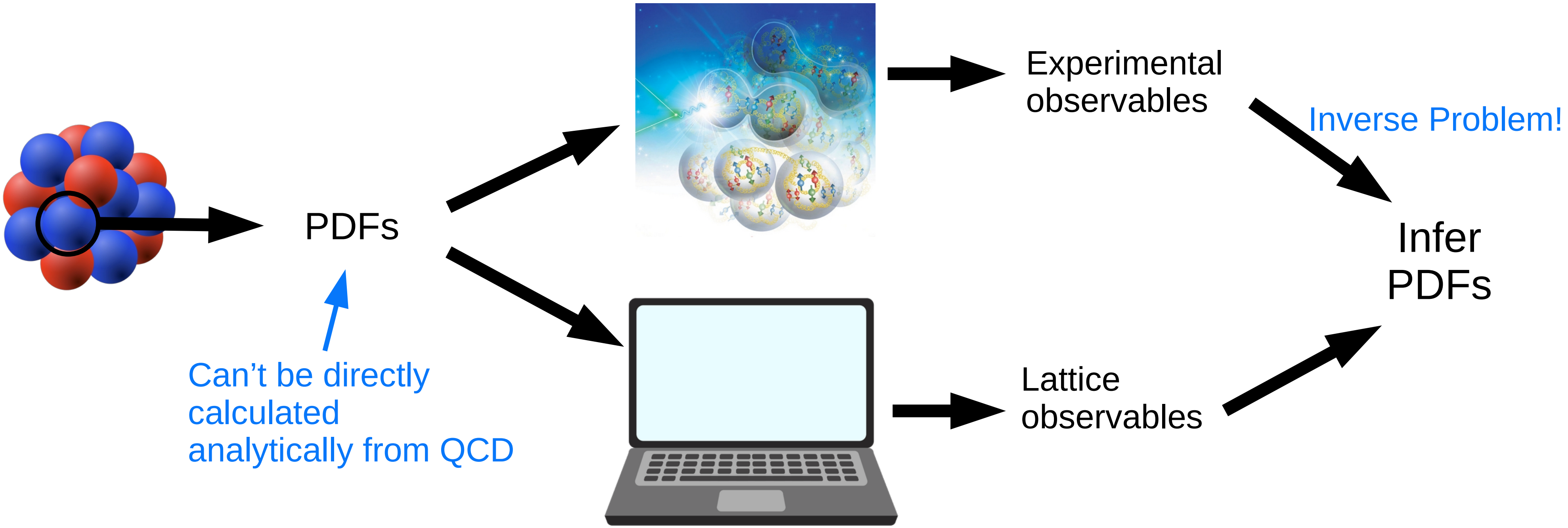
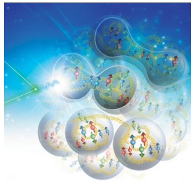


JAM Approach

Assume functional form

$$f(x, \mu_0^2; \vec{a})$$

Fit to observables



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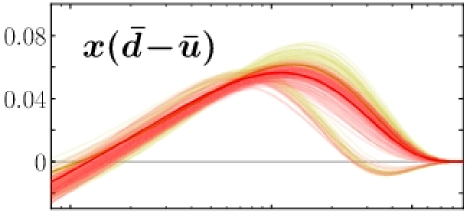

Overview

1. Introduction ✓
2. Lattice methodology
3. JAM framework
4. Numerical Analysis
5. Takeaways and Conclusions

2. Lattice Methodology

We consider the quasi-PDF approach to accessing PDFs from lattice observables:

$$\mathcal{M}_q \xrightarrow{\text{Fourier transform}} \tilde{f}_q(x, \mu, P_3) \xrightarrow{\text{Perturbative matching}} f_q(x, \mu)$$

Nucleon matrix element

Quasi-PDFs

Light cone PDF

Lattice matrix element:

$$\mathcal{M}^q_{[\Gamma]} = Z_\Gamma(z,\mu)\langle N(P_3)|\bar{\psi}_q(0,z)\Gamma W_3(z)\psi_q(0,0)|N(P_3)\rangle$$

