

1

$$8 \text{ Given } F(w) = \frac{1}{1 + e^{-w^T x}}$$

$$10 \text{ - the sigmoid } \sigma(z) = \frac{1}{1 + e^{-z}},$$

$$11 \text{ We have } F(w) = \sigma(w^T x)$$

13 The derivative of the Sigmoid Function w.r.t z is:

$$\frac{d}{dz} \sigma(z) = \sigma(z) \cdot (1 - \sigma(z))$$

14 Applying Chain rule $\frac{d}{dw} F(w)$ is

$$16 \frac{dF}{dw} = \frac{d}{dw} \sigma(w^T x) =$$

$$18 \sigma(w^T x) \cdot (1 - \sigma(w^T x)) \cdot \frac{d}{dw} (w^T x)$$

$$20 = F(w) \cdot (1 - F(w)) \cdot \frac{d}{dw} (w^T x)$$

$$\Rightarrow F(w) \cdot (1 - F(w)) \cdot x$$

9

10

11 Finding the Matrix of Second derivative
12 of $F(w)$ w.r.t w

13

$$\frac{d^2 F}{dw^2} = \frac{d}{dw} \left(\frac{dF}{dw} \right)$$

Applying the chain rule

14

$$15 \frac{d^2 F}{dw^2} = \frac{d}{dw} (F(w) \cdot (1 - F(w)) \cdot x)$$

16

$$17 \Rightarrow \frac{d}{dw} (F(w) \cdot (1 - F(w)) \cdot x) + \frac{d}{dw} (F(w) \cdot (1 - F(w))) \cdot x$$

19

$$20 \Rightarrow \frac{dF}{dw} \cdot (1 - F(w)) \cdot x + F(w) \cdot \left(-\frac{dF}{dw} \right) \cdot x$$

$$8 \Rightarrow F(w) \cdot (1-F(w)) \cdot x \cdot (1-F(w)) \cdot x - F(w) \cdot F(w) \cdot x \cdot x$$

$$9 \Rightarrow F(w) \cdot (1-F(w)) \cdot x \cdot (1-F(w)) \cdot x - F(w)^2 \cdot x \cdot x$$

$$10 \Rightarrow F(w) \cdot (1-F(w)) \cdot x \cdot x^T \cdot (1-F(w)) - F(w)^2 \cdot x \cdot x^T$$

$$11 \Rightarrow F(w) \cdot (1-F(w)) \cdot (x \cdot x^T) \cdot (1-F(w)) - F(w)^2 \cdot (x \cdot x^T)$$

12

$$13 \Rightarrow (1-F(w)) \cdot F(w) \cdot (1-F(w)) \cdot (x \cdot x^T) - F(w)^2 \cdot (x \cdot x^T)$$

$$\Rightarrow (1-F(w)) \cdot F(w) \cdot (1-F(w)) \cdot (x \cdot x^T - F(w) \cdot (x \cdot x^T))$$

$$\Rightarrow (1-F(w)) \cdot F(w) \cdot (1-F(w)) \cdot (x \cdot x^T - F(w) \cdot (x \cdot x^T))$$

$$14 \Rightarrow (1-F(w)) \cdot F(w) \cdot (1-F(w)) \cdot (x \cdot x^T - F(w) \cdot x \cdot x^T)$$

$$15 \Rightarrow (1-F(w)) \cdot F(w) \cdot (1-F(w)) \cdot x \cdot x^T \cdot (1-2F(w))$$

$$16 \Rightarrow F(w) \cdot (1-F(w)) \cdot (1-2F(w)) \cdot x \cdot x^T$$

17

$$18 \text{ therefore } \frac{d^2 F}{dw^2} = F(w) \cdot (1-F(w)) \cdot (1-2F(w)) \cdot x \cdot x^T$$

19