

Notizen - Sitzung 13

Jonas Schulte

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Verwendete Packages

```
library(here)
library(data.table)
library(tidyverse)
library(ggplot2)
library(ggfortify)
library(car)
library(lmtest)
library(sandwich)
```

Datensatz einlesen

```
data_path <- here("data/bip-mortrate.csv")
lifeexp_data <- fread(data_path,
  colClasses = c("character", "integer", rep("double", 5)))
head(lifeexp_data, 3)
```

```
##      country year LIFEEXP GDP_PPPpc MORTRATE GINI_post GINI_pre
## 1:      AFG 2010  61.028  1671.581      88.0      30.2      30.9
## 2:      ALB 2010  76.562  9927.630      13.3      38.4      44.0
## 3:      ARM 2010  73.331  6702.829      18.5      35.9      47.4
```

Regressionen: Lebenserwartung und BIP pro Kopf

Modelle

```
modell1 <- lm(LIFEEXP ~ GDP_PPPpc, lifeexp_data)
summary(modell1)
```

```
##
## Call:
## lm(formula = LIFEEXP ~ GDP_PPPpc, data = lifeexp_data)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -23.865  -3.826   2.087   4.875   9.498
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.521e+01  7.351e-01  88.70  <2e-16 ***
## GDP_PPpC    3.148e-04  2.996e-05  10.51  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.663 on 148 degrees of freedom
## Multiple R-squared:  0.4272, Adjusted R-squared:  0.4233
## F-statistic: 110.4 on 1 and 148 DF,  p-value: < 2.2e-16
```

```
model2 <- lm(LIFEEXP ~ log(GDP_PPpC), lifeexp_data)
summary(model2)
```

```
##
## Call:
## lm(formula = LIFEEXP ~ log(GDP_PPpC), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.3420  -1.7187   0.6502   3.0746   9.7919
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    15.1124     3.0494   4.956 1.95e-06 ***
## log(GDP_PPpC)    6.0748     0.3322  18.287 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.876 on 148 degrees of freedom
## Multiple R-squared:  0.6932, Adjusted R-squared:  0.6911
## F-statistic: 334.4 on 1 and 148 DF,  p-value: < 2.2e-16
```

```
model3 <- lm(log(LIFEEXP) ~ log(GDP_PPpC), lifeexp_data)
summary(model3)
```

```
##
## Call:
## lm(formula = log(LIFEEXP) ~ log(GDP_PPpC), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.31957 -0.02348  0.01000  0.04694  0.15263
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.430276   0.048212  71.15  <2e-16 ***
## log(GDP_PPpC) 0.089613   0.005252  17.06  <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.07709 on 148 degrees of freedom
## Multiple R-squared:  0.663, Adjusted R-squared:  0.6607
## F-statistic: 291.1 on 1 and 148 DF,  p-value: < 2.2e-16

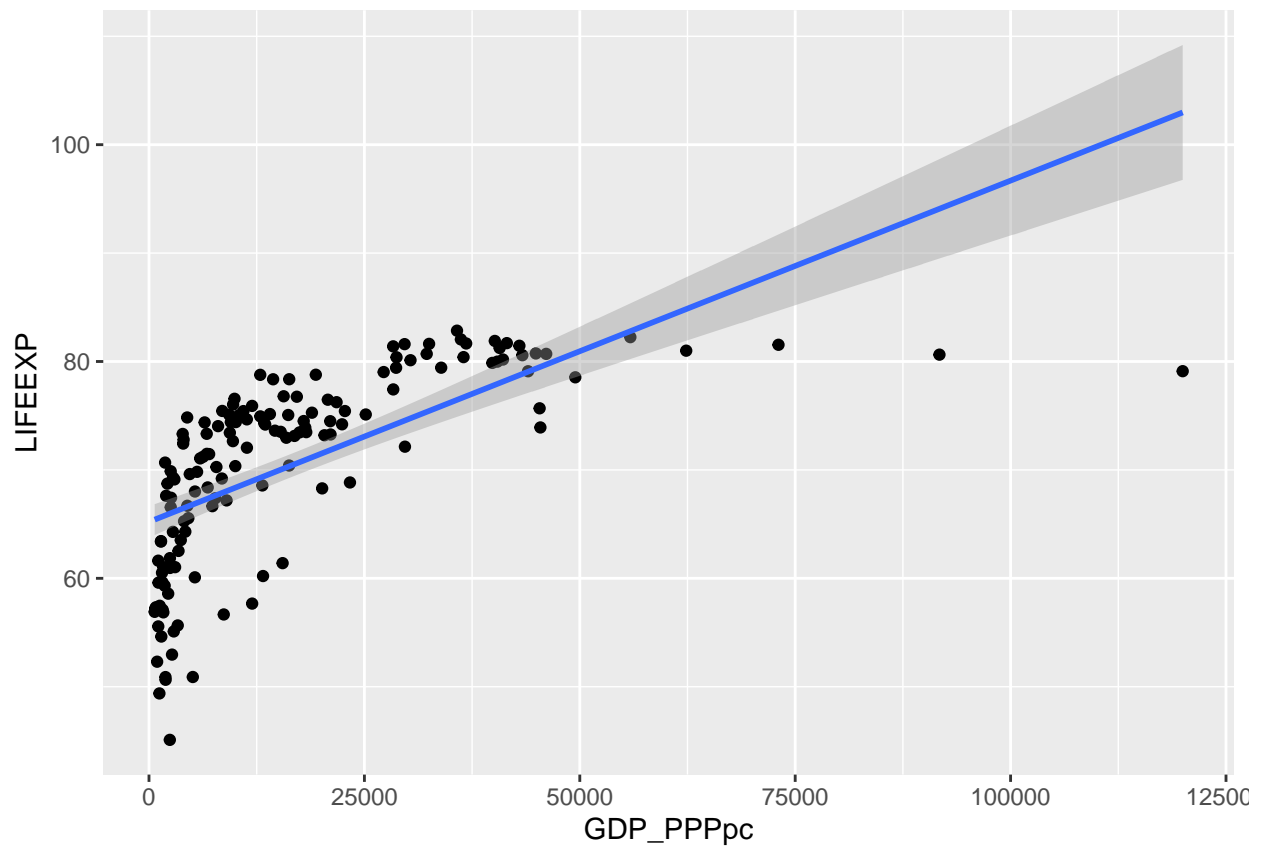
model4 <- lm(LIFEEXP ~ log(GDP_PPPpc) + I(log(GDP_PPPpc)**2), lifeexp_data)
summary(model4)

##
## Call:
## lm(formula = LIFEEXP ~ log(GDP_PPPpc) + I(log(GDP_PPPpc)^2),
##     data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.3626  -1.6936   0.8357   2.9741   9.9662
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -7.8443     21.5501  -0.364   0.7164
## log(GDP_PPPpc)  11.2891     4.8570   2.324   0.0215 *
## I(log(GDP_PPPpc)^2) -0.2907     0.2702  -1.076   0.2837
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.874 on 147 degrees of freedom
## Multiple R-squared:  0.6956, Adjusted R-squared:  0.6915
## F-statistic: 168 on 2 and 147 DF,  p-value: < 2.2e-16
```