Lattice matrix element:

$$\mathcal{M}^{q}_{[\Gamma]} = Z_{\Gamma}(z,\mu)\langle N(P_3)|\bar{\psi}_q(0,z)\Gamma W_3(z)\psi_q(0,0)|N(P_3)\rangle$$

Renormalization
factor

Lattice matrix element:

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Renormalization factor

Nucleon state

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Quark field operator

Lattice matrix element:

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Dirac structure:
γ^0: spin-averaged
$\gamma^3\gamma_5$: spin-dependent

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Wilson line

Lattice matrix element:

$$\mathcal{M}^q_{[\Gamma]} = \underbrace{Z_\Gamma(z,\mu)}_{\text{Renormalization factor}} \underbrace{\langle N(P_3)|}_{\text{Nucleon state}} \underbrace{\bar{\psi}_q(0,z)}_{\text{Quark field operator}} \underbrace{\Gamma}_{\text{Dirac structure}} \underbrace{W_3(z)}_{\text{Wilson line}} \psi_q(0,0)|N(P_3)\rangle$$

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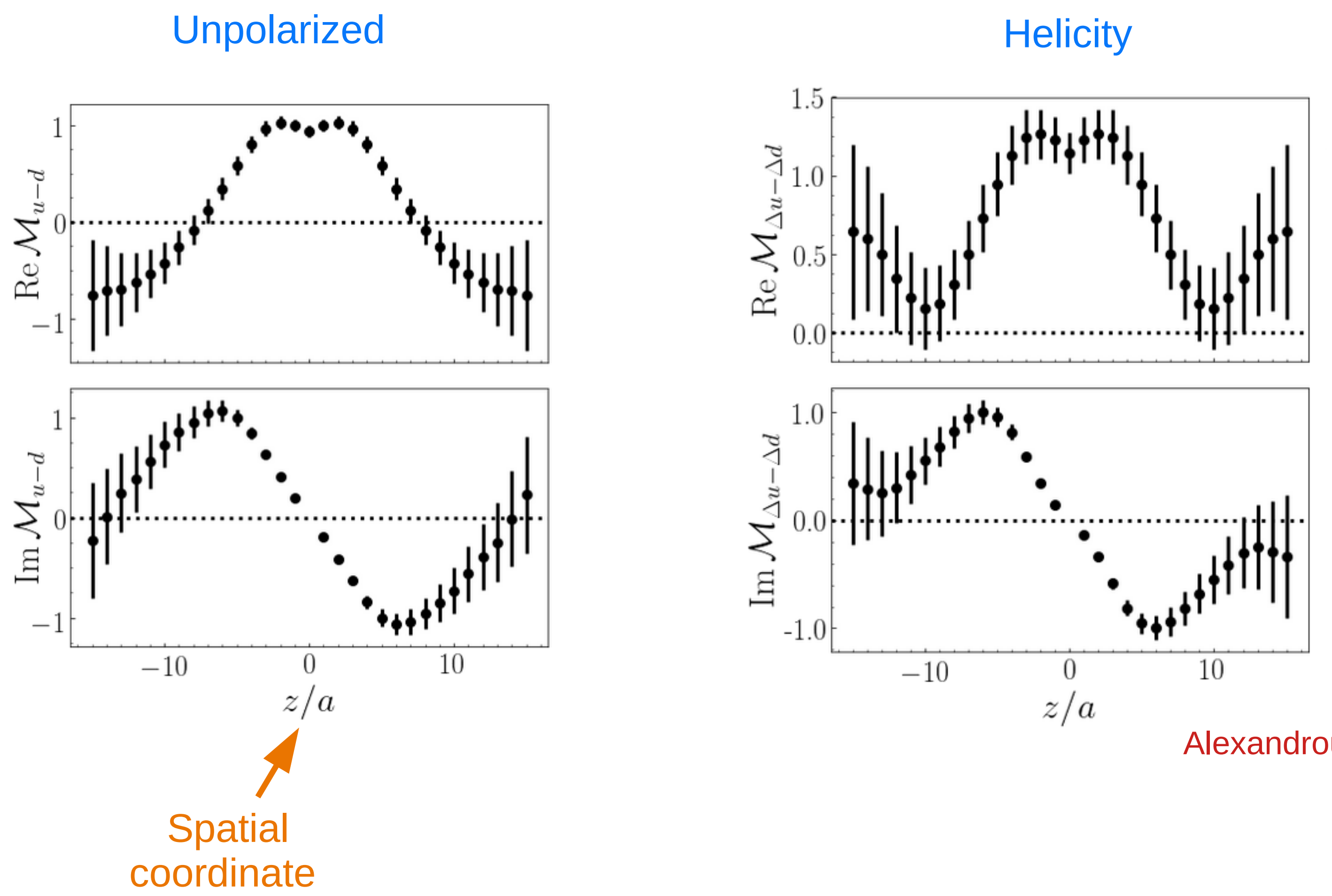
γ^0: spin-averaged
$\gamma^3\gamma_5$: spin-dependent

