Summarizing Data

Computational Mathematics and Statistics

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One Minute Paper Results

What was the most important thing you learned during this class?

What important question remains unanswered for you?

Data Types / Descriptives / Visualizations

Data Type	Descriptive Stats	Visualization
Continuous	mean, median, mode, standard deviation, IQR	histogram, density, box plot
Discrete	contingency table, proportional table, median	bar plot
Categorical	contingency table, proportional table	bar plot
Ordinal	contingency table, proportional table, median	bar plot
Two quantitative	correlation	scatter plot
Two qualitative	contingency table, chi-squared	mosaic plot, bar plot
Quantitative & Qualitative	grouped summaries, ANOVA, t-test	box plot

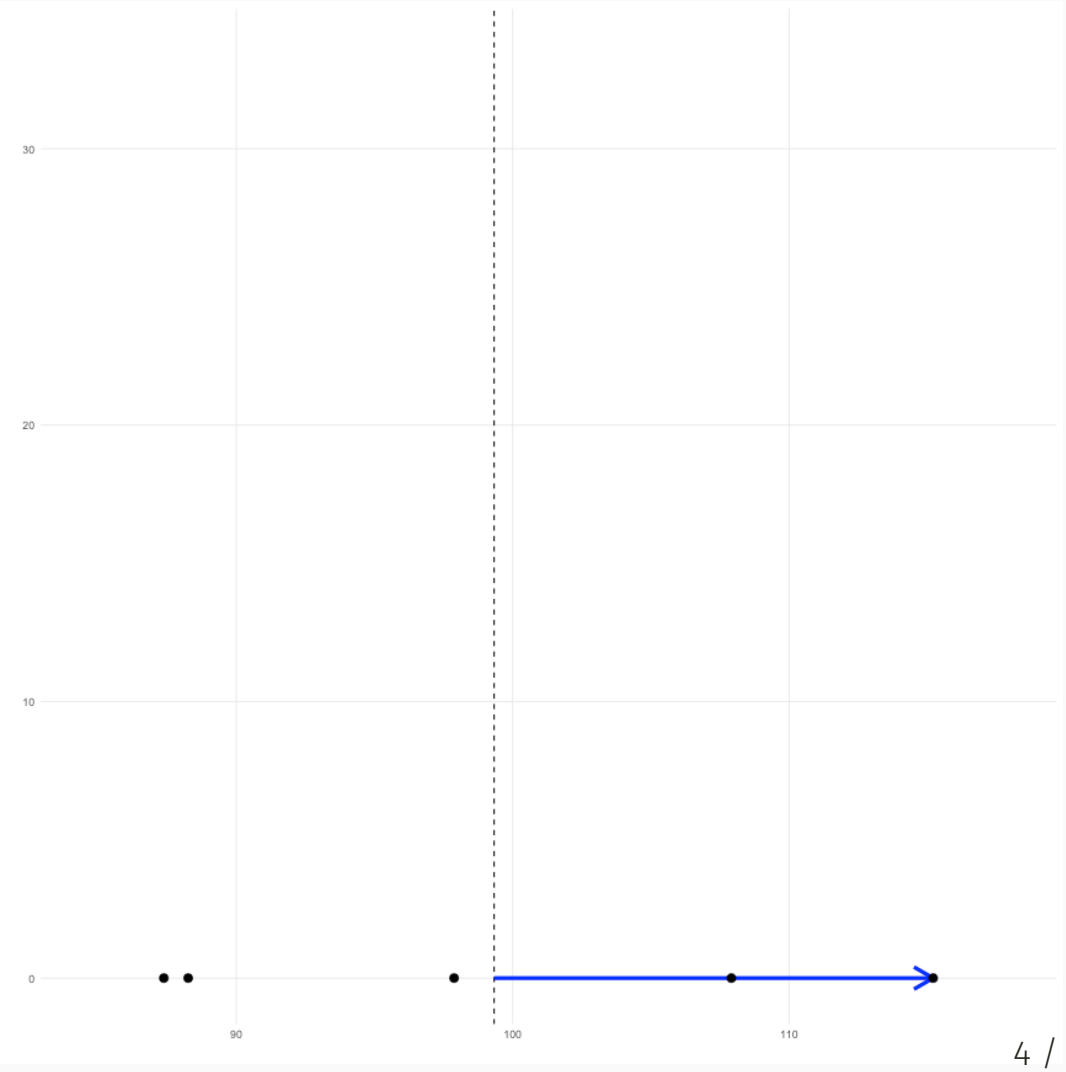
Variance

Population Variance:

$$S^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

Consider a dataset with five values (black points in the figure). For the largest value, the deviance is represented by the blue line ($x_i - \bar{x}$).

See also:
https://shiny.rit.albany.edu/stat/visualizess/
https://github.com/jbryer/VisualStats/

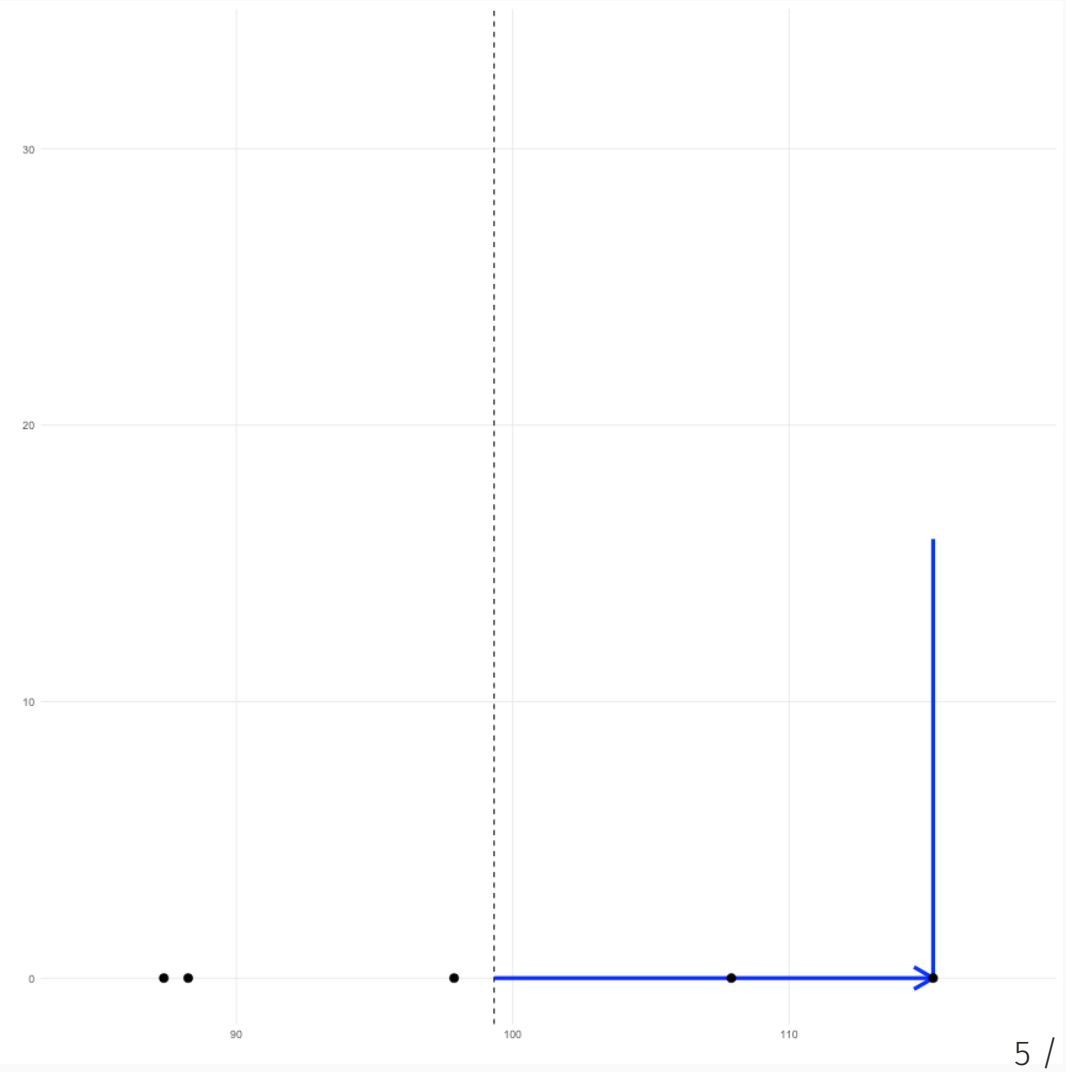


Variance (cont.)

Population Variance:

$$S^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

In the numerator, we square each of these deviances. We can conceptualize this as a square. Here, we add the deviance in the *y* direction.

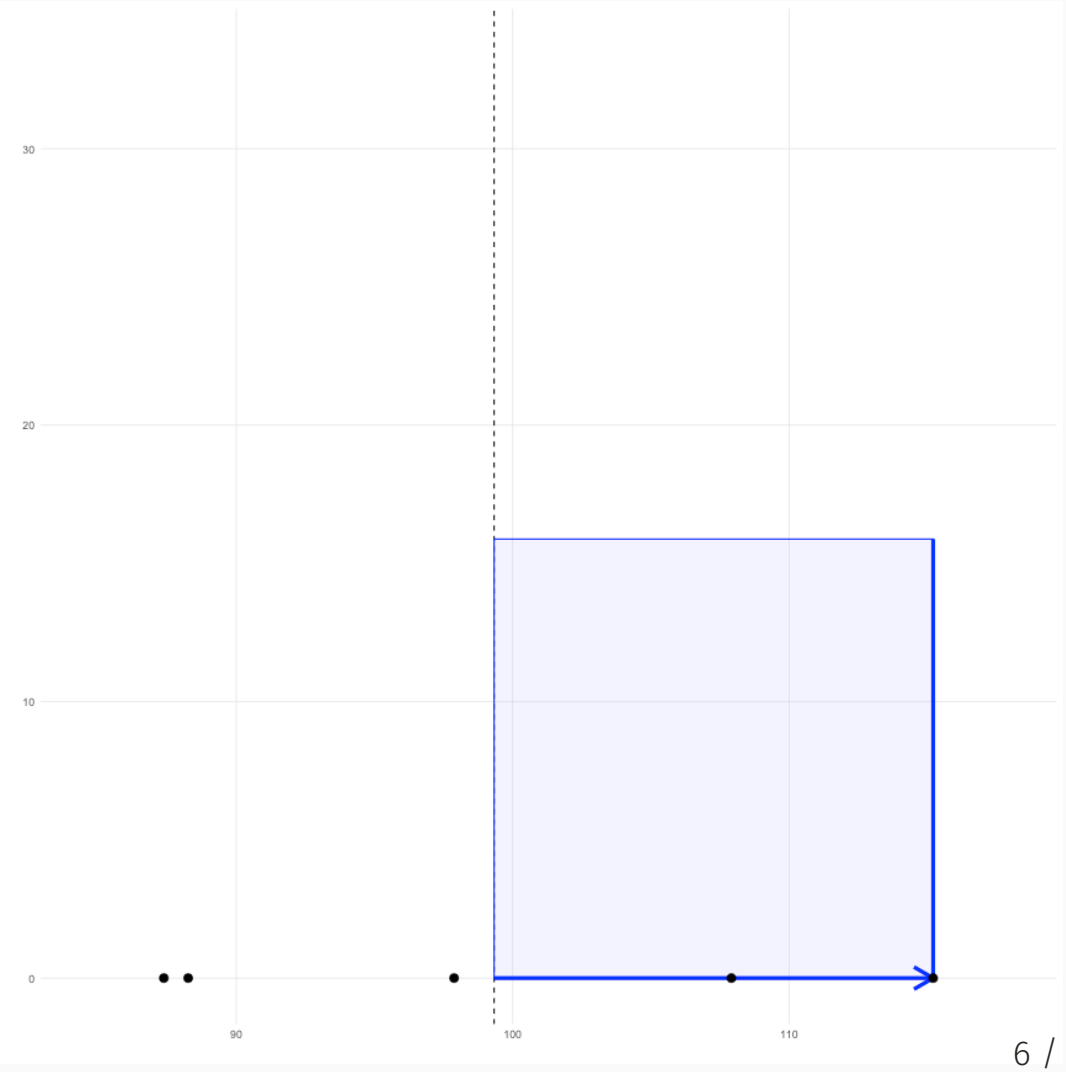


Variance (cont.)

Population Variance:

$$S^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

We end up with a square.

