Linear Mathematical Models are used to Conduct Pre-Development Feasibility Studies Regarding the Doubly-Compensated Hydrostatic Transmission

Preliminary Copy
To be Presented at
National Conference on Fluid Power
Las Vegas, NV
March 2011

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IDAS
Engineering Inc
Technical Publications

Paper Number - To Be Assigned

Linear Mathematical Models are used to Conduct Pre-Development Feasibility Studies Regarding the Doubly-Compensated Hydrostatic Transmission

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ABSTRACT:

Linear, steady-state mathematical models are used to investigate the performance and tuning of the doubly-compensated hydrostatic transmission (HST). A doubly-compensated hydrostatic transmission is one in which both the pump and the motor are outfitted with pressure compensation controls, both sequentially active and simultaneously active. The approach consists of development of a methodology to converge on the compensator settings that will achieve a targeted final performance. The synthesized compensator settings become the input into the performance models.

The models then simulate the expected performance characteristics, including speed-torque characteristics, overall efficiency, insights into the actions inside the transmission along with input and output power. The techniques discussed in this paper are useful for doing pre-development feasibility investigations. It is a theoretical study. The linear models are based upon performance data for the pump and motor and do not rely upon internal mechanical dimensions, so they are practical when planning for specific applications, not for pump and motor design. The models apply to all types of positive displacement pumps and motors and with simple extensions, the models can be adapted to dynamic simulations as well, however, this paper explores only the steady-state models and methods. Graphs of the simulated performance data and all key equations are given.

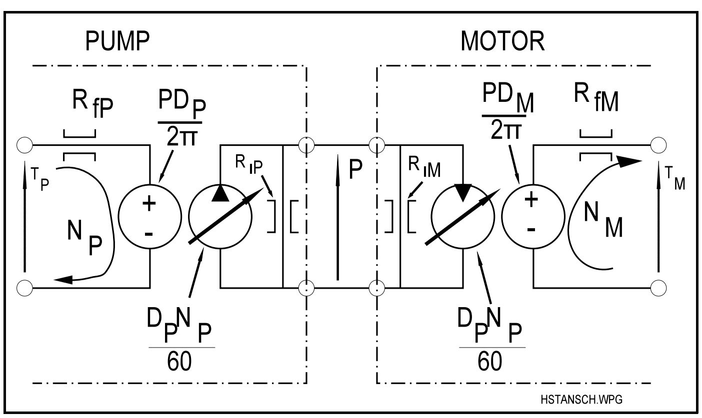

Figure 1 The analytical schematic of the hydrostatic transmission shows a variable displacement pump powering a variable displacement motor using TYPE 1 models for each machine. 8KR

INTRODUCTION:

The jargon of the hydraulic industry has adopted the term "theoretical" to connote that a speaker or writer is discussing pumps and motors that have NO power losses. There is an unfortunate, but chronic, misuse of the term, "theoretical". It's no wonder we don't understand one another. There are better word choices. Merritt 1) refers to the "ideal" machine rather than the inappropriate "theoretical" to describe the actions of pumps and motors that have no leakage or frictional losses. This paper, as well as many others that were reviewed, contain "theoretical analyses", but do not necessarily limit their discussions to lossless machines. This writer will adopt Merritt's more descriptive term, "ideal", also labeled as the TYPE 0 models, to denote machines without losses. TYPE 1 models are used to characterize those machines with losses, ie, frictional and volumetric inefficiencies.

Although many processes in hydraulic fluid power have non-linear relationships among the performance variables, the technical literature is rife with characterizations of machinery with linearized relationships. Kacem et al 2) have modeled using linear friction and leakage coefficients, and used the symbol "R" to signify "resistance", comfortingly consistent with this writer's own electrical engineering background. They do not address the issue of frictional factors, since they chose to concentrate on leakage effects. Burton, et al 3) dealt with dynamics as well as steady-state, and used a linear friction, damping coefficient, B, but used square law, non-linear pressure-flow orifices for the control valves, thus incorporating a mixed linear and non-linear modeling process. Borghi. et al 4), concerned with design of gear pumps for improved efficiency, state that internal "leakage flow rate can be expressed in the form of the Poiseulle equation" in which it is linearly proportional to pressure. Ossyra. et al 5) used multi-order polynomial losses for both leakage and friction and suggest empirically derived polynomial coefficients to characterize leakage and frictional losses in rotating machines. Jung et al 6) used a newly developed non-linear model that requires some knowledge of internal clearances, but still produced up to 2.37% model error in efficiency compared to test result efficiencies. Even when non-linear models are used, they suffer departures from real world hardware.

The issue of anti-lugging of prime movers has been reported extensively over several decades. The Lease patent, 7), disclosed a method of anti-lugging as a consequence of "maintaining the transmission at a constant output power". Similarly, Khalil et al 8) were concerned about controlling output power, but only in the pump. Zarotti and Paoluzzi 9), in a theoretical investigation, simulated the dynamic and steady-state performance in control of both pump and motor displacement and showed dynamically the efficacy of the method as an anti-lugging method, however, the control means they employed were described algorithmically and thus lacked hardware details and implementation methodology. As such, they explored control of the doubly-compensated hydrostatic transmission. Prime mover governor setting, prime mover speed and transmission pressure were used as control variables but did not explore effects adjusting strategies. They did not address the issue of controlling input power, rather, their assessment of control efficacy was based upon their ability to maintain small transient variations in prime mover speed in the face of large load changes.

Three patents by Kuras, B. D., et al 10), 11) and 12) also address anti-lugging, and limiting output power, along with protection of lesser rated transmission elements, while also addressing special responsiveness problems with automatically controlled hydrostatic transmissions.

This paper expands on the steady-state aspects of the hydrostatic transmission and the use of compensation for both the pump and motor, using linear models, both ideal (TYPE 0) and non-ideal (TYPE 1) to characterize the pump and motor performances. But, it also explores the transmission's input power because that is the variable that affects prime mover lugging. Comparisons are made of the results obtained with the two different model types. Furthermore, the TYPE 1 models of this paper will use loss coefficients (resistances) that are totally based upon the standardized, objective test methods contained in ISO 4409, 13) and ISO 8426, 14). Output power has been stressed, but HST input power affects prime mover lugging.

Such models are useful in applications, that is, by users and OEMs, because they rely upon readily available, published performance data and therefore do not compromise the confidential, dimensionally-based models that manufacturers use to design and improve pumps and motors. Yet the results are very useful in simulations, especially when control concepts are being introduced and designers are eager to try all manner of "what if" scenarios and strategies.

There are two "what if" scenarios covered in this paper. Additionally, the control method pursued herein will consist of simple spring-pressure-area compensators on both the pump and motor and will assume no algorithmic or electronic control. It does not, however, preclude more complex, cross-technology control methods.

A primary purpose of this paper is to stress the value of using judiciously evaluated linear models in a non-linear world, and to do so by applying the models to two configurations of the doubly-compensated hydrostatic transmission. The aim of the configurations is to determine how well the two systems achieve anti-lugging characteristics and to explore what features are important.

A secondary aim is to show how the models can be manipulated algebraically (for the most part) to provide insight into how the transmission is adjusted to achieve a specified output performance level, ie, output power and speed ranging while avoiding prime mover lugging, and to provide a compendium of equations that may aid the reader. The equations can be programmed in any scientific language such as BASIC, FORTRAN, C++, or even a spread sheet. The two configurations explored herein are:

1. Sequentially compensated, wherein the motor compensator becomes active and then the pump becomes active only after the motor saturates. This is also the conventional configuration for "constant power output" control strategy, and it can be argued, that it is actually, a singly compensated HST.

2. Simultaneously compensated, wherein both the pump and motor compensators become active at the same pressure, with the pump's saturation pressure set higher than for the motor for reasons that will emerge as the paper progresses.

This study will have two focal points: The first will deal with the strategies for applying the models to work toward a specific goal and the second will be concerned with the results and conclusions to be drawn with the models, more specifically, the sequential compensation and simultaneous compensation will be modeled and compared as to their respective efficacies in achieving prime mover anti-lugging under variable loading.

An example of the Type 1 model of the hydrostatic transmission is shown in Figure 1. Explanations of all variables and algebraic symbols are shown in Table 1. They are used to develop the steady-state, mathematical models of the doubly compensated hydrostatic transmission configurations.

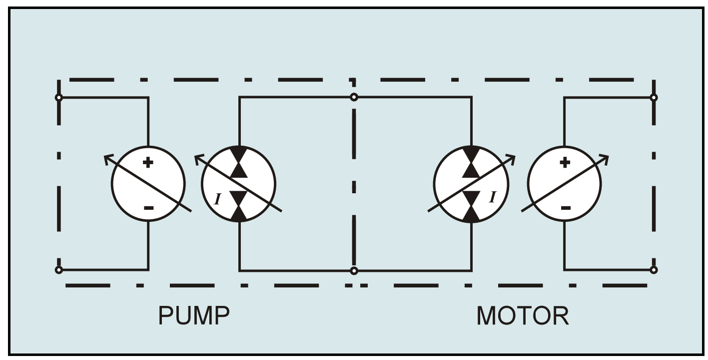

Figure 2 The TYPE 0 model of the hydrostatic transmission is a degenerate form of the TYPE 1 model because it has no loss resistance in either the mechanical or hydraulic circuits. 8NA

Figure 2 shows the analytical schematic of the TYPE 0 model of the hydrostatic transmission. It is merely the TYPE 1 model with the loss elements, that is, the resistance, removed from the circuit. It is therefore referred to as the "ideal" machine, meaning that it enjoys 100% overall efficiency.

The models' analytical schematics make use of hydraulic-mechanical analogy. Pressure is analogous to torque, therefore the mechanical parts of the circuit have "torque generators", one for the motor and another for the pump. Rotational shaft speed is analogous to hydraulic flow, so the pump speed "flows" into the pump's torque generator while the motor output shaft speed "flows" out of the motor's torque generator. Such models submit easily to the conventional circuit analysis techniques used in Direct Current electrical circuits, by perfectly conforming to both Ohm's and Kirchoff's Laws and other circuit theorems as well.

To stress the utility of the linearized models, the Power Lost characteristics of the HST are generated and used to illustrate how they are used to estimate the energy lost in the application, say in a complete work shift. Basic statistical methods are used. Specifically, the Probability of Finding a Speed histogram is applied to the Power Lost data to calculate the lost energy over an arbitrary time period.

NOMENCLATURE FOR THE DOUBLY-COMPENSATED HYDROSTATIC TRANSMISSION

γ_P, Pressure-dependent rate of change of the pump displacement, (in^3/rev)/psi
γ_M, Pressure-dependent rate of change of the motor displacement, (in^3/rev)/psi
η_{mrM}, Mechanical efficiency of the motor under rated conditions, %
η_{mrP}, Mechanical efficiency of the pump under rated conditions, %
η_o, Overall efficiency of the HST, %
η_{vrM}, Volumetric efficiency of the motor under rated conditions, %
η_{vrP}, Volumetric efficiency of the pump under rated conditions, %
D_{0M}, A mathematical construct representing the intercept on the motor's Displacement-Pressure graph, in^3/rev
D_{0P}, A mathematical construct representing the intercept on the pump's Displacement-Pressure graph, in^3/rev
D_M, Displacement of the motor, in^3/rev
D_{NM}, Minimum displacement limit of the variable displacement motor, in^3/rev
D_{MNL}, Motor displacement at no-load condition, in^3/rev
D_{NP}, Minimum displacement limit of the variable displacement pump, in^3/rev
D_{PNL}, Pump displacement at no-load condition, in^3/rev
D_P, Displacement of the pump, in^3/rev
D_{rM}, Rated displacement of the motor, in^3/rev
D_{rP}, Rated displacement of the pump, in^3/rev
D_{XM}, Maximum displacement of the motor, in^3/rev
D_{XP}, Maximum displacement of the pump, in^3/rev
K, A constant for synthesizing constant power curve(s) on the Torque-Speed graph,
N_M, Motor speed at any condition, rpm
N_{MA}, Motor speed at the low speed point on the constant power hyperbola, rpm
N_{MB}, Motor speed at the high speed point on the constant power hyperbola, rpm, rpm
N_{MNL}, Motor speed at no load, rpm
N_P, Pump speed, assumed to be constant in the simulation, rpm
N_{rM}, Motor speed at rated conditions, rpm
N_{rP}, Pump speed at rated conditions, rpm
P, Transmission differential pressure, psid
P_A, Pressure at the low speed point on the constant power hyperbola, rpm
P_B, Pressure at the high speed point on the constant power hyperbola, rpm, rpm
P_{CM}, Cracking pressure of the motor compensator, psi
P_{CP}, Cracking pressure of the pump compensator, psi
P_{NL}, Transmission pressure under no load conditions, psid
P_{rM}, Rated pressure of the motor, psi
P_{rP}, Rated pressure of the pump, psi
P_S, Transmission pressure when the motor is stalled, psid
P_{XM}, Pressure that causes maximum motor displacement, psi
P_{XP}, Pressure that causes zero pump displacement, psi
P_{XTQP}, Pressure that results in the peak value of pump torque in the simultaneously compensated HST, psi
$R_{\iota M}$ or R_{LM}, Leakage resistance coefficient of the motor, psi/(in^3/sec)
$R_{\iota P}$ or R_{LP}, Leakage resistance coefficient of the pump, psi/(in^3/sec)
R_e, Leakage resistance coefficient of the combined effects of pump and motor, psi/(in^3/sec)
R_{fM}, Viscous friction coefficient of the motor, in-lb/rpm
R_{fP}, Viscous friction coefficient of the pump, in-lb/rpm
ΔT_P, the rise in torque above the hyperbolic torque at **Point A** or **Point B** occurring at P_{XTQP} and T_{XP}, in-lb
T_M, Motor torque at any conditions, in-lb
T_{MA}, Motor torque at the low speed point on the constant power hyperbola, in-lb
T_{MB}, Motor torque at the high speed point on the constant power hyperbola, in-lb
T_{PA}, Pump torque at the low speed point on the constant power hyperbola, in-lb
T_{PB}, Pump torque at the high speed point on the constant power hyperbola, in-lb
T_{MNL}, Motor torque at no-load conditions, in-lb
T_P, Pump torque at any conditions, in-lb
T_{XP}, the peak torque value that occurs in the simultaneously compensated HST and occurs at the P_{XTQP} pressure, in-lb
W_d, Desired power when constructing constant power hyperbolas on the torque-speed graph, HP
W_H, alternate notation to W_d. They are the same
W_{in}, Transmission input shaft power to the pump, HP
W_{out}, Transmisssion output shaft power from the motor, HP

TABLE **1** Variable names, algebraic symbology and explanations used in this paper.8JH

BASIC TRANSMISSION EQUATIONS AND PARAMETER EVALUATIONS

Three equations can be written for the transmission, starting with the pump input torque, flow summation in the hydraulic circuit, and lastly, the output torque summation at the motor shaft. **First**, we sum the torques at the pump input shaft:

$$T_P = R_{fP}N_P + \frac{D_P}{2\pi}P \qquad \textbf{(1)}\text{TP1}$$

where the linearize, mechanical viscous friction factor for TYPE 1 pump models, $\boldsymbol{R_{fP}}$, is:

$$R_{fP} = \frac{P_{rP}D_{rP}(1 - \eta_{mrP})}{2\pi N_{rP}\eta_{mrP}} \qquad \textbf{(2)}\text{RFP1}$$

In the inch-pound-second system, $\boldsymbol{R_{fP}}$ has units of in-lb/rpm.

