Predicting the Dominant Resonant Frequency in Hydromechanical Systems Containing Fluid Compressibility, Fixture Compliance and Unequal Area Cylinders

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PREDICTING THE DOMINANT RESONANT FREQUENCY IN HYDROMECHANICAL SYSTEMS CONTAINING FLUID COMPRESSIBILITY, FIXTURE COMPLIANCE AND UNEQUAL AREA CYLINDERS

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ABSTRACT

Most fluid power application engineers have little or no access to advanced computational methods, such as simulation and mathematical modelling. And yet, they are called upon time and again, to design and commission complex industrial machines. Many of these machines are one-of-a-kind, therefore neither funding nor time is available to conduct extensive mathematical verification, eg, simulation, before or after committing to hardware.

Unlike mass produced machines of the large OEMs, who are wise to conduct extensive mathematical modelling, the designers of limited production machinery are required to rely upon less mathematically intense methodologies and rules of thumb. At the same time, clients expect complete success.

The heart of many modern hydraulic machines is the electrohydraulic positional servomechanism, which lends itself admirably to computerized motion control technology. Fortunately, the design of such machines has been reduced to a series of formulas that yield the key quantities that designers need to increase the likelihood of application success. It is well-known that the hydromechanical resonant frequency can have a profound effect on servo system performance, limiting closed loop bandwidth and ultimately, positional accuracy among other things. Just knowing the dominant resonant frequency allows the designer to use simple design tools to predict the suitability of a given design to an application.

There is a very well-known formula for calculating the resonant frequency of a system that is based on the load mass and fluid compressibility. However, experienced machine designers know that mechanical deflection of the housing and fixturing works to lower the resonant frequency, adding to the difficulty in achieving system control and smoothness in a routine way.

This paper outlines a semi-empirical method by which a very simple algebraic formula has been derived that allows calculation of the dominant hydromechanical resonance in the presence of both fluid compressibility and mounting fixture compliance with the commonly used single rod

Keywords: hydraulic servo valve, hydromechanical resonance, mechanical resonance, hydromechanical resonant frequency, dominant frequency, hydraulic capacitance, fluid compressibility, fixture compliance, fixture deflection, positional accuracy, effective bulk modulus, effective capacitance, maximum hydraulic capacitance, two-pump linear model, pressure gain, flow gain, leakage resistance, worst case design.

NOMENCLATURE

$\boldsymbol{A_{BE}}$ = Cylinder blank end area
$\boldsymbol{A_{CYL}}$ = Average area of blank and rod ends
$\boldsymbol{A_{PE}}$ = Area of cylinder powered end
$\boldsymbol{A_{RE}}$ = Wetted rod end hydraulic area
$\boldsymbol{B_{B}}$ = Linear viscous friction coefficient, piston to cylinder body
$\boldsymbol{B_{L}}$ = Linear viscous friction coefficient, load to frame
$\boldsymbol{\beta}$ = Fluid bulk modulus
$\boldsymbol{C_{H}}$ = Hydraulic capacitance
$\boldsymbol{C_{HA}}$ = Hydraulic capacitance on the A-side
$\boldsymbol{C_{HB}}$ = Hydraulic capacitance on the B-side
$\boldsymbol{C_{H,ACTUAL,EQ}}$ = Actual, single, equivalent hydraulic capacitance value effect of the A-side and B-side capacitances combined
$\boldsymbol{C_{H,EQ,MAX}}$ = Actual, single, equivalent hydraulic capacitance value effect of the A-side and B-side capacitances combined at the piston position that results in its maximization
$\boldsymbol{C_{HA,MH}}$ = Composite capacitance on the A-side due to both fluid compressibility and mechanical fixture deflection
$\boldsymbol{C_{HB,MH}}$ = Composite capacitance on the B-side due to both fluid compressibility and mechanical fixture deflection
$\boldsymbol{C_{H,EFF,MH}}$ = Single, composite capacitance due to compressibility and fixture deflection
$\boldsymbol{f_{L}}$ = Load force at the linearizing point
$\boldsymbol{G_{PR}}$ = Valve pressure gain at some reference pressure, eg, flow rating pressure
$\boldsymbol{G_{Q,BE}}$ = Linearized valve flow gain on the blank end of the cylinder at the linearizing point
$\boldsymbol{G_{Q,RE}}$ = Linearized valve flow gain on the rod end of the cylinder at the linearizing point
$\boldsymbol{G_{QR}}$ = Linearized valve flow gain at zero load pressure with supply set to flow rating pressure
$\boldsymbol{G_{V}}$ = Linearized velocity gain of the valve and cylinder at the linearizing point
$\boldsymbol{i}$ = Valve input current
$\boldsymbol{I_{r}}$ = Rated valve input current, ie, saturation current
$\boldsymbol{K_{S}}$ = Fixture deflection stiffness coefficient
$\boldsymbol{K_{VPL}}$ = Flow coefficient, powered land
$\boldsymbol{K_{VRL}}$ = Flow coefficient, return land
$\boldsymbol{L_{S}}$ = Cylinder stroke length

end cylinder. Linear system modelling was used to numerically determine the best estimate of the dominant frequency for a spectrum of likely operating conditions. Curve fitting was then used to synthesize a simple algebraic function. The resulting formula offers the applications engineer a tool to help improve the odds of success in building sophisticated, high performance, one-of-a-kind motion control machines without having to resort to other mathematically intense and unavailable design methodologies.

1 BACKGROUND

The system that is used as the basic model for the investigation is shown in simplified schematic form in Figure 1. It is a typical hydraulic installation and contains hydraulic compliance in the fluid compressibility, and mechanical compliance in the mounting structure, or fixture. The fixture is characterized as having a perfectly rigid part connected to mother earth, while its compliance is characterized by a linear spring, labelled K_S in Figure 1.

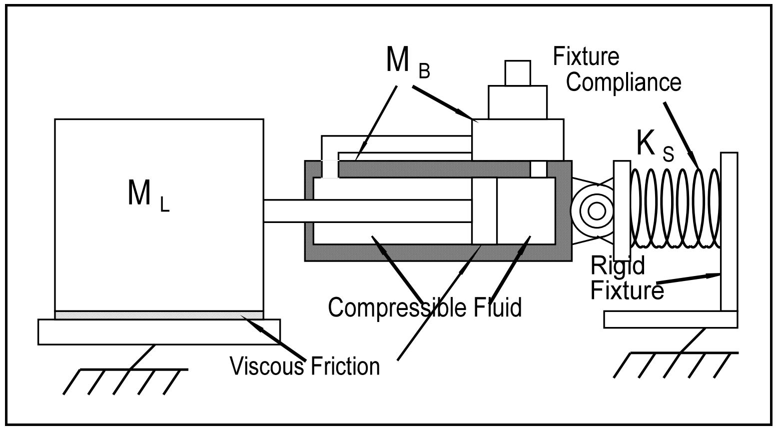

Figure 2 Simplified hydromechanical schematic representation of the system that is modelled. FW^

Given the limited analytical resources available to and at the command of many Hydraulic Applications Engineers, they seek simplifying approximations and rules of thumb to aid them in their designs of one-of-a-kind machines. This is especially true of electrohydraulic servo mechanisms. It is well-known by those who design such systems for a living that hydromechanical resonance produces a near-brick-wall limit as to the maximum performance of modern electrohydraulic motion control systems.

Authors [1], [2], have assisted in filling the pressing needs of applications engineers, and have derived and reported formulas which have been anecdotally verified over several years and countless applications. The differences among them are slight disagreements as to the exact form and the emphasis on which parameters are most likely known or can be easily calculated by the users. For example, in Reference [2], the concept of *hydraulic capacitance* is used and leads to this formula for hydromechanical resonant frequency, $\boldsymbol{\omega_n}$:

$$\omega_n = \frac{A_{CYL}}{\sqrt{MC_H}} \tag{1}$$ FXL

The value for $\boldsymbol{A_{CYL}}$ is one of the points of disagreement among the various authors, and the dispute will continue because there is no exact analytical method for determining the resonant frequency. In the case of Equation (1), above, the value used is the average of the blank end and rod end areas for the ubiquitous, industrial, single rod end cylinder. Later discusion and results in this paper will offer semi-empirical justification for using the average cylinder area. The value for $\boldsymbol{M}$ is the dead weight of the total rigid load that is to be accelerated by the cylinder, divided by gravitational acceleration. $\boldsymbol{C_H}$ is the hydraulic capacitance, a modelling concept that has been used for at least 50 years [3], and is especially preferred by engineers with experience in electrical circuit analysis, such as this author. Hydraulic capacitance is a circuit or system parameter, and is calculated by dividing the total compressed volume of fluid at each circuit node by the fluid's bulk modulus.

One of the side objectives of this paper is to present arguments for the given methods for determining resonant frequency, beyond expanding the analysis to include structure deflection, and there are these issues involved:

1. The choice for piston area value is arguable because of the non-linearities introduced by the unequal areas in the typical industrial cylinder. Most authors use equal area cylinders [7][8], because the equations are intuitively linearized. This paper will introduce a new linear model of the hydraulic system that makes use of unequal cylinder areas and offers analytical justification.

