

**Referee queries on** *Donaldson–Thomas invariants of local elliptic surfaces...*

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### Substantial comments and questions

1. The proofs rely fundamentally on the use of the fpqc topology. This is a technique much of the intended readership of this paper would not be familiar with. It would be important to see a pointer to a source explaining the basic properties of this topology in more detail than it is done in the paper. The only actual reference I can find in the paper is to the “Stacks Project, Tag 0BNH, Ex.50.15.3(5)”, which I have not been able to locate.

This applies especially to “fpqc decent” (p14). The spelling should be corrected, a reference to a clear statement of the fpqc descent theorem should be given, and it should be carefully explained why it applies in the situation at hand.

2. p15 “With these choices, we have the identifications...” - I am confused here. It seems that on the right hand side of these identifications, only combinatorial clusters show up, whereas on the left hand side, there does not seem to be any restriction. Is a step not missing here?

### Suggestions for minor amendments

3. p1 abstract “we get a new derivation of the Katz-Klemm-Vafa formula” - perhaps mention that this is a new derivation for primitive classes.
4. p3 “derivation of the KKV formula, which” should be “derivation of the KKV formula that”; later “general conjecture which...” should also be “general conjecture that...”
5. p4 “Euler characteristics  $e(B)$ , and  $e(S)$ ” - no comma needed here; later “has nice multiplicative properties which allows us” should be “has nice multiplicative properties that allow us”; later “from the ways that embedded points” should be “from the ways in which embedded points”
6. p7, around Theorem 7: I was confused for a time why a curve in class  $[B + dF]$  must have support along  $B$  and a few fibres. This is nicely proved in Lemma 28 but I wonder whether it would make the argument clearer to emphasise this point earlier
7. p7 “partition thickened comb curve”: it is implicit in the notation and discussion that, with respect to a fixed set of local coordinates, the data  $(B, \{x_i\}, \{\lambda_i\})$  defines a unique one-dimensional closed subscheme of  $X$ . This is more-or-less obvious but should be more clearly stated and briefly argued.
8. p28 “straight forward” should be “straightforward”

The referee