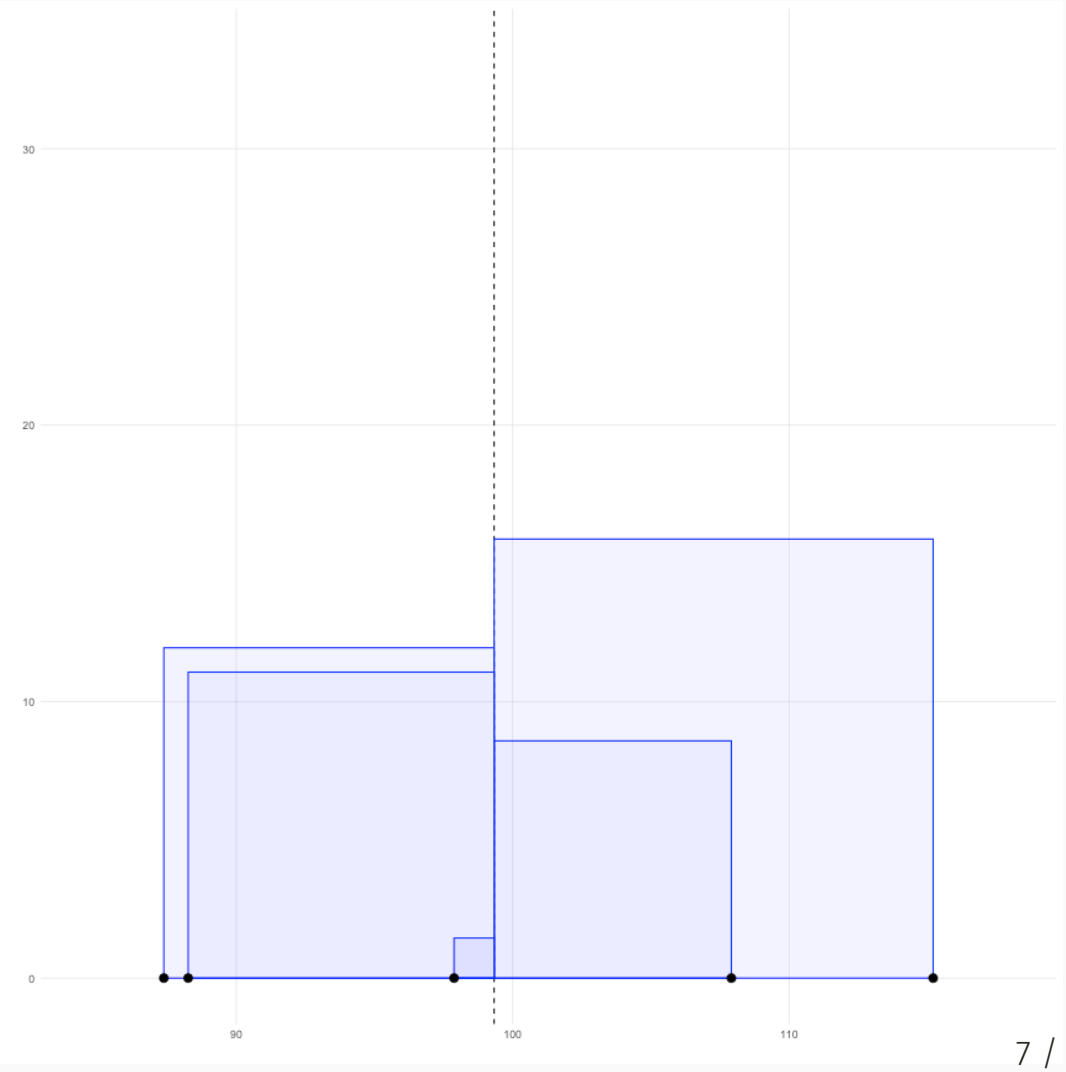


Variance (cont.)

Population Variance:

$$S^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

We can plot the squared deviance for all the data points. That is, each component in the numerator is the area of each of these squares.

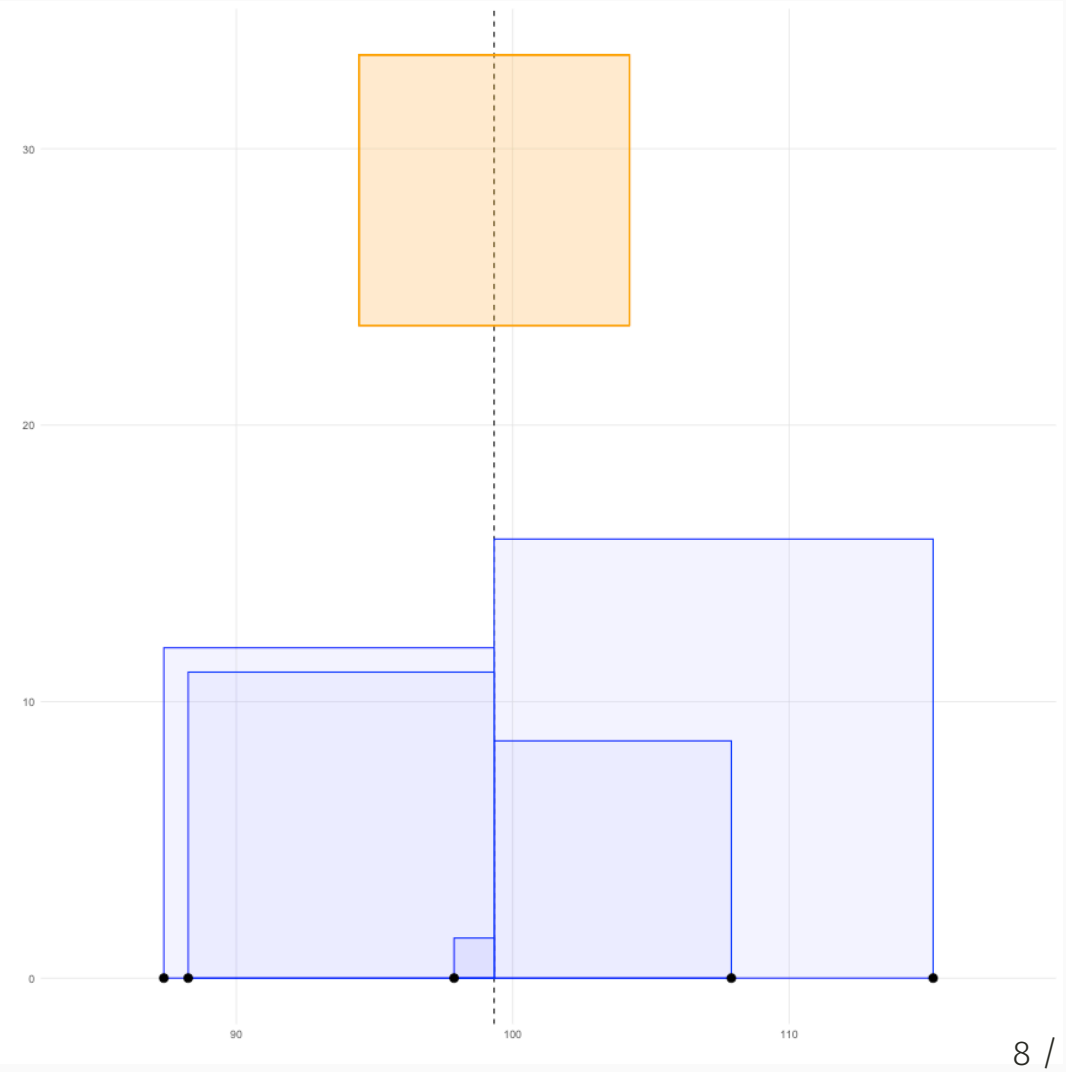


Variance (cont.)

Population Variance:

$$S^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

The variance is therefore the average of the area of all these squares, here represented by the orange square.



Population versus Sample Variance

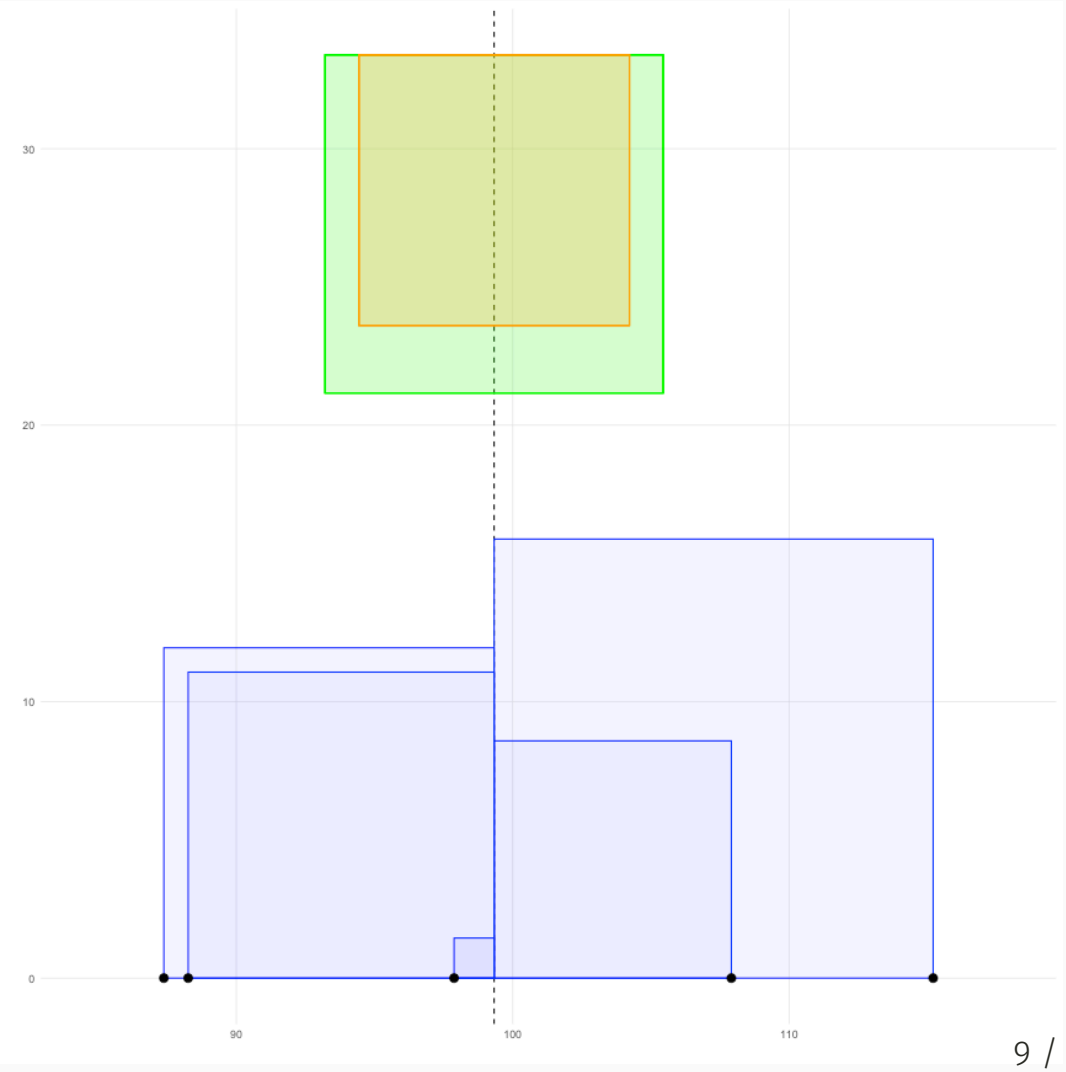
Typically we want the sample variance. The difference is we divide by $n-1$ to calculate the sample variance. This results in a slightly larger area (variance) then if we divide by n.

Population Variance (yellow):

$$S^2 = \frac{\Sigma(x_i - \bar{x})^2}{N}$$

Sample Variance (green):

$$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1}$$



Robust Statistics

Consider the following data randomly selected from the normal distribution:

```
set.seed(41)
x <- rnorm(30, mean = 100, sd = 15)
mean(x); sd(x)
```

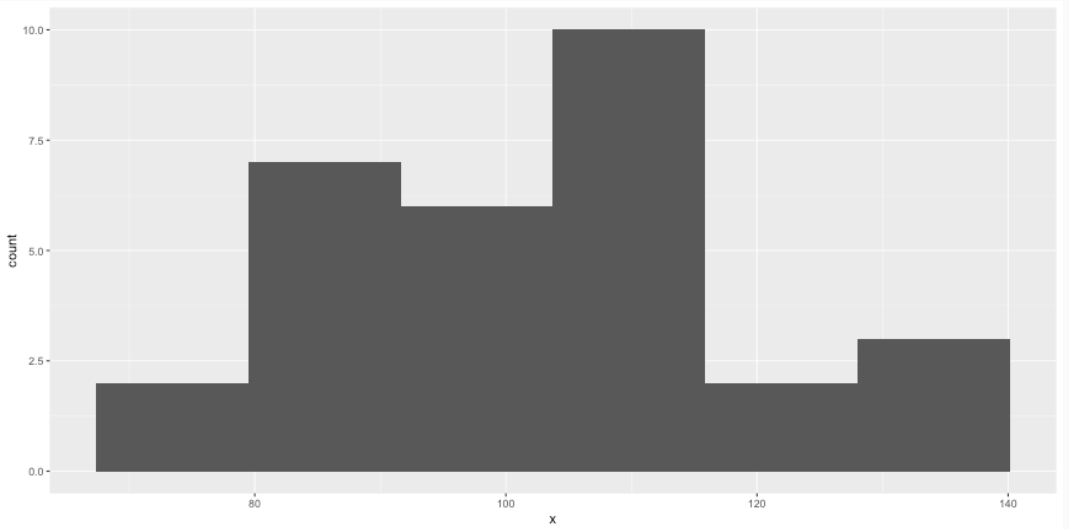
```
## [1] 103.1934
```

```
## [1] 16.8945
```