We consider a proton with:

$$q = u - d$$
$$\mu = 2\,\mathrm{GeV}$$



Alexandrou et. al. (2018)

Matrix elements to quasi-PDFs:

$$\tilde{f}_q(x, \mu, P_3) = P_3 \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP_3z} \mathcal{M}_q(z, \mu)$$

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Quasi-PDFs and light cone PDFs related by perturbative matching procedure:

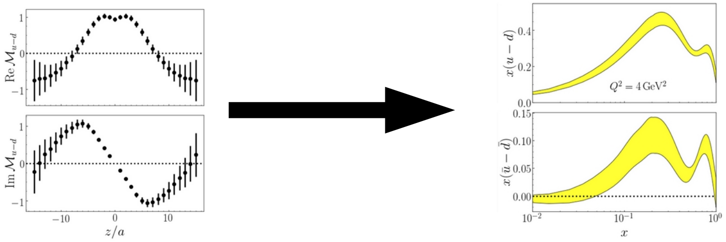
$$\widetilde{f}_q(x,\mu,P_3) = \int_{-1}^{1} \frac{d\xi}{|\xi|} C_q\left(\frac{x}{\xi}, \frac{\mu}{\xi P_3}\right) f_q(\xi,\mu) + \mathcal{O}\left(\frac{m^2}{P_3^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_3^2}\right)$$

Matching kernel

Light cone PDF

Valid for sufficiently large nucleon longitudinal momentum

Standard approach: Truncated discrete FT of calculated lattice elements



In JAM framework:

$$\mathcal{M}_q(z,\mu) = \int_{-\infty}^{\infty} dx\, e^{-ixP_3 z} \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C_q\!\left(\frac{x}{\xi}, \frac{\mu}{\xi P_3}\right) f_q(\xi,\mu) \;+\; \cdots$$

Insert parameterized PDF functional form here

To put in JAM framework:

$$\mathcal{M}_q(z,\mu) = \int_{-\infty}^{\infty} dx\, e^{-ixP_3 z} \int_{-1}^{1} \frac{d\xi}{|\xi|}\, C_q\left(\frac{x}{\xi}, \frac{\mu}{\xi P_3}\right) f_q(\xi,\mu) \;+\; \cdots$$

Insert parameterized PDF functional form here

Equivalent to how we connect experimental observables to PDFs:

e.g. $$\sigma_{\mathrm{DIS}}(x_B, Q^2) = \sum_i \left[H^i_{\mathrm{DIS}} \otimes f_i\right](x_B, Q^2),$$

Flavor label

Short-distance partonic cross section

Split:

- PDFs into quark and antiquark components
- Matrix elements into real and imaginary parts

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Then to leading order:

$$\mathrm{Re}\,\mathcal{M}_q(z,\mu) = -\int_0^1 dy\ \cos(yP_3z)\left[q(y) - \bar{q}(y)\right] + \mathcal{O}(\alpha_s^2)$$

$$\mathrm{Im}\,\mathcal{M}_q(z,\mu) = \int_0^1 dy\ \sin(yP_3z)\left[q(y) + \bar{q}(y)\right] + \mathcal{O}(\alpha_s^2)$$

Split:

- PDFs into quark and antiquark components
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Then to leading order:

Real part sensitive to valence only (unpolarized)!