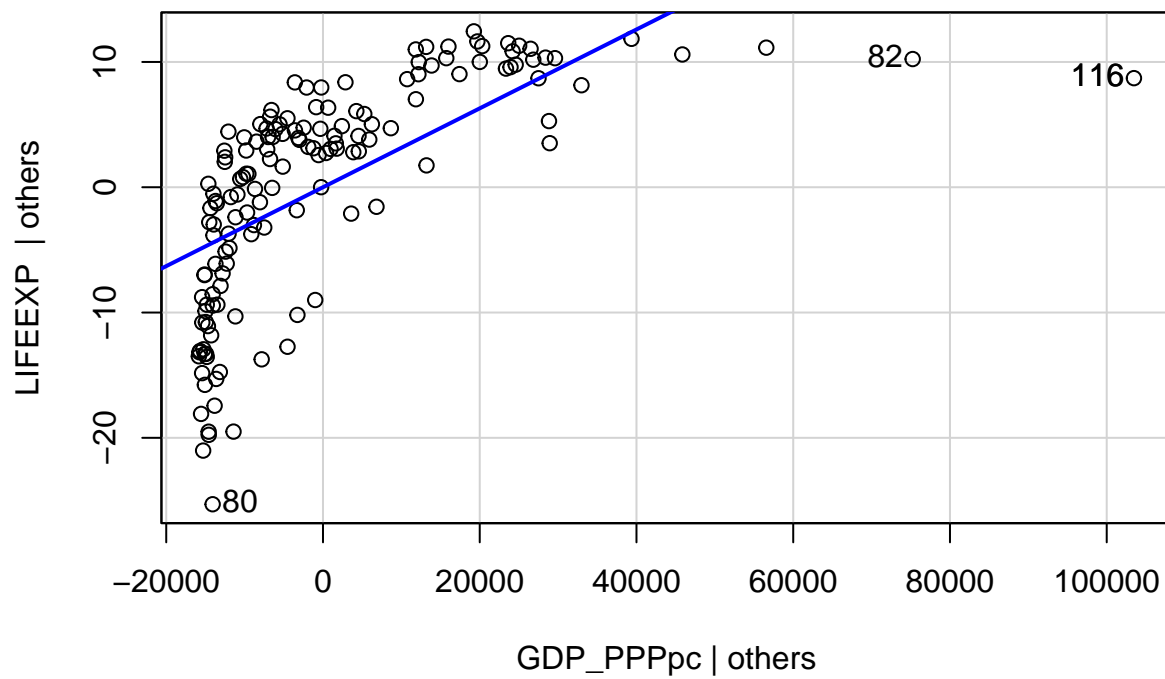
Regressionsdiagnostik

Scatter-Plots

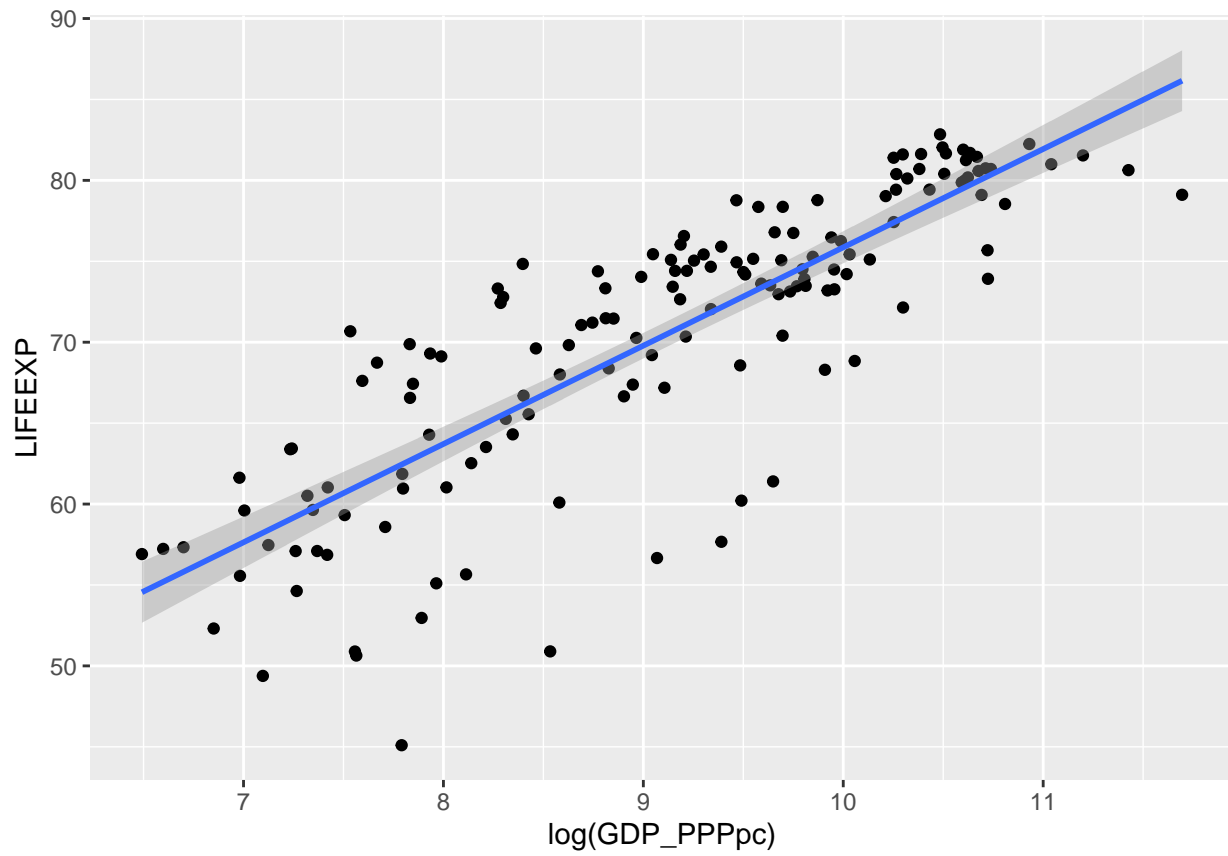
```
# Modell 1
ggplot(lifeexp_data, aes(x = GDP_PPPpc, y = LIFEEXP))+
  geom_point()+
  geom_smooth(method = "lm")
```