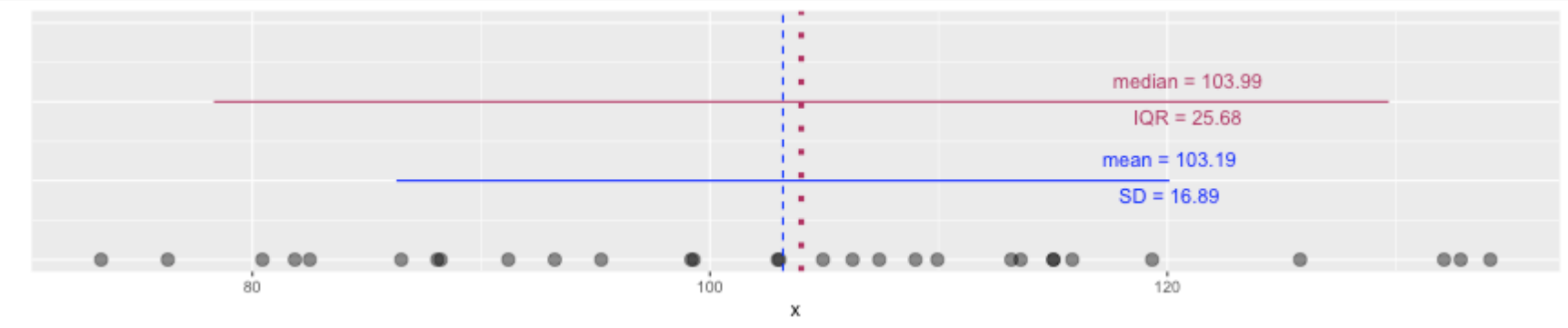
```
median(x); IQR(x)
```

```
## [1] 103.9947
```

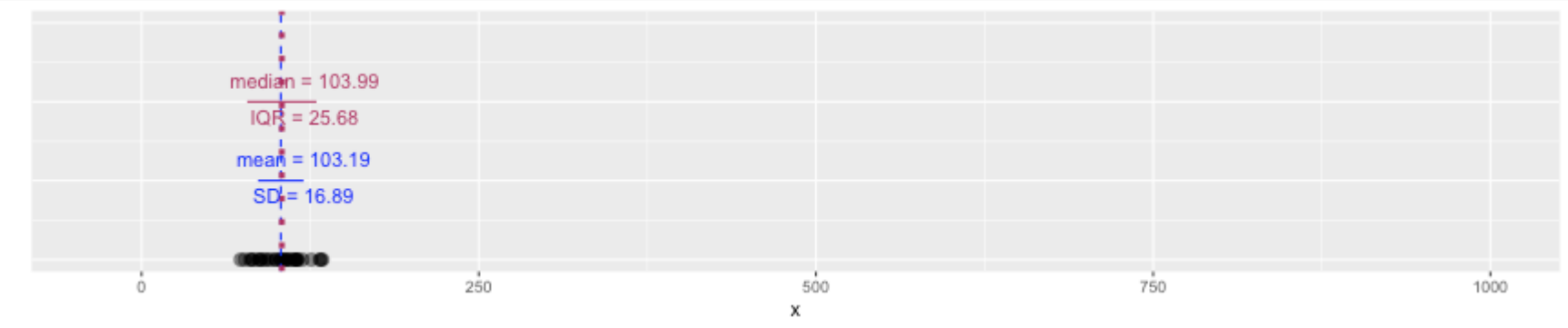
```
## [1] 25.68004
```



Robust Statistics

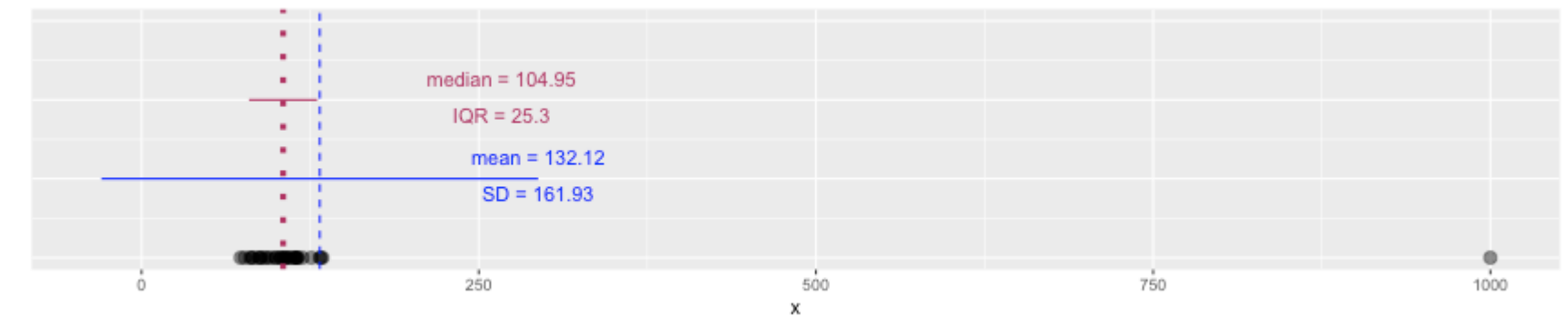


Robust Statistics



Let's add an extreme value:

```
x <- c(x, 1000)
```



Robust Statistics

Median and IQR are more robust to skewness and outliers than mean and SD. Therefore,

- for skewed distributions it is often more helpful to use median and IQR to describe the center and spread
- for symmetric distributions it is often more helpful to use the mean and SD to describe the center and spread

Grammer of Graphics

Data Visualizations with ggplot2

- `ggplot2` is an R package that provides an alternative framework based upon Wilkinson's (2005) Grammar of Graphics.
- `ggplot2` is, in general, more flexible for creating "prettier" and complex plots.
- Works by creating layers of different types of objects/geometries (i.e. bars, points, lines, polygons, etc.) `ggplot2` has at least three ways of creating plots:
    1. `qplot`
    2. `ggplot(...) + geom_XXX(...) + ...`
    3. `ggplot(...) + layer(...)`
- We will focus only on the second.

Parts of a `ggplot2` Statement

- Data

  ```
  ggplot(myDataFrame, aes(x=x, y=y))
  ```

- Layers

  ```
  geom_point(), geom_histogram()
  ```

- Facets

  ```
  facet_wrap(~ cut), facet_grid(~ cut)
  ```

- Scales

  ```
  scale_y_log10()
  ```

- Other options

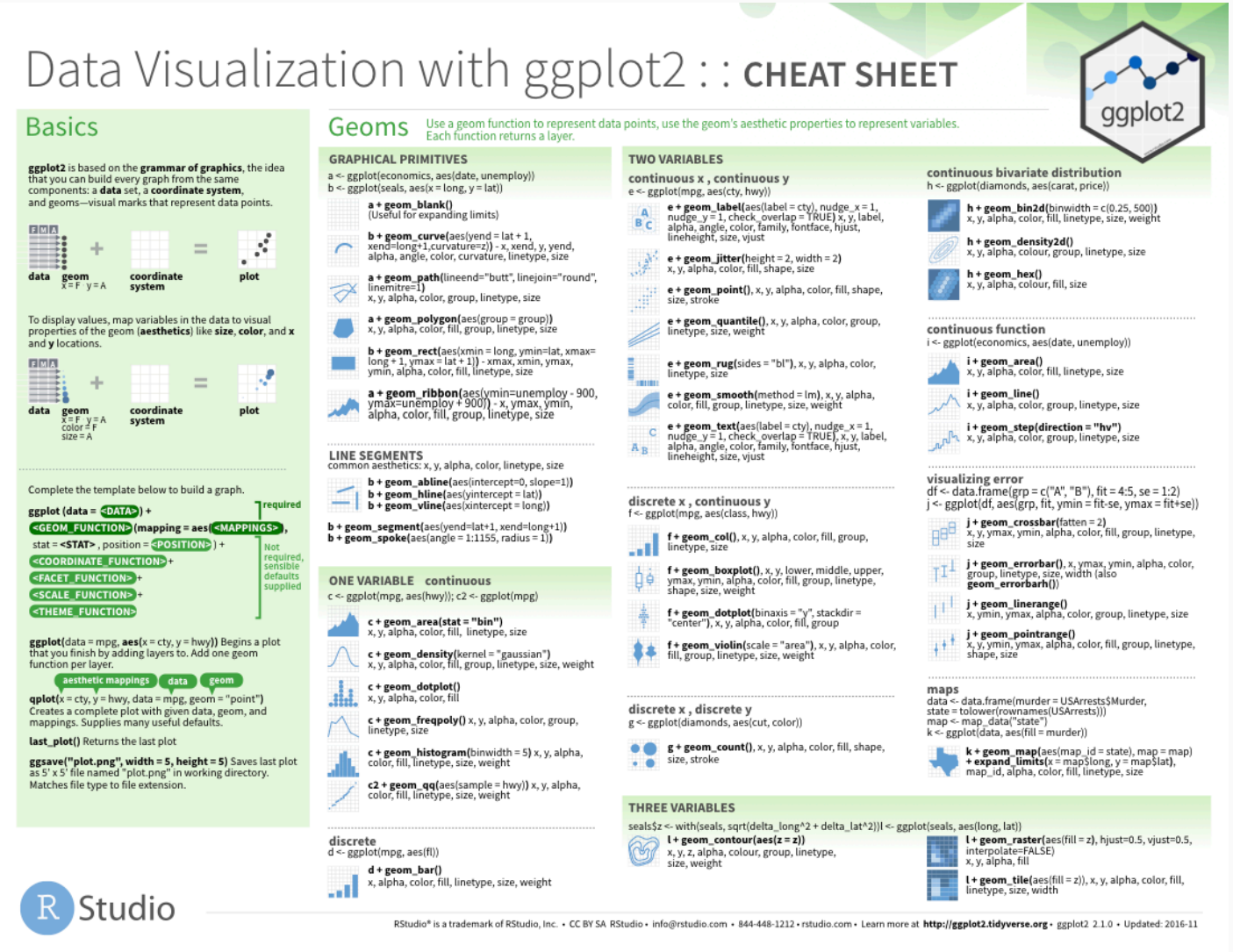
  ```
  ggtitle('my title'), ylim(c(0, 10000)), xlab('x-axis label')
  ```

Lots of geoms

```
ls('package:ggplot2')[grep('^geom_', ls('package:ggplot2'))]
```

```
##  [1] "geom_abline"           "geom_area"             "geom_bar"              "geom_bin_2d"
##  [5] "geom_bin2d"            "geom_blank"            "geom_boxplot"          "geom_col"
##  [9] "geom_contour"          "geom_contour_filled"   "geom_count"            "geom_crossbar"
## [13] "geom_curve"            "geom_density"          "geom_density_2d"       "geom_density_2d_filled"
## [17] "geom_density2d"        "geom_density2d_filled" "geom_dotplot"          "geom_errorbar"
## [21] "geom_errorbarh"        "geom_freqpoly"         "geom_function"         "geom_hex"
## [25] "geom_histogram"        "geom_hline"            "geom_jitter"           "geom_label"
## [29] "geom_line"             "geom_linerange"        "geom_map"              "geom_path"
## [33] "geom_point"            "geom_pointrange"       "geom_polygon"          "geom_qq"
## [37] "geom_qq_line"          "geom_quantile"         "geom_raster"           "geom_rect"
## [41] "geom_ribbon"           "geom_rug"              "geom_segment"          "geom_sf"
## [45] "geom_sf_label"         "geom_sf_text"          "geom_smooth"           "geom_spoke"
## [49] "geom_step"             "geom_text"             "geom_tile"             "geom_violin"
## [53] "geom_vline"
```

Data Visualization Cheat Sheet



Data Visualization with ggplot2 : : CHEAT SHEET

Basics

ggplot2 is based on the **grammar of graphics**, the idea that you can build every graph from the same components: a **data** set, a **coordinate system**, and geoms—visual marks that represent data points.

data | geom (x = F, y = A) | coordinate system | plot

To display values, map variables in the data to visual properties of the geom (**aesthetics**) like **size**, **color**, and **x** and **y** locations.

data | geom (x = F, y = A, color = F, size = A) | coordinate system | plot

Complete the template below to build a graph.

```
ggplot (data = <DATA>) +                                   required
<GEOM_FUNCTION> (mapping = aes(<MAPPINGS>),
  stat = <STAT> , position = <POSITION> ) +                Not required, sensible defaults supplied
<COORDINATE_FUNCTION> +
<FACET_FUNCTION> +
<SCALE_FUNCTION> +
<THEME_FUNCTION>
```

ggplot(data = mpg, **aes**(x = cty, y = hwy)) Begins a plot that you finish by adding layers to. Add one geom function per layer.

aesthetic mappings | data | geom

qplot(x = cty, y = hwy, data = mpg, geom = "point") Creates a complete plot with given data, geom, and mappings. Supplies many useful defaults.

last_plot() Returns the last plot

ggsave("plot.png", width = 5, height = 5) Saves last plot as 5' x 5' file named "plot.png" in working directory. Matches file type to file extension.

Geoms

Use a geom function to represent data points, use the geom's aesthetic properties to represent variables. Each function returns a layer.