Second, we note that the pump flow and motor flow must be equal, because they are connected together. Taking into account the internal leakages, one sees that the continuity equation in the hydraulic circuit is:

$$\frac{D_P N_P}{60} = \frac{P}{R_{\iota P}} + \frac{P}{R_{\iota M}} + \frac{D_M N_M}{60} \qquad \textbf{(3)}\text{QEQ1}$$

Where the internal leakage resistances for TYPE 1 models are found from:

$$R_{\iota P} = \frac{60P_{rP}}{D_{rP}N_{rP}(1 - \eta_{vrP})} \qquad \textbf{(4)}\text{RLP1}$$

and, again referring to the TYPE 1 theories:

$$R_{\iota M} = \frac{60\,\eta_{vrM}P_{rM}}{D_{rM}N_{rM}(1 - \eta_{vrM})} \qquad \textbf{(5)}\text{RLM1}$$

In the inch-pound-second system, the units on $\boldsymbol{R_{\iota P}}$ and $\boldsymbol{R_{\iota M}}$ are psi/(in^3/sec). $\boldsymbol{R_{\iota M}}$ and $\boldsymbol{R_{\iota P}}$ are the linearized leakage coefficients for the motor and pump respectively, and based upon the assumption that internal leakage is primarily laminar. They can be found from Equations 5 and 4. It is convenient for analytical purposes to transpose all terms to one side of the equation, hence:

$$0 = \frac{D_p N_p}{60} - P\left[\frac{1}{R_{\iota P}} + \frac{1}{R_{\iota M}}\right] - \frac{D_M N_M}{60} \qquad \textbf{(6)}\text{QEQ2}$$

Further, it will ease the algebraic manipulations if a new, equivalent leakage coefficient is defined, ie:

$$\frac{1}{R_e} = \frac{1}{R_{\iota P}} + \frac{1}{R_{\iota M}} \qquad \textbf{(7)}\text{RE1}$$

Now, Equation **(6)** becomes:

$$0 = \frac{D_P N_P}{60} - \frac{P}{R_e} - \frac{D_M N_M}{60} \qquad \textbf{(8)}\text{QEQ3}$$

And **third**, we see that the motor output torque is:

$$T_M = \frac{PD_M}{2\pi} - N_M R_{fM} \qquad \textbf{(9)}\text{TM1}$$

where:

$$R_{fM} = \frac{P_{rM}D_{rM}(1 - \eta_{mrM})}{2\pi N_{rM}} \qquad \textbf{(10)}\text{RFM1}$$

$\boldsymbol{R_{fP}}$ and $\boldsymbol{R_{fM}}$ are the frictional torque loss coefficients for the pump and motor, respectively. They can be calculated from Equations 2 and 10.

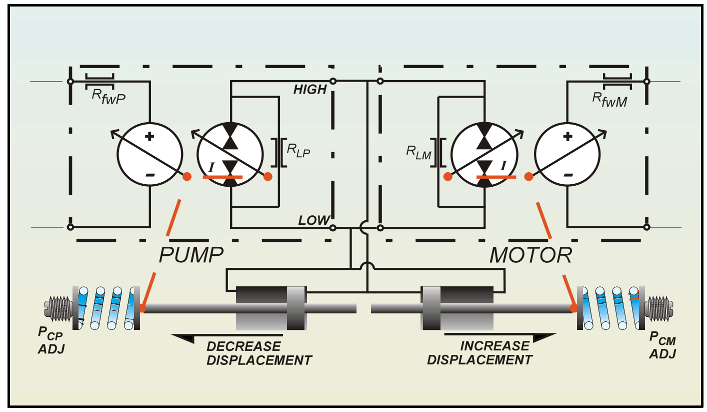

Figure 3 The TYPE 1 analytical schematic of the doubly compensated transmission wherein pump displacement decreases with rising pressure while motor displacement increases. PMWEQK

TERMS AND DEFINITIONS - PUMP AND MOTOR COMPENSATORS

Terminology and Definitions Regarding the Compensators

Five terms are used throughout the following discussion. Refer to Figure 4 for a graphical interpretation of these terms:

1. Cracking pressures (P_{CP} & P_{CM}), or **cracking point:** This is a term borrowed from relief valves, and it refers to that pressure which just causes the compensator to lift off its bottom-most condition, that is, its low pressure condition. The term applies to both the pump and motor. Symbolically, this pressure is identified with the subscript ***C***, derived from ***C***racking, to distinguish it from the saturation (ma**X**imum) pressure, elaborated below.

2. Active zone, or **active region:** The active zone occurs when the pressure is such that the sensing piston is being balanced between the pressure and the spring force. In a way, the piston is "floating", and the compensator is actively controlling the displacement.

3. Saturation, or **Maximum pressures (P_{XP} & P_{XM}):** This term refers to the condition in which the pressure is so high that it forces the compensator sensing piston into some mechanical limit. Further pressure increases will not change displacement. Symbolically, the saturation pressure is identified with the subscript ***X***, derived from ma***X***imum, to distinguish it from the cracking pressure. This is NOT to be construed as the maximum pressure sustainable by the transmission, but rather, the maximum pressure for a compensator to be active.

4. Inactive zone: This is the condition where the pressure is below the cracking pressure such that the spring forces the piston into its lower mechanical limit. It is the zone that exists when the pressure is below the cracking pressure.

5. Saturation zone, or **saturation region:** It is the zone that exists when the pressure is at or above the saturation pressure. This is also an inactive zone or region.

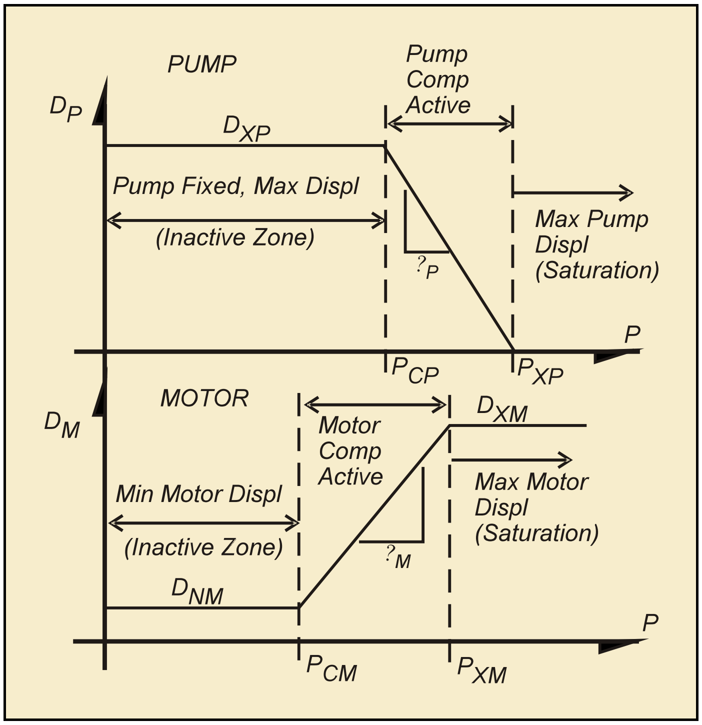

Figure 4 Generalized displacement characteristics and nomenclature for the pump and motor compensators and parameter identification. 2DISPL

Any degree of compensator overlap or underlap can exist in any given combination at any given moment, depending upon the pressure and the settings for the compensators.

The compensators are assumed to be adjustable in the following ways:

1. The cracking pressure is adjustable (tunable) by means of an adjusting screw, suggested in Figure 3.

2. The rate at which the pressure changes the displacement is controlled by the spring rate. In this respect, the rate is not tunable in the same sense that cracking pressure is, because it requires a physical change of the spring, and perhaps, even the spring cavity dimensions. Spring sizing requires dimensional details of the machines' construction, details that are avoided by the approach used in this methodology.

3. The mechanical, physical travel limit of the sensing pistons is adjustable with adjusting screws, which are NOT shown in Figure 3.

These compensator parameters are all inputs to the mathematical models that follow and will be required in order to carry out the simulation. However, converging on a workable set of values can be, in itself, a daunting task. Trying to converge on them by successive simulations is many times no more expeditious than building the hardware and doing trial-and-error in the lab. But, by using the models, it is also possible to prepare equations and computer programs that will calculate the compensator parameter settings that are required to achieved a targeted performance characteristic based upon the specific needs of a given application. That will be done later. Compensators are next.

MODELING THE COMPENSATORS

Compensator Models, Alternate 1, the Pump

It can be seen from **Figure 4**, in its active region, that:

$$D_P = D_{XP} + \gamma_P(P - P_{CP}) \qquad \textbf{(11)}\text{DXP1}$$

Where the compensator gain, γ_P, is the displacement change divided by the pressure change that caused it:

$$\gamma_P = -\frac{D_{XP}}{P_{XP} - P_{CP}} \qquad \textbf{(12)}\text{GAMP1}$$

which evaluates to a negative value under the condition that:

$$P_{CP} \le P \le P_{XP} \qquad \textbf{(13)}\text{PINEQ1}$$

which is also the condition for the pump compensator to be active.

When $P < P_{CP}$:

$$D_P = D_{XP} \qquad \textbf{(14)}\text{DPDPX1}$$

when $P > P_{XP}$:

$$D_P = 0 \qquad \textbf{(15)}\text{DP0}$$

Note then, that P_{XP} is defined, NOT as the pump deadhead pressure, but rather as the pressure that drives the pump displacement to zero.

Compensator Models, Alternate 1, the Motor

For the active zone of the motor compensator:

$$D_M = D_{NM} + \gamma_M(P - P_{CM}) \qquad \textbf{(16)}\text{DXM1}$$

where:

$$\gamma_M = \frac{D_{XM} - D_{NM}}{P_{XM} - P_{CM}} \qquad \textbf{(17)}\text{GAMM1}$$

under the condition that:

$$P_{CM} \le P \le P_{XM} \qquad \textbf{(18)}\text{PINEQ2}$$

in order for the motor compensator to be active.

When $P \le P_{CM}$, the motor must be at minimum displacement:

$$D_M = D_{NM} \qquad \textbf{(19)}\text{DMIN1}$$

and when $P \ge P_{CM}$ then the motor must be at maximum displacement:

$$D_M = D_{XM} \qquad \textbf{(20)}\text{DMAX1}$$

Compensator Models, Alternate 2, the Motor

It is at times convenient to have the pump and motor compensator models in a little bit different algebraic form, to wit, starting with the motor's active zone:

$$D_M = D_{0M} + \gamma_M P \qquad \textbf{(21)}\text{8KK}$$

where (see Figure 4 for graphical interpretation):

$$\gamma_M = \frac{\Delta D_M}{\Delta P} = \frac{D_{XM} - D_{NM}}{P_{XM} - P_{CM}} \qquad \textbf{(22)}\text{8KL}$$

And, with a bit of algebra:

$$D_{0M} = \frac{D_{NM}P_{XM} - D_{XM}P_{CM}}{P_{XM} - P_{CM}} \qquad \textbf{(23)}\text{8KM}$$

Compensator Models, Alternate 2, the Pump

Similarly, in its active region, the pump alternate 2 model is:

$$D_P = D_{0P} + \gamma_P P \qquad \textbf{(24)}\text{8KN}$$

Where:

$$\gamma_P = \frac{\Delta D_P}{\Delta P} = \frac{D_{XP}}{P_{CP} - P_{XP}} \qquad \textbf{(25)}\text{8KP}$$

Again, after some algebra:

$$D_{0P} = \frac{D_{XP} P_{XP}}{P_{XP} - P_{CP}}$$ **(26)**8KQ

It's worth repeating that γ_P evaluates to a negative number in normal operation, and $\boldsymbol{D_{0P}}$ evaluates to a positive number. In the motor model, just the opposite occurs.

PERFORMANCE CHARACTERISTICS OF THE SEQUENTIALLY COMPENSATED TYPE 0 HYDROSTATIC TRANSMISSION

A sequentially compensated HST uses compensator tuning such that the pump compensator becomes active at the same pressure that causes motor compensator saturation. The performance of the TYPE 0 transmission is completely characterized by converting Equations 1, 3 and 9 to their idealized configurations, that is, by eliminating all of the frictional and leakage losses. The results are the equations for the TYPE 0 model:

$$T_P = 0N_P + \frac{D_P}{2\pi}P + 0N_M$$ **(27)**8KS

$$0 = \frac{D_P}{60}N_P - 0P - \frac{D_M}{60}N_M$$ **(28)**8KT

$$T_M = 0N_P + \frac{D_M}{2\pi}P - 0N_M$$ **(29)**8KU

Be reminded, when operating in the hyperbolic region of the speed-torque curve, only the motor compensator is active in the sequentially compensated HST. Additionally, it is necessary to define the motor's compensator, originally defined in Equation 21:

$$D_M = D_{0M} + \gamma_M P$$ **(30)**8KV

which can be substituted into Equation 28 from which we can solve for the pressure, $\boldsymbol{P}$:

$$P = \frac{D_P N_P - D_{0M} N_M}{\gamma_M N_M}$$ **(31)**8KW

Next, Equation 30 is substituted into Equation 29 producing:

$$T_M = \frac{D_{0M} + \gamma_M P}{2\pi} P$$ **(32)**8KX

Now, Equation 31 can be used to eliminate the pressure in the motor torque Equation 32 which expresses the motor torque in terms of the motor speed instead of pressure:

$$T_M = \frac{D_{0M} + \gamma_M \dfrac{D_P N_P - D_{0M} N_M}{\gamma_M N_M}}{2\pi} x \frac{D_P N_P - D_{0M} N_M}{\gamma_m N_M}$$ **(33)**8KY

After some simplifying:

$$T_M = \frac{D_P^2 N_P^2}{2\pi \gamma_M N_M^2} - \frac{D_P N_P D_{0M}}{2\pi \gamma_m N_M}$$ **(34)**8KZ

At this point it's worthwhile to make a brief summary. First, it's necessary to realize that both the pump displacement, $\boldsymbol{D_P}$, and the pump shaft rotational speed, $\boldsymbol{N_P}$, are constant. $\boldsymbol{D_P}$ is constant because this is the sequentially compensated hydrostatic transmission, and only the motor compensator is active in the constant power hyperbolic portion of the speed-torque curve. Transmission pressure is below the pump compensator's cracking pressure, so the pump is at its maximum displacement. $\boldsymbol{N_P}$ is constant because we are simulating the transmission only, much as it would be controlled while being tested in the laboratory.