2. Some authors use a spring model for fluid compressibility [4], and is very descriptive in showing how the load mass resonates with the compressing fluid. Refer to Figure 2. However, this author, in [2] has argued that it is valid only when the piston is allowed to move. Clearly, compressibility is at play in the hydraulic system even when there is no mechanical movement whatsoever. This fact argues in favor of the capacitance model, detailed later, which illustrates the effects of compressibility in the absence of mechanical movement.

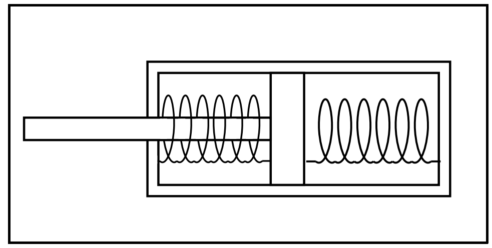

Figure 3 The two-spring model for compressibility requires that the piston move in order to explain compressibility effects. FW&

3. But this author has also shown [2] that the use of the capacitance modelling method is problematic, because the calculation of resonance requires that the two capacitances, ie, one on each side of the moving piston, must be combined to form a single equivalent capacitance, however, this cannot be rigorously done except with numerical methods. The model that is usually implemented views the two capacitances as being in series, but, this conclusion is valid only at the *worst-case design* condition in which the dominant hydromechanical resonant frequency is at its lowest value [2]. This condition will be discussed shortly. We know that most real hydromechanical systems have a single dominant resonant frequency, and there must be therefore, a single equivalent capacitance in resonance with a single equivalent mass. The method of choice, in spite of incongruities, remains as the treating of the two capacitances as being in series. They are then combined by the familiar product over sum method so common in electrical circuit analysis textbooks at all technical levels. Results are offered later that support this ploy. In the case of the two-spring model, the method of combining them is indisputable.

4. Most authors also suggest, whether using capacitance models or others, that the user determine the *effective bulk modulus*, sometimes also called the *apparent bulk modulus*, of the system under study. Both Keller [1] and Merritt [5] have provided detailed analytical methods for including effects of entrained air in the hydraulic fluid and expansion of pressure containing envelopes. Keller has also reported a significant reduction in fluid bulk modulus with temperature rise, and this author has used Keller's data to produce a crude math model of that temperature dependence [6]. The added tedium of including these analytical details leads most practitioners to simply multiply the published fluid bulk modulus by a factor, sometimes as little as 0.3, in order to account for all the modulus-reducing effects. The factor applied is based upon the experiences and preferences of the particular designer. There is no effort in this paper to open up discussions or offer rationale on the best value for bulk modulus, it is mentioned only as background information.

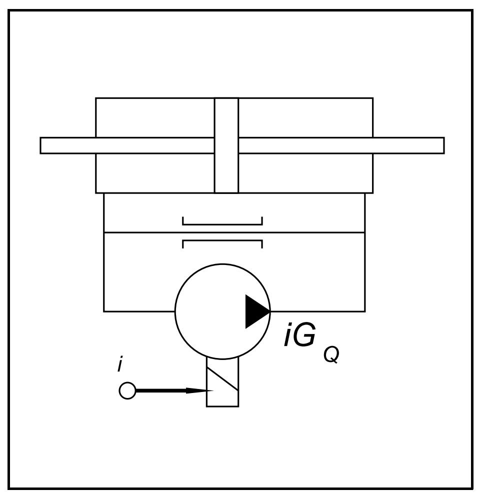

Figure 4 The one pump model for the servo/proportional valve requires an equal area cylinder. FW*

2 TWO-PUMP LINEAR MODEL OF A VALVE CONTROLLED CYLINDER

For many years authors have been using equal area cylinders for derivations [1], [7] , as drawn in Figure 3, however, the unequal, or differential area cylinder is the rule in industrial hydraulic systems. The equal area, double rod end cylinder is the exception. It is essential that there be a simple linear model for the ever-present, unequal area machine.

The classical approach to the hydraulic valve controlled cylinder, shown in Figure 3, is that the control valve is replaced by an *ideal* current-dependent pump whose output is directed into the equal area cylinder. In this idealized scenario, the cylinder return flow is exactly equal to the powered flow, so the cylinder effluent is simply directed back to the pump inlet. There is no need for a reservoir in this ideal machine. The model has some instructional value, to be sure, however, it is impossible to extend the model into more realistic and complex configurations. This model will NOT correctly predict cylinder pressures.

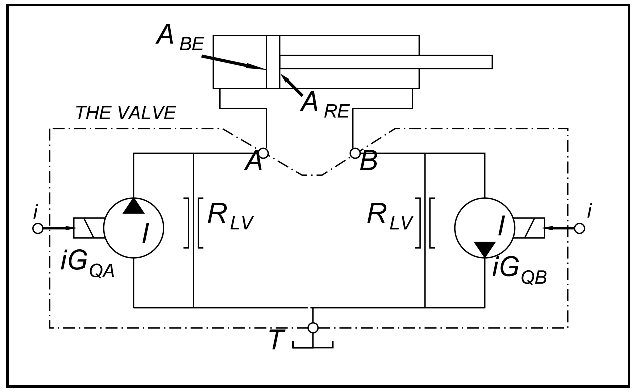

Figure 5 Modelling the valve as two ideal pumps allows unequal area cylinders to be used and yet have a mathematically linear system. FW+

The key to characterizing unequal areas while maintaining linear and soluble models is to look at the valve, not the actuator. Refer to Figure 4. Instead of replacing the valve with a single pump, it is replaced by two pumps, one at each valve outlet port. Furthermore, the flow gain of the rod end pump is lower than the blank end pump by the area ratio of the cylinder. Note that each pump connects to the reservoir, a practical situation, and both pumps are plumbed to assist flow propulsion (an optional configuration is to have each pump flow out of its respective valve port, but then the flow gain on one has to have a negative value). Now there are unequal areas with unequal flows in the two ports, and the model supports internal leakages and compressibility flows because of the reservoir connection. Valve internal leakage (hydraulic damping) is modelled as laminar hydraulic orifices, one at each pump, labelled $\boldsymbol{R_{LV}}$. Leakages, therefore, are directly proportional to the gage pressures at the respective valve work ports.

A linear model is the best first method of analyzing a new design problem provided the aim of the control system is not pressure. The pressures in the hydraulic system are substantially a child of the system non-linearities, that is, the square-square root pressure-flow relationship of the valve's metering orifices. Pressures can be handled with piece-wise linear models which are valid only over a very small pressure range. On the other hand, if the output of the system is cylinder force, cylinder speed or cylinder position, linear modeling is the best first choice, because the pressure is unobservable in these outputs. The two-pump model, like the one-pump model, does not correctly predict cylinder pressures. This paper does not offer analytical or experimental proof of the validity of these assertions, nor is there any intent to imply that linear analysis is the only approach required. It is merely asserted that it is the best *first* method. Justification is the author's 30-plus years of experience in practical application of electrohydraulic control technology to industrial needs.

Actual construction of the linear, unequal area model requires finding the following component and system parameters:

1. The valve's full-open turbulent flow coefficient, $\boldsymbol{K_{VPL}}$.
2. The valve's internal laminar leakage resistance, $\boldsymbol{R_{LV}}$
3. The valve's orifice coefficient ratio $\boldsymbol{\rho_V}$

4. The valve's flow gain at some reference supply pressure and zero load, G_{QR}.
5. The valve's blocked port pressure gain at the same reference pressure, G_{PR}.
3. Velocity gain at the output load in steady-state, G_V

In Reference [2] I have made the following empirical definitions. The practical flow coefficient value of the valve's *powered land* empirically, that is, the value is based upon actual test data for the specific valve, or, alternatively, is based upon the manufacturer's rated flow data measured at the standard *flow rating pressure* [ISO 10770-1]:

$$K_{VPL} \triangleq \frac{Q_r}{\sqrt{\Delta P_{Qr,PL}}} \qquad \textbf{(2)}\text{FXM}$$

Where Q_r is the rated flow of the valve at the standard flow rating pressure and $\Delta P_{Qr,PL}$ is the differential pressure drop across the powered land when operated with rated flow through the valve land. For symmetrical valves, that is, valves that have equal powered and return land flow areas, the rated per-land differential pressure drops, are 3.5 MPa and 0.5 MPa for servo and proportional valves, respectively per ISO standard 10770-1 [9]. This standard calls for 7 MPa and 1MPa for the two valve types, therefore, the symmetrical land pressure drops are simply half of the total pressure values for both lands in series.

Given the definition in Equation (2), it is then derived in [2] the resulting steady state output speed at a specified load, supply pressure and valve powered land coefficient:

$$V_{ss} = K_{VPL} \sqrt{\frac{(P_S A_{PE} - f_L)}{A_{PE}^3 \left(1 + \frac{\rho_v^2}{\rho_c^3}\right)}} \qquad \textbf{(3)}\text{FXN}$$

It is in Equation (3) where the user must decide on the value of load force, f_L, that is to be used at this, the linearizing speed, v_{SS}. It is normally interpreted as that load that exists at constant velocity, that is, the acceleration force is normally removed at linearizing time. I have further shown in [2] that the internal laminar leakage resistance of the valve at one work port is:

$$R_{LV} = \frac{1}{2}\frac{G_{PR}}{G_{QR}} \qquad \textbf{(4)}\text{FXO}$$

For the *return land*:

$$K_{VRL} \triangleq \frac{Q_r}{\sqrt{\Delta P_{Qr,RL}}} \qquad \textbf{(5)}\text{FX\^{}}$$

Where the valve coefficient of the return land is evaluated empirically with rated flow through the valve and measured pressure drop across the land. The valve ratio is defined as the ratio of the empirically derived powered land coefficient over the return land coefficient:

$$\rho_C \triangleq \frac{K_{VPL}}{K_{VRL}} \qquad \textbf{(6)}\text{FX\&}$$

In a so-called *symmetrical valve*, the ratio is unity.