GRAPHICAL PRIMITIVES

a <- ggplot(economics, aes(date, unemploy))
b <- ggplot(seals, aes(x = long, y = lat))

a + geom_blank() (Useful for expanding limits)

b + geom_curve(aes(yend = lat + 1, xend=long+1,curvature=z)) - x, xend, y, yend, alpha, angle, color, curvature, linetype, size

a + geom_path(lineend="butt", linejoin="round", linemitre=1) x, y, alpha, color, group, linetype, size

a + geom_polygon(aes(group = group)) x, y, alpha, color, fill, group, linetype, size

b + geom_rect(aes(xmin = long, ymin=lat, xmax= long + 1, ymax = lat + 1)) - xmax, xmin, ymax, ymin, alpha, color, fill, linetype, size

a + geom_ribbon(aes(ymin=unemploy - 900, ymax=unemploy + 900)) - x, ymax, ymin, alpha, color, fill, group, linetype, size

LINE SEGMENTS

common aesthetics: x, y, alpha, color, linetype, size

b + geom_abline(aes(intercept=0, slope=1))
b + geom_hline(aes(yintercept = lat))
b + geom_vline(aes(xintercept = long))

b + geom_segment(aes(yend=lat+1, xend=long+1))
b + geom_spoke(aes(angle = 1:1155, radius = 1))

ONE VARIABLE continuous

c <- ggplot(mpg, aes(hwy)); c2 <- ggplot(mpg)

c + geom_area(stat = "bin") x, y, alpha, color, fill, linetype, size

c + geom_density(kernel = "gaussian") x, y, alpha, color, fill, group, linetype, size, weight

c + geom_dotplot() x, y, alpha, color, fill

c + geom_freqpoly() x, y, alpha, color, group, linetype, size

c + geom_histogram(binwidth = 5) x, y, alpha, color, fill, linetype, size, weight

c2 + geom_qq(aes(sample = hwy)) x, y, alpha, color, fill, linetype, size, weight

discrete

d <- ggplot(mpg, aes(fl))

d + geom_bar() x, alpha, color, fill, linetype, size, weight

TWO VARIABLES

continuous x , continuous y

e <- ggplot(mpg, aes(cty, hwy))

e + geom_label(aes(label = cty), nudge_x = 1, nudge_y = 1, check_overlap = TRUE) x, y, label, alpha, angle, color, family, fontface, hjust, lineheight, size, vjust

e + geom_jitter(height = 2, width = 2) x, y, alpha, color, fill, shape, size

e + geom_point(), x, y, alpha, color, fill, shape, size, stroke

e + geom_quantile(), x, y, alpha, color, group, linetype, size, weight

e + geom_rug(sides = "bl"), x, y, alpha, color, linetype, size

e + geom_smooth(method = lm), x, y, alpha, color, fill, group, linetype, size, weight

e + geom_text(aes(label = cty), nudge_x = 1, nudge_y = 1, check_overlap = TRUE), x, y, label, alpha, angle, color, family, fontface, hjust, lineheight, size, vjust

discrete x , continuous y

f <- ggplot(mpg, aes(class, hwy))

f + geom_col(), x, y, alpha, color, fill, group, linetype, size

f + geom_boxplot(), x, y, lower, middle, upper, ymax, ymin, alpha, color, fill, group, linetype, shape, size, weight

f + geom_dotplot(binaxis = "y", stackdir = "center"), x, y, alpha, color, fill, group

f + geom_violin(scale = "area"), x, y, alpha, color, fill, group, linetype, size, weight

discrete x , discrete y

g <- ggplot(diamonds, aes(cut, color))

g + geom_count(), x, y, alpha, color, fill, shape, size, stroke

continuous bivariate distribution

h <- ggplot(diamonds, aes(carat, price))

h + geom_bin2d(binwidth = c(0.25, 500)) x, y, alpha, color, fill, linetype, size, weight

h + geom_density2d() x, y, alpha, colour, group, linetype, size

h + geom_hex() x, y, alpha, colour, fill, size

continuous function

i <- ggplot(economics, aes(date, unemploy))

i + geom_area() x, y, alpha, color, fill, linetype, size

i + geom_line() x, y, alpha, color, group, linetype, size

i + geom_step(direction = "hv") x, y, alpha, color, group, linetype, size

visualizing error

df <- data.frame(grp = c("A", "B"), fit = 4:5, se = 1:2)
j <- ggplot(df, aes(grp, fit, ymin = fit-se, ymax = fit+se))

j + geom_crossbar(fatten = 2) x, y, ymax, ymin, alpha, color, fill, group, linetype, size

j + geom_errorbar(), x, ymax, ymin, alpha, color, group, linetype, size, width (also **geom_errorbarh()**)

j + geom_linerange() x, ymin, ymax, alpha, color, group, linetype, size

j + geom_pointrange() x, y, ymin, ymax, alpha, color, fill, group, linetype, shape, size

maps

data <- data.frame(murder = USArrests$Murder, state = tolower(rownames(USArrests)))
map <- map_data("state")
k <- ggplot(data, aes(fill = murder))

k + geom_map(aes(map_id = state), map = map) **+ expand_limits**(x = map$long, y = map$lat), map_id, alpha, color, fill, linetype, size

THREE VARIABLES

seals$z <- with(seals, sqrt(delta_long^2 + delta_lat^2))l <- ggplot(seals, aes(long, lat))

l + geom_contour(aes(z = z)) x, y, z, alpha, colour, group, linetype, size, weight

l + geom_raster(aes(fill = z), hjust=0.5, vjust=0.5, interpolate=FALSE) x, y, alpha, fill

l + geom_tile(aes(fill = z)), x, y, alpha, color, fill, linetype, size, width

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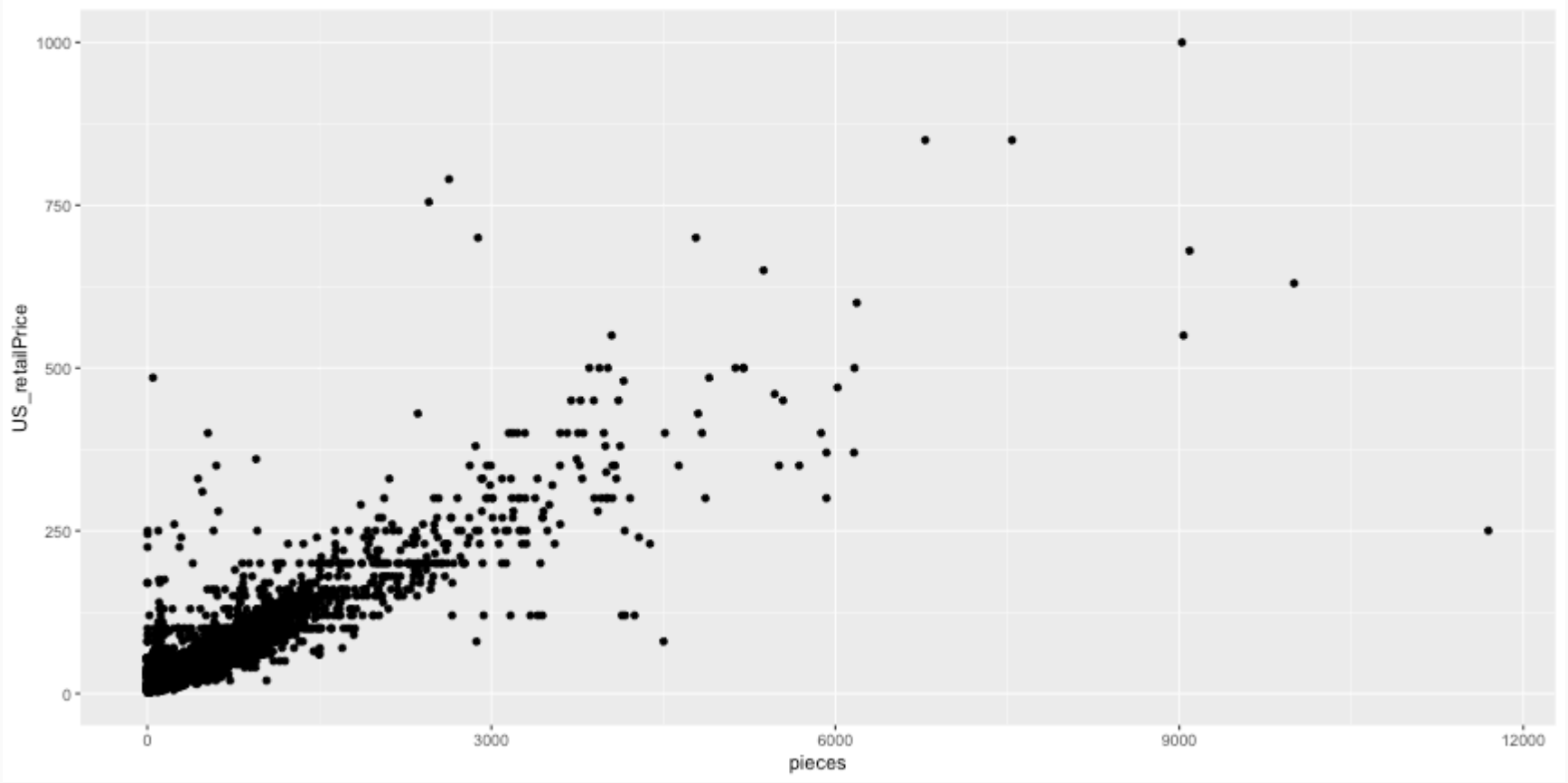
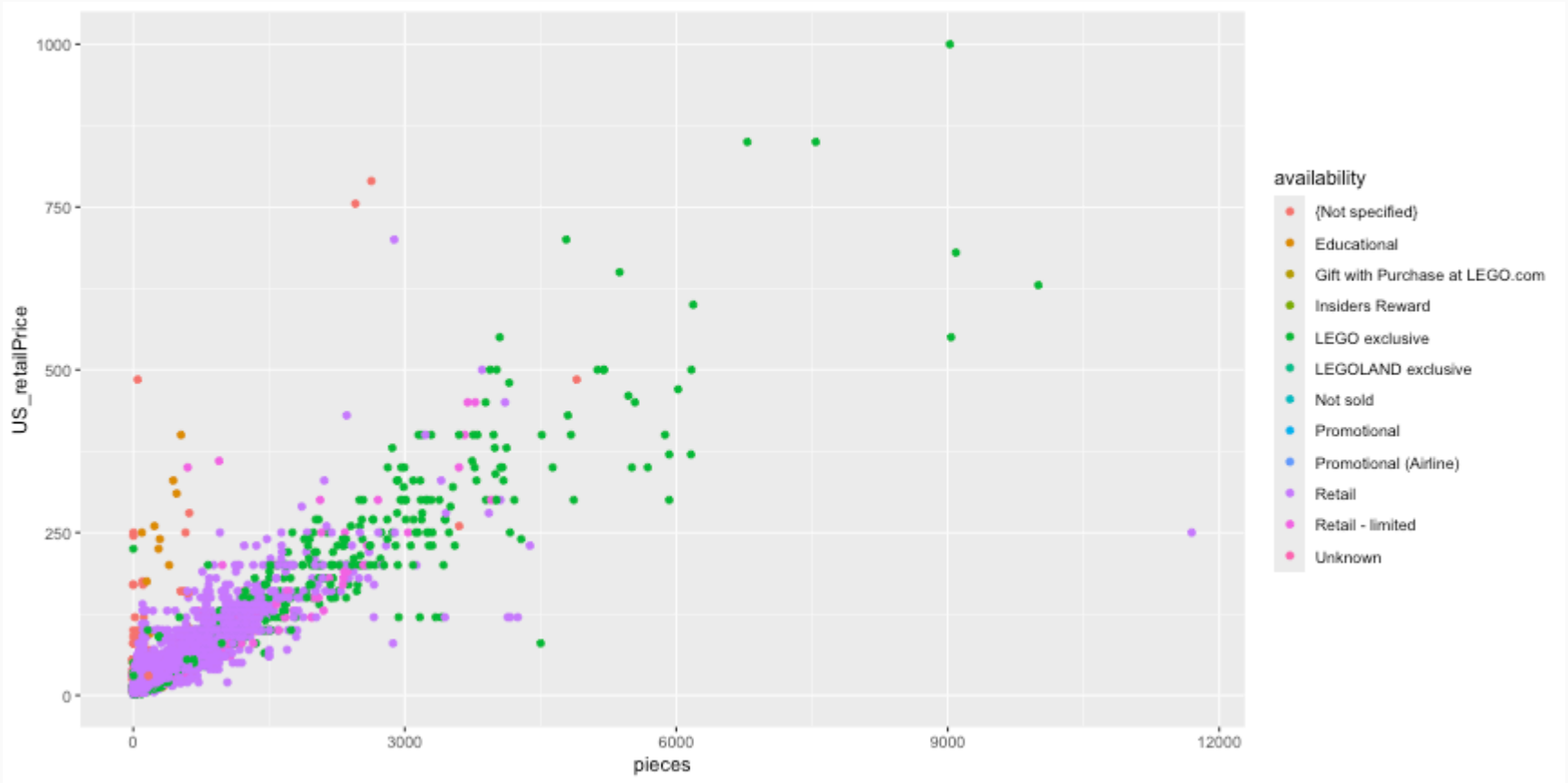
Scatterplot

```
ggplot(legosets, aes(x=pieces, y=US_retailPrice)) + geom_point()
```