Second, we note that motor output torque, $\boldsymbol{T_M}$, and motor speed, $\boldsymbol{N_M}$, are NOT constant, but instead rise and fall, in inverse proportion to one another, to meet the needs of the load. More specifically, it is desired to have the output torque and speed follow a constant power hyperbola.

Equation 34 does NOT describe a hyperbola, however, the motor speed in the denominator of the second term gives a clue as to how to achieve a hyperbolic characteristic. For instance, if γ_M, the pressure compensator gain, is made very large, the first term becomes increasingly insignificant. The second term remains a problem. It must be realized, however, that both γ_M and the mathematical term for the displacement axis intercept, that is, $\boldsymbol{D_{0M}}$, are dependent upon the very same adjustment/tuning parameters, ie, motor displacement, compensator cracking pressure and motor compensator saturation pressure. From the derivations involving the modeling of the compensators, it's possible to form the ratio that will provide insight into the transmission's speed-torque characteristics. Specifically, that ratio is formed using Equations 22 and 23:

$$\frac{D_{0M}}{\gamma_M} = \frac{D_{NM} P_{XM} - D_{XM} P_{CM}}{D_{XM} - D_{NM}}$$ **(35)**8K1

Equation 35 is substituted into the second term of Equation 34, producing:

$$T_M = \frac{D_P^2 N_P^2}{2\pi \gamma_M N_M} \;.....$$
$$- \frac{D_P N_P}{2\pi N_M} x \frac{D_{NM} P_{XM} - D_{XM} P_{CM}}{D_{XM} - D_{NM}}$$ **(36)**8K2

Even in this, the lossless, idealized case, Equation 36 does not describe a constant output power drive. But it is possible to interpret it to see what is required for a constant power output speed-torque characteristic. Basically, constant output power requires that the compensator gain, γ_M, be very large. To achieve this phenomenon, there must be little difference between the cracking pressure and the saturation pressure. When the pressures approach one another ($\boldsymbol{P_{XM} = P_{CM} = P}$), γ_M approaches infinity and causes the first term to approach zero, a desired result. But also, when the two pressures approach one another, they can each be replaced with the generalized transmission pressure, $\boldsymbol{P}$, and it can be factored out of the numerator of the second term, producing:

$$T_M = \frac{D_P N_P}{2\pi} P \; x \; \frac{1}{N_M}$$ **(37)**8K3

This is the desired end, where it must be recognized that $\boldsymbol{N_P}$, $\boldsymbol{D_P}$ and $\boldsymbol{P}$ are all constants and the motor torque is inversely proportional to the reciprocal of its speed. The motor torque, then, lies on a constant power hyperbola. When a graph is made of Equation 37, it looks very similar to the generic hyperbolic graph in Figure 5.

But consider what this means: Constancy of the pressure is concluded by having required that the cracking and saturation pressures of the motor be the same. In order for the pressure to be constant along the hyperbola, the motor compensator must be ideal . . . perfect might be a better term. So, not only must the pump and motor be perfectly lossless, the motor's compensator must produce perfectly constant pressure! This is a worthwhile goal, but, alas, it is not achievable in any practical sense. The so-called constant pressure drive is a myth, an illusion, which can only be approximated. This does not by any means imply that the machine is worthless, rather, it requires more investigation to more fully understand its function. This also leads us to the conclusion that the sequentially compensated HST cannot simultaneously pass through two different points on the targeted constant power hyperbola.

TUNING/ADJUSTING THE SEQUENTIALLY COMPENSATED HST

Data needed to complete the investigation consists of the following known or given values:

1. Expected prime mover speed, N_P, which is also the pump speed
2. Desired motor output speed at **Point A**, N_{MA}
3. Desired motor output speed at **Point B**, N_{MB}
4. Cracking pressure of the motor compensator, P_{CM}
5. Saturation pressure of the motor compensator, P_{XM}
6. Cracking pressure of the pump compensator, P_{CP}, which is also the same as motor saturation pressure, P_{XM}
7. Saturation pressure of the pump, P_{XP}
8. The pump leakage and friction loss coefficients must be known or calculable, ie, R_{fP} and R_{LP}
9. The motor leakage and friction loss coefficients must be known or calculable, ie, R_{fM} and R_{LM}
10. And last, but not least, the targeted, constant, output power, W_H

The key unknowns that need to be calculated from the models and the derivations are:

Maximum motor displacement, D_{XM}
Minimum motor displacement, D_{NM}
Maximum pump displacement, D_{XP}

When Point A on the Constant Power Hyperbola is Favored - Sequential Compensation

Because sequential compensation does not allow passing through two targeted points (refer to Figure 5 for the general interpretation of the hyperbolic power curve), one of the two points must be favored. When **Point A** is favored, it means that the motor's speed torque characteristic will pass through that point. The next section develops the pertinent equations.

Recall that at **Point A** both the pump and motor are at maximum displacement and the pressure is, at the same time, the motor saturation pressure and the pump cracking pressure. It will not be possible to have the speed-torque curve also pass through **Point B**, a reality that can be checked with the derivations that follow. For **Point A** preference, the three HST equations, starting from Equations 1, 3 and 9, after modifying the subscripts to conform to **Point A** conditions, are:

$$T_{PA} = \frac{D_{XP}}{2\pi} P_{XM} + R_{fP} N_P \qquad \textbf{(38)}$$
8K5

$$\frac{D_{XP}}{60} N_P = \frac{P_{XM}}{R_e} + \frac{D_{XM}}{60} N_{MA} \qquad \textbf{(39)}$$
8K6

$$T_{MA} = \frac{D_{XM}}{2\pi} P_{XM} - R_{fM} N_{MA} \qquad \textbf{(40)}$$
8K7

We see from the list of givens, above, that the only unknown in Equation 40 is D_{XM}, so it can be solved for:

$$D_{XM} = \frac{2\pi (T_{MA} + R_{fM} N_{MA})}{P_{XM}} \qquad \textbf{(41)}$$
8K8

where it can be recognized that T_{MA} is the desired output torque on the constant power hyperbola and can be calculated from the targeted output power, W_H :

$$T_{MA} = \frac{W_H K_U}{N_{MA}} \qquad \textbf{(42)}$$
8K9

where K_U is a constant for the engineering units. The result of Equation 41 is the maximum motor displacement required to get the motor's speed-torque characteristic to pass through **Point A**.

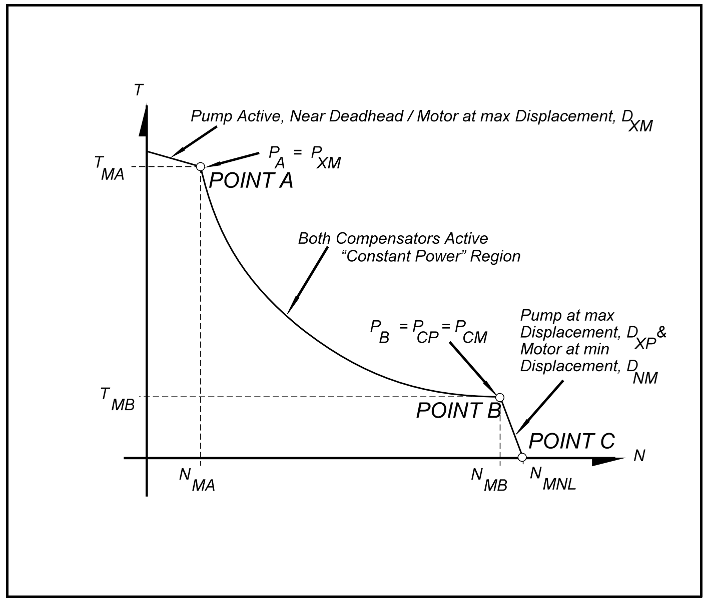

Figure 5 Double, simultaneous compensation requires that points "A" and "B" be design targets for optimal performance. 8II

Once the maximum motor displacement, D_{XM}, is known, the maximum pump displacement, D_{XP}, can be calculated by solving for it in Equation 39:

$$D_{XP} = \frac{60}{N_P}\left(\frac{P_{XM}}{R_e} + \frac{N_{MA} D_{XM}}{60} \right) \qquad \textbf{(43)}$$
8K!

It's still necessary to determine the minimum motor displacement, D_{NM}, which does not appear in Equations 38, 39 or 40. It will be necessary to use the conditions of **Point B** in order to complete the tuning of the HST. The 3 transmission equations under conditions at **Point B** become:

$$T_{PB} = \frac{D_{XP}}{2\pi} P_{CM} + R_{fP} N_P \qquad \textbf{(44)}$$
8K@

$$\frac{D_{XP}}{60} N_P = \frac{P_{CM}}{R_e} + \frac{D_{NM}}{60} N_{MB} \qquad \textbf{(45)}$$
8K#

$$T_{MB} = \frac{D_{NM}}{2\pi} P_{CM} - R_{fM} N_{MB} \qquad \textbf{(46)}$$
8K$

At this point there may be a temptation to impose T_{MB} and N_{MB} on Equation 46, that is, to use the values for both speed and torque that lie perfectly on the constant power hyperbola. This cannot be done when correct values are sought. Doing so will give conflicts in values, for example, conditions where the cracking pressure of the motor must be higher than the saturation pressure, and so on. It's repeated here that it is not physically possible to have the speed-torque curve pass through both **Points A** and **B**. It is possible to have the curve pass through either the motor speed at **B**, ie, N_{MB}, OR the torque at **B**, ie, T_{MB}, but not both, when favoring **Point A**. This leads to two possible strategies that we can use to proceed: Calculate the D_{NM} that will achieve the speed, N_{MB}, OR Calculate the D_{NM} that will achieve the torque, T_{MB}. The approach here will be to target the speed at **Point B**, because it is often the case that the power limiting must function over a required/specified speed range.

Inspection of Equation 45 reveals that the only unknown is D_{NM}. It is a simple matter to solve for it to complete this exercise:

$$D_{NM} = \frac{60}{N_{MB}}\left(\frac{D_{XP}N_P}{60} - \frac{P_{CM}}{R_e}\right) \qquad \textbf{(47)}\text{8K\%}$$

This value of D_{NM} will assure that the speed, N_{MB}, will be achieved when the pump is at maximum displacement, D_{XP}, the pump is being driven at N_P and the pressure is at the motor's cracking point, P_{CM}, the desired result. Now it's possible to use Equation 46 to calculate the motor torque when operating at N_{MB}, but note that the value will NOT lie on the constant power hyperbola.

When Point B on the Constant Power Hyperbola is Favored

The development is similar to that which was just made, above, except it starts, obviously, with the equations for the conditions at **Point B** on the constant power hyperbola. They are Equations 44, 45 and 46 where it can be seen that in Equation 46 we can solve for D_{NM}:

$$D_{NM} = \frac{2\pi\,(T_{MB} + R_{fM}N_{MB})}{P_{CM}} \qquad \textbf{(48)}\text{8K\^{}}$$

Then Equation 45 can be used to solve for D_{XP}:

$$D_{XP} = \frac{60}{N_P}\left(\frac{P_{CM}}{R_e} + \frac{D_{NM}N_{MB}}{60}\right) \qquad \textbf{(49)}\text{8K\&}$$

To calculate the maximum motor displacement, D_{XM}, we have to go to **Point A** equations, specifically, to Equation 39 where we see that it's the only unknown. Solving for D_{XM}:

$$D_{XM} = \frac{60}{N_{MA}}\left(\frac{D_{XP}N_P}{60} - \frac{P_{XM}}{R_e}\right) \qquad \textbf{(50)}\text{8K*}$$

Now, Equation 38 can be used to calculate the actual torque required to drive the pump under the conditions that the motor is operating at N_{MA} when the motor saturates and will be operating at N_{MB} when the motor compensator reaches its cracking pressure.

SIMULTANEOUS COMPENSATION

When both the pump and motor compensators are sized and tuned to be simultaneously active, it's possible to have the speed -torque curve pass through both **Point A** and **Point B**.

In this development, the need is to arrive at a set of formulas which will aid in adjusting the compensators to coincide with target points on the hyperbola.

In the name of manageability, some constraints must apply. The first regards the compensator. The developments will proceed under the condition that both compensators crack at the same pressure. This condition has been given the special name of simultaneous compensation. The reason, already stated, for choosing this design is because with both compensators simultaneously active it's possible to force the output speed-torque curve to pass through both **Point A** and **Point B**. Recall that this condition is NOT possible with sequential compensation. Other strategies are not pursued in this paper.

The second constraint deals with the data which are known at the outset. That is, an assumption is made regarding what specific performance requirements must be met, and which quantities must be known at the outset. Figure 5 will help lend insight into the performance goals.

It's required (according to the constraints and assumptions) that the speed torque characteristic of the properly sized and adjusted constant power transmission will have to pass through two targets, labeled "**A**" and "**B**" in Figure 5 . Also, it will be required that a certain no-load speed not be exceeded, as indicated by the intercept point, labeled "**C**" in Figure 5. Additionally, it will be assumed that the maximum motor displacement is constrained by virtue of the motor's selection, and is therefore known from facts outside the specific development which follows.

The following items, then, are specified, known, or are knowable, at sizing/design time. Here is a brief description of the several quantities to help in navigating through the mathematical derivations that follow:

1. Targeted motor output power, W_H, at **Point A**, is required to be given at the outset. It is the first design target, because a maximum output power limit will be specified as a design target value, and it will be used to determine the speed and torque at **Point A**. The value of W_H is based, firstly, upon the required job that needs to be done, but, secondly, tempered by the limitations of the prime mover capability and, thirdly, affected by the losses in the transmission. This is somewhat problematic, because the losses are not necessarily known until after the process under discussion right now has been completed. Like all design processes, some trial and error may have to be used to converge on the final design. The current development is intended to expedite the convergence.

2. Motor output power, W_H, at **Point B**, the second design target, at **Point B,** is the same as the targeted power at **Point A**, because both points lie on the constant power hyperbola.

