

⁸ Let the number of wires $N = 5$

⁹ Since each wire has double ends

¹⁰ $\Rightarrow 2N = 10$

¹¹ Let X be the number of loops formed

¹²

¹³ Case 1: One Long Loop

The possibility of this happening is when all 10 ends are paired together

$$\text{Pr}(\text{Case 1}) = \frac{1}{\binom{10}{2}}$$

¹⁴

¹⁵ The number of loops for case 1 is one (1)

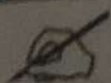
¹⁶

¹⁷ Case 2: Two loops

¹⁸ This is possible when 8 ends are paired together

¹⁹ Leaving 2 loose end that form a separate loop

$$\text{Pr}(\text{Case 2}) = \frac{\binom{10}{2} \binom{8}{2}}{\binom{10}{4}}$$



8 Case 3: There could be 3 loops

9 This is possible when 6 ends are paired together

10 Leaving 4 loose end that form separate loops

11

$$12 \Pr(\text{Case 3}) = \frac{\binom{10}{2} \cdot \binom{8}{2} \cdot \binom{6}{2}}{\binom{10}{6}}$$

13

Case 4: Four loops

14

$$15 \Pr(\text{Case 4}) = \frac{\binom{10}{2} \cdot \binom{8}{2} \cdot \binom{6}{2} \cdot \binom{4}{2}}{\binom{10}{8}}$$

16

17

18 Case 5: Five loops

19

$$20 \Pr(\text{Case 5}) = \frac{\binom{10}{2} \cdot \binom{8}{2} \cdot \binom{6}{2} \cdot \binom{4}{2} \cdot \binom{2}{2}}{\binom{10}{10}}$$

~~21~~

$$E(x) = 1 \cdot \Pr(x=1) + 2 \cdot \Pr(x=2) + 3 \cdot \Pr(x=3) + 4 \cdot \Pr(x=4) + 5 \cdot \Pr(x=5)$$

$$\Rightarrow = 1 \cdot \frac{1}{\binom{10}{2}} + 2 \cdot \frac{\binom{10}{2} \cdot \binom{8}{2}}{\binom{10}{4}} + 3 \cdot \frac{\binom{10}{2} \cdot \binom{8}{2} \cdot \binom{6}{2}}{\binom{10}{6}}$$

$$+ 4 \cdot \frac{\binom{10}{2} \cdot \binom{8}{2} \cdot \binom{6}{2} \cdot \binom{4}{2}}{\binom{10}{8}} + 5 \cdot \frac{\binom{10}{2} \cdot \binom{8}{2} \cdot \binom{6}{2} \cdot \binom{4}{2} \cdot \binom{2}{2}}{\binom{10}{10}}$$

$$\Rightarrow \frac{1}{45} + 2 \cdot \frac{45 \times 28}{210} + 3 \cdot \frac{45 \times 28 \times 15}{210} +$$

$$4 \cdot \frac{45 \times 28 \times 15 \times 6}{45} + 5 \cdot \frac{45 \times 28 \times 15 \times 6 \times 1}{1}$$

$$\Rightarrow \frac{1}{45} + 12 + 63 + 504 + 3780$$

$$\Rightarrow 4860 + \frac{1}{45} \approx 4857.8$$



The expected Number of loops For Five wires is approximately 4857.8