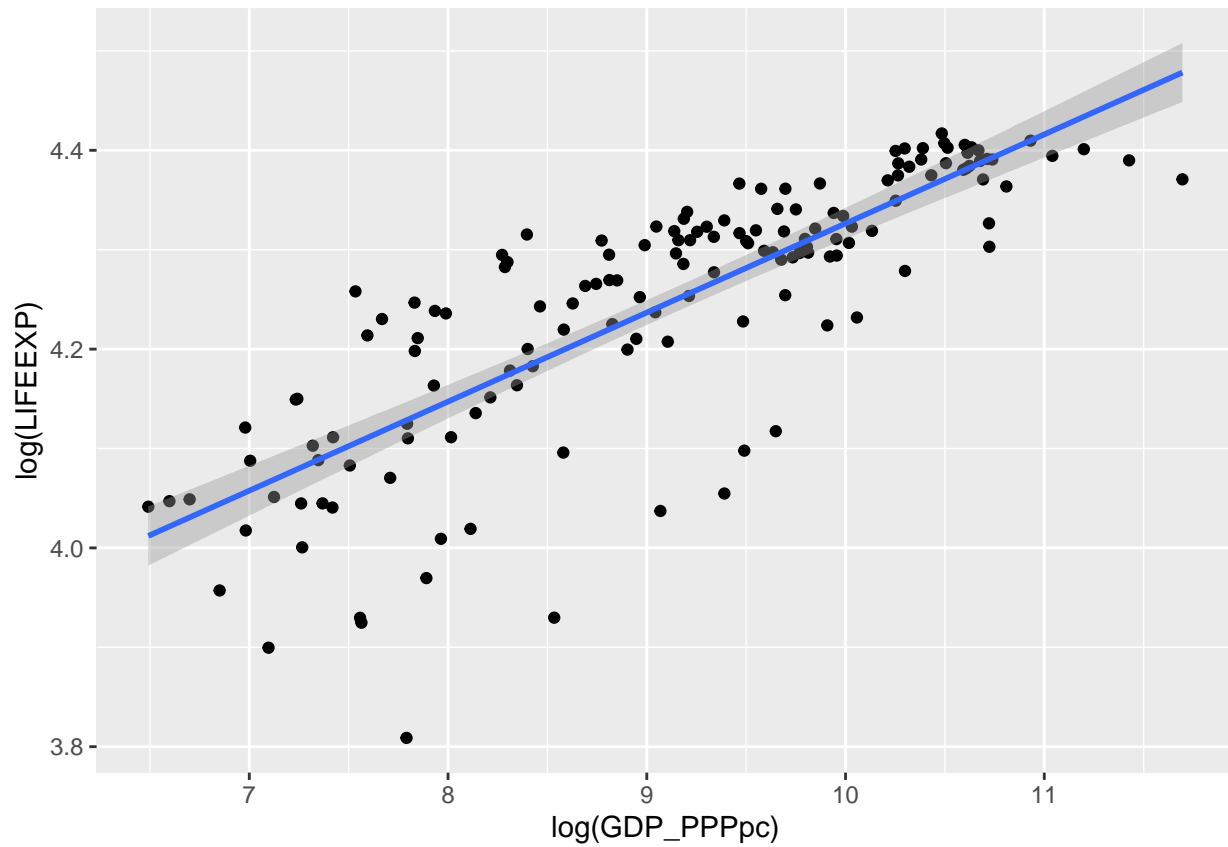
```
avPlots(model1)
```



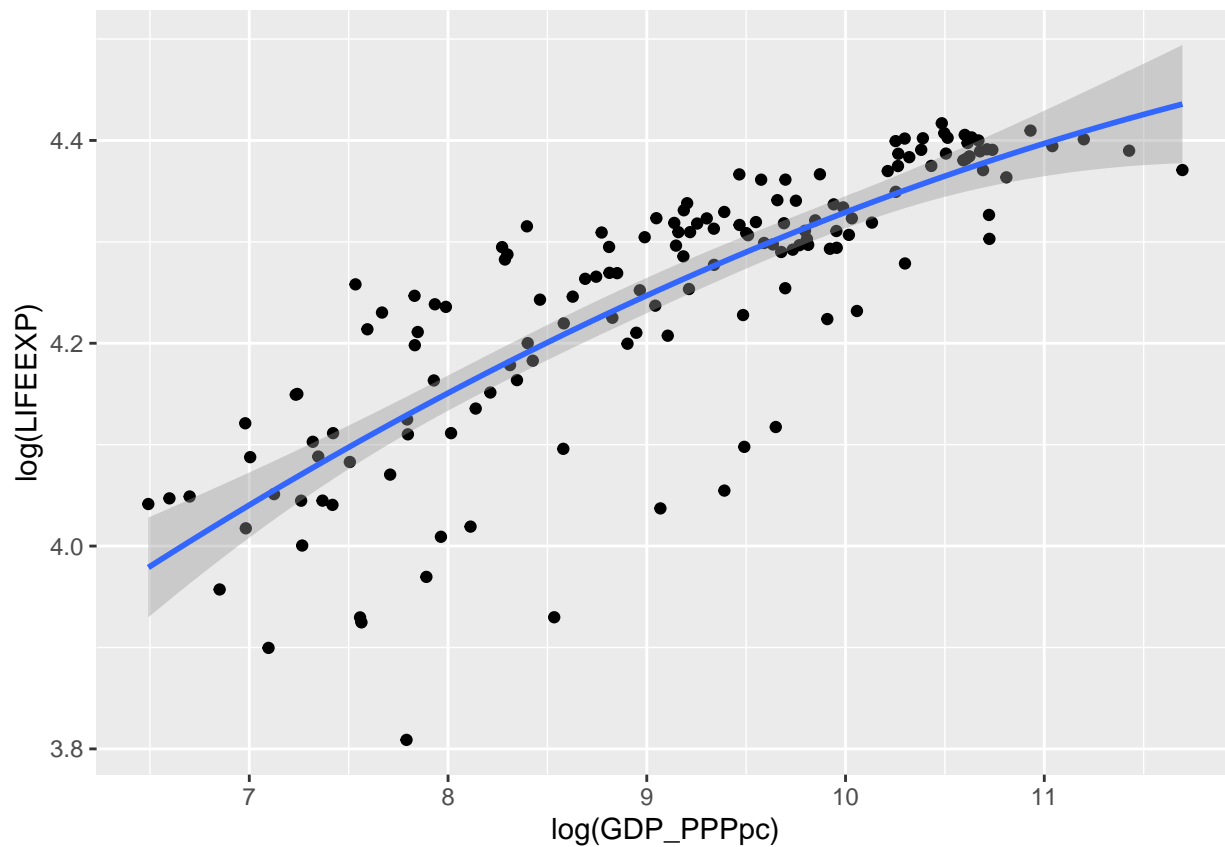
```
# Modell 2
ggplot(lifeexp_data, aes(x = log(GDP_PPPpc), y = LIFEEXP))+
  geom_point()+
  geom_smooth(method = "lm")
```



```
# Modell 3
ggplot(lifeexp_data, aes(x = log(GDP_PPPpc), y = log(LIFEEXP)))+
  geom_point()+
  geom_smooth(method = "lm")
```



```
# Modell 4
ggplot(lifeexp_data, aes(x = log(GDP_PPPpc), y = log(LIFEEXP)))+
  geom_point()+
  geom_smooth(method = "lm", formula = y ~ x + I(x**2))
```



RESET-Test

```
library(lmtest)
resettest(model1) # Mit dem Power Argument, kann man festlegen, für welche Potenzen getestet werden soll
```

```
##
## RESET test
##
## data: model1
## RESET = 47.873, df1 = 2, df2 = 146, p-value < 2.2e-16
```

```
resettest(model2)
```

```
##
## RESET test
##
## data: model2
## RESET = 1.3622, df1 = 2, df2 = 146, p-value = 0.2593
```

```
resettest(model3)
```

```
##
## RESET test
```