3. It will be required that the user specify the motor output speeds at **Points A** and **B**, respectively, call them N_{MA} and N_{MB}. Now, when the power, W_H, is given as well as the two speeds, the two respective motor torques, T_{MA} and T_{MB} are calculable very simply.

4. No Load Speed is known, N_{MNL}, at **Point C** (also referred to as the no-load condition). The no-load speed must be greater than that of **Point B**, and there are some issues regarding its realistic selection, because there are other consequences when hastily specified.

5. Prime mover speed, which is also the pump speed, N_P, is fixed and must be specified at the outset.

6. One of three parameters, maximum motor displacement, maximum pump pressure, or maximum motor pressure has to be known during the current process, because there are fewer equations than there are unknowns. The details will unfold as the derivations proceed.

7. The leakage and friction loss coefficients must be known or reasonably estimated. This fact is what prevents the following derivations from being a component selection process. It is a tuning/adjusting procedure for existing (already selected) hardware. That is, there must have been some procedure for selecting the components (pump and motor) so that their rated displacements and frictional and leakage loss coefficients are known at the outset of the following procedure.

8. On the subject of displacements, it is important to realize that the so-called maximum displacements, $\boldsymbol{D_{XP}}$ and $\boldsymbol{D_{XM}}$ are NOT the same as the rated displacements, $\boldsymbol{D_{rM}}$ and $\boldsymbol{D_{rP}}$. The former must ALWAYS be less than the latter, and if the sizing/tuning procedure that follows, here, violates that rule, then a new pump and/or motor will have to be selected with bigger displacement(s). $\boldsymbol{D_{XP}}$ and $\boldsymbol{D_{XM}}$ are normally limited to less than the respective rated displacements by means of adjusting screws on the compensators.

ADJUSTING THE SIMULTANEOUSLY COMPENSATED HST FOR CONSTANT OUTPUT POWER

The starting point for the development is the family of equations for the generalized linearized hydrostatic transmission, ie, Equations 1, 3 and 9 are very general, and can be adapted to nearly any manner of control conceivable. The specific approach will be to impose the known conditions on the equations and then solve them simultaneously to arrive at the necessary values to meet the imposed requirements. The three conditions for which we have knowledge are:

1. **Point A**
2. **Point B**
3. No-Load Speed, **Point C** in Figure 5.

No-Load Conditions

We commence with the no-load condition at **Point C:**

$$T_{MNL} = 0$$ **(51)**8IM

$$N_M = N_{MNL}$$ **(52)**8IN

Recall that N_{MNL} is a known quantity, that is, it is specified as a design goal. Now, Equations 3 and 9 become:

$$\frac{D_{PNL}N_P}{60} - \frac{P_{NL}}{R_e} - \frac{D_{MNL}N_{MNL}}{60} = 0$$ **(53)**8IO

$$T_{MNL} = 0 = \frac{D_{MNL}}{2\pi}P_{NL} - R_{fM}N_{MNL}$$ **(54)**8IP

But, at no load, we are imposing the condition that both compensators are relaxed, because pressure is below cracking. That is, the pump is at maximum displacement and the motor is at minimum displacement:

$$D_{PNL} = D_{XP}$$ **(55)** 8IQ

$$D_{MNL} = D_{NM}$$ **(56)**8IR

Now, Equation 53 becomes:

$$\frac{D_{XP}N_P}{60} - \frac{P_{NL}}{R_e} - \frac{D_{NM}N_{MNL}}{60} = 0$$ **(57)**8IS

The torque, Equation 54, becomes:

$$\frac{D_{NM}P_{NL}}{2\pi} - R_{fM}N_{MNL} = 0$$ **(58)**8IT

Equation 58 can be used to solve for no-load pressure and then substituted into Equation 57 to eliminate the pressure therefrom:

$$P_{NL} = \frac{2\pi R_{fM}N_{MNL}}{D_{NM}}$$ **(59)**8IU

So now, after substituting Equation 59 into it, Equation 57 becomes:

$$\frac{D_{XP}N_P}{60} - \frac{1}{R_e} * \frac{2\pi R_{fM}N_{MNL}}{D_{NM}} - \frac{D_{NM}N_{MNL}}{60} = 0$$ **(60)**8IV

This is the first equation. Note that it has two unknowns, $\boldsymbol{D_{XP}}$ and $\boldsymbol{D_{MIN}}$, maximum pump and minimum motor displacements, respectively. We will obviously need more information, which can be found by using **Point B**.

Point B Conditions

Consider now, conditions at **Point B**, where speed and torque are known:

$$T_M = T_{MB}$$ **(61)**8IW

$$N_M = N_{MB}$$ **(62)**8IX

Pressure is low, so both compensators are inactive, but on the verge of becoming active:

$$D_P = D_{XP}$$ **(63)**8IY

$$D_M = D_{NM}$$ **(64)**8IZ

And, because **Point B** is the point where both compensators are about to become active:

$$P = P_B = P_{CM} = P_{CP}$$ **(65)**8I1

Note that Equation 65 is where we are imposing equal cracking pressures on both the pump and motor displacements, respectively, because they are just on the verge of going active at **Point B**. Now, Equations 61 through 65 are used in Equations 3 and 9:

$$\frac{D_{XP}N_P}{60} - \frac{P_{CM}}{R_e} - \frac{D_{NM}N_{MB}}{60} = 0$$ **(66)**8I2

$$T_{MB} = \frac{D_{NM}}{2\pi}P_{CM} - R_{fM}N_{MB}$$ **(67)**8I3

Equation 67 is used to solve for motor cracking pressure. Now:

$$P_{CM} = (T_{MB} + R_{fM}N_{MB})\frac{2\pi}{D_{NM}}$$ **(68)**8I4

and, after substituting into Equation 66:

$$\frac{D_{XP}N_P}{60} - \frac{(T_{MB} + R_{fM}N_{MB})2\pi}{R_e D_{NM}} - \frac{D_{NM}N_{MB}}{60} = 0$$ **(69)**8I5

At this point we note that Equations 60 and 69 have the same two unknowns, $\boldsymbol{D_{XP}}$, the maximum pump displacement and $\boldsymbol{D_{NM}}$, the minimum motor displacement. This, of course, is a happy state of affairs, having two equations and two unknowns. The equations are solvable.

The specific approach is to solve for $\boldsymbol{D_{XP}}$ in each, then set the results equal to one another. This equation has only one unknown, $\boldsymbol{D_{NM}}$, which can be solved for. After some algebraic manipulations, and after reworking Equation 60:

$$D_{XP} = \frac{120\pi R_{fM}N_{MNL}}{N_P R_e D_{NM}} + \frac{D_{NM}}{N_P}N_{MNL}$$ **(70)**8I6

and likewise, using Equation 69:

$$D_{XP} = \frac{60(T_{MB} + R_{fM}N_{MB})2\pi}{N_P R_e D_{NM}} + \frac{D_{NM}}{N_P}N_{MB}$$ **(71)**8I7

After setting Equations 70 and 71 equal to one another and solving:

$$D_{NM} = \sqrt{\frac{120\pi(T_{MB} + R_{fM}N_{MB} - R_{fM}N_{MNL})}{R_e(N_{MNL} - N_{MB})}} \qquad \textbf{(72)}$$ 8I8

Once $\boldsymbol{D_{NM}}$ is evaluated, $\boldsymbol{D_{XP}}$ can be calculated from either Equation 70 or 71. Use either one to calculate $\boldsymbol{D_{XP}}$.

The cracking pressures for both the pump and motor compensators can be calculated using Equation 68. The no-load running pressure can be calculated, too, from Equation 59.

Note that we now know two displacement values, $\boldsymbol{D_{XP}}$ and $\boldsymbol{D_{NM}}$, one each for the pump and motor, plus, we also know where to set the cracking pressure for both the pump and motor, $\boldsymbol{P_{CP}}$ and $\boldsymbol{P_{CM}}$. The no-load pressure is calculable, however, it is helpful to know the value, but not critical to the tuning/adjusting process.

The next step will be the use of conditions at **Point A**, from which the design will be completed. But before investigating the needs of that point, we now have enough information to go "shopping for a transmission". That is, there is enough data to start scanning commercial catalogs for a suitable set of hardware, if this is being used as a sizing/selection procedure.

Point A Conditions

At **Point A**, due to meeting specific load requirements, both the speed and torque are known. Now, Equation 9 becomes

$$T_{MA} = \frac{D_{XM}P_{XM}}{2\pi} - R_{fM}N_{MA} \qquad \textbf{(73)}$$ 8I$

Note that the pressure is $\boldsymbol{P_{XM}}$, because **Point A** is defined as that operating point where the motor just reaches its maximum displacement. Equation 73 can be solved for the pressure, recalling that $\boldsymbol{N_{MA}}$ and $\boldsymbol{T_{MA}}$ are design targets:

$$P_{XM} = \frac{2\pi T_{MA}}{D_{XM}} + \frac{2\pi R_{fM}N_{MA}}{D_{XM}} \qquad \textbf{(74)}$$ 8I%

Knowing $\boldsymbol{P_{XM}}$, the pressure that causes the motor displacement to maximize, we can impose "XM conditions" on transmission flow equation, Equation 3, becoming:

$$\frac{D_P N_P}{60} - \frac{P_{XM}}{R_e} - \frac{D_{XM}}{60}N_{MA} = 0 \qquad \textbf{(75)}$$ 8I^

In this case, we know that the pump compensator is still in its active region, therefore, we have no specific value for it. It's only known to be between zero and maximum stroke. The general expression for pump displacement in the active region will have to be substituted. Equation 11 is repeated here for convenience:

$$D_P = D_{XP} + \gamma_P(P - P_{CP}) \qquad \textbf{(76)}$$ 8I&

where it is understood that γ_p, the pump stroke rate, is actually a negative value. It was first defined in Equation 12.Now, Equation 76 becomes:

$$D_P = D_{XP} - \frac{D_{XP}}{P_{XP} - P_{CP}}(P - P_{CP}) \qquad \textbf{(77)}$$ 8I+

Note here that $\boldsymbol{D_{XP}}$, maximum pump displacement, is known, having been found from either Equation 70 or Equation 71. Furthermore, because the two compensators are adjusted for equal cracking pressures:

$$P_{CP} = P_{CM} \qquad \textbf{(78)}$$ 8I=

and, we know the motor's cracking point, from Equation 68.

Now, we substitute Equation 77 into Equation 75:

$$\left(D_{XP} - \frac{D_{XP}}{P_{XP} - P_{CP}}(P_{XM} - P_{CP})\right)\frac{N_P}{60} \;\ldots\; - \frac{P_{XM}}{R_e} - \frac{D_{XM}}{60}N_{MA} = 0 \qquad \textbf{(79)}$$ 8JA

Note that the generalized system pressure, $\boldsymbol{P}$, in Equation 77 has been replaced with the system pressure that exists at **Point A**, namely, $\boldsymbol{P_{XM}}$, because we are identifying that point with the saturation of the motor compensator.

At this time it's recommended that an "inventory" be taken of what is known, and what is not known:

A. We know the following:

1. Minimum motor displacement, $\boldsymbol{D_{NM}}$, however, it does not appear in the equations at hand, and therefore does not immediately help us.

2. Cracking pressures, $\boldsymbol{P_{CM}}$ and $\boldsymbol{P_{CP}}$, which are the same.

3. Maximum pump displacement $\boldsymbol{D_{XP}}$.

4. $\boldsymbol{N_P}$, prime mover speed.

5. Torque at **Point A**, $\boldsymbol{T_{MA}}$.

6. Motor output speed at **Point A**, $\boldsymbol{N_{MA}}$.

B. The following are still unknown (assuming we know or are able to estimate or know the leakage and friction coefficients):

1. The saturation pressure of the pump, $\boldsymbol{P_{XP}}$.

2. Maximum motor displacement, $\boldsymbol{D_{XM}}$.

3. Saturation pressure of the motor, $\boldsymbol{P_{XM}}$.

Since there are three unknowns remaining and only two equations (Equation 74 and Equation 79) there are now two possibilities to proceed to a final solution:

A. Seek or impose an additional requirement, such as the stall pressure, and write the equations which describe the condition, or,

B. Arbitrarily choose one of the three unknowns, then solve for the other two.

Strong arguments can be put forth for either situation. Each would have its valid points. In this instance, method B, above, will be adopted, and again, strong arguments can be made for the arbitrary selection of any one of the remaining parameters. It will be explored next, and there are three possible situations in which one of the three quantities is known.

Situation 1 - Solving for P_{XM} and P_{XP} When D_{XM} is Given or Specified

So, given that $\boldsymbol{D_{XM}}$ is going to be specified/known, based upon preliminary knowledge of the transmission (motor, more specifically), the saturation pressure, $\boldsymbol{P_{XM}}$, can be calculated from Equation 74 directly. Once that is done, then Equation 79 can be solved for the saturation pressure of the pump. After some algebraic manipulation:

$$P_{XP} = \frac{\dfrac{D_{XP}P_{XM}N_P}{60} - \dfrac{P_{CP}P_{XM}}{R_e} - \dfrac{P_{CP}D_{XM}N_{MA}}{60}}{\dfrac{D_{XP}N_P}{60} - \dfrac{P_{XM}}{R_e} - \dfrac{D_{XM}N_{MA}}{60}} \qquad \textbf{(80)}$$ 8JB

Thus, Equations 74 and 80 are the final evaluators of the transmission's key parameters. Equation 74 is repeated here for convenience:

$$P_{XM} = \frac{2\pi T_{MA}}{D_{XM}} + \frac{2\pi R_{fM}N_{MA}}{D_{XM}} \qquad \textbf{(81)}$$ 8L5

With this, all three remaining parameters have been evaluated.

