REPORT FOR "DONADSON–THOMAS INVARIANTS OF LOCAL ELLIPTIC SURFACES VIA THE TOPOLOGICAL VERTEX" BY BRYAN AND KOOL

This is an interesting and well-written paper. The results seem to be correct as far as I checked. I recommend its publication to Sigma.

This paper consists of two parts. In the first, the authors study Euler numbers of certain moduli spaces associated to elliptic surfaces. By analyzing stratification of the moduli spaces and using a combination of toric and motivic methods, they express these Euler numbers in terms of the topological vertex. Nice product formulas are deduced for the generating series, which are closely connected to Jacobi forms.

Their calculations are motivated by Donaldson–Thomas theory for certain Calabi–Yau 3-folds. The method introduced in this paper also has applications in other geometries, including abelian 3-folds and the product of a K3 surface and an elliptic curve. These results provide the first non-trivial evidence for previously conjectured formulas in the mentioned cases.

In the second part, the authors relate their formulas of Euler numbers to Donaldson–Thomas invariants. They propose a conjecture concerning the Behrend function, which in my opinion may be of independent interest. Assuming this conjecture, they obtain the Donaldson–Thomas invariants from the Euler numbers calculated in the first part.

Here are a few minor comments:

Abstract and Page 3: My understanding of the KKV formula is the expression of curve counting invariants of K3 surfaces in terms of certain Jacobi form. However, the DT calculations in this paper are still conditional, which rely on a (possibly very difficult) conjecture. I think it is a little confusing to claim a new derivation of the KKV formula for primitive classes.

Page 15: The notation "prime of a partition" should be explained when it first appeared in line 5, not in line -1.

Page 20: line 4-5, it would be better to give the reference for the definition of the normalized volume here.

Page 24: Remark 19. For toric Calabi–Yau, do you need any assumption on the curve C (e.g. torus fixed)? Also it would be better to explain a little more how to deduce Conjecture 18 from MNOP (e.g. how to evaluate $\nu([C])$).