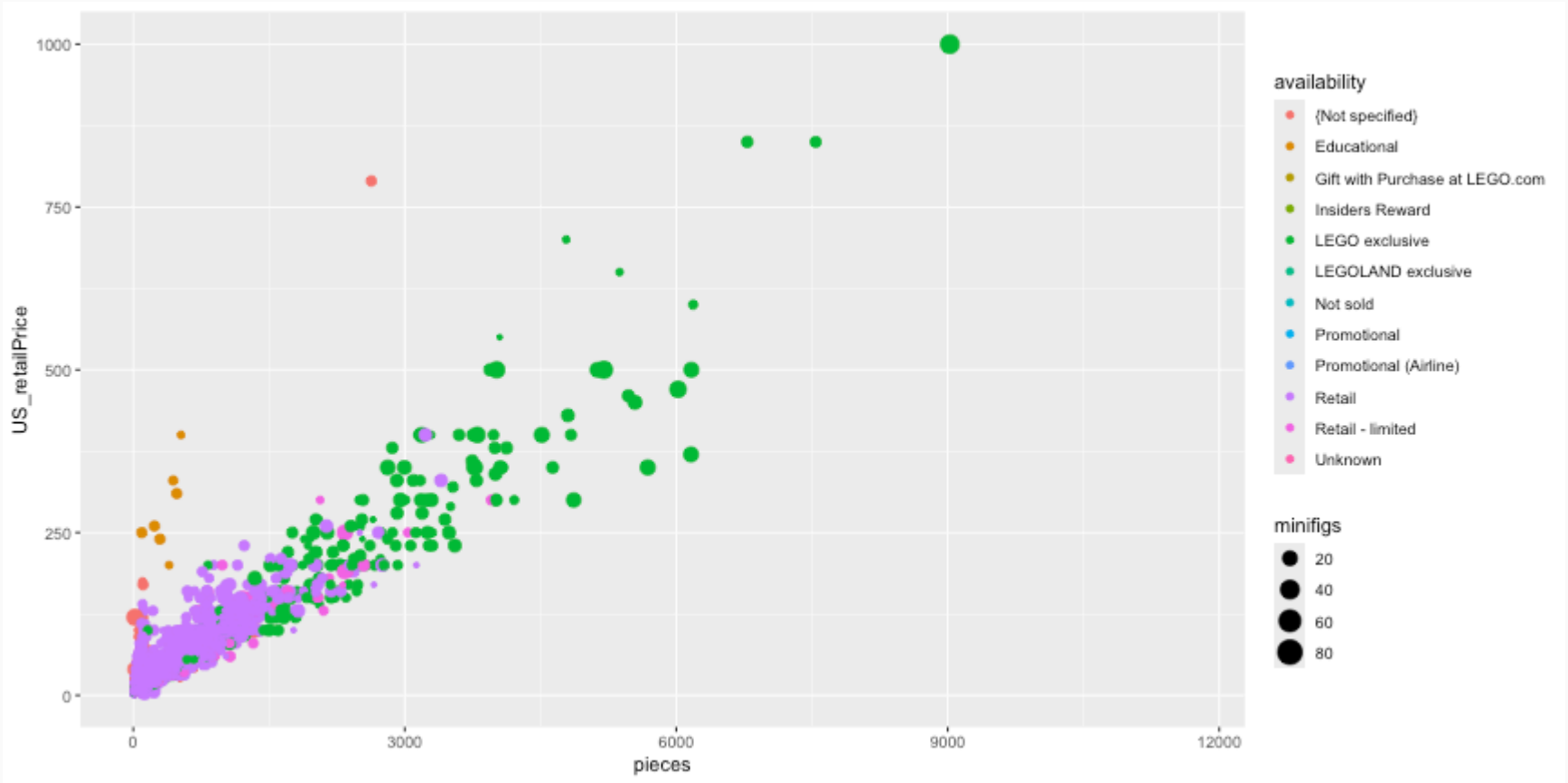
Scatterplot (cont.)

```
ggplot(legosets, aes(x=pieces, y=US_retailPrice, color=availability)) + geom_point()
```



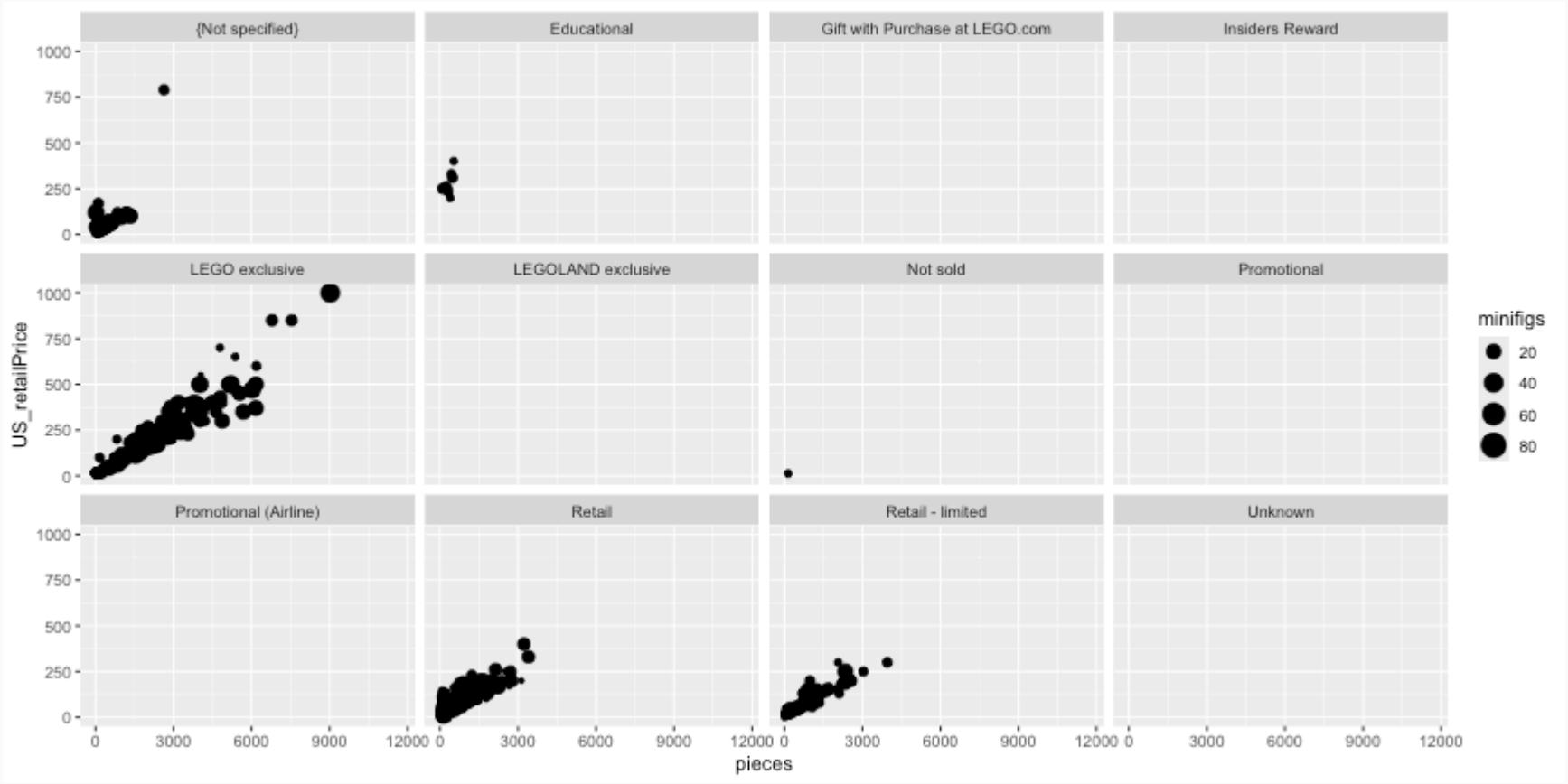
Scatterplot (cont.)

```
ggplot(legosets, aes(x=pieces, y=US_retailPrice, size=minifigs, color=availability)) + geom_point()
```



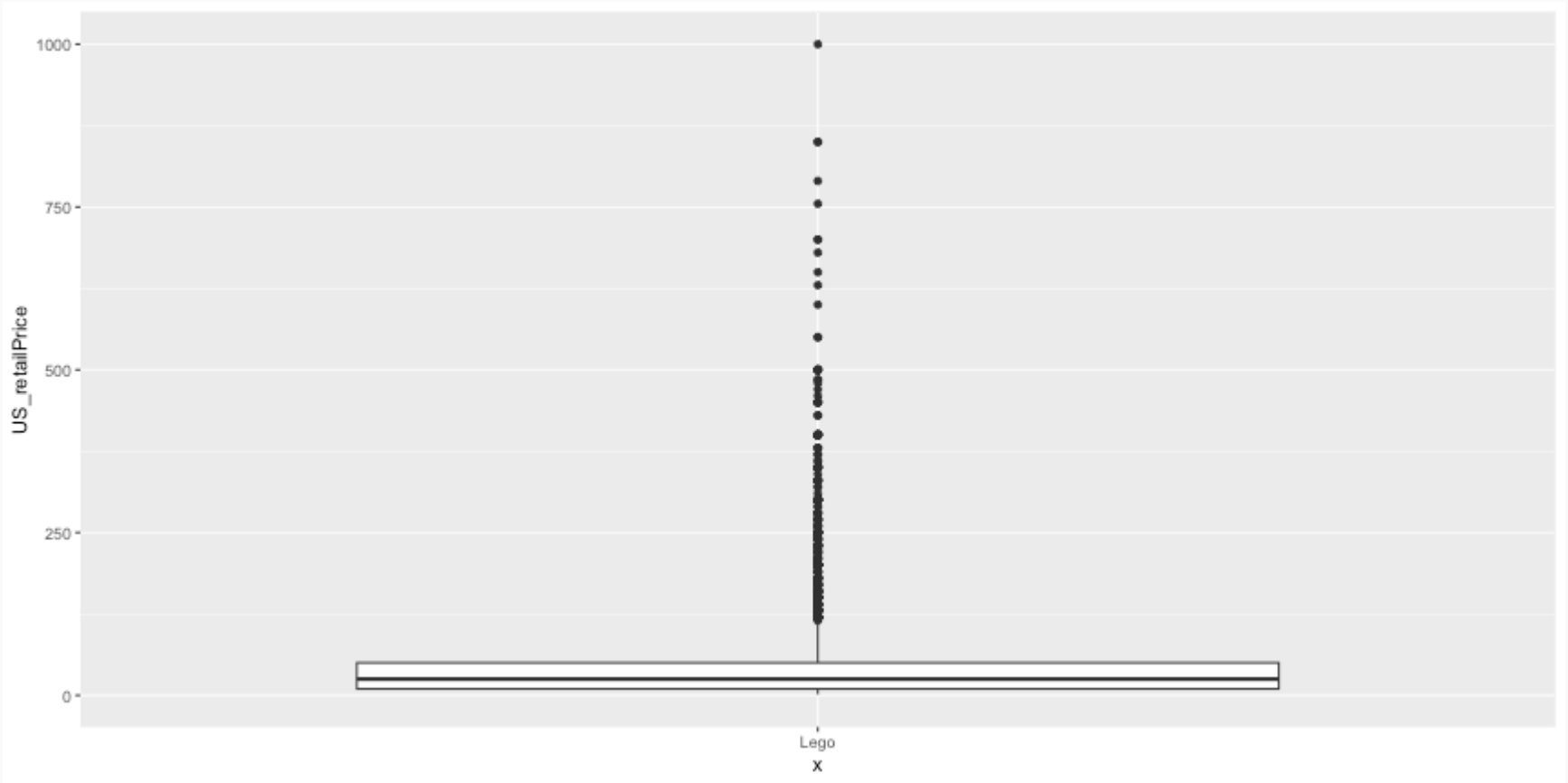
Scatterplot (cont.)

```
ggplot(legosets, aes(x=pieces, y=US_retailPrice, size=minifigs)) + geom_point() + facet_wrap(~ availability)
```