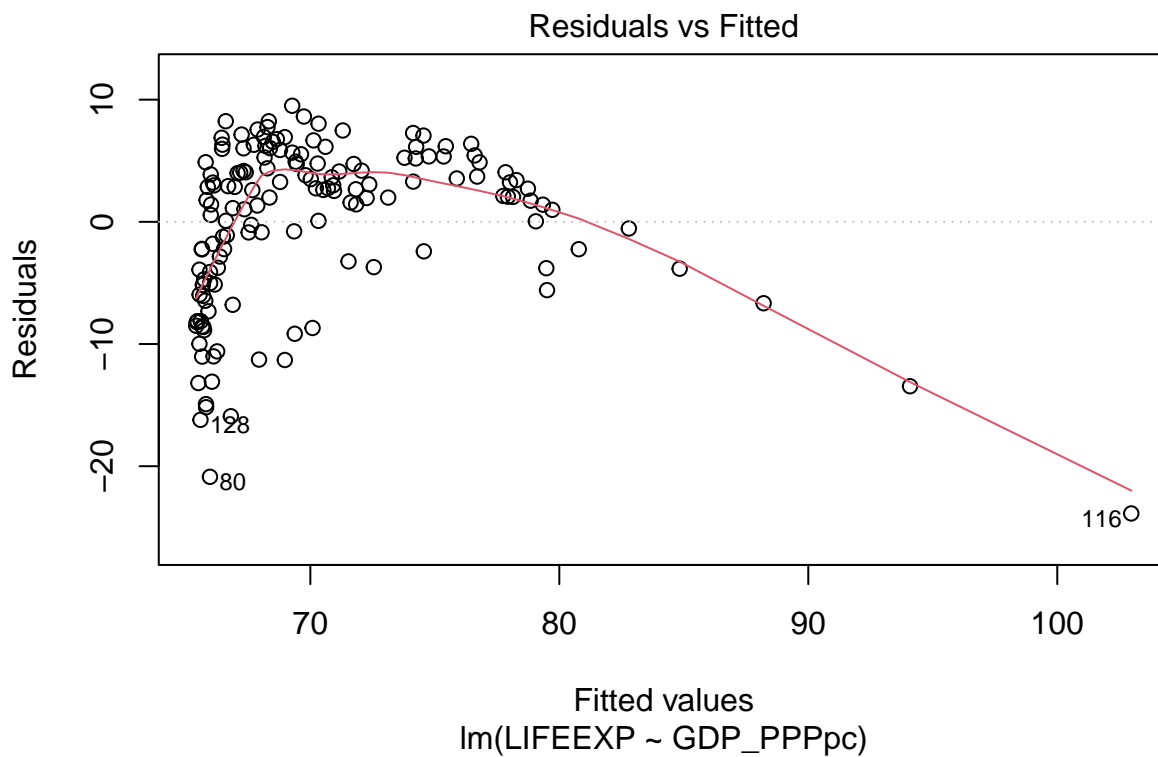
```
##
## data: model3
## RESET = 2.055, df1 = 2, df2 = 146, p-value = 0.1318
```

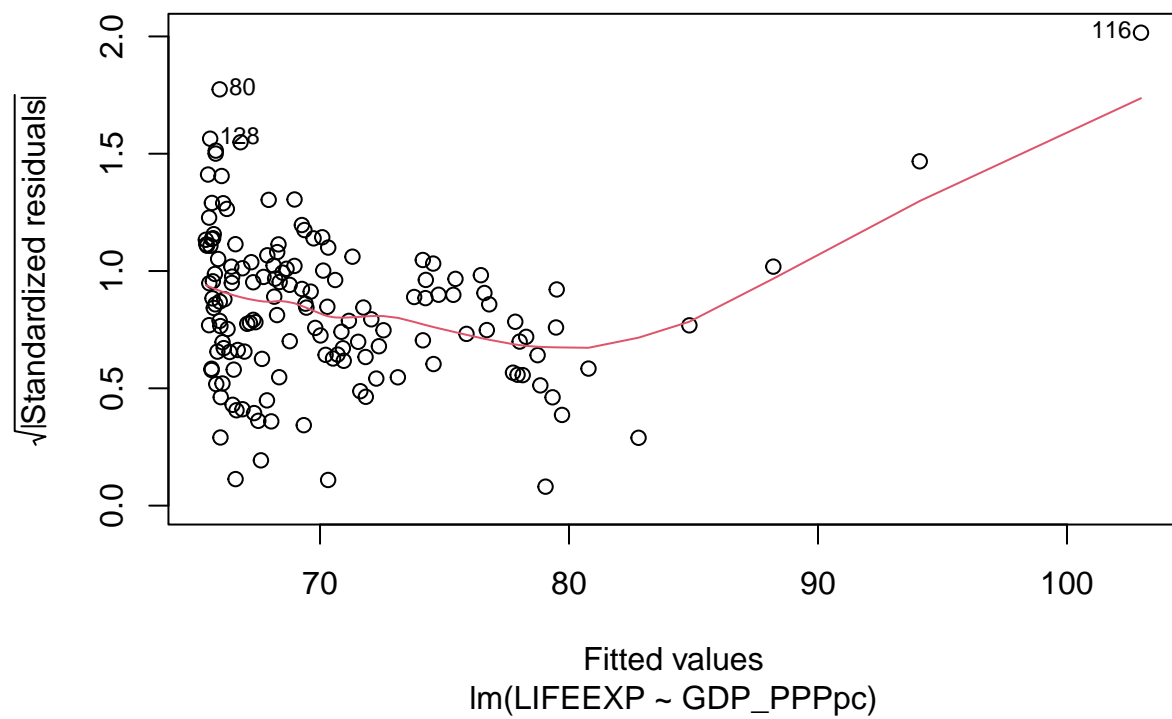
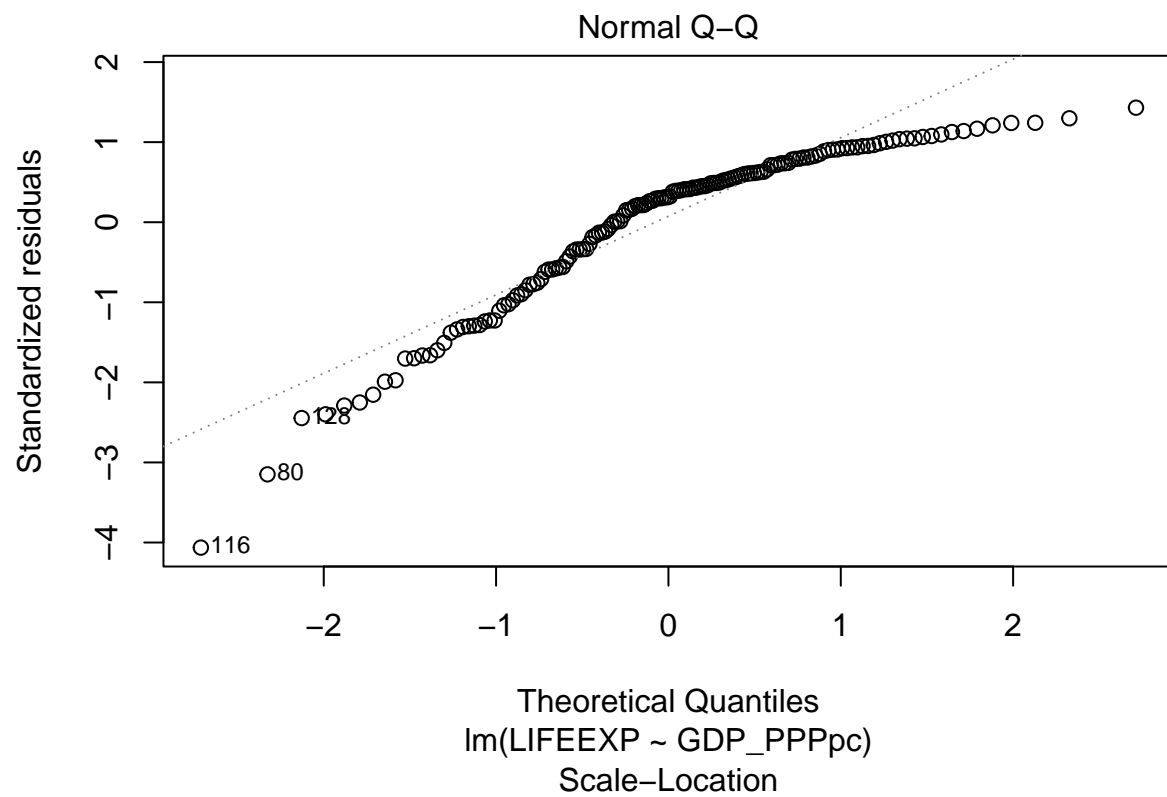
```
resettest(model4, power = 3)
```