Situation 2 - Solving for D_{XM} and P_{XP} When P_{XM} is Given or Specified

The first step is to solve for D_{XM} in Equation 81:

$$D_{XM} = \frac{2\pi T_{MA}}{P_{XM}} + \frac{2\pi R_{fM} N_{MA}}{P_{XM}} \qquad \textbf{(82)}$$ 8L6

Knowing D_{XM}, it can be substituted into Equation 80 which will give the value for P_{XP}

Situation 3 - Solving for D_{XM} and P_{XM} When P_{XP} is Given or Specified

This requires a bit more algebraic manipulation, but the situation definitely has a solution. It starts by going back to Equation 79, clearing the denominator on the right hand side. Reworked Equation 79 now looks like this as the first step:

$$\frac{P_{XP}D_{XP}N_P}{60} - \frac{P_{XP}P_{XM}}{R_e} - \frac{P_{XP}D_{XM}N_{MA}}{60} \ldots\ldots$$

$$= \frac{D_{XP}P_{XM}N_P}{60} - \frac{P_{CP}P_{XM}}{R_e} - \frac{P_{CP}D_{XM}N_{MA}}{60} \qquad \textbf{(83)}$$ 8L7

Now, substituting Equation 74 into it in order to eliminate D_{XM} and clearing P_{XM} from the denominator produces a quadratic in P_{XM} :

$$\left(\frac{D_{XP}N_P}{60} - \frac{P_{CP}}{R_e} + \frac{P_{XP}}{R_e}\right)P_{XM}^2 - \left(\frac{P_{XP}D_{XP}N_P}{60}\right)P_{XM} \ldots\ldots$$

$$+ \frac{2\pi N_{MA}(P_{XP} - P_{CP})(T_{MA} + R_{fM}N_{MA})}{60} = 0 \qquad \textbf{(84)}$$ 8L8

which can be solved with the quadratic equation, recognizing that:

$$a = \frac{D_{XP}N_P}{60} - \frac{P_{CP}}{R_e} + \frac{P_{XP}}{R_e} \qquad \textbf{(85)}$$ 8L9

and:

$$b = -\frac{P_{XP}D_{XP}N_P}{60} \qquad \textbf{(86)}$$ 8L!

and, also:

$$c = \frac{2\pi N_{MA}(P_{XP} - P_{CP})(T_{MA} + R_{fM}N_{MA})}{60} \qquad \textbf{(87)}$$ 8L@

Of course, the quadratic formula solved for P_{XM} is:

$$P_{XM} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad \textbf{(88)}$$ 8L#

Of the normal two roots to the quadratic, the subtractive term does not apply. This completes the analysis directed toward determining the adjustments and settings for the compensators in the simultaneously compensated hydrostatic transmission.

Commentary on the Sizing/Adjusting Process - Simultaneous Compensation

The foregoing discussion on adjusting the transmission compensators has elements in the process that are helpful in the initial sizing of the transmission components. However, there are special challenges at sizing time. When the applications engineer is faced with a totally new system, in actual matter of fact, it is difficult to come up with good *a priori* estimates of the loss coefficients, especially those who are inexperienced in this analytical methodology. The actual sizing equations have their greatest value in helping with the final system adjustments after specific components are chosen. They will also give an excellent estimate as to whether or not the selected components are feasible. It is not uncommon to simply assume 100%, TYPE 0 models, size the pump and motor and then make allowances for the inevitable inefficiencies in the real components. This method will be explored next.

SIZING USING THE TYPE 0 MODEL - SIMULTANEOUS COMPENSATION

When lossless conditions are imposed upon Equations 3 and 9, note that they degenerate to the TYPE 0 (ideal) model:

$$\frac{D_P N_P}{60} - \frac{P}{\infty} - \frac{D_M}{60}N_M = 0 \qquad \textbf{(89)}$$ 8JC

$$T_M = \frac{D_M}{2\pi}P - (0)N_M \qquad \textbf{(90)}$$ 8JD

At **Point B**, for instance (the motor is at minimum displacement), using Equation 90:

$$D_{NM} = \frac{2\pi T_{MB}}{P_B} \qquad \textbf{(91)}$$ 8JE

which can be substituted into Equation 89, and also evaluated at **Point B:**

$$\frac{D_{XP}N_P}{60} - \frac{2\pi T_{MB}}{60P_B}N_{MB} = 0 \qquad \textbf{(92)}$$ 8JF

Of course, T_{MB} and N_{MB} are known, because they represent the desired operating point. It will be necessary to guess at the operating pressure at **Point B**. Recall, that at **B**, the pressure is low, with both compensators being relaxed. One should expect about a 3- or 4- to -1 pressure change between cracking and saturation on the motor. Be aware that the actual running pressure at **B** will be greater than what is now guessed for the ideal model, because the real system has friction and leakage losses. At any rate, one can estimate the pressure, P_B, based upon maximum system pressure. The four-to-one scaling is probably the proper one. Now:

$$D_{XP} = \frac{2\pi T_{MB}N_{MB}}{N_P(0.25xP_{MAX,SYSTEM})} \qquad \textbf{(93)}$$ 8JG

To complete the basic sizing process, estimation of the maximum motor displacement, D_{XM}, needs to be calculated:

$$T_{MA} = \frac{D_{XM}}{2\pi}P_A \qquad \textbf{(94)}$$ 8LV

It is also known that the pressure at **Point A** is the saturation pressure of the motor in the case of simultaneous compensation. It is reasonable to estimate it as being about 80% of the maximum system pressure, so this produces:

$$D_{XM} = \frac{2\pi T_{MA}}{0.8xP_{MAX,SYSTEM}} \qquad \textbf{(95)}$$ 8LW

Recognizing that the pressure at **Point B** is the same as the pressure in Equation 93, that is, 25% of the maximum system pressure, then with some algebra, Equation 91 evaluates the minimum motor displacement, D_{NM}:

$$D_{NM} = \frac{2\pi T_{MB}}{0.25 x P_{MAX,SYSTEM}} \qquad \textbf{(96)}$$ 8LX

Equations 93, 95 and 96 comprise the basic sizing equations that can be used to get a first estimate of pump and motor sizes, provided the pump speed, N_P, and the expected system pressure, $P_{MAX,SYSTEM}$, are known or specified. Also, it should be clear that the values of the required motor torques and motor speeds at **Points A** and **B** are specified for the constant power hyperbola, and are therefore known.

With the values derived from these equations, the system designer can go shopping for real components, knowing that they are undersized, requiring a bump up in displacements before converging on actual pump and motor with their internal losses, ie, inefficiencies.

A summarizing note: The 25% and 80% adjustments in the above equations, may be re-examined by the experienced system designer. The minimum pump displacement in the doubly compensated hydrostatic transmission is always zero. If, during the execution of the adjustment equation, there ever comes a condition where the tuning displacement exceeds the rated displacement, then it is evidence that the affected component is under-sized, requiring a different component selection.

Simulation Program Structure In Brief

The simulation requires that all the pump and motor loss coefficients be known (resistance values), as well as the pump and prime mover shaft speed and all compensator parameters. Then the transmission equations are solved for the remaining unknowns and they are calculated while the system pressure is iterated from the no-load pressure to the stall pressure. Briefly, the program looks like this (this is NOT computer code, it is only similar, but contains the key steps of a program):

```
      PROGRAM OUTLINE - DBLCOHST.EXE
1. CALCULATE TRANSMISSION COEFFICIENTS
2. CALCULATE COMPENSATOR COEFFICIENTS
3. SET PUMP INPUT SPEED
4. CALCULATE DESIRED PRESSURE INCREMENT
5. INITIALIZE THE PRESSURE AT P = P_NL

   TOP_OF_THE_LOOP:
      IF P  ≤ P_CM THEN
         D_M = D_NM
      ELSEIF P > P_CM AND P ≤ P_XM THEN
         D_M = D_OM + γ_M P
      ELSE
         D_M = D_XM
      END IF

      IF P  ≤ P_CP THEN
         D_P = D_XP
      ELSEIF P > P_CP AND P ≤ P_XP THEN
         D_P = D_OP + γ_P P
      ELSE
         D_P = D_XP
      END IF
```

$$T_P = R_{fP} N_P + \frac{D_P}{2\pi} P + 0 N_M \qquad \textbf{(97)}$$ 8M*

$$N_M = \frac{D_P}{D_M} N_P - \frac{60}{D_M R_e} P \qquad \textbf{(98)}$$ 8M+

$$T_M = 0 \cdot N_P + \frac{D_M}{2\pi} P - R_{fM} N_M \qquad \textbf{(99)}$$ 8M=

```
6. SAVE ALL DATA AS DESIRED/REQUIRED
7. INCREASE P BY ONE INCREMENT
         IF P ≤ P_STALL THEN
            GOTO TOP_OF_THE_LOOP
         ELSE
            END
         END IF

   END OF OUTLINE LISTING -DBLCOHST.EXE
```

In all, three different programs were written in the study for this paper:

1. **DBLCOHST.EXE**, already explained, in the outline, above. It is the program that performs the actual simulation of the steady-state model.
2. **ODBSEQT1.EXE**, a program specifically for tuning and adjusting the TYPE 1, sequentially compensated HST. It calculates applicable Equations 41 through 50.
3. **ODBSIMT1.EXE**, a program specifically for tuning and adjusting the TYPE 1, simultaneously compensated HST. It calculates applicable Equations 51 through 88.

The second two calculate the settings needed by the first.

Program Input Data - Configurations #1, #2 & #3

LISTING OF THE 19 INPUT PARAMETERS NEEDED TO PERFORM A STEADY-STATE SIMULATION OF THE DOUBLY-COMPENSATED HYDROSTATIC TRANSMISSION DBLCOHST.EXE INPUT DATA

1.	Prime Mover Speed	1800 rpm
2.	Rated Pump Speed	2400 rpm
3.	Rated Pump Disp	7.95 in^3/rev
4.	Rated Pump Pressure	3500 psi
5.	Rated Mech. Efficiency	97 & 100 %
6.	Rated Volumetric Eff.	97 & 100 %
7.	Rated Motor Speed	2400 rpm
8.	Rated Motor Pressure	3500 psi
9.	Rated Motor Disp	24.8 in^3/rev
10.	Rated Motor Mech Eff	97 & 100 %
11.	Rated Motor Vol Eff.	97 & 100 %
12.	Min Motor Disp	2.674 in^3/rev
13.	Max Motor Disp	16.047 in^3/rev
14.	Max Pump Disp	4.457 in^3/rev
15.	Motor Cracking Pressure	3050 psi
16.	Motor Saturation Pressure	3060 psi
17.	Pump Cracking Pressure	3060 psi
18.	Pump Saturation Pressure	3070 psi
19.	Minimum Pump Disp	0 in^3/rev

Table **2** Input data for the ideal case for a 62hp, sequentially compensated HST 8L$

The three configurations of a sequentially compensated HST are:
1. Ideal, TYPE 0 models with 100% efficiencies.
2. TYPE 1 models with all efficiencies at 97% and favoring **Point A**

3. TYPE 1 models with all efficiencies at 97% and favoring **Pont B**.

Table 2 is a listing of the 19 input data parameters to the program that simulates the steady-state performance of a sequentially compensated HST. The pump and motor efficiencies are iterated between 97% and 100% in order to compare the TYPE 0 to the TYPE 1 model results. Not shown is the fact that the simulation was carried out for both **Favoring A** conditions as well as **Favoring B** conditions, again for the sake of comparing results.

INVESTIGATION INTO THE PERFORMANCE OF THE SEQUENTIALLY COMPENSATED HYDROSTATIC TRANSMISSION

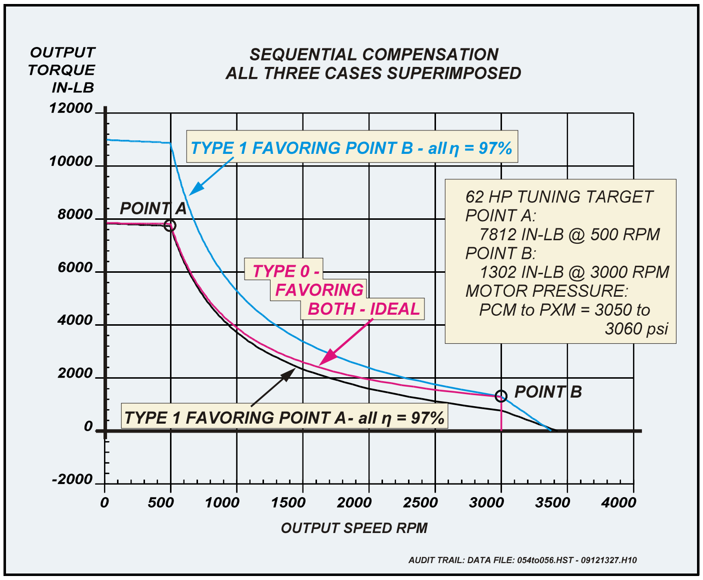

Figure 6 Speed-Torque characteristics of all three cases overlaid on one set of axes to facilitate objective comparisons. 8MA

Note that the difference between the cracking and saturation pressures of the motor is only 10 psi (see items 15, 16, 17 & 18, Table 2)! This is not practical, however it is the mathematical means by which an ideal compensator can be approximated. 0 psi difference would cause a program abort on DIVIDE BY ZERO error. Also, the program cannot handle 100% efficiencies. The actual value was set to 99.9999%.

Each one of the three cases (100% efficiency - no favoring needed, 97% efficiencies - Favoring **Point A**, 97% efficiencies - Favoring **Point B**) has its own set of Min and Max Pump and Motor Displacements. They were calculated for each case using Equations 41 through 50 (using **ODBSEQT1.EXE**). The values shown in Table 2 apply to the 100% efficiency case. All four efficiencies were always set to equal values, ie, all at 97% or all at 100%. No studies were made regarding relative differences in efficiencies between mechanical and volumetric and between pump and motor.

Figure 6 shows the combined results of the three different configurations. Only the TYPE 0 case passes through both **Point A** and **Point B**. Favoring **Point A** misses **Point B** on the low side and Favoring **Point B** misses **Point A** on the high side.

Figure 7 is a graph of the simulated Motor Output Power for all three cases. It clearly illustrates that the output power is NOT constant, except in the case of 100% efficiency. Recall that the two non-ideal cases were generated with pump and motor models that had 97% volumetric and mechanical efficiencies for all four parameters.