If the value for $\boldsymbol{K_{VPL}}$ in equation 3 is the full-open value, then by definition the current is rated current, therefore it follows immediately that the steady-state flow gain of the blank end pump is the steady-state flow divided by the current that holds the valve fully open:

$$G_{Q,BE} = \frac{v_{ss} A_{BE}}{I_r} \quad \textbf{(7)}\text{FXP}$$

And it further follows that:

$$G_{Q,RE} = \frac{v_{ss} A_{RE}}{I_r} \quad \textbf{(8)}\text{FXQ}$$

Note that the aim is to produce a linear model, therefore it is necessary that the valve metering characteristics be reasonably linear, that is, the metering windows must be essentially rectangular in shape, and further, there can be no substantial center overlap in the valve's metering characteristics. This, of course, is a brief description of the typical industrial hydraulic servo valve.

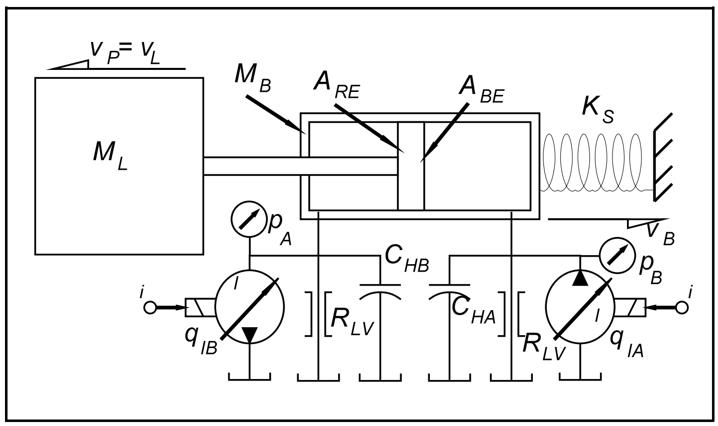

Figure 6 The analytical schematic of the doubly-compliant system facilitates the writing of the node and free-body dynamic equations. FW=

Now, all the values required of the unequal area, linear model have been evaluated. The result is shown in Figure 5 with one addition: The cylinder is shown to have <u>electrical electrolytic capacitors</u> connected to the tube! This absurdity has a meaning. It is a personal preference that I have used for years to indicate that fluid compressibility is important to the study at hand. Compressibility effect, hydraulic capacitance, acts like very stiff, fairly linear accumulators, one in each end, with values that vary with the instantaneous position of the piston. If the circuit schematic is drawn with accumulators, it clearly implies that hardware has been added to the cylinders. That is emphatically not the case. We call the capacitance of compressibility a *parasitic capacitance*. This means that it comes along without being asked or being designed into the system. The cylinder has volume, the volume is filled with compressible fluid, therefore, there is capacitance. I prefer to show that parasitic effect in unmistakable form, with electrical capacitors attached. It also facilitates the writing of the node equations.

3 WORST CASE DESIGN SCENARIO

Clearly, the hydraulic capacitances of the cylinder vary with piston position, because the nodal volumes are changing. One volume increases while the other decreases. There comes a piston position where the effective capacitance maximizes [2], [4], and this is the position that represents the worst-case design scenario, because the hydromechanical resonant frequency is minimized. The lower the frequency, the more difficult it is to control the closed loop system because of the limited closed loop bandwidth. In applications where the mass does not vary, or the actuation geometry does not change, the position of maximum capacitance occurs when the piston is slightly closer to the blank end than to the rod end [2] when the plumbing geometry does not contribute significantly to the capacitances. Unfortunately, this worst case design point is near the piston mid-position, an almost universal operating point [2]:

$$X_{max,C} = \frac{(A_{RE}L_S + V_{PB})\sqrt{\rho_c} - V_{PA}}{A_{BE} + A_{RE}\sqrt{\rho_c}} \qquad \textbf{(9)}\text{FXA}$$

The spring model for compressibility [4], Figure 2, places the point of maximum capacitance, $\boldsymbol{X_{max,C}}$, (with the spring model, the actual goal is minimum stiffness) slightly toward the rod end relative to piston mid-position. Because the spring model has been used for such a long time, it has prompted system designers to place the control valve at the rear of the cylinder so that the longer plumbing volume is added to the rod side. In spite of this technical disagreement with the capacitance-based model, the resulting calculated resonant frequencies from the two methods yield similar values. This paper does not address the differences beyond what has already been stated.

Further adding to the disagreement is the difficulty in experimentally confirming the resonant frequency because of the difficulty in controlling and knowing the fluid bulk modulus at the specific operating point. This difficulty contributes to the common practice of experienced designers to use a correction factor for the bulk modulus, mentioned earlier. The tendency is to err on the safe side, therefore, a substantially reduced bulk modulus value is assumed at design time. Few designers of real industrial systems have the luxury of documenting the exact system conditions after achieving applications success. Therefore, there is no large body of verified application data, even though there are countless successful (some failures, too) applications. There are mostly thousands of personal experiences modulated by whatever biases may befit the designer.

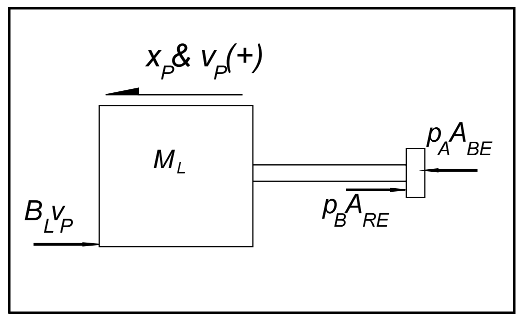

Figure 7 Free-body diagram of the rigid piston-rod-load mass moving at velocity v_p. FXB

The actual value of the maximum, worst-case, capacitance, occurring when the piston is in the position given in Equation (9), was derived in Reference [2]:

$$C_{H,EQ,MAX} = \frac{A_{BE}L_S + \rho_C V_{P,RE} + V_{P,BE}}{\beta(1 + \sqrt{\rho_C})^2} \qquad \textbf{(10)}\text{FYA}$$

4 CONFIGURATION OF THE MODELLED SYSTEM

A hydromechanical analytical schematic of the modelled system is shown in Figure 5. The valve is characterized in its two-pump configuration according to earlier discussion. It is assumed that the driving function is current into the coil of a servo or proportional valve, and the same current drives both model pumps. As discussed earlier, it is a further requirement of the linearizing process that the values of flow gains for each pump be in the same ratio as the cylinder areas:

$$G_{Q,RE} = G_{Q,BE}\frac{A_{RE}}{A_{BE}} \qquad \textbf{(11)}\text{FXR}$$

Fluid compressibility is characterized with capacitor symbols, one at each flow node, representing the compressibility effects of each total volume between the valve spool and each respective piston face. Fixture deflection is characterized by a linear spring, K_s,Figure 5, connected between the valve-cylinder body mass and infinitely rigid mother earth. Please note that the fixture is characterized as being only a spring, not a distributed spring-mass system which it clearly is in any real sense. The justification is two-fold: First, it is a simplifying approximation, and second, the aim is to model systems that are lacking in fixture stiffness, concomittantly suggesting that they are light on mass. The result is a fifth order system:

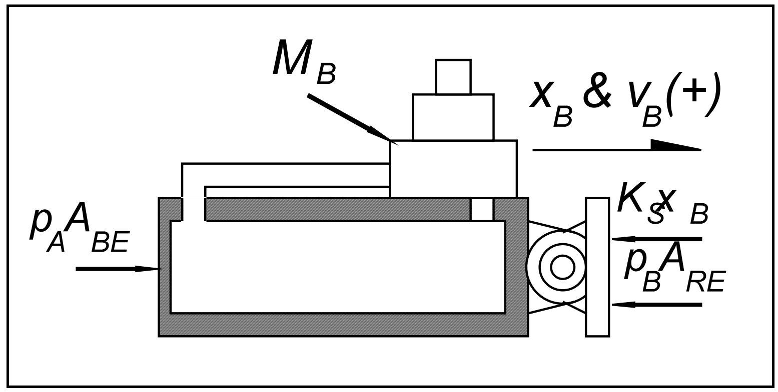

Figure 8 Free-body diagram of the cylinder body-valve-plumbing mass moving at velocity v_B. FXC

1. Flow node at the blank end inlet port of the cylinder
2. Flow node at the rod end inlet port of the cylinder
3. Velocity node at the piston-load mass lump
4. Velocity node at the body-valve mass lump
5. One integration of the body velocity because of the fixture deflection spring.