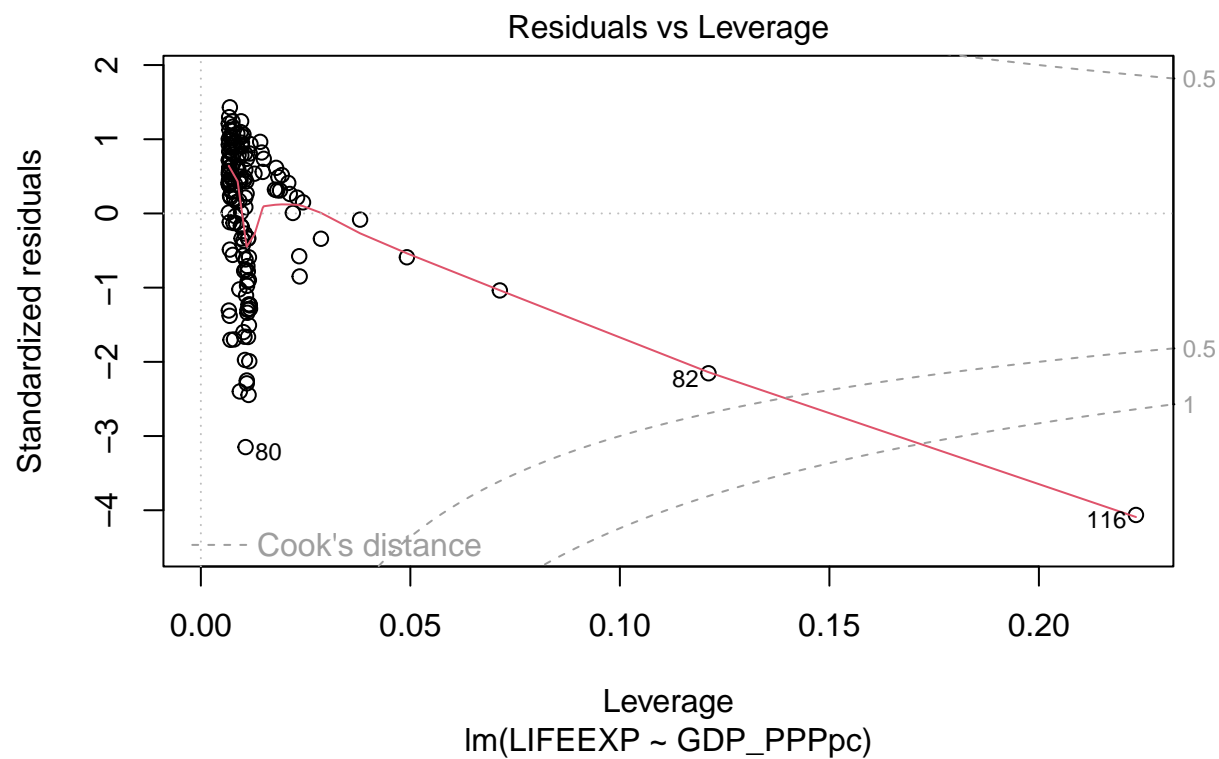
```
##
## RESET test
##
## data: model4
## RESET = 1.4865, df1 = 1, df2 = 146, p-value = 0.2247
```

TA-Plot, QQ_plot, etc.