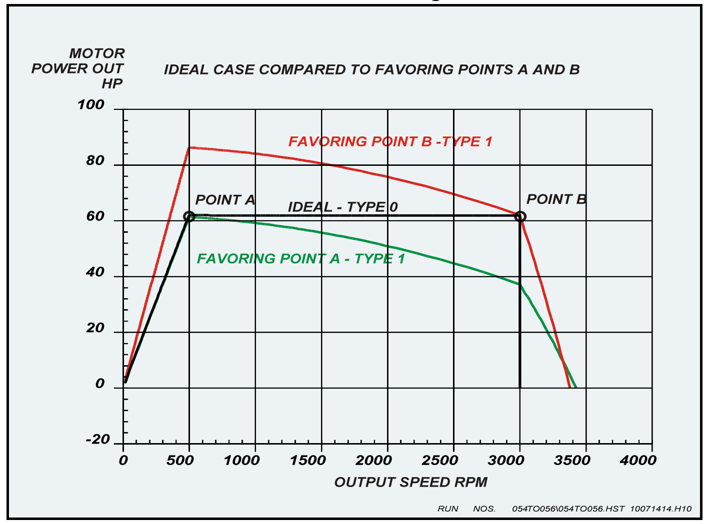

Figure 7 Motor output power curves for all three cases again shows **Point A** on one TYPE 1 curves connects to **Point B** on the other TYPE 1 and the connecting line is the TYPE 0, ideal case. 8MB

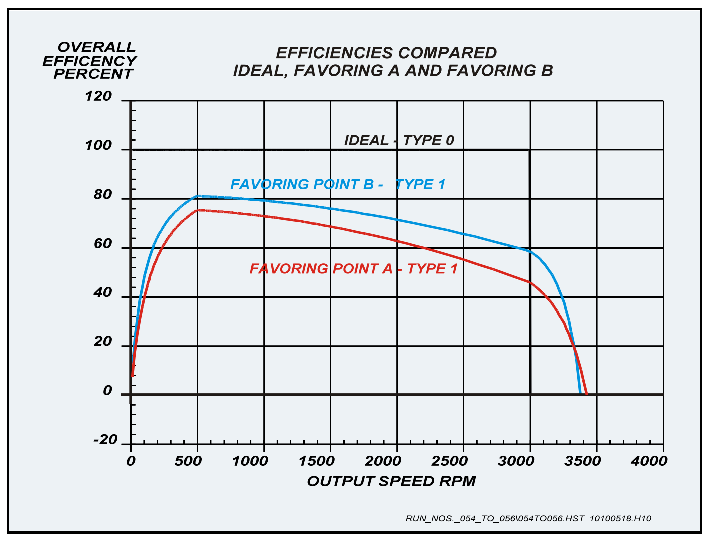

Figure 8 Efficiencies for the three cases shows 100% for the Ideal case, but drops to about 82% and 75% peak values, respectively, for the TYPE 1 cases. 8ME

Figure 8 contains the overall efficiency graphs of all three cases superimposed. Again it can be seen that the ideal, TYPE 0, case is the only one that produces constant output power. In both non-ideal cases, there is an unmistakable rise in output power with decreasing speed, and maximum efficiency occurs near the **Point A** speed, N_{MA}. The speed that results in maximum efficiency depends upon th relative values of rated pressures, speeds and efficiencies of the pump and motor, however, the specific trends and conditions are not explored in this paper.

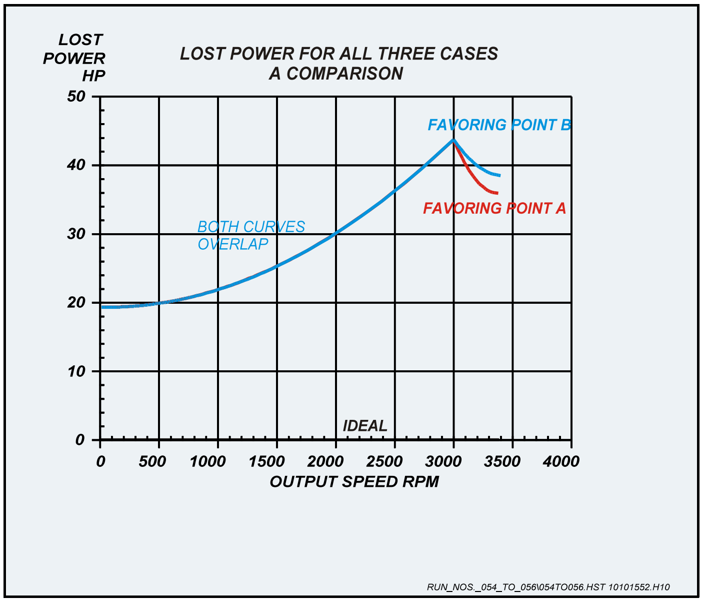

Figure 9 The Lost Power diagram for the transmission of Example Problem #? shows significant power loss with increase in motor output speed. 8MF

Lost Power diagrams for the three cases are shown in

Pump Input Power

The emphasis to this point has been on the output power and the degree to which it can be held constant. It has been shown that it cannot be held constant in the sequentially compensated HST. But, if the aim is prevention of prime mover lugging, then a more important characteristic is the amount of Input Power.

Figure 9 shows the composite Input Power graphs for all three cases under study. The results are dramatic. Even though the imperfect cases (Favoring A and B in the figure) reflect less than 100% efficiency, the input power is essentially constant. There is a slight upward tilt of the non-ideal cases at the lower speeds caused by the 10psi difference between the motor cracking pressure and its saturation pressure. As a constant output power strategy, the sequentially compensated HST is a failure. As an anti-lugging strategy, the sequentially compensated HST is a success. This is doubtless the reason for its continued implementation in industrial mobile machines, but it has been mis-named as a constant output power machine.

Lost Power

Figure 10, the Lost Power graphs, are more useful than overall efficiency diagrams, because they can be related directly to energy lost in the application. That will be discussed shortly. For the moment it is sufficient to observe, first, that the Ideal case is buried along the speed axis, because there is zero lost power with TYPE 0 models. Curiously, though, the amount of lost power in the normal active compensator region (0 to 3000 rpm) is independent of the hyperbolic point being favored.

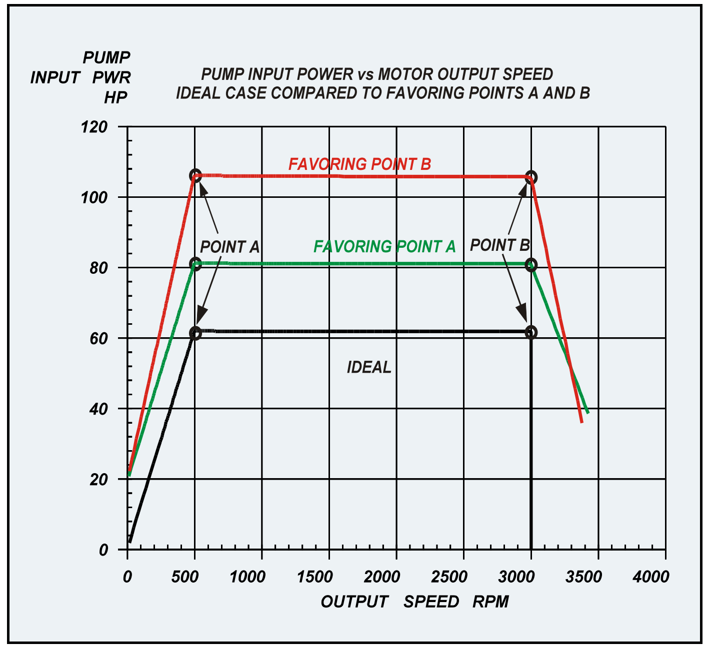

Figure 10 Pump input power characteristics show how the three cases are separated over the entire speed range and how the the singular **Points A** and **B** in the output characteristics each maps into three different points in the input characteristics. 8MC

Application Histogram of Output Speed

If a histogram of the probability of finding a given speed is acquired for the application of the HST (mobile vehicle), it is a straightforward process to estimate the total lost energy over a work shift or day or longer. The histogram is a statistical process that collects speed data at regular intervals over an appropriate time period in the application and then organizes the results into the **Probability of a Speed Occurring** graph, shown in Figure 11.

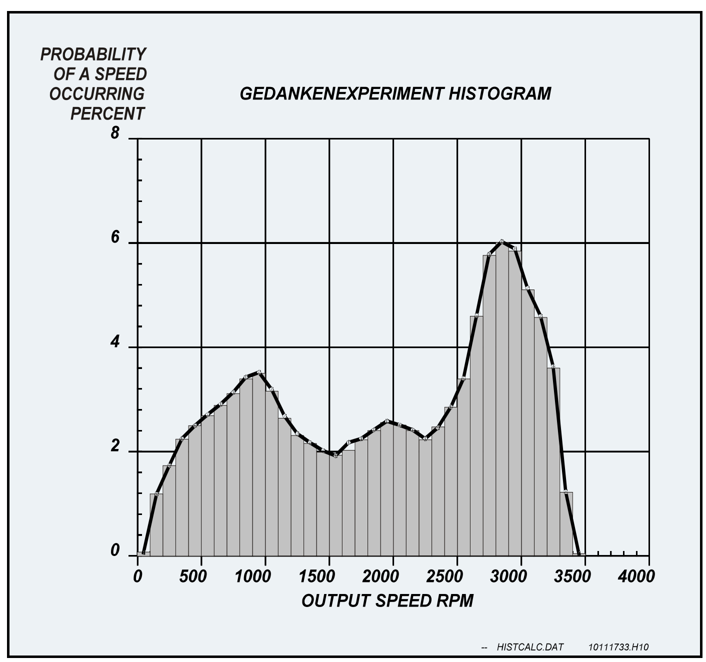

Figure 11 The Probability of a Speed Occurring is useful in calculating expected energy losses when used to operate on the Lost Power data of the transmission. 8MH

The application of Figure 11 is synthesized, it is NOT real application data, but it could depict the statistics of a wheel mounted front end loader that has a significant distance to travel between load pickup and load dump. For that reason, there is a greater probability of there being high speeds than low speeds. Here is the formula for determining the total energy lost during an arbitrary period of time:

$$E_{LT} = T\sum_{i=1}^{K_N} P_i W_{Li} \qquad \textbf{(100)}$$
8MK

Where:

$\boldsymbol{T}$ = the total time interval over which the lost energy is to be evaluated, units of time are arbitrary

$\boldsymbol{P_i}$ = the probability of a speed occurring in the ***ith*** probability histogram interval, numeric

$\boldsymbol{W_{Li}}$ = the power lost when operating at the midpoint speed of the ***ith*** probability histogram interval, hp

$\boldsymbol{K_N}$ = the total number of speed intervals in the probability histogram, numeric

$\boldsymbol{E_{LT}}$ = the total energy lost during the operational time interval of $\boldsymbol{T}$, units depend upon those used for $\boldsymbol{T}$ and $\boldsymbol{W_{Li}}$

When Equation 100 is applied to the data in Figures 10 and 11 for an 8 hour work shift evaluates to 251 hp-hrs. But, if the application process is changed, for example, if the load pickup and load dump points are closer together, so that the vehicle and transmission never gets up to full speed, the total lost energy for that same 8 hour shift reduces to 223 hp-hrs, which is about a 10% reduction in fuel consumption. More applied research and data collection is needed to apply the full potential of statistical methods in hydraulic machines.

Effects of Larger Compensator Pressure Differences

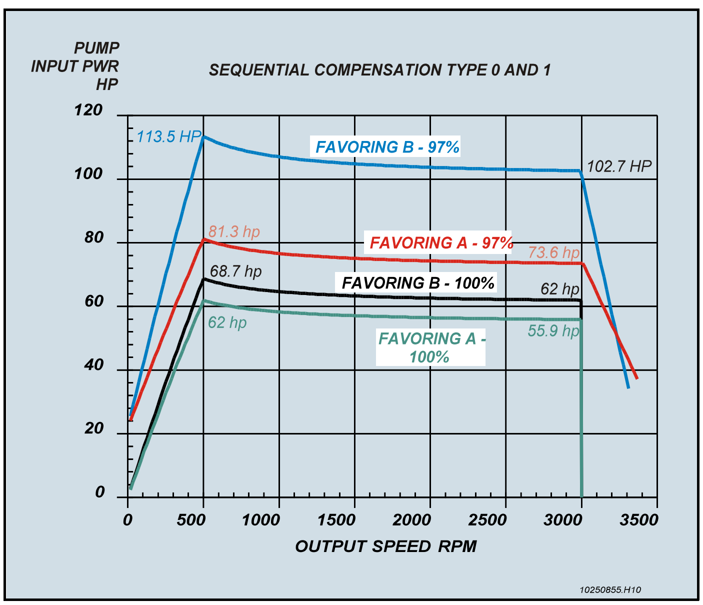

Figure 12 Pump input power characteristics show how the required power increases with reduction in speed, and the increase occurs when efficiencies are 100%. It is caused by imperfect pressure compensation in the motor. 8MY

To see the effects of having a larger pressure variation in the motor pressure compensator, the cracking and saturation points of the motor were set to 2760 and 3060 psi, respectively. Figure 12 is a composite of all four computer simulation configurations and all four design cases, where it is easy to compare one to the other. It contains the graphs of Pump Input Power. The imperfect pressure regulation causes the torque to rise as the speed decreases, simply because of the increase in pressure, even when the efficiencies are all 100%. In all cases, the input power required rises as the speed decreases. The reason is that there is a 300 psi increase in motor pressure as load increases, even when the pump and motor are perfectly efficient.

In spite of the rise in required input power and its inability to deliver constant output power to the load, the sequentially compensated HST does do a fairly good job of limiting input power and in the process, prevent prime mover lugging.

Additionally, the simulation shows that the efficiency suffers, especially at the higher speeds. It is postulated that this is a result of the required 6-to-1 motor displacement change required in the name of regulating output power. In an attempt to improve efficiency and to more closely follow a true constant power hyperbola, the simultaneously compensated HST will be investigated with the same linear models already introduced.