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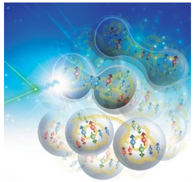
3. JAM Framework

Recall our basic model:

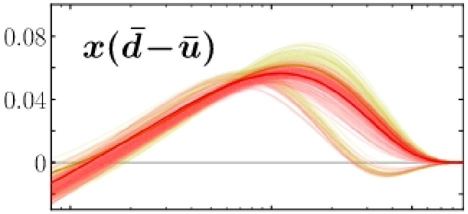
Assume functional form

$$f(x, \mu_0^2; \vec{a})$$

Fit to observables



PDFs



1) Parameterize PDFs at input scale μ_0

$$\mathrm{T}(x, \mu_0^2; a) = \frac{a_0}{\mathcal{N}(a)}\, x^{a_1} (1-x)^{a_2}$$

We parameterize:

$$u_v = u - \bar{u}$$

$$d_v = d - \bar{d}$$

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$$\bar{u}$$

$$s = \bar{s}$$

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3) Determine Bayesian posterior distribution

$$\rho(a|\mathrm{data}) \sim \mathcal{L}(a, \mathrm{data})\, \pi(a) \qquad \mathcal{L}(a, \mathrm{data}) = \exp\left[-\frac{1}{2}\chi^2(a, \mathrm{data})\right]$$

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4) Sample posterior distribution using data resampling and maximize likelihood function

4. Numerical Analysis

Data sets:

- Inclusive DIS:
  - Proton and deuteron targets: BCDMS, SLAC, NMC
  - e^+/e^- - proton cross sections: HERA
- Drell-Yan (pp, pd): E866
- Polarized inclusive DIS:
  - EMC, SMC, COMPASS, SLAC, HERMES
- ETMC lattice data (unpolarized and polarized)

Fits:

- Unpolarized
  - Experimental data sets alone
  - Experimental data sets + lattice
- Polarized
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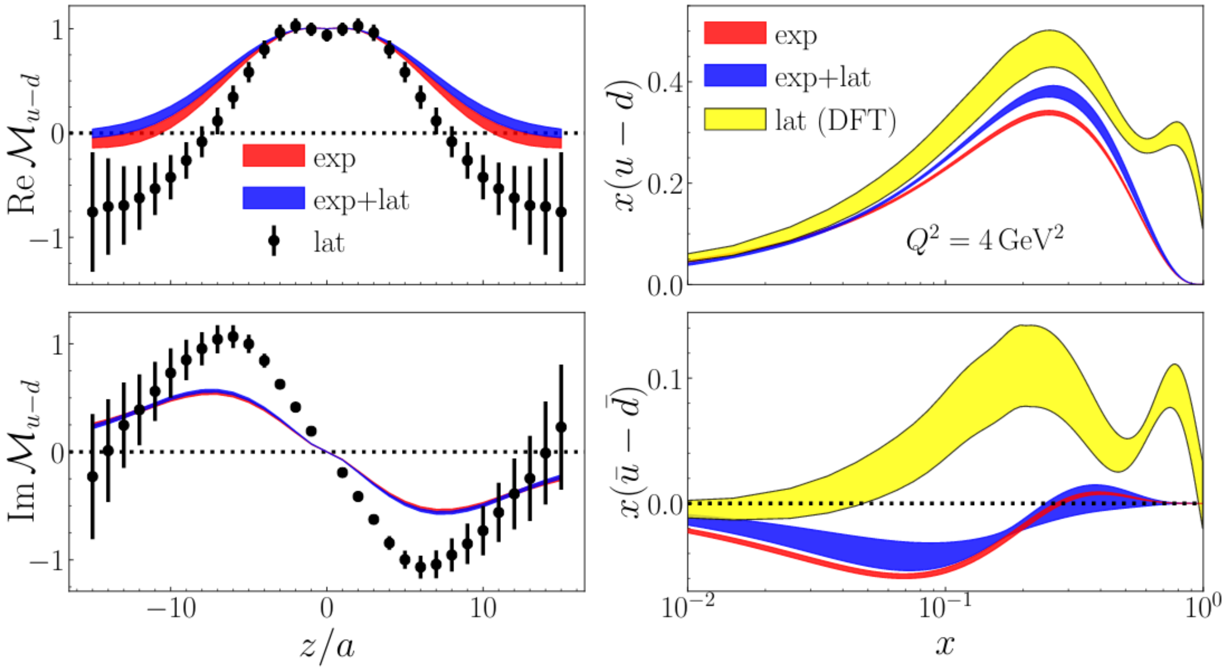
Why not lattice alone?