An algebraic sign convention has been established as follows:

1. Flow entering the blank end of the cylinder is positive
2. Flow leaving the rod end of the cylinder is positive
3. Piston velocity is positive when the piston extends
4. Body velocity is positive when motion compresses the fixture flexure spring.

The linear differential equations are listed now:

I. Flow node at the blank end port of the cylinder, $\boldsymbol{p_A}$ node in Figure 5:

$$q_{IA} = \frac{p_A}{R_L} + C_H\frac{dp_A}{dt} + v_L A_{BE} + v_B A_{BE} \qquad \textbf{(12)}\text{FXS}$$

II. Flow node at the rod end port of the cylinder, $\boldsymbol{p_B}$ node in Figure 5:

$$v_P A_{RE} - v_B A_{RE} = C_{HB}\frac{dp_B}{dt} + \frac{p_B}{R_L} + q_{IB} \tag{13}$$ FXTFXU

III. Free body force summation on the piston-load mass body, Figure 6:

$$p_A A_{BE} - p_B A_{RE} - B_B v_P - B_B v_B = M_L\frac{dv_P}{dt} \tag{14}$$ FXV

IV. Free body force summation on the valve-cylinder mass body, Figure 7:

$$p_A A_{BE} - p_B A_{RE} - K_S x_B - B_B v_B - B_B v_P = M_B\frac{dv_B}{dt} \tag{15}$$ FXW

V. The differentiation of the valve-cylinder body position to get speed:

$$v_B = \frac{dx_B}{dt} \tag{16}$$ FXX

METHOD OF THE INVESTIGATION

The aim of the investigation was to seek a simple function that would produce a reasonable estimate of the *dominant resonant frequency* in a system that has an unequal area cylinder, fluid compressibility effects on both sides of the piston, plus some degree of fixture deflection, or flexure. A three-dimensional matrix of values was formulated that produced resonant frequencies whose range extend from being dominated by fluid compressibility and extending to frequencies that were dominated by fixture deflection. The hydraulic capacitances were set to those values that are consistent with the point of maximum equivalent capacitance [2]:

$$C_{HA} = \sqrt{\rho_c}\, C_{HB} \tag{17}$$ FXZ

Cylinder Length Range

HYD CAP A-SIDE cm^5/n	CYL LENGTH STROKE mm
0.000130	12.299950
0.001300	122.999504
0.013000	1229.995117
0.130000	12299.948242
1.300000	122999.500000

Table 1 The equivalent cylinder lengths for the investigation ranged from 12 mm to an absurd 123 Meters. FXY

This coincides with the worst-case design scenario.

To achieve an extensive map of values, the data was generated with combinations of the three main system parameters:

1. Hydraulic capacitance on the cylinder blank end was iterated in four one-decade steps (five values) from $130\text{x}10^{-6}$ to $1300000\text{x}10^{-6}$ cm^5/N while the rod end capacitance was iterated dependently according to the relationship required by Equation (17).

2. The fixture deflection linear spring coefficient was iterated in four one-decade steps (five

values) from 525 to 5250000 N/cm.

3. The weight of the cylinder body-valve lumped body assembly was iterated in two one-decade steps (three values) from 90 to 9000 N.

4. In order to expand the range of applicability, two additional data sets were generated. The first used an increased rod diameter (raised from 25 mm to 37.5 mm), and the second doubled both the cylinder bore and the rod diameter to 100 mm and 75 mm respectively. This produced a total of five different data sets containing a total of 125 different system configurations. The five data sets are contained in the Appendix of this paper.

For the example numerical evaluation of the system that is defined by Equations (18) to (24), the following key values were used in the model:

1. Load weight = 9000 N.
2 Cylinder bore = 50 mm.
3. Rod Diameter = 25 mm.
4. Valve internal leakage resistance = 125 N-sec/cm^5.
5. Piston-to-bore linear friction coefficient = 17.5 N-sec/cm

The internal leakage and mechanical friction are deliberately kept low in order to reduce their influences on the resonant frequencies.

These parametric values produced dominant resonant frequencies that ranged from a low of just over 0.7 Hz to a maximum of slightly under 84 Hz. Synthesis of the equivalent cylinder length based upon the iterated hydraulic capacitances reveals that it varies from a very short 12 mm to an improbable maximum of 123 meters Table Table 1.

Additionally, the extremities of minimum capacitance with minimum fixture stiffness produce a system that has essentially infinite hydraulic stiffness, while the other extreme of maximum capacitance and maximum fixture stiffness represents a system that has essentially infinite mechanical stiffness. The former case represents a system whose dominant resonant frequency is solely dependent upon the sum of the load mass and the cylinder body mass and the fixture deflection. The latter represents a system whose dominant resonant frequency is dependent solely upon the mass of the load and the hydraulic capacitance. The investigation matrix, as chosen, covers a suitable range so as to include all reasonable application scenarios for the cylinder load and dimensions as simulated. That is, the data range is sufficiently inclusive so as to meet the requirement that any resulting empirical formula will adequately apply to most industrial machine scenarios.

EXAMPLE MATH MODEL OF ONE SYSTEM CONFIGURATION

Of the 125 different configurations that were analyzed, one is selected here as being both representative and typical. It is presented in both state variable and transfer function forms. The specific parameter values that were used to enumerate the matrix elements and coefficients are summarized in the Table above. Fortunately, all five of the real variables are also state variables, so only a little rearrangement is needed to put the equations into the standard state variable form. The state variable matrices for one example system are given here. The variable definitions:

$$[Z] = \begin{bmatrix} BLANK\ END & PRESSURE & (Pa) \\ ROD\ END & PRESSURE & (Pa) \\ PISTON & SPEED & (M/SEC) \\ CYLINDER & SPEED & (M/SEC) \\ CYLINDER & POSITION & (METER) \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{bmatrix} \quad \textbf{(18)}\text{FX1}$$

The A-Matrix:

$$[A] = \begin{bmatrix} -.6091776 & 0 & -1.51038E+09 & -1.51038E+09 & 0 \\ 0 & -.7034177 & 1.308027E+09 & 1.308027E+09 & 0 \\ 2.141203E-06 & -1.605902E-06 & -1.910533 & -1.910533 & 0 \\ 2.141203E-04 & -1.605902E-04 & -191.0533 & -191.0533 & -572516 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \textbf{(19)}\text{FX2}$$

The B-Matrix:

$$[B] = \begin{bmatrix} 5.305779E+15 \\ -4.59494E+15 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(20)}\text{FX3}$$

The C-matrix:

$$[C] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \textbf{(21)}\text{FX4}$$

The D-Matrix:

$$[D] = \begin{bmatrix} 0 \end{bmatrix} \quad \textbf{(22)}\text{FX5}$$

Where the C-Matrix is formulated so as to use the piston speed as the output with servo valve current as input. The transfer function has a fifth order polynomial as the characteristic equation which cannot be factored in general terms. It is necessary to use numerical values in the matrices, and then use a numerical algorithm to obtain the roots of the characteristic equation.
The UNFACTORED form of the transfer function for the example system is:

$$G(s) = \frac{(1.873978E+10)s^3 + (1.248649E+10)s^2 + (1.072882E+16)s + (7.148715E+15)}{(1)s^5 + (194.2765)s^4 + (1111564)s^3 + (2204379)s^2 + (3.055821E+09)s + (2.035471E+09)} \qquad \textbf{(23)}_{\text{FXE}}$$

The FACTORED form of the transfer function for the example system is:

$$G(s) = \frac{(1.873978E+10)(s + -4.806338E-13 \pm j756.6478)(s + .6663095)}{(s + .420647 \pm j52.49399)(s + .666309)(s + 96.38442 \pm j1048.44)} \qquad \textbf{(24)}_{\text{FXD}}$$

RESULTS

There are five different frequencies that come into play in the investigation. They are summarized as follows and refer to the data tables in the Appendix:

1. The *hydromechanical resonant frequency* which is evaluated with Equation (1). It is sometimes referred to as the *hydraulic resonant frequency*. Note that this formula is valid in the case that <u>the fixture is infinitely stiff</u>. Equation (1) is repeated here with applicable subscripts:

$$\omega_h = \omega_n = \frac{A_{CYL}}{\sqrt{M_L C_H}} \qquad \textbf{(25)}_{\text{FX*}}$$

2. The well-known *mechanical resonance* that is the value obtained by considering only the load mass and the spring deflection of the fixture. It is evaluated with

$$\omega_m = \sqrt{\frac{K_S}{M_T}} \qquad \textbf{(26)}_{\text{FX6}}$$

$\boldsymbol{K_S}$ as the linear spring coefficient of the flexible fixturing and $\boldsymbol{M_T}$ is the total mass, ie, load mass plus the valve-cylinder body mass. Note that this formula is valid in the case that the <u>hydraulic fluid is infinitely stiff</u>.

$$\omega_{p/s} = \frac{\omega_h \omega_m}{\sqrt{\omega_h^2 + \omega_m^2}} \qquad \textbf{(27)}_{\text{FX7}}$$

3. The formulated dominant frequency that was derived based upon the data generated in the instant investigation, which is calculated with the following based upon excellent correlation. It is referred to as the *product over sum* in this paper and is seen in abbreviation as *PROD/SUM RESONANCE* in the

Appendix. The verbal description is abbreviated to the point of being possibly misleading, but is done so in the name of brevity. The value is calculated from:
4. The *dominant model resonance*, also abbreviated as *DOM MODEL RESONANCE*, which results from factoring the fifth-order characteristic equation, and because of its use of no analytical assumptions other than the linearizing methods already discussed, is considered to be the most reliable estimate of the system's actual resonant frequency. It is found by reducing the matrix representation to a transfer function using a modified Leverrier algorithm, and then factoring the numerator and denominator polynomials using the Newton-Raphson method. The fifth-order system yields one real pole and two complex pole pairs due to the low damping in the model. Of the two complex conjugate pairs, the lower frequency is the dominant frequency. It is the value that will produce the greater challenge to the control system designer. It is found merely by inspecting the factored transfer function. Its value is 52.49 rad/sec in Equation (24)

5. The *non-dominant resonant frequency*, also referred to as *NON-DOM RESONANCE*, and is also derived by scanning the factored transfer function. It is the greater of the two frequencies from the fifth-order system. It is not critical to the instant investigation, but has been logged only to get a qualitative insight into the extent to which the dominant and non-dominant frequencies are separated from one another. Its value is 1048 rad/sec in Equation (24). The non-dominant resonance is dismissed here as being unimportant to the discussion at hand.