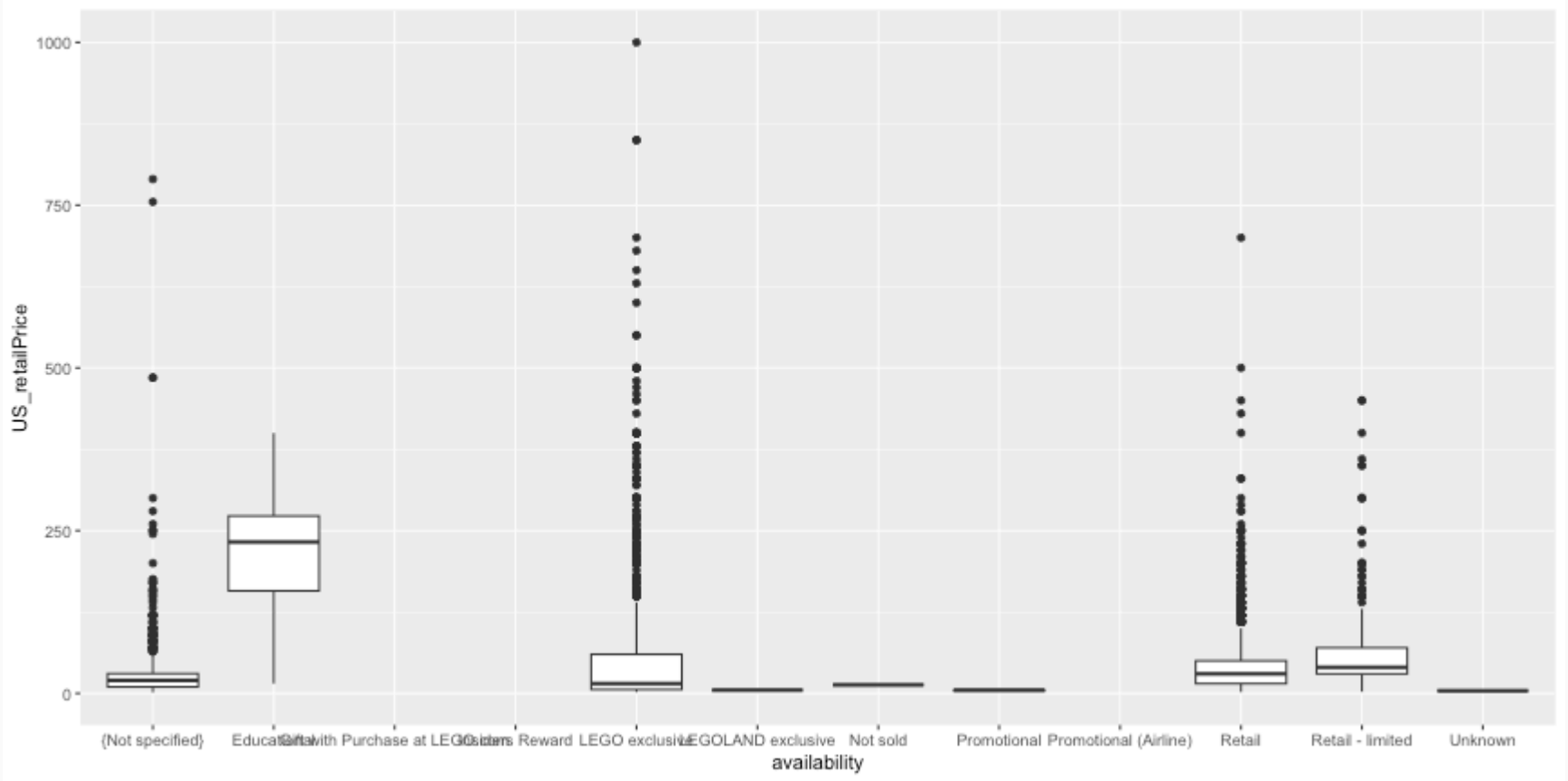
Boxplots

```
ggplot(legosets, aes(x='Lego', y=US_retailPrice)) + geom_boxplot()
```



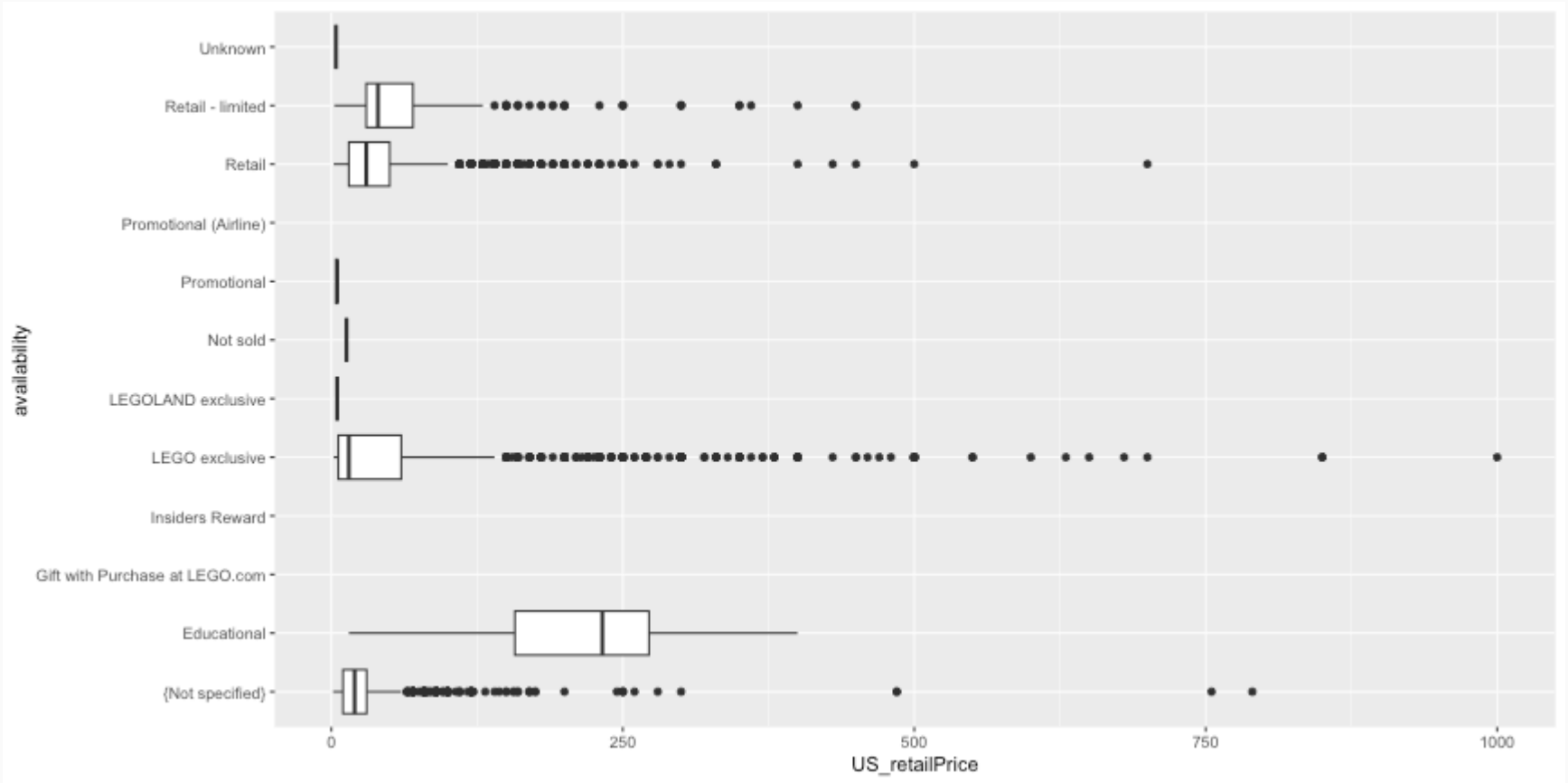
Boxplots (cont.)

```
ggplot(legosets, aes(x=availability, y=US_retailPrice)) + geom_boxplot()
```



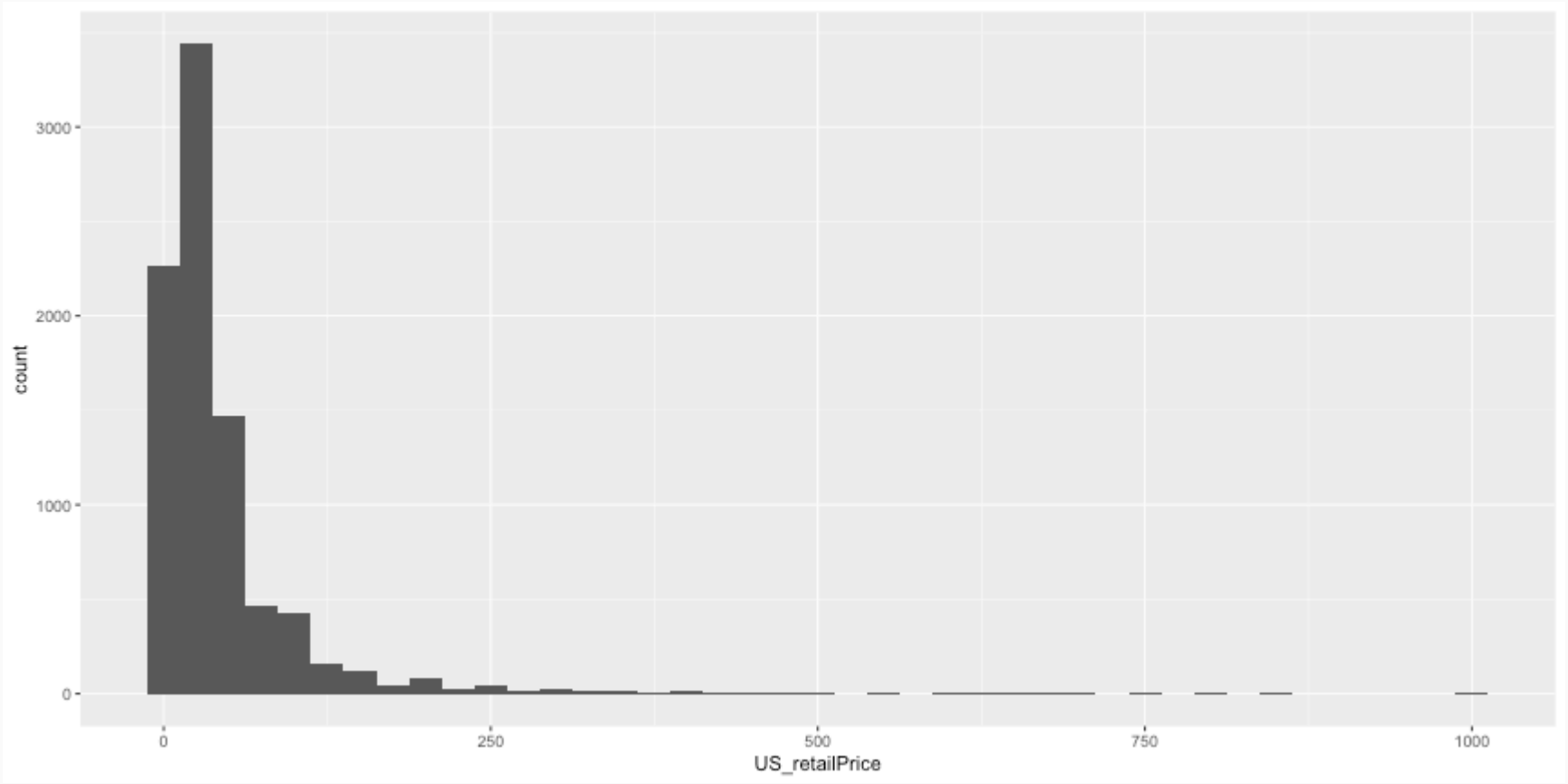
Boxplot (cont.)

```
ggplot(legosets, aes(x=availability, y=US_retailPrice)) + geom_boxplot() + coord_flip()
```



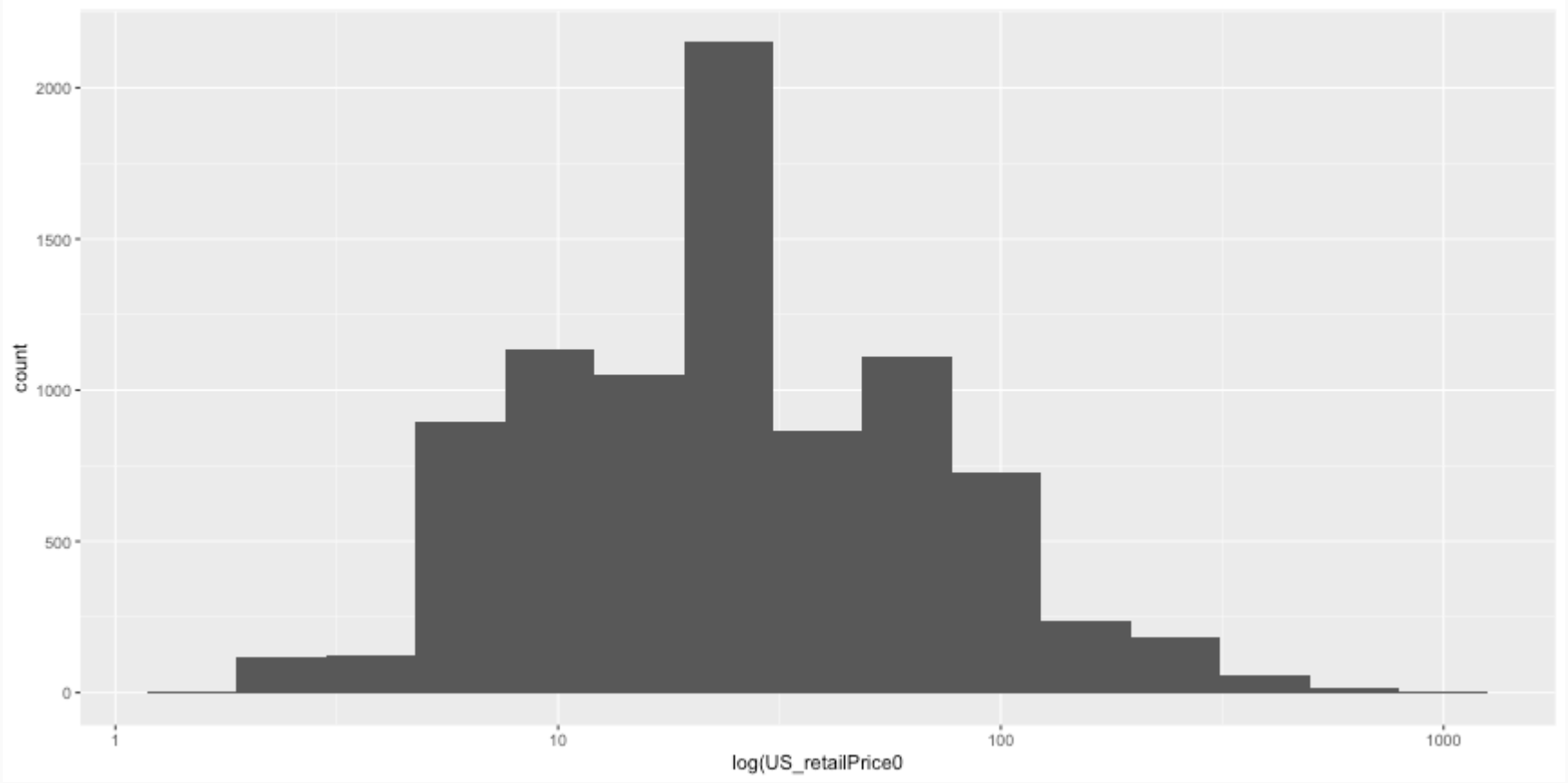
Histograms

```
ggplot(legosets, aes(x = US_retailPrice)) + geom_histogram(binwidth = 25)
```