Inverse problem! In practice, not well constrained – results depend strongly on choice of prior function.

Unpolarized Results

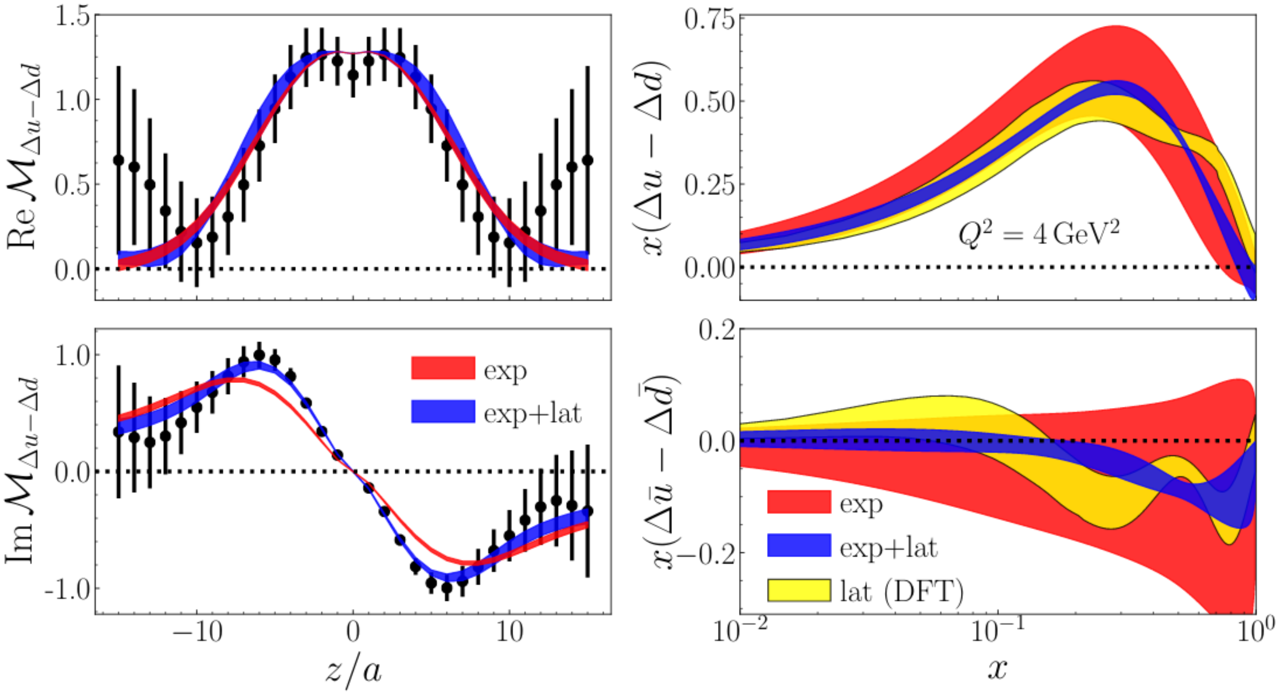
Observable	# data points	χ^2/datum exp	χ^2/datum exp+lat
BCDMS F_2^p [23]	348	1.1	1.1
BCDMS F_2^d [24]	254	1.1	1.2
SLAC F_2^p [25]	218	1.4	1.4
SLAC F_2^d [25]	228	1.0	1.1
NMC F_2^p [26]	273	1.9	1.9
NMC F_2^d/F_2^p [27]	174	1.1	1.2
HERA $\sigma_{\rm NC}^{e^+p}$ (1) [30]	402	1.6	1.6
HERA $\sigma_{\rm NC}^{e^+p}$ (2) [30]	75	1.2	1.2
HERA $\sigma_{\rm NC}^{e^+p}$ (3) [30]	259	1.0	1.0
HERA $\sigma_{\rm NC}^{e^+p}$ (4) [30]	209	1.1	1.1
HERA $\sigma_{\rm NC}^{e^-p}$ [30]	159	1.7	1.7
HERA $\sigma_{\rm CC}^{e^+p}$ [30]	39	1.4	1.2
HERA $\sigma_{\rm CC}^{e^-p}$ [30]	42	1.4	1.4
E866 $\sigma_{\rm DY}^{pp}$ [28]	121	1.3	1.3
E866 $\sigma_{\rm DY}^{pd}$ [28]	129	1.7	1.8
ETMC19 Re $\mathcal{M}_{u-d}$ [8]	31		4.7
ETMC19 Im $\mathcal{M}_{u-d}$ [8]	30		22.7
Total (exp)	2,930	1.3	—
(exp+lat)	2,991	—	1.6