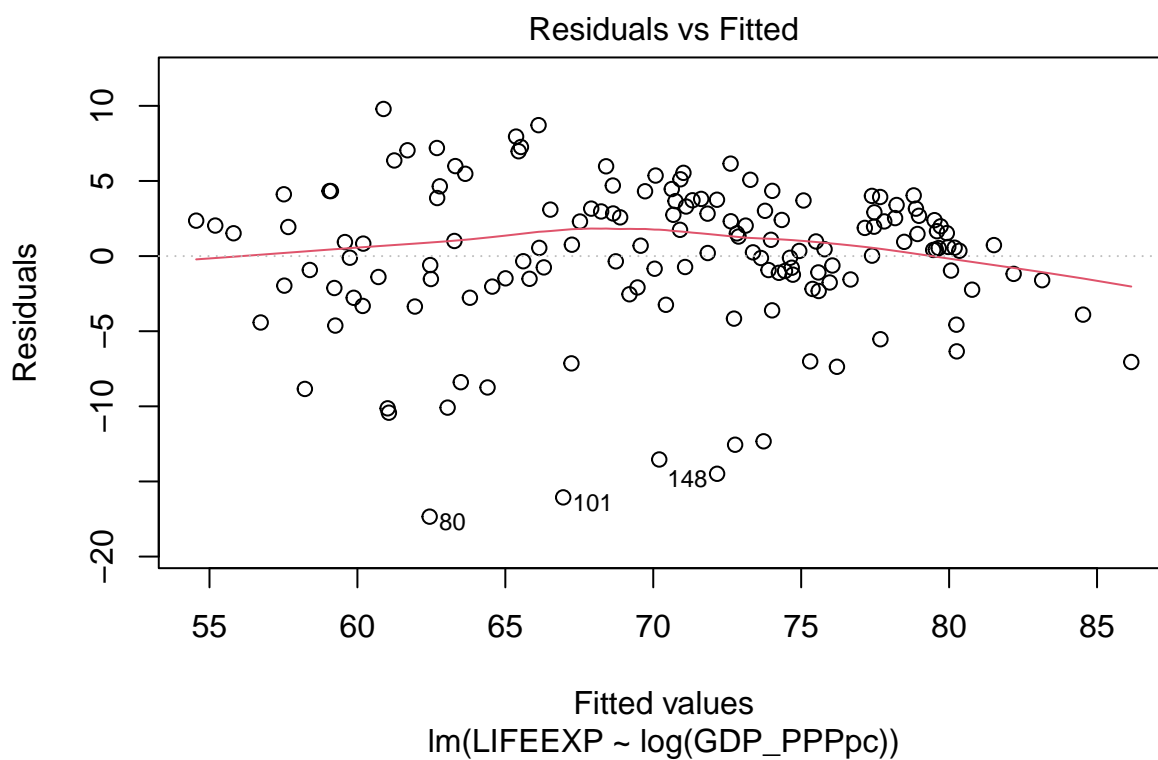
```
plot(model1)
```

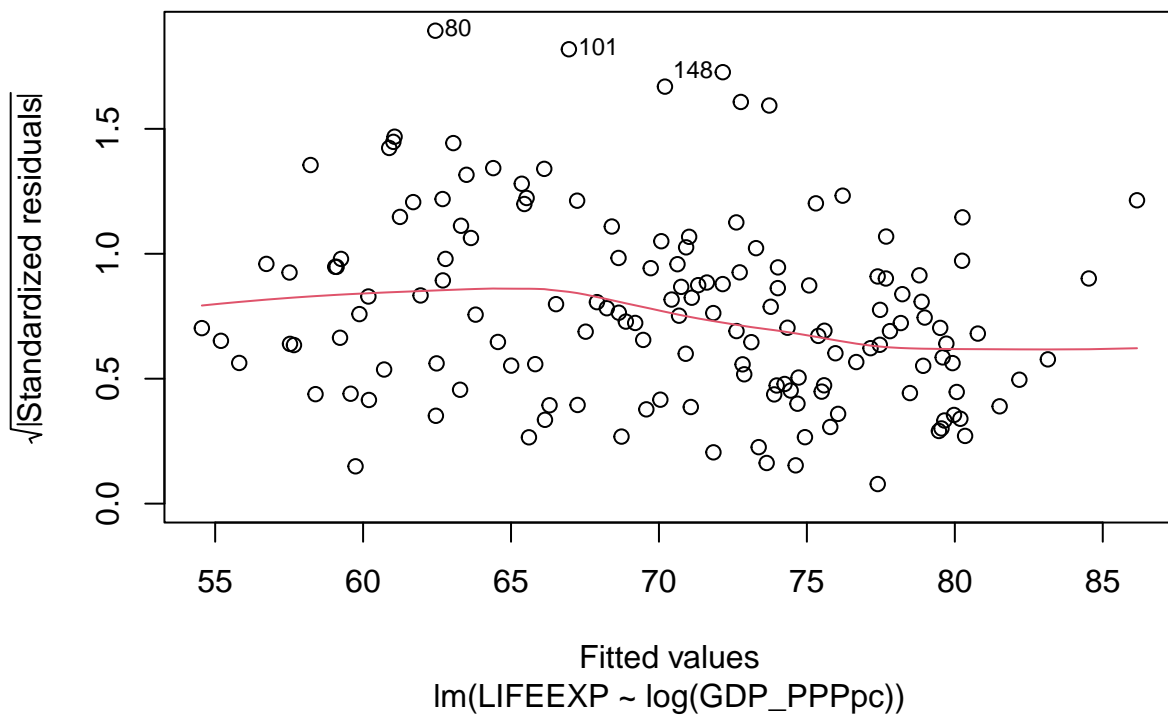
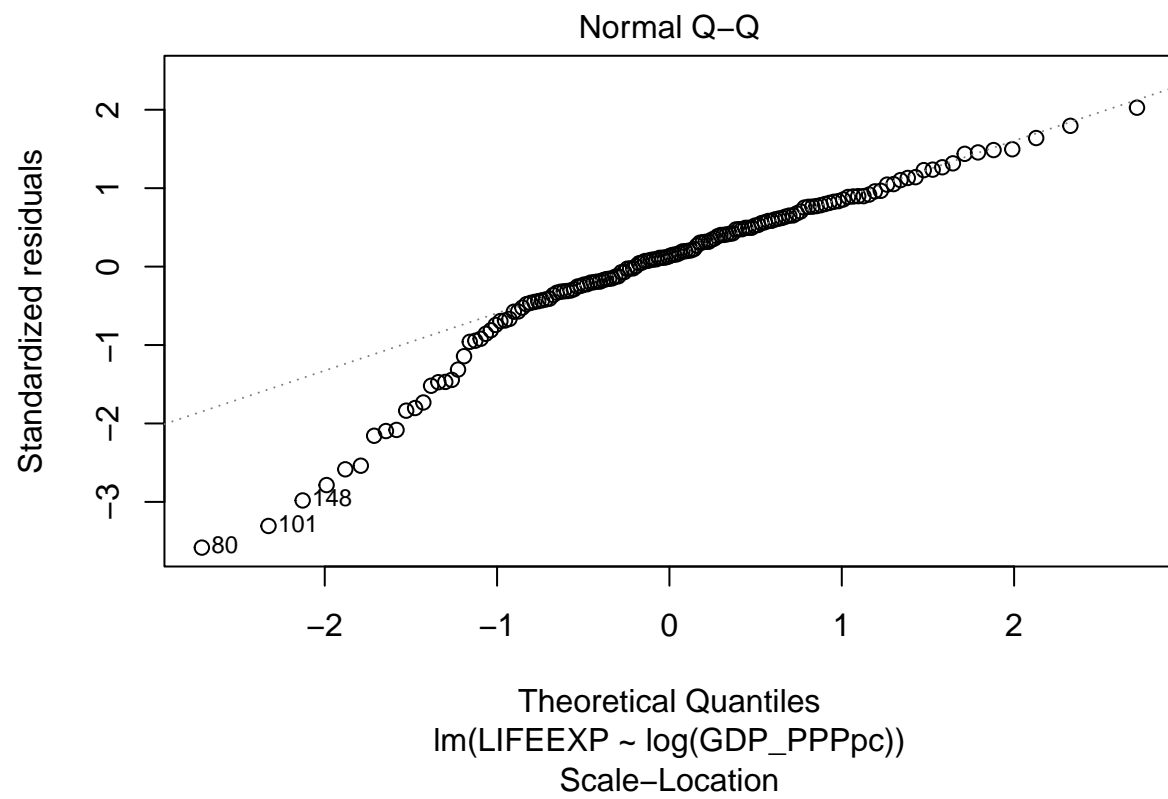