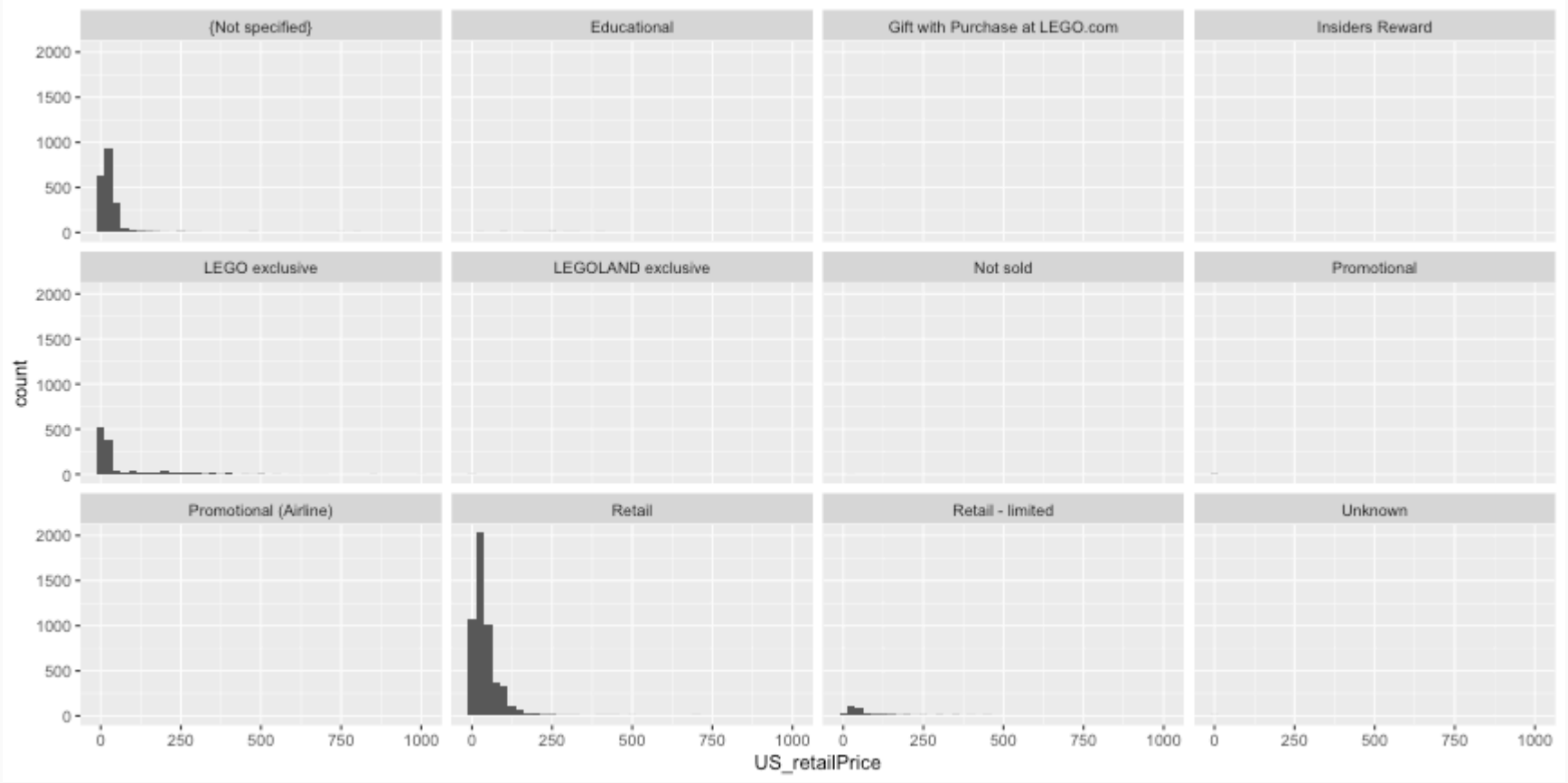
Histograms (cont.)

```
ggplot(legosets, aes(x = US_retailPrice)) + geom_histogram(bins = 15) + scale_x_log10() + xlab('log(US_retailPrice0')
```



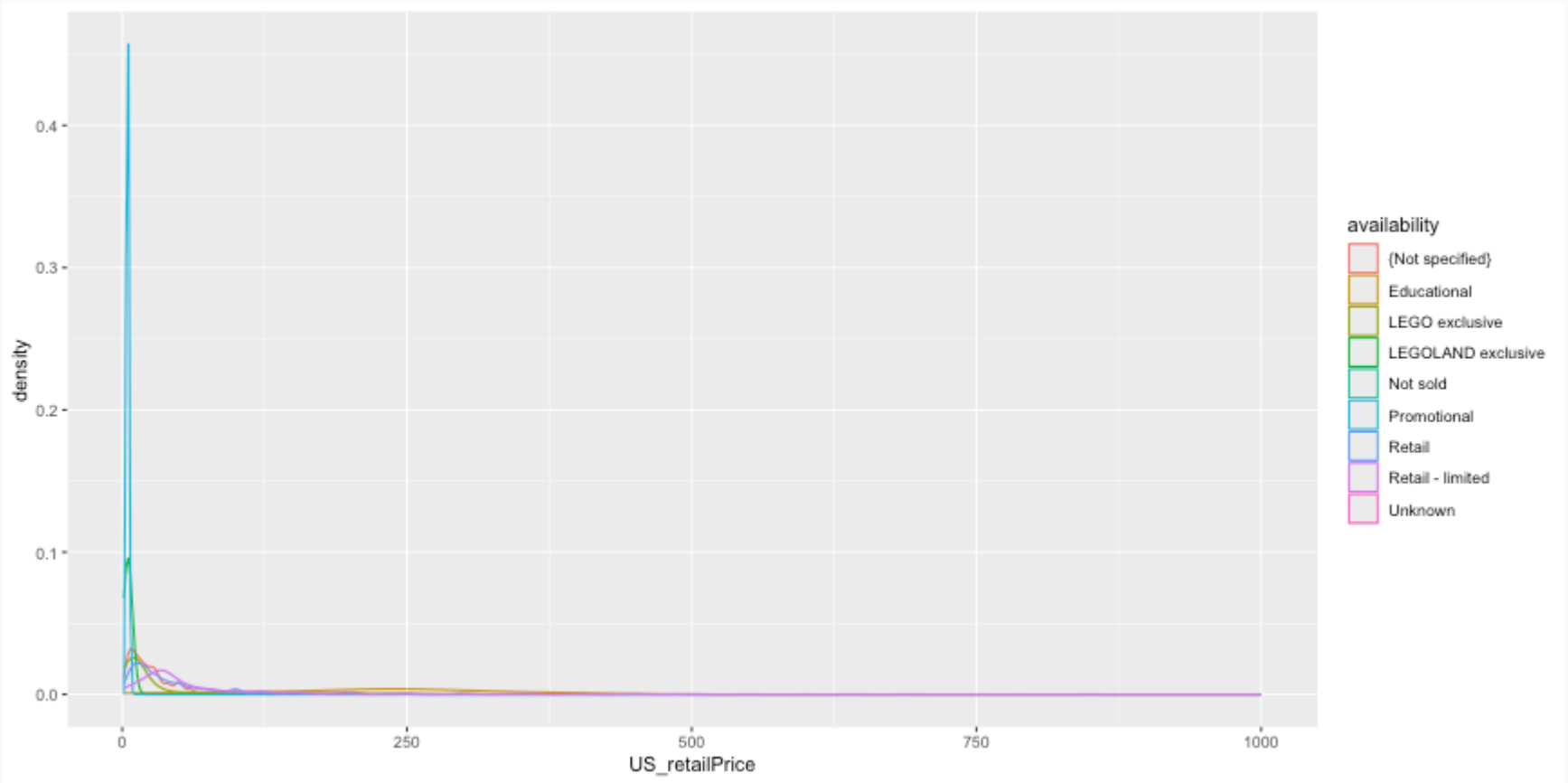
Histograms (cont.)

```
ggplot(legosets, aes(x = US_retailPrice)) + geom_histogram(binwidth = 25) + facet_wrap(~ availability)
```



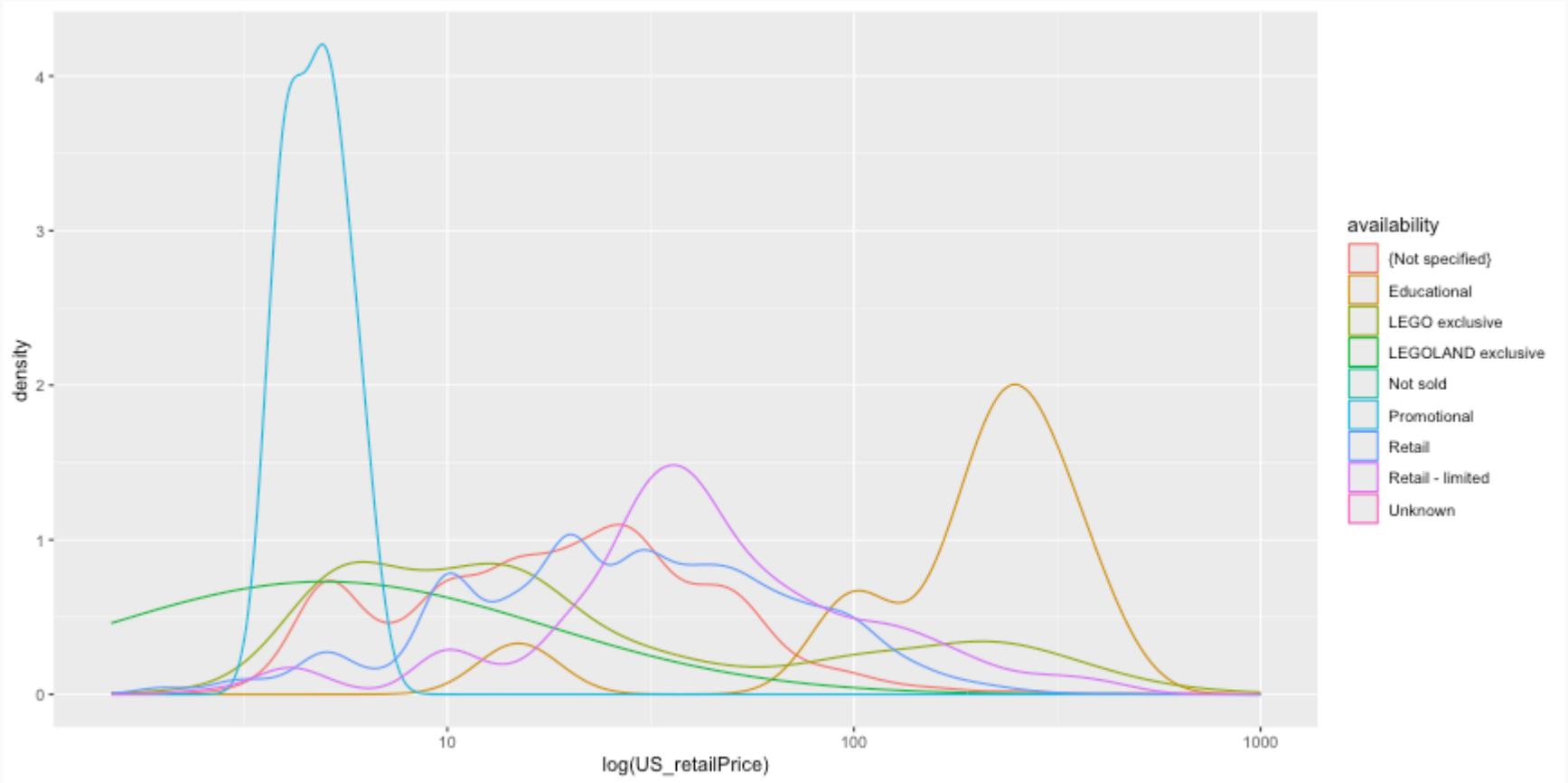
Density Plots

```
ggplot(legosets, aes(x = US_retailPrice, color = availability)) + geom_density()
```