BASIC SIMULATION OF THE SIMULTANEOUSLY COMPENSATED HYDROSTATIC TRANSMISSION

```
  SIZING RESULTS FOR THE TYPE 1 NON-IDEAL MACHINES
      USING SIMULTANEOUSLY ACTIVE COMPENSATORS


INPUT DATA IN 'ODBINPUT.OM1' USED BY
'ODBSIMT1.EXE':

    62.00  WP - HYPERBOLA POWER (HP)
  1800.00  NP - PRIME MOVER SPEED (RPM)
   500.00  NMA - MOTOR SPEED AT POINT A (RPM)
  3000.00  NMB - MOTOR SPEED AT POINT B (RPM)
  2400.00  NrP - RATED PUMP SPEED (RPM)
  3500.00  PrP - RATED PUMP PRESSURE (PSI)
    16.35  DrP - RATED PUMP DISPL'T (IN^3/REV)
    97.00  ETAvrP - RATED PUMP VOL EFF (%)
    97.00  ETAmrP - RATED PUMP MECH EFF (%)
  2400.00  NrM - RATED MOTOR SPEED (RPM)
  3500.00  PrM - RATED MOTOR PRESSURE (PSI)
    24.80  DrM - RATED MOTOR DISPL'T (IN^3/REV)
    97.00  ETAvrM - RATED MOTOR VOL EFF (%)
    97.00  ETAmrM - RATED MOTOR MECH EFFY (%)
  3085.00  NMNL - NO-LOAD  MOTOR SPEED (RPM)

   Pxp, Dxm, Pxm Option set to Pxp = 3360 (PSI)
```

Table **3** The sizing/tuning equations require 16 different input parameters. These are the values needed when the optional input parameters is the pump saturation pressure, $\boldsymbol{P_{XP}}$, set to ***3360 PSI***.8M2

It is desired to compare the performance of the simultaneously compensated HST. In the name of objective comparison to the sequentially compensated HST, the pump and motor efficiencies will all be set to 97%. In order to establish the components required to make a valid comparison, the sizing/tuning program, **ODBSIMT1.EXE** was used. This program evaluates Equations 51 to 88. The 17 input data parameters are listed in Table 3. The sizing/tuning procedure generated the 19 simulation input parameters listed in Table 4

LISTING OF THE 19 INPUT PARAMETERS NEEDED TO PERFORM A STEADY-STATE SIMULATION OF THE DOUBLY-COMPENSATED HYDROSTATIC TRANSMISSION DBLCOHST.EXE INPUT DATA

1. Prime Mover Speed	1800 rpm
2. Rated Pump Speed	2400 rpm
3. Rated Pump Disp	**16.35** in^3/rev
4. Rated Pump Pressure	3500 psi
5. Rated Mech. Efficiency	97%
6. Rated Volumetric Eff.	97%
7. Rated Motor Speed	2400 rpm
8. Rated Motor Pressure	3500 psi
9. Rated Motor Disp	**24.8** in^3/rev
10. Rated Motor Mech Eff	97%
11. Rated Motor Vol Eff.	97%
12. Min Motor Disp	**9.06** in^3/rev
13. Max Motor Disp	**20.247** in^3/rev
14. Max Pump Disp	**15.705** in^3/rev
15. Motor Cracking Pressure	**1262.5** psi
16. Motor Saturation Pressure	**2452.0** psi
17. Pump Cracking Pressure	**1262.5** psi
18. Pump Saturation Pressure	**3360** psi
19. Minimum Pump Disp	0 in^3/rev

Table **4** Results of running **ODBSIMT1.EXE** program become the input to **DBLCOHST.EXE**. These values allow a comparison with sequential compensation. **8M3**

It's worth explaining that using the sizing/tuning program for component selection is somewhat a trial and error process, in part caused by the fact that it is necessary to know the machine efficiencies before selecting the machines themselves. In this "what if" scenario, the efficiencies are not the problem, because they are being forced to all be 97% for purposes of comparative analysis. Also, there is interplay among the specified no-load motor speed, N_{MNL} in Table 3, and the resulting pump displacement and compensators' cracking pressures. That is, when the no-load motor speed is set to a value that is only slightly above the required motor speed at **POINT B**, N_{MB}, the cracking pressures go down, but the required pump displacement goes up. Of course, N_{MB} is a required performance parameter at which the motor must deliver 62 HP to the load, and N_{MB} is set to 3000 rpm in Table 3. That point has got to be met, mathematically speaking. The results of the trial and error (successive computer runs as N_{MNL} was changed) are shown in Table 4. They represent what is a reasonable compromise between resulting pump displacement and the cracking pressure. It is also possible to specify the pump and motor cracking pressure and let the no-load motor speed be dependent. Although it's not difficult, the derivations are not developed in this paper.

The entries in **bold** in Table 4 are stressed specifically to make a comparison with the case of the sequentially compensated HST. First, we note that the pump displacement in simultaneous compensation has to be increased to 16.35 in^3/rev from 7.95 in^3/rev. This must be of concern, because the larger pump will carry with it a higher cost. But the required change in motor displacement is just over 2-to-1, a large improvement over the 6-to-1 change required with sequential compensation. The point is this: The comparison is not exactly matching cases because of the significant differences due to compensation configurations. The quest, remains, however, to anser the question, "Will system efficiency be improved when both compensators are simultaneously active"?

Figure 13 is a composite figure containing the simulated Motor Output Torque, Pump and Motor Displacements and the Motor Output Power. The aim is to provide verification that the design targets have been met as well as to show key output performance data.

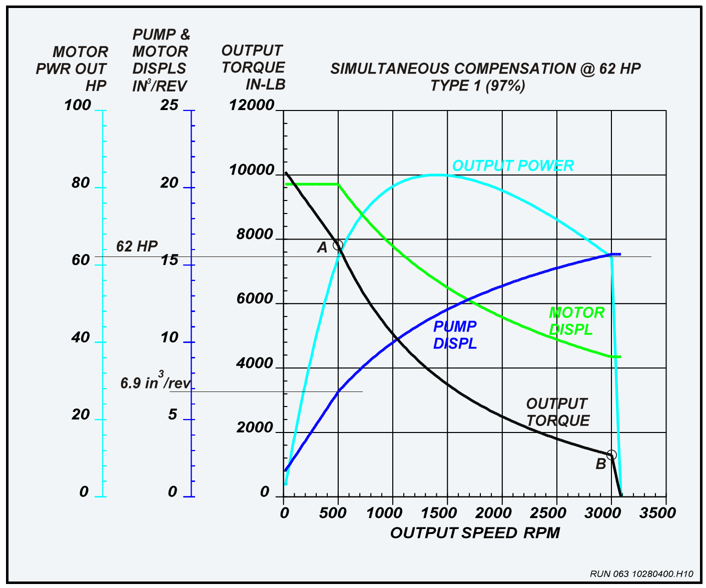

Figure 13 Four performance curves are overlaid in order to gain insight into the simultaneously compensated HST. **8M4**

The graphs confirm the design goals. There is a 62 HP horizontal line that intersects the Motor Power Output curve at the targeted speeds of 500 and 3000 rpm. **POINTS A** and **B** on the Output Torque curve coincide with 7814 and 1302 in-lb of torque and occur at 500 and 3000 rpm, as expected, and required.

Regarding the pump and motor displacements, it helps to recall that there is a reversed relationship between the motor speed and the transmission pressure. That is, the pressure is lowest at the highest speed and highest at the lowest speed. This results in a non-linear relationship between motor speed and displacement, even though the displacements are linear functions of the pressure. The pressure will be graphed, later. For the moment, however, Figure 13 shows unmistakably that both the pump and motor displacements crack at the same place, ie, **POINT B**. It is shown that the motor displacement saturates at speeds below 500 rpm, again, as expected, and required. The exact point of saturation is expected to occur at the motor saturation pressure which, recall, was calculated to be 2452 psi.

Pump displacement is interesting. Recall that the pump compensator saturation pressure, $\boldsymbol{P_{XP}}$, was specified as 3360 psi in order to make a better match for comparing to the sequential compensation results. At the point where the motor saturates, 500 rpm, the pump displacement is 6.9 in^3/rev, and at stall, the pump displacement is about 2 in^3/rev. Like any pressure-compensated pump, the resulting flow is required to supply internal leakage through both the pump and motor leakage resistances, $\boldsymbol{R_{LP}}$ and $\boldsymbol{R_{LM}}$. The pump displacement range is now calculable, and it is normally done over the compensation range, that is, from 500 to 3000 rpm. The displacement change ratio (15.7/6.9) is just a little over 2-to-1. In this respect, it is similar to the displacement change ratio of the motor. This is desirable, because it means that the ranges are equitably and evenly distributed over both machines. Neither one is operating at conditions that diverge greatly from the rated conditions.

Here are some facts that can be gleaned from Figure 13: First, the Speed-Torque characteristic <u>does</u> intersect with both **POINTS A** and **B**. Second, the output power curve <u>does</u> pass through the targeted 62 hp point, and at 500 and 3000 rpm, respectively. Third, both pump and motor displacements are changing at the same time. Fourth, in spite of hitting **A** and **B**, more or less perfectly, the resulting Speed-Torque curve deviates substantially from the ideal hyperbola.

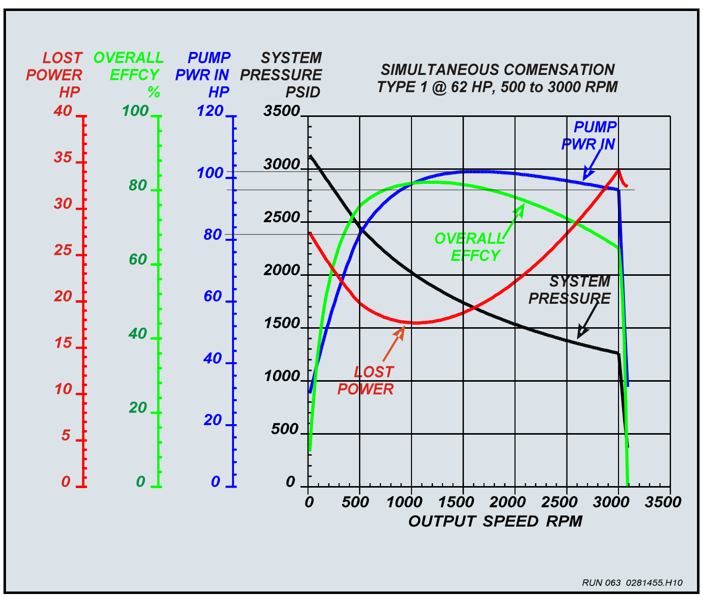

Figure 14 Lost Power, Overall Efficiency, Pump Input Power and System Pressure are key performance quantities that aid in comparisons between sequential and simultaneous compensation. **8M5**

This last point is most obvious from the Output Power curve. Clearly, the simultaneous compensation scheme does NOT produce constant output power. The aim was 62 HP, but the power peaks at about 84 HP at a speed just below 1500 rpm. If the purpose of the compensation is to prevent prime mover lugging, then it is essential that the system be designed to accommodate the peak power, not the targeted power at **POINT A** or **B**. The sequentially compensated case, likewise, does not deliver constant output power, but it does a fairly good job of requiring constant input power. And it is the input power that is important in controlling prime mover lugging. It will be necessary to look at other performance characteristics of this, the simultaneously compensated HST.

Figure 14 shows additional simulated performance data from the simultaneously compensated HST. Four different variables are plotted: Lost Power, Overall Efficiency, Pump Power In and System Pressure.

Of interest at the moment is the Pump Input Power. Given the rise in Motor Output Power between the two targeted points, as shown earlier in Figure 13, it should come as no surprise that there is also a rise in Pump Input Power. At the lower speed target, 500 rpm, the input power is about 82 HP, it peaks at about 102 HP at just over 1500 rpm and drops to about 96 HP at the higher target speed, 3000 rpm. Again, this is the quantity that will affect prime mover lugging. The prime mover will have to be sized and selected to deliver 102 HP if lugging is to be prevented.

Overall Efficiency peaks at about 82% and about 1200 rpm. This, too is interesting. The hope was that the peak efficiency with simultaneous compensation would improve when compared to sequential compensation because each machine would be operating nearer its respective rated conditions. In both transmissions, the peak efficiency is essentially the same, 82%. The anticipated efficiency advantage did not materialize. There is a difference in the speed at which the peak efficiency and minimum lost power occur, however. Therefore, the simultaneously compensated transmission of Figure 14 would be enhanced if there exists an application histogram that peaks at the mid-range speeds.

System Pressure, clearly, is NOT constant when displacements of both machines are changing. But this, too, is NOT unexpected. The cracking pressures for the pump and motor are both set to 1252 psi and the motor does not saturate until pressure reaches 2452 psi. The pump saturates A large pressure swing is required in order to effect the required changes in displacements.

Figure 15 is a graph of the Lost Power, Input and Output Powers and the Output Speed, but they are plotted against System Pressure. This is given in order to show the flexibility of math models, as well as to expand, as it were, the fixed displacement region of the characteristics, that is, the region between zero and 1252 psi, the cracking pressure. In earlier graphs, this is the near vertical portion of the Speed-Torque curves between no-load speed and **POINT B**. It shows us that the Lost Power minimizes at about 2000 psi, the Pump Input Power peaks at about 1700 psi, while the Motor Output Power maximizes around 1750 psi.

This method of displaying the Lost Power data can be very important in applications where the operation is biased heavily to the high speed, low pressure end of operation. It is completely correct to assemble a histogram of the probability of there being a pressure instead of probability of there being a speed. In fact, it would not be difficult for an on-board computer to assemble both histogram types, and then the one that makes the most common sense can be applied. The point is, that there are a number of ways to evaluate cumulative energy losses; the only requirement is that the horizontal axis of the histogram be the same as the horizontal axis in the Lost Power graph.

Summary Comments Regarding the Simultaneously Compensated Hydrostatic Transmission

The promise of some significant improvement in overall efficiency in the simultaneously compensated transmission relative to the sequentially compensated method did not materialize. The overall efficiencies were similar, if not the same. There is a different shape to the Lost Power curve that suggests that the simultaneous method could produce energy savings in an application that is biased toward the mid-range speeds. To draw that conclusion requires an application histogram that differs from those thus far shown in the foregoing example problems.

A concern with simultaneous compensation is the rise in both the motor output power and the pump input power in the region of compensation. It begs the questions, “Where does the maximum power occur and what parameters affect it”, and “By how much does the peak exceed the targeted output power at **POINTS A** and **B**”? With the mathematical models for the pump and motor, it’s possible to derive answers to these questions and furthermore, it’s relatively straightforward. In fact, it will be done in the next section.

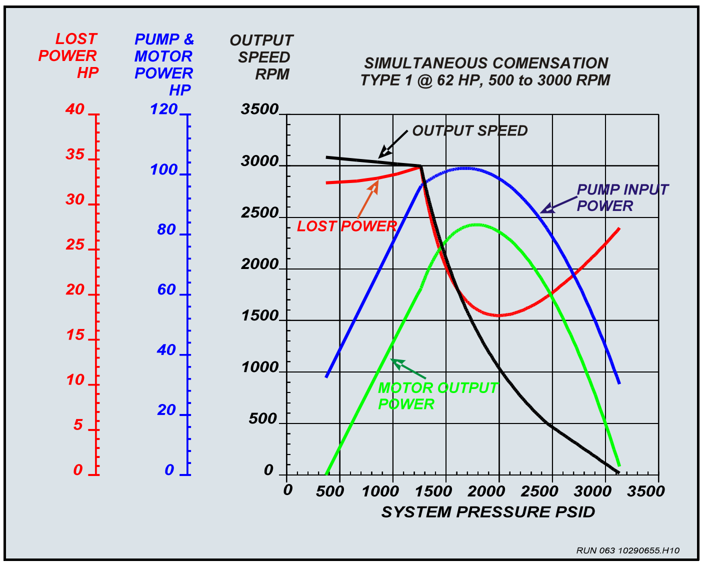

Figure 15 Lost Power, Input and Output Powers and Output Speed are plotted against Pressure in order to present the data in a different way for Example Problem #?. **8M6**

DEALING WITH THE RISE IN PUMP POWER WITH SIMULTANEOUS COMPENSATION

The Problem

The previous discussion demonstrates clearly that the output power in the simultaneously compensated HST is NOT constant, and furthermore, the input power is likewise NOT constant. But it also points the larger issue, namely, it is not the output power that it is necessary to hold to a limit, but rather the input power. Only by limiting input power to the transmission can prime mover lugging be controlled. This begs for a change in design/sizing strategy.