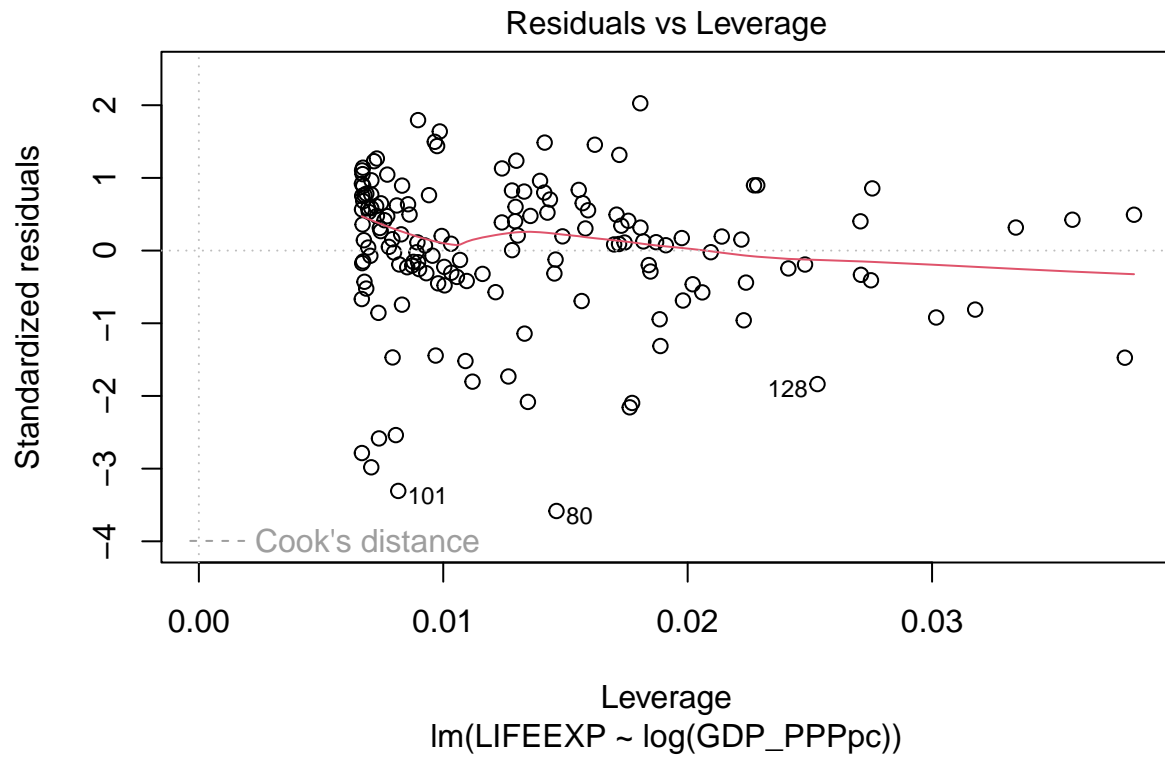




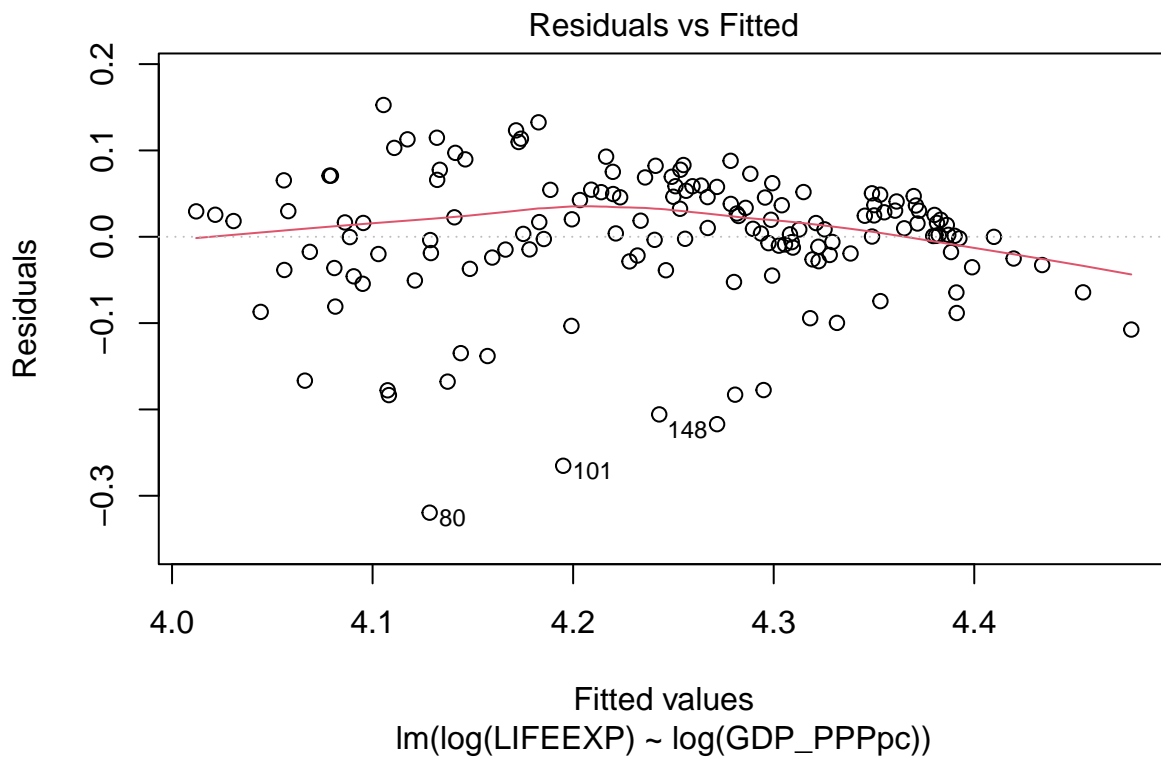
```
plot(model2)
```