Density Plots (cont.)

```
ggplot(legosets, aes(x = US_retailPrice, color = availability)) + geom_density() + scale_x_log10() + xlab('log(US_ret
```



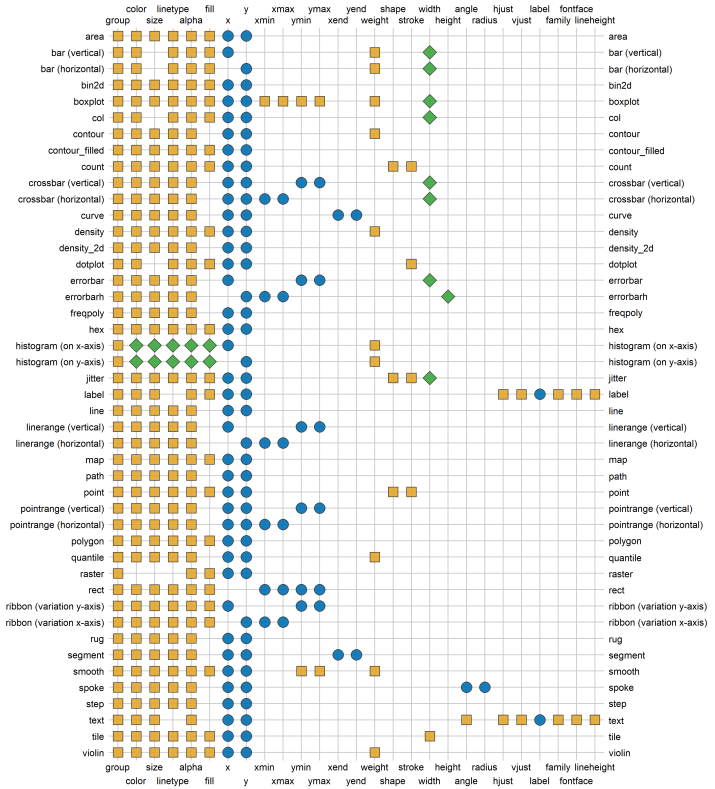
ggplot2 aesthetics

ggplot2 aesthetics cheat sheet

Use this table to find the right aesthetics for your geoms:

Aesthetics that usually must be mapped to the data: use inside aes()
Aesthetics that can be mapped to the data: use in or outside aes()
Aesthetics that cannot be mapped to the data: use outside aes()

e.g., ggplot(mpg, aes(x = class, y = displ)) + geom_col(aes(fill = class), width = .9)



idea and design: Christian Burkhart
design advice: Ida Aarnio

Likert Scales

Likert scales are a type of questionnaire where respondents are asked to rate items on scales usually ranging from four to seven levels (e.g. strongly disagree to strongly agree).

```
library(likert)
library(reshape2)
data(pisaitems)
items24 <- pisaitems[,substr(names(pisaitems), 1,5) == 'ST24Q']
items24 <- items24 |> dplyr::rename(
             "I read only if I have to." = ST24Q01,
             "Reading is one of my favorite hobbies." = ST24Q02,
             "I like talking about books with other people." = ST24Q03,
             "I find it hard to finish books." = ST24Q04,
             "I feel happy if I receive a book as a present." = ST24Q05,
             "For me, reading is a waste of time." = ST24Q06,
             "I enjoy going to a bookstore or a library." = ST24Q07,
             "I read only to get information that I need." = ST24Q08,
             "I cannot sit still and read for more than a few minutes." = ST24Q09,
             "I like to express my opinions about books I have read." = ST24Q10,
             "I like to exchange books with my friends." = ST24Q11)
```

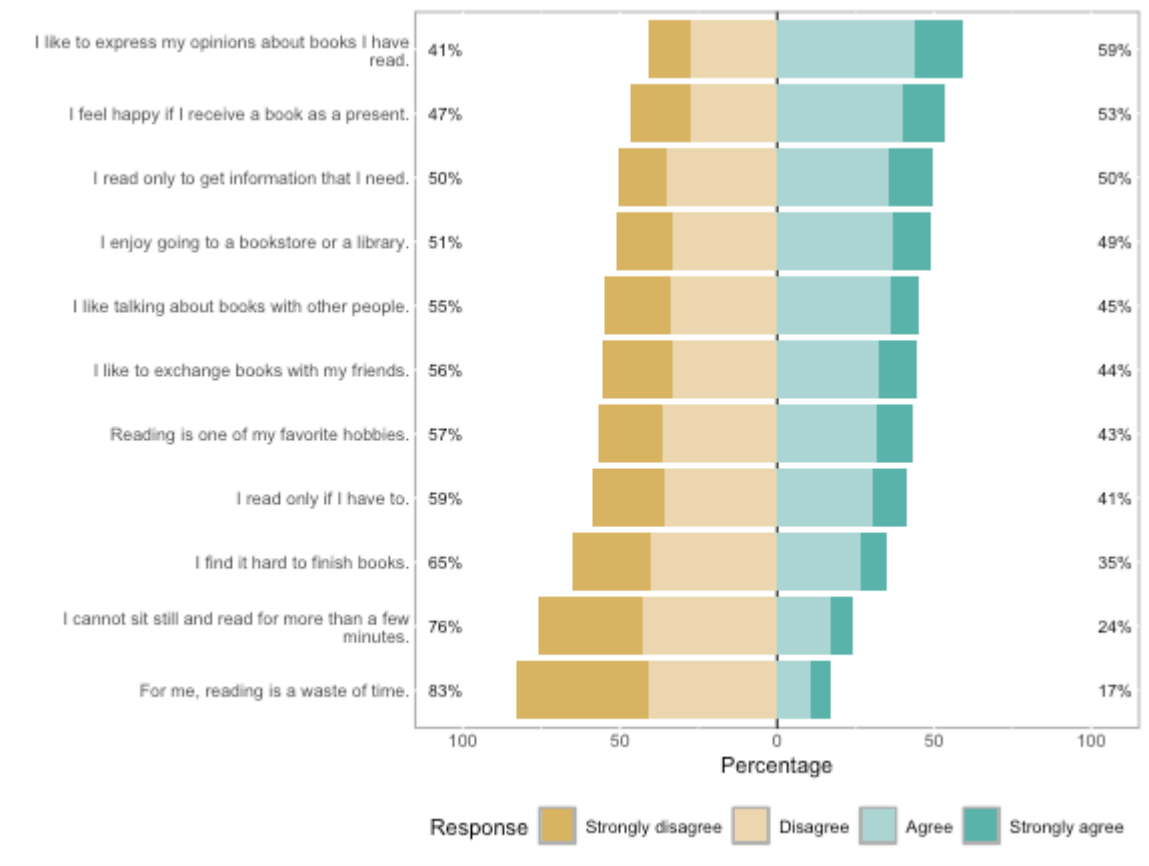
likert R Package

```
l24 <- likert(items24)
summary(l24)
```

```
##                                                      Item      low neutral     high     mean        sd
## 10   I like to express my opinions about books I have read. 41.07516       0 58.92484 2.604913 0.9009968
## 5            I feel happy if I receive a book as a present. 46.93475       0 53.06525 2.466751 0.9446590
## 8               I read only to get information that I need. 50.39874       0 49.60126 2.484616 0.9089688
## 7                I enjoy going to a bookstore or a library. 51.21231       0 48.78769 2.428508 0.9164136
## 3             I like talking about books with other people. 54.99129       0 45.00871 2.328049 0.9090326
## 11               I like to exchange books with my friends. 55.54115       0 44.45885 2.343193 0.9609234
## 2                   Reading is one of my favorite hobbies. 56.64470       0 43.35530 2.344530 0.9277495
## 1                                I read only if I have to. 58.72868       0 41.27132 2.291811 0.9369023
## 4                          I find it hard to finish books. 65.35125       0 34.64875 2.178299 0.8991628
## 9  I cannot sit still and read for more than a few minutes. 76.24524       0 23.75476 1.974736 0.8793028
## 6                       For me, reading is a waste of time. 82.88729       0 17.11271 1.810093 0.8611554
```

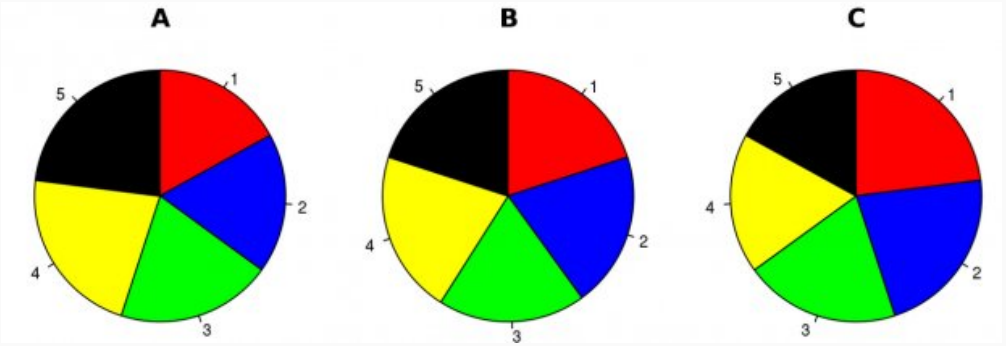
likert Plots

```
plot(l24)
```



Pie Charts

There is only one pie chart in *OpenIntro Statistics* (Diez, Barr, & Çetinkaya-Rundel, 2015, p. 48). Consider the following three pie charts that represent the preference of five different colors. Is there a difference between the three pie charts? This is probably a difficult to answer.



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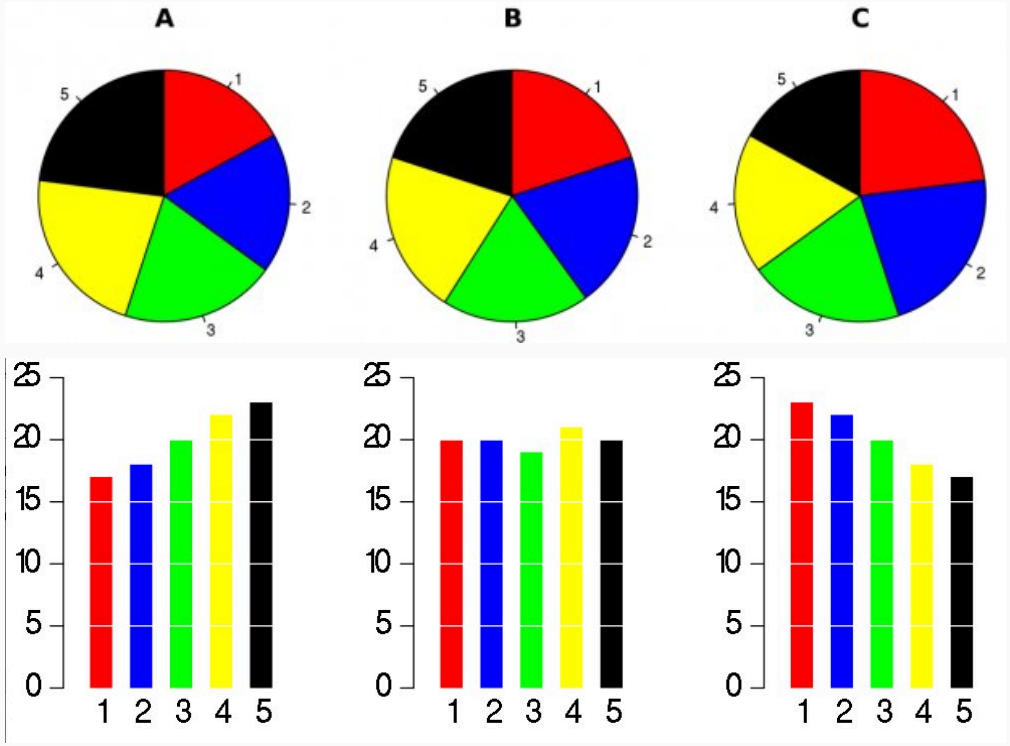

Source: https://en.wikipedia.org/wiki/Pie_chart.

"There is no data that can be displayed in a pie chart that cannot better be displayed in some other type of chart"

John Tukey

Additional Resources

- `ggplot2` website: https://ggplot2.tidyverse.org
- R for Data Science book: https://r4ds.had.co.nz/data-visualisation.html
- R Graphics Cookbook: https://r-graphics.org
- Data visualization cheat sheet: https://github.com/rstudio/cheatsheets/raw/master/data-visualization-2.1.pdf

One Minute Paper

1. What was the most important thing you learned during this class?
2. What important question remains unanswered for you?

https://forms.gle/hWUktVZ4Fyiia3xy6