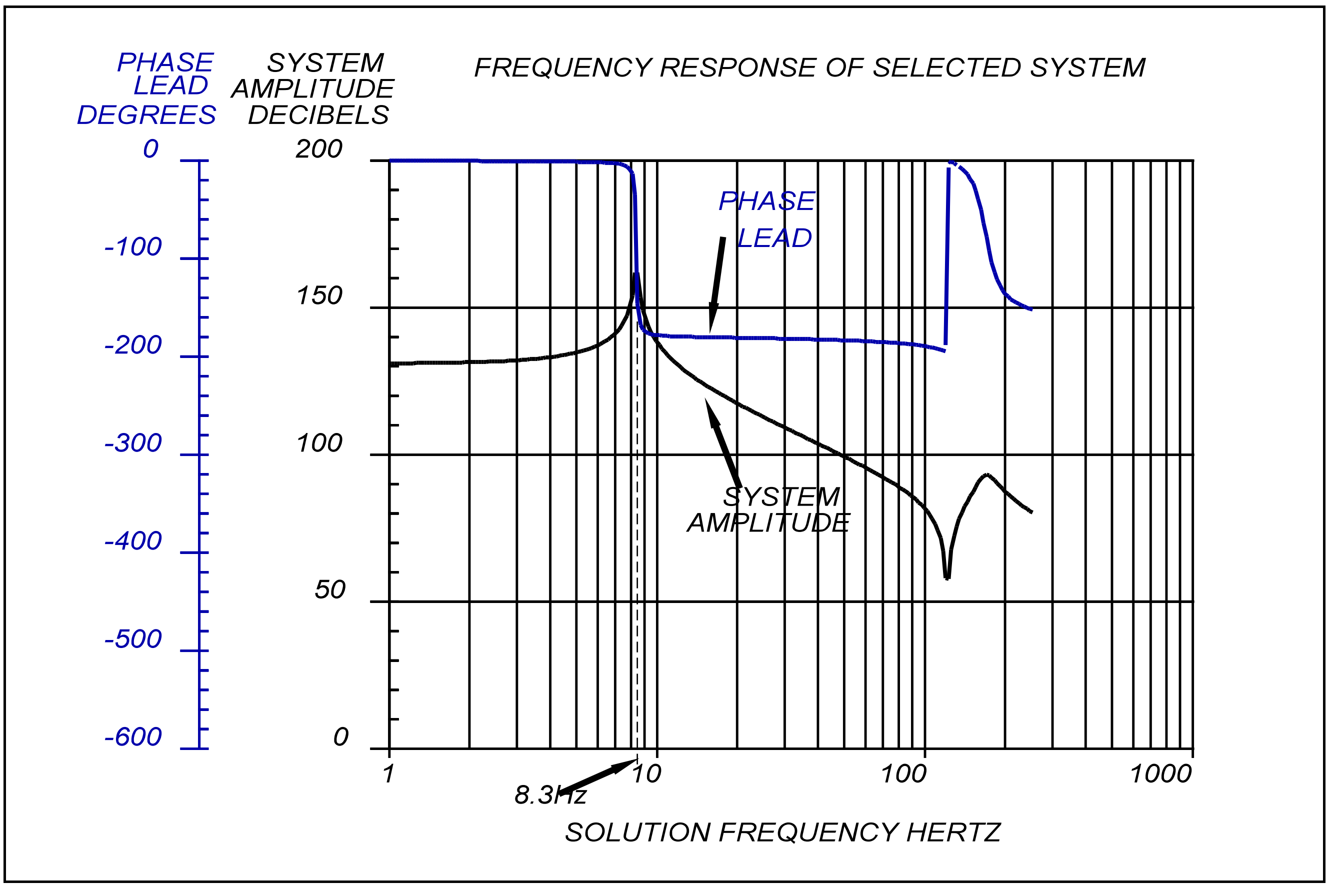

Figure 9 Frequency response of the selected system. This plot uses the model example given by the transfer function in Equation (24). The dominant resonance can be clearly seen at about 8.3 Hz (52.49 rad/sec). FXF

The frequency response graph of Figure 8 applies to the transfer function given in Equations (23) and (24).

The dominant frequency is the one that causes the first resonant rise and it presents the greatest limitation to system performance and causes the greatest problem for the designer. Because hydraulic systems inherently have very low friction and low internal leakage, that is, there is very little damping, it is not unusual that the maximum closed loop bandwidth can be no greater than one-tenth, or even one-twentieth, of the dominant system open loop resonance. The price of a low hydromechanical resonant frequency is great indeed. Sometimes internal leakage paths are deliberately incorporated in order to increase the meager inherent damping.

This investigation was aimed at using the two very well-known Equations (25) and (26) in such a way that they could be combined to predict the new dominant frequency in the system that had both hydraulic capacitance and mechanical compliance of the fixture. Each combination of capacitance, fixture spring coefficient and valve-cylinder body mass is another system and each has its own model. For each combination, the transfer function was formulated, it was scanned for the low and high resonances and logged along with the input data combination. There were, therefore, 125 different systems that were analyzed, representing ranges that have already been discussed. The results are tabulated in five tables in the Appendix.

Each table carries all 25 capacitance-spring coefficient combinations (5 capacitances times 5 springs), while each individual table applies to the three valve-cylinder body mass values, 90, 900 and 9000 Newton weights. As stated earlier, two additional data sets were generated using different cylinder and other parameter values. These and other values are tabulated on the respective data page in the Appendix.

The chosen expression for predicting the dominant system frequency in the presence of both hydraulic and mechanical compliances was given earlier in Equation (27). The efficacy of the method can be seen in the Appendix by comparing the column headed ***PROD/SUM RESONANCE*** to the column headed ***DOM MODEL RESONANCE*** and the error graphs in Figure 9 to Figure 13. Recall that the former was calculated from the individual resonances but the latter was found by the more rigorous method of factoring the system characteristic equation. The amount of correlation between the two is so great that it was deemed valueless to graph the data. Instead, tabulated numerical data are given in the five tables in the Appendix. The percent error between the two calcualtion methods is depicted in Figure 9 to Figure 13.

The error graphs are plotted with error on the vertical axis and the ratio of the simple hydraulic resonant frequency to the simple mechanical frequency on the logarithmic horizontal axis. An interesting effect takes place when the ratio is unity where a trainsition seems to occur. At times the error peaks, in anotehr case it minimizes, and in some cases it is a region of significant slope. There is no apparent explanation for the behavior of the data, nor has any been sought. The most significant result is that the maximum error is about 7.5% and it can be seen in Figure 11, an unlikely scenario. That it is unlikely can be realized by the fact that the ratio of the mass of the valve-cylinder body is equal to the mass of the load. The applications where this might occur are most likely those where the load dead weight is relatively small, meaning that both the mechanical and hydraulic resonances are high, and the challenges to designing a well-behaved servo system are minimal. In all of the more problematic situations, that is those where the load mass is greater than the valve-cylinder mass, the errors are small, indicating that the simple formula is a reliable predictor in the most troublesome applications.

The procedure for calculating the dominant resonant frequency in the presence of both hydraulic fluid compressibility and fixture compliance is now quite simple:

1. Calculate the simple hydromechanical resonant frequency under the assumption that the structure is infinitely rigid using Equation (25).

2. Calculate the simple mechanical resonant frequency under the assumption that the hydraulic

system has infinite rigidity using Equation (26).

3. Calculate the resulting dominant system resonant frequency (product over sum frequency) using Equation (27).

The results above can be directly useful, however, it is interesting to calculate the effect upon the equivalent capacitance in a manner that was used by [1] and [5] to include the effects of line expansion and entrained gas. It starts by setting the simple hydromechanical resonant frequency using Equation (25) (with a new effective capacitance) equal to the product-over-sum frequency as determined in Equation (27), forming a new and useful relationship:

$$\frac{A_{CYL}^2}{\sqrt{MC_{H,EFF,MH}}} = \frac{\omega_h \omega_m}{\sqrt{\omega_h^2 + \omega_m^2}} \tag{28}$$ FX!