arXiv:2010.00548

Polarized Results

Observable	# data points	χ^2/datum exp	χ^2/datum exp+lat
EMC A_1^p [31]	10	0.3	0.3
SMC A_1^p [32]	11	0.6	0.7
SMC A_1^d [32]	11	2.4	2.3
SMC A_1^p [33]	7	1.3	1.3
SMC A_1^d [33]	7	0.7	0.7
COMPASS A_1^p [34]	11	1.0	0.9
COMPASS A_1^d [35]	11	0.5	0.5
COMPASS A_1^p [36]	35	1.0	1.0
SLAC E80/E130 $A_\parallel^p$ [37]	10	0.8	0.8
SLAC E143 $A_\parallel^p$ [39]	39	0.9	0.8
SLAC E143 $A_\parallel^d$ [39]	39	1.0	1.0
SLAC E143 $A_\perp^p$ [39]	33	1.0	1.0
SLAC E143 $A_\perp^d$ [39]	33	1.2	1.2
SLAC E155 $A_\parallel^p$ [41]	59	1.5	1.4
SLAC E155 $A_\parallel^p$ [42]	59	1.1	1.1
SLAC E155 $A_\perp^p$ [43]	46	0.8	0.8
SLAC E155 $A_\perp^d$ [43]	46	1.5	1.5
SLAC E155x $\tilde{A}_\perp^p$ [44]	69	1.3	1.3
SLAC E155x $\tilde{A}_\perp^d$ [44]	69	0.9	0.9
HERMES A_1^n [45]	5	0.3	0.3
HERMES A_1^p [46]	16	0.6	0.6
HERMES A_1^p [46]	16	1.3	1.3
HERMES A_2^p [47]	9	1.1	1.1
ETMC19 Re $\mathcal{M}_{\Delta u - \Delta d}$ [8]	31		0.5
ETMC19 Im $\mathcal{M}_{\Delta u - \Delta d}$ [8]	30		0.3
Total (exp)	651	1.1	—
(exp+lat)	712	—	1.0



