

**Truth Definitions:**

$\neg\mathbf{T}$ : A negation of the form  $\lceil\neg A\rceil$  is true in an interpretation  $\mathcal{I}$  just in case  $A$  is false in  $\mathcal{I}$ .

$\wedge\mathbf{T}$ : A conjunction of the form  $\lceil(A \wedge B)\rceil$  is true in an interpretation  $\mathcal{I}$  just in case  $A$  is true in  $\mathcal{I}$  and  $B$  is true in  $\mathcal{I}$ .

$\vee\mathbf{T}$ : A disjunction of the form  $\lceil(A \vee B)\rceil$  is true in an interpretation  $\mathcal{I}$  just in case either  $A$  is true in  $\mathcal{I}$  or  $B$  is true in  $\mathcal{I}$ .

$\rightarrow\mathbf{T}$ : A material conditional of the form  $\lceil(A \rightarrow B)\rceil$  is true in an interpretation  $\mathcal{I}$  just in case either  $A$  is false in  $\mathcal{I}$  or  $B$  is true in  $\mathcal{I}$ .

$\leftrightarrow\mathbf{T}$ : A material bi-conditional of the form  $\lceil(A \leftrightarrow B)\rceil$  is true in an interpretation  $\mathcal{I}$  just in case either ( $A$  is true in  $\mathcal{I}$  and  $B$  is true in  $\mathcal{I}$ ) or ( $A$  is false in  $\mathcal{I}$  and  $B$  is false in  $\mathcal{I}$ ).

**Derivative Falsity Definitions:**

$\neg\mathbf{F}$ : A negation of the form  $\lceil\neg A\rceil$  is false in an interpretation  $\mathcal{I}$  just in case  $A$  is true in  $\mathcal{I}$ .

$\wedge\mathbf{F}$ : A conjunction of the form  $\lceil(A \wedge B)\rceil$  is false in an interpretation  $\mathcal{I}$  just in case either  $A$  is false in  $\mathcal{I}$  or  $B$  is false in  $\mathcal{I}$ .

$\vee\mathbf{F}$ : A disjunction of the form  $\lceil(A \vee B)\rceil$  is false in an interpretation  $\mathcal{I}$  just in case both  $A$  is false in  $\mathcal{I}$  and  $B$  is false in  $\mathcal{I}$ .

$\rightarrow\mathbf{F}$ : A material conditional of the form  $\lceil(A \rightarrow B)\rceil$  is false in an interpretation  $\mathcal{I}$  just in case  $A$  is true in  $\mathcal{I}$  and  $B$  is false in  $\mathcal{I}$ .

$\leftrightarrow\mathbf{F}$ : A material bi-conditional of the form  $\lceil(A \leftrightarrow B)\rceil$  is false in an interpretation  $\mathcal{I}$  just in case either ( $A$  is true in  $\mathcal{I}$  and  $B$  is false in  $\mathcal{I}$ ) or ( $A$  is false in  $\mathcal{I}$  and  $B$  is true in  $\mathcal{I}$ ).

Basic Inference Rules for  $\mathcal{F}$ 

<b>Hyp</b>	$\left  \begin{array}{l} i. A \\ \hline \end{array} \right $		<b><math>\wedge</math>Elim</b>	$\left  \begin{array}{l} i. A \wedge B \\ \hline j. A/B \end{array} \right $	<b><math>\wedge</math>Elim:</b> $i$
			<b><math>\vee</math>Intro</b>	$\left  \begin{array}{l} i. A/B \\ \hline j. A \vee B \end{array} \right $	<b><math>\vee</math>Intro:</b> $i$
<b><math>\neg</math>Elim</b>	$\triangleright \left  \begin{array}{l} i. \neg\neg A \\ \hline j. A \end{array} \right $	<b><math>\neg</math>Elim:</b> $i$	<b><math>\rightarrow</math>Elim</b>	$\left  \begin{array}{l} i. A \rightarrow B \\ \hline j. A \\ \hline k. B \end{array} \right $	<b><math>\rightarrow</math>Elim:</b> $i, j$
<b><math>\wedge</math>Intro</b>	$\triangleright \left  \begin{array}{l} i. A \\ j. B \\ \hline k. A \wedge B \end{array} \right $	<b><math>\wedge</math>Intro:</b> $i, j$	<b><math>\leftrightarrow</math>Elim</b>	$\left  \begin{array}{l} i. A \leftrightarrow B \\ \hline j. A/B \\ \hline k. B/A \end{array} \right $	<b><math>\leftrightarrow</math>Elim:</b> $i, j$

Conditional Inference Rules for  $\mathcal{F}$ 

<b><math>\rightarrow</math>Intro</b>	$\triangleright \left  \begin{array}{l} i. A \\ \hline \vdots \\ j. B \\ \hline k. A \rightarrow B \end{array} \right $	<b><math>\rightarrow</math>Intro:</b> $i, j$	<b><math>\leftrightarrow</math>Intro</b>	$\triangleright \left  \begin{array}{l} i. A \\ \hline \vdots \\ j. B \\ \hline k. B \\ \hline \vdots \\ l. A \end{array} \right $	<b><math>\leftrightarrow</math>Intro:</b> $(i, j), (k, l)$
--------------------------------------	---	--	--	--	--