With some algebra, it is easy to solve for the effective hydraulic capacitance in the presence of both fluid compressibility and fixture deflection:

$$C_{H,EFF,MH} = \frac{A_{CYL}^2}{K_S} + \frac{A_{CYL}^2}{M\omega_h^2} \tag{29}$$ FX@

This is very interesting, because the first term is the contribution to the capacitance due to the structure compliance while the second term is the actual, effective hydraulic capacitance as determined in its single-capacitance equivalence and as determined on the hydraulic side of the piston, including line expansion and entrained air. That is, Equation (29) can be replaced as follows:

$$C_{H,EFF,MH} = \frac{A_{CYL}^2}{K_S} + C_{H,ACTUAL,EQ} \tag{30}$$ FX#

where the second term, on the right hand side, represents the conventionally determined hydraulic capacitance.

The relationship can be further extended to reflect the structure compliance into the hydraulic circuit. It is necessary to maintain the maximum capacitance ratio required of Equation (17) and then further impose the requirement that the actual equivalent capacitance be found from the familiar product over sum formula for capacitors in series. After some algebra, the capacitances on the blank and rod sides of the cylinder can be evaluated. For the blank end (A-side):

$$C_{HA,MH} = \frac{A_{CYL}^2(1 + \sqrt{\rho_C})}{K_S} + C_{H,A} \tag{31}$$ FX$

$$C_{HB,MH} = \frac{A_{CYL}^2(1 + \sqrt{\rho_C})}{K_S\sqrt{\rho_C}} + C_{H,B} \tag{32}$$ FX%

The first term, on the right hand side, is the capacitive equivalent due to the fixture deflection, K_S, after it is referred to the blank side of the cylinder. The second term is simply the actual hydraulic capacitance by conventional methods. For the rod end (B-side):

CONCLUSIONS AND RECOMMENDATIONS

Based upon the results of this investigation, applications engineers have a simple method for predicting the dominant hydromechanical resonant frequency in the presence of both fluid compressibility and structure compliance. Additionally, a new system model is offered for the unequal area system that remains linear and is easily soluble by classical and state space methods.

The data arrays in the Appendix also offer semi-empirical justification for using the average cylinder area to calculate the hydromechanical resonant frequency using Equation (1). It is only necessary to look at the very last entry in each data array. This is the situation where the fixture is stiffest, and therefore has the least effect on the resonance, that is, resonance is caused only by fluid compressibility, and cylinder area enters into the analysis. Compare the HYDRAULIC RESONANCE to the DOMINANT FREQUENCY (7.304 vs 7.2412, for example in Data Table ARRAY 191).

It is recommended that the linear, fifth order model be manipulated to derive an "exact" equation for the dominant resonance. The exact model can be compared to the semi-empirical, simple result in Equation (27). An exact model can also help in more specifically and broadly defining the limits of applicability. Although the results of this paper are felt to have broad applicability, the semi-empirical method is only limited proof.

REFERENCES

[1] Keller, George R, (1970), **Hydraulic System Analysis**, Hydraulics & Pneumatics Magazine, Cleveland, OH, USA
[2] Johnson, Jack L, (1995), **Design of Electrohydraulic Systems for Industrial Motion Control**, Parker Hannifin, Cleveland, OH, USA
[3] Wilson, Warren E, (1950), **Positive Displacement Pumps and Fluid Motors**, Pitman Publishing Co, New York, NY, USA
[4] Tonyan, Michael J, (1985), **Electronically Controlled Proportional Valves**, Marcel Dekker, New York, NY, USA
[5] Merritt, Herbert E, (1967), **Hydraulic Control Systems**, John Wiley, New York, NY, USA
[6] Johnson, Jack L, (1998, 1999, 2000), **Designers' Handbook for Electrohydraulic Servo and Proportional Systems**, IDAS Engineering Inc, East Troy, WI, USA
[7] Harrison, John G and Bollinger, John G, (1963), **Introduction to Automatic Controls**, International Textbook Co, Scranton, PA, USA
[8] Dorf, Richard C, (1989), **Fifth Edition - Modern Control Systems**,
Addison-Wesley Publishing Co, Reading, MA, USA
[9] International Standards Organization, (1999), **Proportional and Servo Valve Test Methods (10770-1),** ISO, Geneva, Switzerland

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APPENDIX
DATA ARRAY TABLES

The following data array tables contain the results of the 125 different system configurations on five different pages. To save space the column headings are at times abbreviated. Here is an interpretation of each heading:

HYDRAULIC CAPANCE CM^5/M is the hydraulic capacitance representing the compressed volume on the A-Side (blank end) of the cylinder. It was iterated from minima to maxima as shown in the table and discussed elsewhere. Note that the capacitance of the B-Side (rod end) IS NOT shown in the table. Its value was set in compliance with this column and Equation (17).

SPRING STIFFNESS N/CM is the equivalent linearized spring stiffness coefficient representing fixture compliance. It was iterated over the ranges indicated.

HYDRAULIC RESONANCE RAD/SEC is the simple resonance based upon only the fluid compressibility and load mass calculated in the "classical" hydromechanical sense using Equation (1) and under the condition that the piston is positioned in the position that produces maximum single equivalent capacitance due to both A- and B-side effects.

MECH RESONANCE RAD/SEC is the simple resonance based upon only fixture compliance and the sum of the load mass and the valve body-cylinder mass and calculated using Equation (26).

PROD/SUM RESONANCE RAD/SEC is the predicted dominant resonance calculated using the product of hydraulic and mechanical frequencies over the square root of the sum of their squares, that is, using Equation (27).

DOMINANT FREQUENCY RAD/SEC is the value deterimined by factoring the characteristic equation and selecting the lesser of the two imaginary roots.

DOM FREQ ERROR RAD/SEC is merely the difference between the dominant frequency from the fifth order model and the product over sum frequency.

DOM FREQ ERROR % OF VALUE is the previous data column converted to percent.

Other controlling pertinent data values are summarized at the bottom of each data page.

DATA TABLE: 'A:ARRAY191.PRT' (See Figure 9)

HYDRAULIC CAPANCE CM^5/N	SPRING STIFFNESS N/CM	HYDRAULIC RESONANCE RAD/SEC	MECH RESONANCE RAD/SEC	PROD/SUM RESONANCE RAD/SEC	DOMINANT FREQUENCY RAD/SEC	DOM FREQ ERROR RAD/SEC	DOM FREQ ERROR % OF VALU
0.000130	525.	730.4220	7.5290	7.5290	7.5285	0.000471	0.000063
0.000130	5250.	730.4220	23.8090	23.7960	23.7960	0.000030	0.000001
0.000130	52500.	730.4220	75.2890	74.8920	74.8949	-0.002910	-0.000039
0.000130	525000.	730.4220	238.0860	226.3640	226.4403	-0.076300	-0.000337
0.000130	5250001.	730.4220	752.8930	524.2500	524.6538	-0.403800	-0.000770
0.001300	525.	230.9800	7.5290	7.5250	7.5250	0.000037	0.000005
0.001300	5250.	230.9800	23.8090	23.6830	23.6843	-0.001280	-0.000054
0.001300	52500.	230.9800	75.2890	71.5830	71.6137	-0.030670	-0.000428
0.001300	525000.	230.9800	238.0860	165.7820	165.9880	-0.206000	-0.001243
0.001300	5250001.	230.9800	752.8930	220.8210	220.8904	-0.069400	-0.000314
0.013000	525.	73.0420	7.5290	7.4890	7.4896	-0.000645	-0.000086
0.013000	5250.	73.0420	23.8090	22.6360	22.6465	-0.010510	-0.000464
0.013000	52500.	73.0420	75.2890	52.4250	52.4940	-0.068990	-0.001316
0.013000	525000.	73.0420	238.0860	69.8300	69.8548	-0.024840	-0.000356
0.013000	5250001.	73.0420	752.8930	72.7010	72.6981	0.002910	0.000040
0.130000	525.	23.0980	7.5290	7.1580	7.1616	-0.003641	-0.000509
0.130000	5250.	23.0980	23.8090	16.5780	16.6050	-0.027000	-0.001629
0.130000	52500.	23.0980	75.2890	22.0820	22.0821	-0.000070	-0.000003
0.130000	525000.	23.0980	238.0860	22.9900	22.9728	0.017220	0.000749
0.130000	5250001.	23.0980	752.8930	23.0870	23.0677	0.019320	0.000837
1.300000	525.	7.3040	7.5290	5.2430	5.2665	-0.023506	-0.004483
1.300000	5250.	7.3040	23.8090	6.9830	6.9569	0.026106	0.003739
1.300000	52500.	7.3040	75.2890	7.2700	7.2119	0.058060	0.007986
1.300000	525000.	7.3040	238.0860	7.3010	7.2385	0.062452	0.008554
1.300000	5250001.	7.3040	752.8930	7.3040	7.2412	0.062780	0.008595

OTHER MODEL INPUT VALUES FOR THIS DATA ARRAY:

Cylinder Bore = 50 (mm)
Rod Diameter = 25 (mm)
Valve Leak Resistance = .001263 (N-sec/mm^5)
Load Dead Weight = 9000 (N)

Valve-Cylinder Body Wt = 90 (N)
Viscous Fric Coeff = 1.752 (N-sec/mm)
Bulk Modulus = 862 (MPa)

DATA TABLE: 'A:ARRAY192.PRT' (See Figure 10)

HYDRAULIC CAPANCE CM^5/N	SPRING STIFFNESS N/CM	HYDRAULIC RESONANCE RAD/SEC	MECH RESONANCE RAD/SEC	PROD/SUM RESONANCE RAD/SEC	DOMINANT FREQUENCY RAD/SEC	DOM FREQ ERROR RAD/SEC	DOM FREQ ERROR % OF VALU
0.000130	525.	730.4220	7.2140	7.2140	7.2140	0.000033	0.000457
0.000130	5250.	730.4220	22.8140	22.8030	22.8036	0.000630	0.002763
0.000130	52500.	730.4220	72.1440	71.7940	71.8245	0.030540	0.042520
0.000130	525000.	730.4220	228.1380	217.7630	218.5637	0.800700	0.366346
0.000130	5250001.	730.4220	721.4350	513.2790	519.1508	5.871800	1.131039
0.001300	525.	230.9800	7.2140	7.2110	7.2112	0.000151	0.002094
0.001300	5250.	230.9800	22.8140	22.7030	22.7132	0.010200	0.044908
0.001300	52500.	230.9800	72.1440	68.8630	69.1201	0.257060	0.371904
0.001300	525000.	230.9800	228.1380	162.3130	164.2360	1.923000	1.170876
0.001300	5250001.	230.9800	721.4350	219.9800	220.8177	0.837700	0.379363
0.013000	525.	73.0420	7.2140	7.1790	7.1826	0.003557	0.049523
0.013000	5250.	73.0420	22.8140	21.7760	21.8579	0.081860	0.374511
0.013000	52500.	73.0420	72.1440	51.3280	51.9396	0.611620	1.177560
0.013000	525000.	73.0420	228.1380	69.5640	69.8319	0.267890	0.383621
0.013000	5250001.	73.0420	721.4350	72.6710	72.6978	0.026810	0.036879
0.130000	525.	23.0980	7.2140	6.8860	6.9122	0.026209	0.379170
0.130000	5250.	23.0980	22.8140	16.2310	16.4298	0.198810	1.210057
0.130000	52500.	23.0980	72.1440	21.9980	22.0749	0.076940	0.348540
0.130000	525000.	23.0980	228.1380	22.9800	22.9727	-0.007300	-0.031777
0.130000	5250001.	23.0980	721.4350	23.0860	23.0677	-0.018320	-0.079418
1.300000	525.	7.3040	7.2140	5.1330	5.2115	0.078518	1.506624
1.300000	5250.	7.3040	22.8140	6.9560	6.9551	-0.000922	-0.013257
1.300000	52500.	7.3040	72.1440	7.2670	7.2119	-0.055080	-0.763736
1.300000	525000.	7.3040	228.1380	7.3000	7.2385	-0.061452	-0.848955
1.300000	5250001.	7.3040	721.4350	7.3040	7.2412	-0.062780	-0.866981

OTHER MODEL INPUT VALUES FOR THIS DATA ARRAY:

Cylinder Bore = 50 (mm)
Rod Diameter = 25 (mm)
Valve Leak Resistance = .001263 (N-sec/mm^5)
Load Dead Weight = 9000 (N)

Valve-Cylinder Body Wt = **900 (N)**
Viscous Fric Coeff = 1.752 (N-sec/mm)
Bulk Modulus = 862 (MPa)

DATA TABLE: 'A:ARRAY193.PRT' (See Figure 11)

HYDRAULIC CAPANCE CM^5/N	SPRING STIFFNESS N/CM	HYDRAULIC RESONANCE RAD/SEC	MECH RESONANCE RAD/SEC	PROD/SUM RESONANCE RAD/SEC	DOMINANT FREQUENCY RAD/SEC	DOM FREQ ERROR RAD/SEC	DOM FREQ ERROR % OF VALU
0.000130	525.	730.4220	5.3500	5.3500	5.3502	0.00024	0.004411
0.000130	5250.	730.4220	16.9190	16.9150	16.9169	0.00189	0.011172
0.000130	52500.	730.4220	53.5030	53.3600	53.4313	0.07129	0.133424
0.000130	525000.	730.4220	169.1920	164.8270	166.9140	2.08700	1.250344
0.000130	5250001.	730.4220	535.0310	431.6240	463.0221	31.39810	6.781123
0.001300	525.	230.9800	5.3500	5.3490	5.3496	0.00059	0.011010
0.001300	5250.	230.9800	16.9190	16.8740	16.8965	0.02245	0.132868
0.001300	52500.	230.9800	53.5030	52.1230	52.7815	0.65846	1.247521
0.001300	525000.	230.9800	169.1920	136.4910	146.4035	9.91250	6.770671
0.001300	5250001.	230.9800	535.0310	212.0620	220.0358	7.97380	3.623865
0.013000	525.	73.0420	5.3500	5.3360	5.3431	0.00713	0.133424
0.013000	5250.	73.0420	16.9190	16.4830	16.6909	0.20793	1.245766
0.013000	52500.	73.0420	53.5030	43.1620	46.2976	3.13557	6.772645
0.013000	525000.	73.0420	169.1920	67.0600	69.5848	2.52479	3.628365
0.013000	5250001.	73.0420	535.0310	72.3710	72.6950	0.32401	0.445711
0.130000	525.	23.0980	5.3500	5.2120	5.2782	0.06616	1.253411
0.130000	5250.	23.0980	16.9190	13.6490	14.6446	0.99560	6.798410
0.130000	52500.	23.0980	53.5030	21.2060	21.9984	0.79235	3.601861
0.130000	525000.	23.0980	169.1920	22.8860	22.9718	0.08583	0.373632
0.130000	5250001.	23.0980	535.0310	23.0760	23.0677	-0.00833	-0.036111
1.300000	525.	7.3040	5.3500	4.3160	4.6438	0.32778	7.058494
1.300000	5250.	7.3040	16.9190	6.7060	6.9357	0.22972	3.312129
1.300000	52500.	7.3040	53.5030	7.2370	7.2117	-0.02528	-0.350568
1.300000	525000.	7.3040	169.1920	7.2970	7.2385	-0.05845	-0.807552
1.300000	5250001.	7.3040	535.0310	7.3040	7.2412	-0.06278	-0.866981

OTHER MODEL INPUT VALUES FOR THIS DATA ARRAY:

Cylinder Bore = 50 (mm)
Rod Diameter = 25 (mm)
Valve Leak Resistance = .001263 (N-sec/mm^5)
Load Dead Weight = 9000 (N)

Valve-Cylinder Body Wt = **9000 (N)**
Viscous Fric Coeff = 1.752 (N-sec/mm)
Bulk Modulus = 862 (MPa)

DATA TABLE: 'A:ARRAY291.PRT' (See Figure 12)

HYDRAULIC CAPANCE CM^5/N	SPRING STIFFNESS N/CM	HYDRAULIC RESONANCE RAD/SEC	MECH RESONANCE RAD/SEC	PROD/SUM RESONANCE RAD/SEC	DOMINANT FREQUENCY RAD/SEC	DOM FREQ ERROR RAD/SEC	DOM FREQ ERROR % OF VALU
0.000130	525.	647.8100	7.5290	7.5280	7.5284	0.000408	0.005419
0.000130	5250.	647.8100	23.8090	23.7920	23.7922	0.000180	0.000757
0.000130	52500.	647.8100	75.2890	74.7860	74.7801	-0.005950	-0.007957
0.000130	525000.	647.8100	238.0860	223.4710	223.4353	-0.035700	-0.015978
0.000130	5250001.	647.8100	752.8930	491.0560	490.3159	-0.740100	-0.150944
0.001300	525.	204.8550	7.5290	7.5240	7.5238	-0.000199	-0.002645
0.001300	5250.	204.8550	23.8090	23.6490	23.6496	0.000550	0.002326
0.001300	52500.	204.8550	75.2890	70.6680	70.6776	0.009640	0.013639
0.001300	525000.	204.8550	238.0860	155.2860	155.1900	-0.096000	-0.061860
0.001300	5250001.	204.8550	752.8930	197.6690	197.1317	-0.537300	-0.272559
0.013000	525.	64.7810	7.5290	7.4790	7.4788	-0.000225	-0.003009
0.013000	5250.	64.7810	23.8090	22.3470	22.3511	0.004050	0.018120
0.013000	52500.	64.7810	75.2890	49.1060	49.0816	-0.024410	-0.049734
0.013000	525000.	64.7810	238.0860	62.5080	62.3434	-0.164650	-0.264102
0.013000	5250001.	64.7810	752.8930	64.5420	64.3382	-0.203840	-0.316826
0.130000	525.	20.4860	7.5290	7.0670	7.0684	0.001384	0.019580
0.130000	5250.	20.4860	23.8090	15.5290	15.5268	-0.002180	-0.014040
0.130000	52500.	20.4860	75.2890	19.7670	19.7038	-0.063180	-0.320648
0.130000	525000.	20.4860	238.0860	20.4100	20.3268	-0.083240	-0.409509
0.130000	5250001.	20.4860	752.8930	20.4780	20.3922	-0.085800	-0.420749
1.300000	525.	6.4780	7.5290	4.9110	4.9284	0.017433	0.353723
1.300000	5250.	6.4780	23.8090	6.2510	6.1952	-0.055783	-0.900420
1.300000	52500.	6.4780	75.2890	6.4540	6.3673	-0.086722	-1.361995
1.300000	525000.	6.4780	238.0860	6.4760	6.3850	-0.091042	-1.425883
1.300000	5250001.	6.4780	752.8930	6.4780	6.3867	-0.091270	-1.429057