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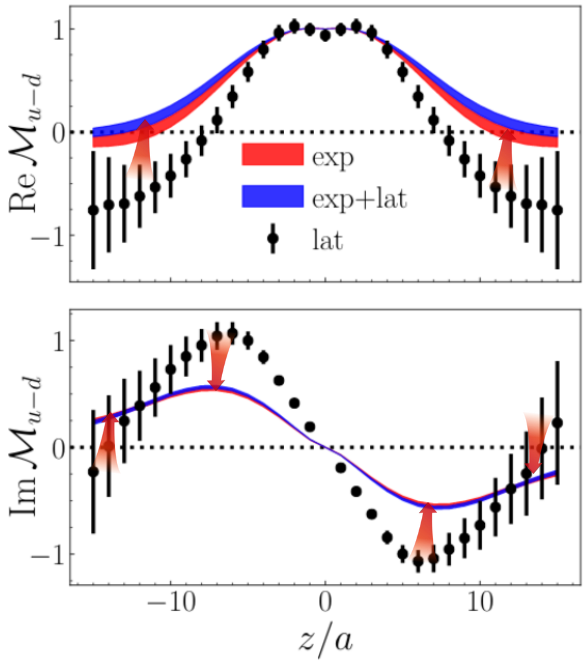
Pseudo-data analysis

Suppose disagreement between lattice and experimental data is resolved.

What impact do the current precision lattice data have on the PDFs?



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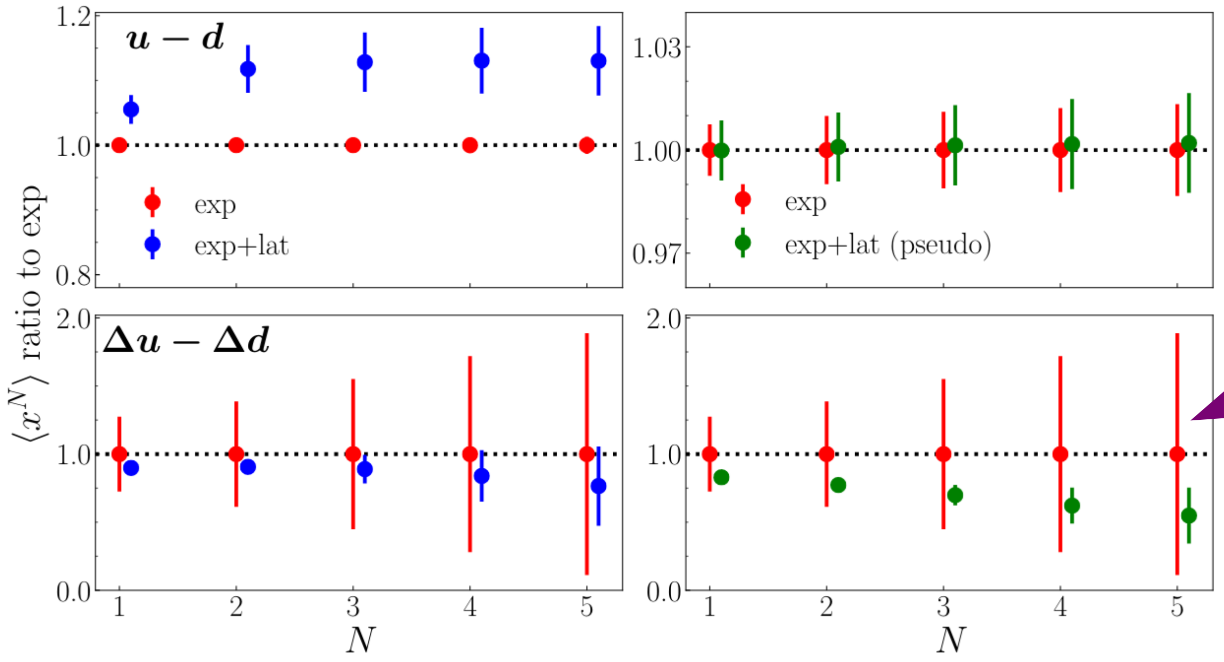
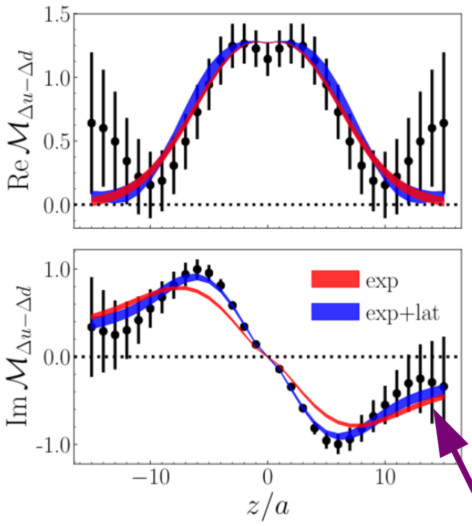