It is not difficult to see that there needs to be a strategy that will provide the requisite power to perform the load handling duties of the HST, but there must also be a strategy to limit input power at the same time. Toward that end, the following development was undertaken in order to determine the conditions under which the HST input power maximizes, and then it proceeds to determine the amount by which the power exceeds the power at the key design targets of **Point A** and **Point B**. Because both the pump power and torque have the same values at both points, it is necessary to analyze only one of them, that is, the torque. Be awae that the pump input power can be determined merely by multiplying the pump input torque by the pump input speed, (N_P/K_U), which is assumed to constant throughout this entire analysis. Also, we can evaluate the torque rise relative to either **Point A** or **Point B**. The latter will be used in the development that follows.

Pressure that Maximizes Pump Torque, TYPE 1, Simultaneously Compensated HST

The brief outline of the process is to write the equation for the pump Torque, T_P, in the hyperbolic region where both compensators are active, then differentiate the expression with respect to the pressure, P, then solve for the pressure, P_{XTQP}, that causes the peak torque. We start with Equation 1:

$$T_P = \frac{D_P}{2\pi} P + R_{fP} N_P \qquad \mathbf{(101)}\text{8K+}$$

But the pump compensator is active, so the pump displacement is dependent upon the pressure and the compensator adjustments, first given in Equation 24, and repeated here for convenience:

$$D_P = D_{0P} + \gamma_P P \qquad \mathbf{(102)}\text{8K=}$$

After substituting Equation 102 into Equation 101, and expanding a bit:

$$T_P = \frac{D_{0P} P}{2\pi} + \frac{\gamma_P P^2}{2\pi} + R_{fP} N_P \qquad \mathbf{(103)}\text{8LA}$$

The equation is ready to be differentiated, recognizing that the speed, N_P, is constant, and in the **TYPE 1** model, R_{fP} is assumed to be constant, as are D_{0P}, γ_P, and, of course, π. The only variables are the torque, T_P, and pressure, P, so, differentiating, we get:

$$\frac{dT_P}{dP} = \frac{D_{0P}}{2\pi} + \frac{\gamma_P P}{\pi} \qquad \mathbf{(104)}\text{8LB}$$

To calculate the pressure that maximizes the torque, P_{XTQP}, we set Equation 104 equal to zero and solve for that pressure:

$$0 = \frac{D_{0P}}{2\pi} + \frac{\gamma_P P_{XTQP}}{\pi} \qquad \mathbf{(105)}\text{8LC}$$

Now, solving for the pressure that creates maximum pump torque:

$$P_{XTQP} = -\frac{D_{0P}}{2\gamma_P} \qquad \mathbf{(106)}\text{8LD}$$

Recalling that γ_P evaluates to a negative number in normal circumstances. Equation 106 is useful in the form shown, however, it is perhaps more revealing if substitutions are made for D_{0P} and γ_P. Specifically, they were defined earlier, and repeated here for convenience, first, Equation 26:

$$D_{0P} = \frac{D_{XP} P_{XP}}{P_{XP} - P_{CP}} \qquad \mathbf{(107)}\text{8LE}$$

And now, γ_P, which was described in Equation 25:

$$\gamma_P = \frac{\Delta D_P}{\Delta P} = \frac{D_{XP}}{P_{CP} - P_{XP}} \qquad \mathbf{(108)}\text{8LF}$$

After substituting Equations 107 and 108, Equation 106 simplifies to:

$$P_{XTQP} = \frac{P_{XP}}{2} \qquad \textbf{(109)}_{\textbf{8LG}}$$

Which is worth remembering if one is inclined to design a simultaneously compensated hydrostatic transmission. It is also worth remembering, if only for its profound simplicity.

Pump Torque at the Peak Value, TYPE 1, Simultaneously Compensated HST

The pressure of Equation 109 will produce the peak value of pump input torque, T_{XP}. It can be substituted into Equation 103 to calculate the maximum pump input torque:

$$T_{XP} = \frac{D_{0P}}{2\pi}\frac{P_{XP}}{2} + \frac{\gamma_P}{2\pi}\frac{P_{XP}^2}{4} + R_{fP}N_P \qquad \textbf{(110)}_{\textbf{8LH}}$$

Of course, Equation 110 is useful as is, however, Equations 107 and 108 can be substituted into Equation 110 in order to produce a different form:

$$T_{XP} = \frac{D_{XP}P_{XP}}{2\pi(P_{XP} - P_{CP})}\frac{P_{XP}}{2} \;....$$

$$+ \frac{D_{XP}}{2\pi(P_{CP} - P_{XP})}\frac{P_{XP}^2}{4} + R_{fP}N_P \qquad \textbf{(111)}_{\textbf{8LI}}$$

With some algebra, this reduces to:

$$T_{XP} = \frac{D_{XP}P_{XP}^2}{8\pi(P_{XP} - P_{CP})} + R_{fP}N_P \qquad \textbf{(112)}_{\textbf{8LJ}}$$

Recall that the approach to the design and adjustment of the simultaneously compensated HST is to do so around the two key points on the constant power hyperbola, labeled **Point A** and **Point B**, with the former being the high pressure, high torque point and the latter the lower. This assures that the load receives the required amount of power, however, whether or not the prime mover is over-torqued depends upon the amount of torque and power needed by the pump at its peak power and torque point. Equation 112 calculates that peak torque, and by multiplying by the pump speed, the pump input power can be calculated. Now, that having been said, it is, arguably, more important to know how much the peak torque exceeds the values at **Point A** and **Point B**. That is, we are now interested in determining the difference between the peak torque and the hyperbolic torque at **Point B**, which is calculated merely by subtracting the torque at **Point B** from that of Equation 112:

$$\Delta T_P = T_{XP} - T_{PB} \qquad \textbf{(113)}_{\textbf{8LK}}$$

Recognizing that at **Point B** the pump is at its maximum displacement and is also at its cracking pressure, then that set of facts and Equation 112 can be imposed on Equation 113, resulting in:

$$\Delta T_P = \frac{D_{XP}P_{XP}^2}{8\pi(P_{XP} - P_{CP})} + R_{fP}N_P \;.....$$

$$- \frac{D_{XP}P_{CP}}{2\pi} - R_{fP}N_P \qquad \textbf{(114)}_{\textbf{8LL}}$$

Immediately we see that the frictional loss terms cancel, and the remaining terms can be reduced to:

$$\Delta T_P = \frac{D_{XP}(P_{XP} - 2P_{CP})^2}{8\pi(P_{XP} - P_{CP})} \qquad \textbf{(115)}_{\textbf{8LM}}$$

It is important to bear in mind that Equation 115 is referenced to the pump INPUT torque at **Point B**, NOT the motor output torque at **Point B**. But Equation 115 is the quantity we need in order to prevent prime mover lugging. Since the pump power at **Point B** includes the effects of all internal HST losses, it is an estimate of the actual amount of power needed by the pump in order to satisfy those internal losses AND provide the requisite power to the load at both design points, **A** and **B**.

Perhaps of greater importance is the realization that Equation 115 suggests another tuning strategy for the pump compensator. That is, it begs the question, "What happens if the pump compensator is tuned so that its cracking pressure, P_{CP} is exactly one-half its saturation pressure, P_{XP}"? Immediately we see that ΔT goes to zero, and indeed, that's what can be made to occur, tuning and adjusting tolerances notwithstanding. The actual speed-torque curve at its maximum can be made to go right through **Point B**! However, **Point A** must necessarily be sacrificed. That is, it is not possible to force the peak to occur at **Point B** and also pass through **Point A**. The latter will be missed. The amount of input power at **Point A** will be less than that at **Point B**.

Calculating Peak Pump Torque Relative to Motor Output Conditions

Equation 115 calculates the amount by which the peak pump input torque exceeds the pump input torque at the equi-power **Points A** and **B**. At system design time, it can be more helpful if the torque is evaluated relative to targeted motor output conditions. That development will be undertaken, now.

We know from Equation 109 that the pressure that causes peak pump torque is simply one-half the pump compensator's saturation pressure, regardless of any volumetric or frictional losses. Now, we will calculate the peak pump torque relative to the motor output conditions at **Points A** and **B**. Either point will suffice, let it be **Point B**. The conditions that exist at **Point B** are:

1. The pressure is the cracking pressure of the motor, P_{CM}.
2. Motor displacement is at its minimum value, D_{NM}.
3. The pump is at its maximum displacement, D_{XP}.

To emphasize the relativity to conditions at **POINT B**, the torque will be designated as $T_{XP/B}$. We can relate the maximum pump torque to the pump torque at **Point B** plus the ΔT_P from Equation 115, that is:

$$T_{XP/B} = T_{PB} + \Delta T_P \qquad \textbf{(116)}_{\textbf{8LY}}$$

Where it is known from the transmission equations that:

$$T_{PB} = R_{fP} + \frac{D_{XP}}{2\pi} P_{CM} \qquad \textbf{(117)}_{8LZ}$$

Now Equations 115 and 117 can be substituted into Equation 116, (producing the amount by which pump torque at its peak exceeds pump torque at **POINT B** conditions) yielding:

$$T_{XP/B} = R_{fP} N_P + \frac{D_{XP} P_{CM}}{2\pi} \;.... + \frac{D_{XP}(P_{XP} - 2P_{CP})^2}{8\pi(P_{XP} - P_{CP})} \qquad \textbf{(118)}_{8L1}$$

This equation is useable in its form, above, however, the motor cracking pressure, P_{CM}, can be expressed as a function of the targeted motor output torque at **Point B** by solving for it from the motor output transmission equation . That is:

$$P_{CM} = \frac{2\pi T_{MB}}{D_{NM}} + \frac{2\pi R_{fM} N_{MB}}{D_{NM}} \qquad \textbf{(119)}_{8L2}$$

This can be substituted into Equation 118:

$$T_{XP} = R_{fP} N_P + \frac{D_{XP} T_{MB}}{D_{NM}} + \frac{D_{XP} R_{fM} N_{MB}}{D_{NM}} \;..... + \frac{D_{XP}(P_{XP} - 2P_{CP})^2}{8\pi(P_{XP} - P_{CP})} \qquad \textbf{(120)}_{8L3}$$

Again, this equation is useful in its present form, however, it may be desirable to express the motor torque, T_{MB}, in terms of the hyperbola power at **Point B** which is:

$$T_{XP} = R_{fP} N_P + \frac{D_{XP}}{D_{NM}} \frac{W_H K_U}{N_{MB}} \;..... + \frac{D_{XP} R_{fM} N_{MB}}{D_{NM}} + \frac{D_{XP}(P_{XP} - 2P_{CP})^2}{8\pi(P_{XP} - P_{CP})} \qquad \textbf{(121)}_{8L4}$$

where it should be recalled that $K_U = 63024$ when power is in horsepower.

CONCLUSIONS

About Linear Pump, Motor and HST Models

Linear models are userful tools at the system concept stage because they yield a great deal of information with a minimal effort. In the vernacular of the day, "They give the most bang for the buck", and are a powerful means of conducting "what if" and feasibility studies at concept time.

Linear models can provide useful conclusions regarding system performance, especially when considering the feasibility of a proposed control method. It is this writer's experience(admittedly anecdotal) that if control cannot be effectively applied to the linearized models, one should not expect to get lucky when the real hardware non-linearities are in play in the laboratory or application.

The linear models used in this paper are based upon the published performance characteristics of the modeled pumps and motors and do not rely at all upon interior dimensional and other proprietary data.

Because the models used in this paper are based upon measured performance, they are useful for those who are required to apply them.

Sizing of the HST components is problematic because it is necessary to know the efficiencies of the pump and motor before they have been selected. This results in a selection/sizing process that is necessarily trial-and-error, however, convergence on viable components usually requires no more than two or three iterations.

Linear models for the pump and motor, because of mathematical tractability, allow the key steady-state performance points to be predicted at design time so that the need for extensive trial-and-error searches for suitable settings, during the simulation phase, is essentially eliminated. Although many of the sizing and tuning equations are non-linear, they are all solvable by straightforward algebraic means. Equations for key parameter values are derived and presented in this paper.

About Constant Power Hydrostatic Transmissions

Hydrostatic transmissions with pressure compensated motors are touted as delivering constant output power, however, the results of this study lead to a contrary conclusion except in the most ideal circumstances.

The sequentially compensated hydrostatic transmission, in spite of not delivering constant output power, can be adjusted to require constant input power, which is essential to provide anti-lugging of the prime mover.

The simultaneously compensated hydrostatic transmission, at first postulated to be more efficient than the sequentially compensated configuration because neither machine has to undergo large displacement changes, is proven to be untrue. Both machines have approximately the same overall efficiency when designed for the same load goals.

The results of comparing the sequentially compensated HST to the simultaneously compensated HST showed that the hoped-for improvement in efficiency was not achieved. This points out the usefulness of the models, because it is better to learn this before committing to hardware than after.

The simultaneously compensated hydrostatic transmission can be designed and adjusted to meet two points on the constant power hyperbola, and in this respect, is more flexible than is the sequentially compensated HST.

The simultaneously compensated hydrostatic transmission does not deliver either constant input or output power. There is a significant peaking of the input power that must be taken into account when anti-lugging capability is the goal.

The simultaneously compensated hydrostatic transmission reaches peak input power when the system pressure is one-half the pump's saturation pressure, but the amount of the power rise is predictable using the equations presented in this paper, and therefore can be taken into account at design/selection time.

The Power Loss graph, along with a histogram of the Probability of a Speed Occurring, is useful in determining the total energy lost in an application.

RECOMMENDATIONS

All the models in this paper used the unlikely efficiency values of either all being set to 100% or all being set to 97%. More studies should be done using unbalances between volumetric and mechanical efficiencies as well as unbalances between the pump and motor efficiencies, which would be useful in drawing conclusions about power lost and energy lost in the application.

All studies in this paper used simple, linear spring-area-pressure compensation methods. Other control configurations should be investigated, especially methods that show promise of delivering either constant input or output power.

This paper has confined itself to steady-state performance studies only. Dynamic models should be developed so that transient performance and stability can be determined.

Laboratory studies are needed to evaluate the degree to which the models predict actual transmission performance.

ACKNOWLEDGEMENTS

I want to thank Prof Ron Jorgensen, PhD, Math Department, MSOE, for his very valuable help in formulating the histograms for the evaluation of total lost energy.

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