OTHER MODEL INPUT VALUES FOR THIS DATA ARRAY:

Cylinder Bore = 50 (mm)
Valve-Cylinder Body Wt = **90 (N)**
Rod Diameter = **37.5 (mm)**
Viscous Fric Coeff = 1.752 (N-sec/mm)
Valve Leak Resistance = .001263 (N-sec/mm^5)
Bulk Modulus = 862 (MPa)
Load Dead Weight = 9000 (N)

DATA TABLE 'A:ARRAY391.PRT' (SeeFigure 13)

HYDRAULIC CAPANCE CM^5/N	SPRING STIFFNESS N/CM	HYDRAULIC RESONANCE RAD/SEC	MECH RESONANCE RAD/SEC	PROD/SUM RESONANCE RAD/SEC	DOMINANT FREQUENCY RAD/SEC	DOM FREQ ERROR RAD/SEC	DOM FREQ ERROR % OF VALU
0.000520	2100.	647.8100	7.5290	7.5280	7.5284	0.000410	0.005446
0.000520	21000.	647.8100	23.8090	23.7920	23.7924	0.000390	0.001639
0.000520	210000.	647.8100	75.2890	74.7860	74.7867	0.000670	0.000896
0.000520	2100000.	647.8100	238.0860	223.4710	223.4981	0.027100	0.012125
0.000520	21000000.	647.8100	752.8930	491.0560	490.7234	-0.332600	-0.067777
0.005200	2100.	204.8550	7.5290	7.5240	7.5239	-0.000143	-0.001901
0.005200	21000.	204.8550	23.8090	23.6490	23.6499	0.000890	0.003763
0.005200	210000.	204.8550	75.2890	70.6680	70.6777	0.009730	0.013767
0.005200	2100000.	204.8550	238.0860	155.2860	155.1891	-0.096900	-0.062440
0.005200	21000000.	204.8550	752.8930	197.6690	197.1442	-0.524800	-0.266201
0.052000	2100.	64.7810	7.5290	7.4790	7.4788	-0.000239	-0.003196
0.052000	21000.	64.7810	23.8090	22.3470	22.3503	0.003310	0.014810
0.052000	210000.	64.7810	75.2890	49.1060	49.0755	-0.030530	-0.062210
0.052000	2100000.	64.7810	238.0860	62.5080	62.3428	-0.165220	-0.265019
0.052000	21000000.	64.7810	752.8930	64.5420	64.3406	-0.201380	-0.312990
0.520000	2100.	20.4860	7.5290	7.0670	7.0678	0.000811	0.011475
0.520000	21000.	20.4860	23.8090	15.5290	15.5194	-0.009610	-0.061923
0.520000	210000.	20.4860	75.2890	19.7670	19.7138	-0.053160	-0.269658
0.520000	2100000.	20.4860	238.0860	20.4100	20.3451	-0.064880	-0.318897
0.520000	21000000.	20.4860	752.8930	20.4780	20.4116	-0.066420	-0.325404
5.200000	2100.	6.4780	7.5290	4.9110	4.9088	-0.002208	-0.044981
5.200000	21000.	6.4780	23.8090	6.2510	6.2319	-0.019137	-0.307083
5.200000	210000.	6.4780	75.2890	6.4540	6.4299	-0.024071	-0.374359
5.200000	2100000.	6.4780	238.0860	6.4760	6.4508	-0.025243	-0.391318
5.200000	21000000.	6.4780	752.8930	6.4780	6.4529	-0.025150	-0.389750

OTHER MODEL INPUT VALUES FOR THIS DATA ARRAY:

Cylinder Bore = **100 (mm)**
Rod Diameter = **75 (mm)**
Valve Leak Resistance = .001263 (N-sec/mm^5)
Load Dead Weight = **36000 (N)**

Valve-Cylinder Body Wt = **360 (N)**
Viscous Fric Coeff = 1.752 (N-sec/mm)
Bulk Modulus = 862 (MPa)

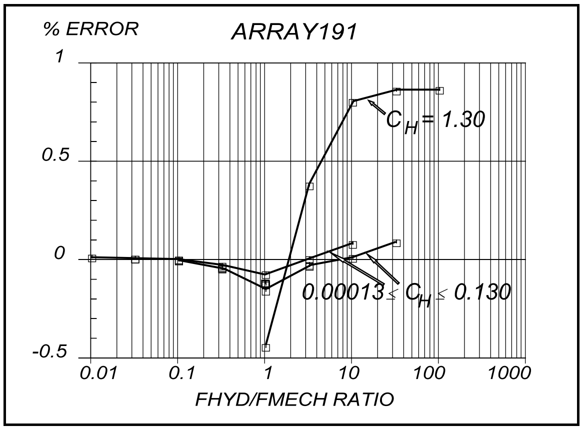

Figure 10 Error results with 100-to-1 mass ratio FXH

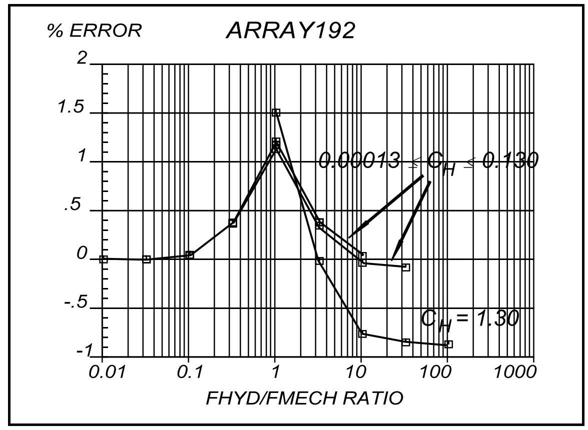

Figure 11 Error results with 10-to-1 mass ratio. FXI

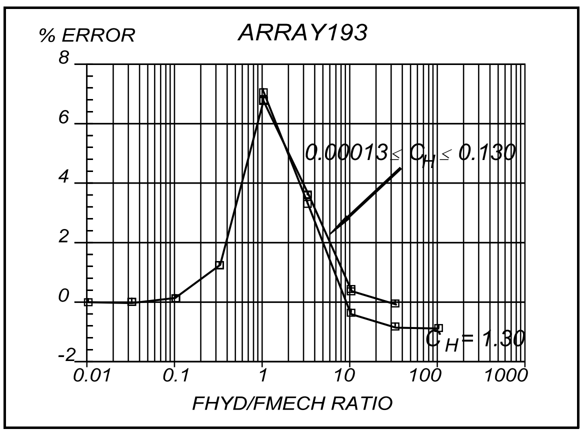

Figure 12 Error results with 1-to-1 mass ratio. FXJ

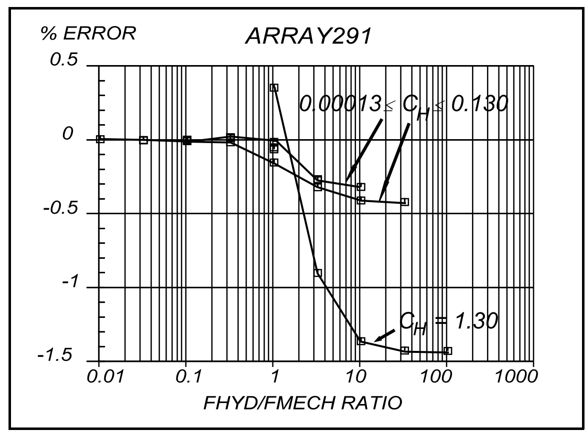

Figure 13 Error results with 100-to-1 mass ratio and rod diameter raised to 37.5mm. FXG

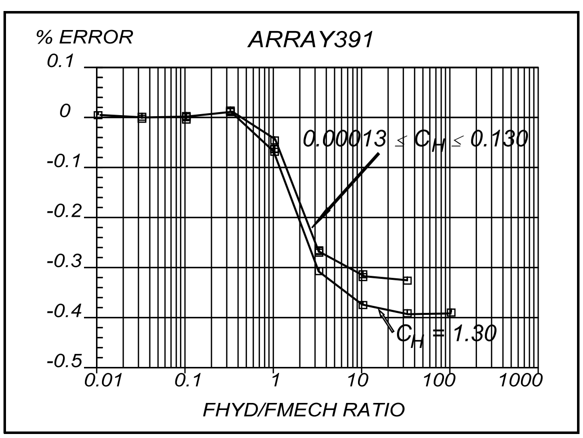

Figure 14 Error results with 100-to-1 mass ratio, rod diameter bore set to 100 and 75 mm. FXK