Helicity case: lattice has a big impact due to sensitivity to anti-quark distribution

Importance of large-|z| matrix elements?



Large uncertainties at large |z|
- power-law divergences, sensitivity to power corrections, systematic uncertainties

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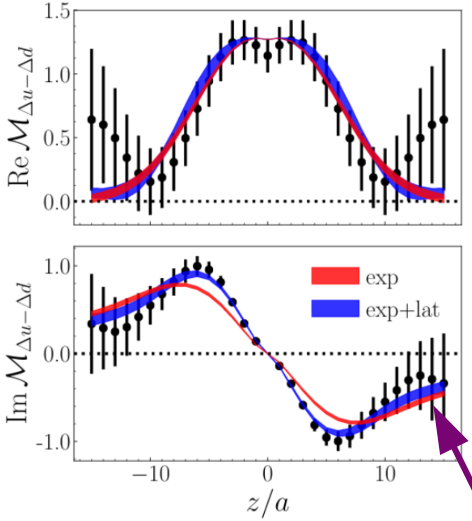

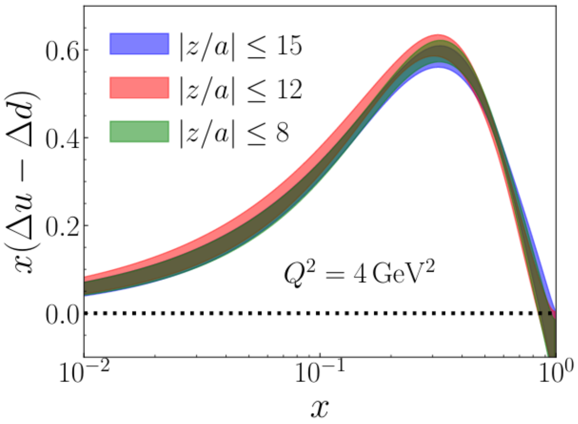

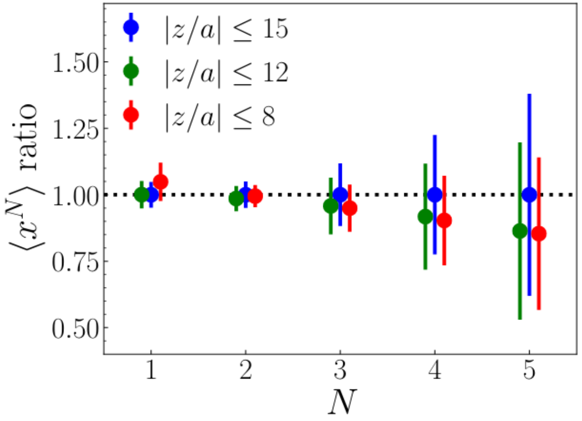

Large uncertainties at large |z|
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5. Takeaways and Conclusions

- We performed first combined MC-based global QCD analysis of spin-averaged and spin-dependent PDFs using both experimental and lattice observables
  - Unpolarized: significant tension
  - Polarized: Lattice data provides significant constraints
- Smaller lattice spacing – larger z, larger P_3, better constraints on x-dependence of PDFs
- Feasible to include lattice data in standard QCD global analysis!