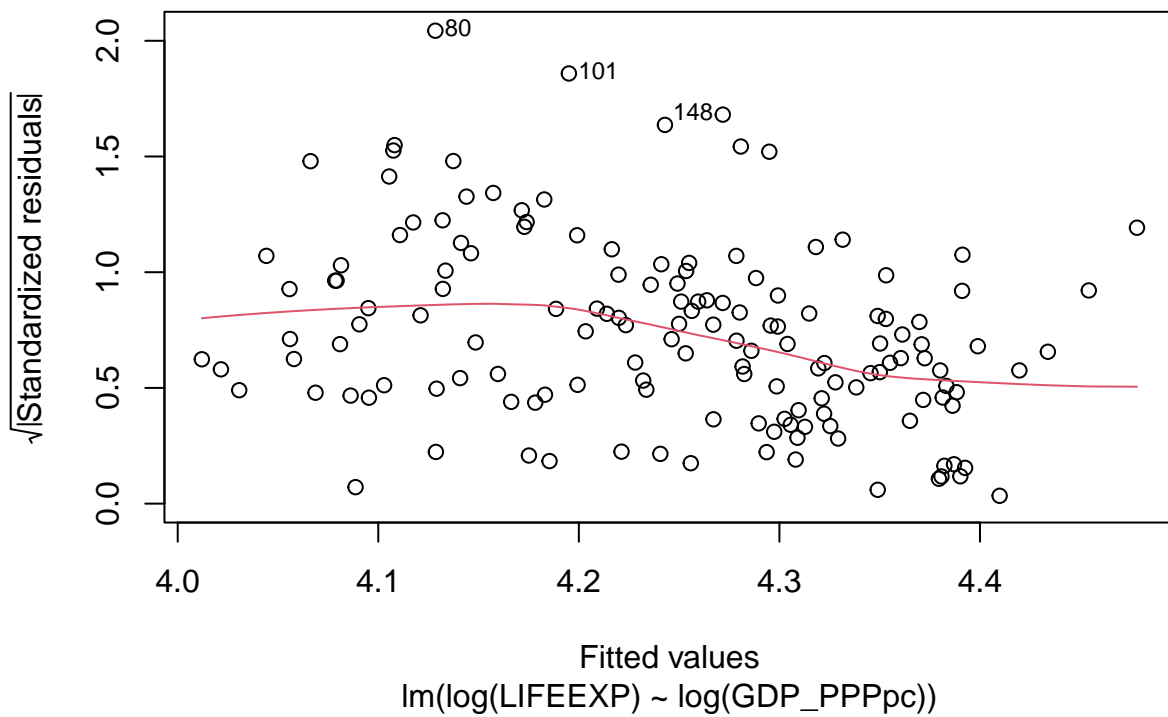
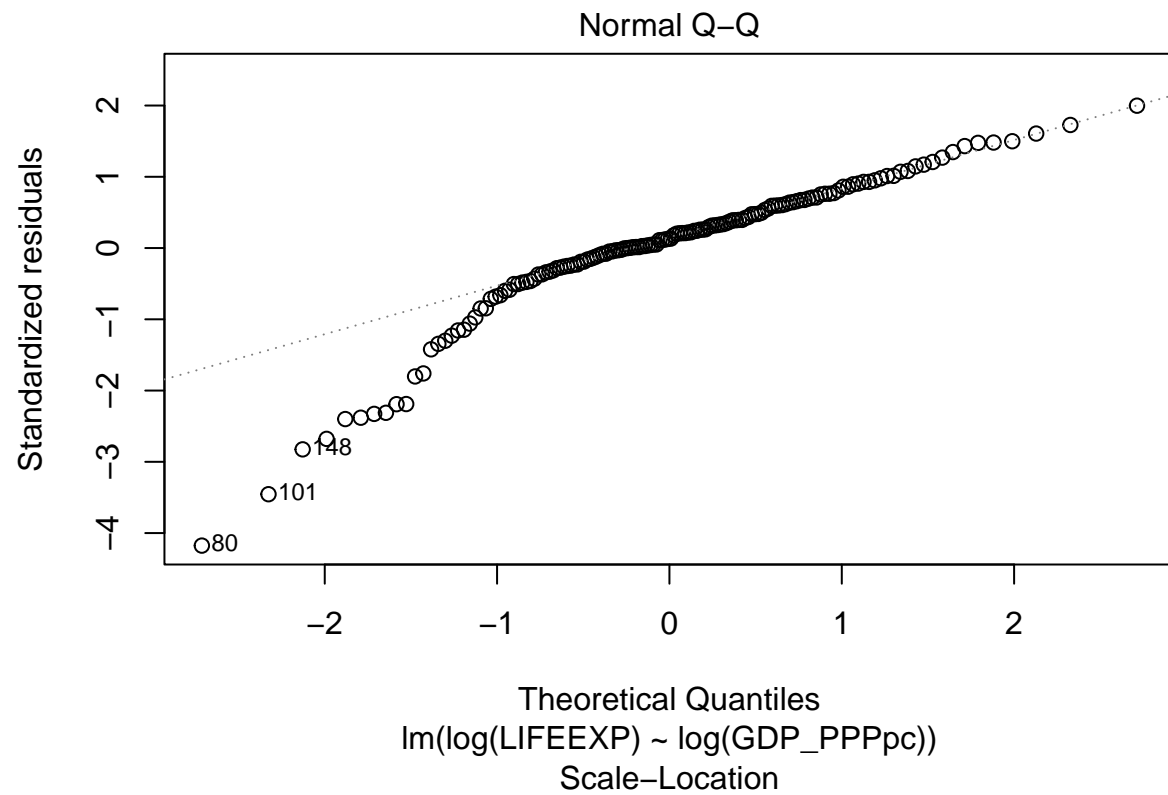


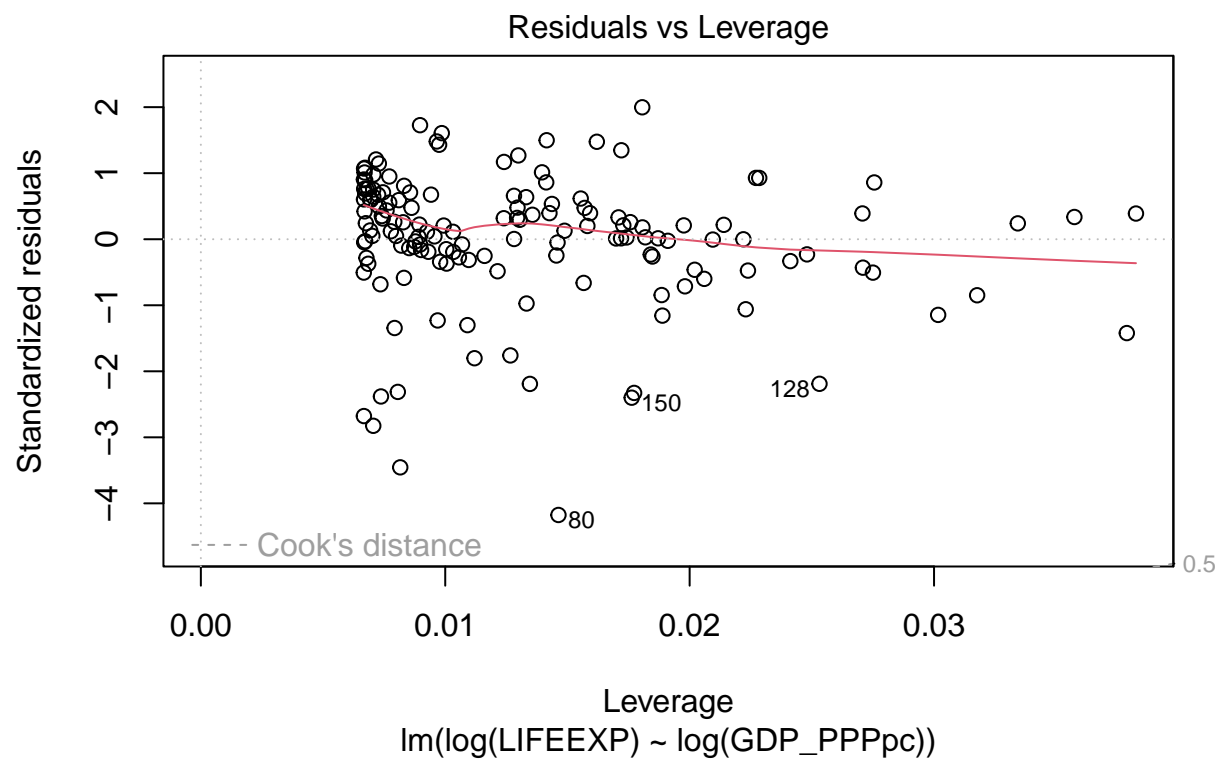




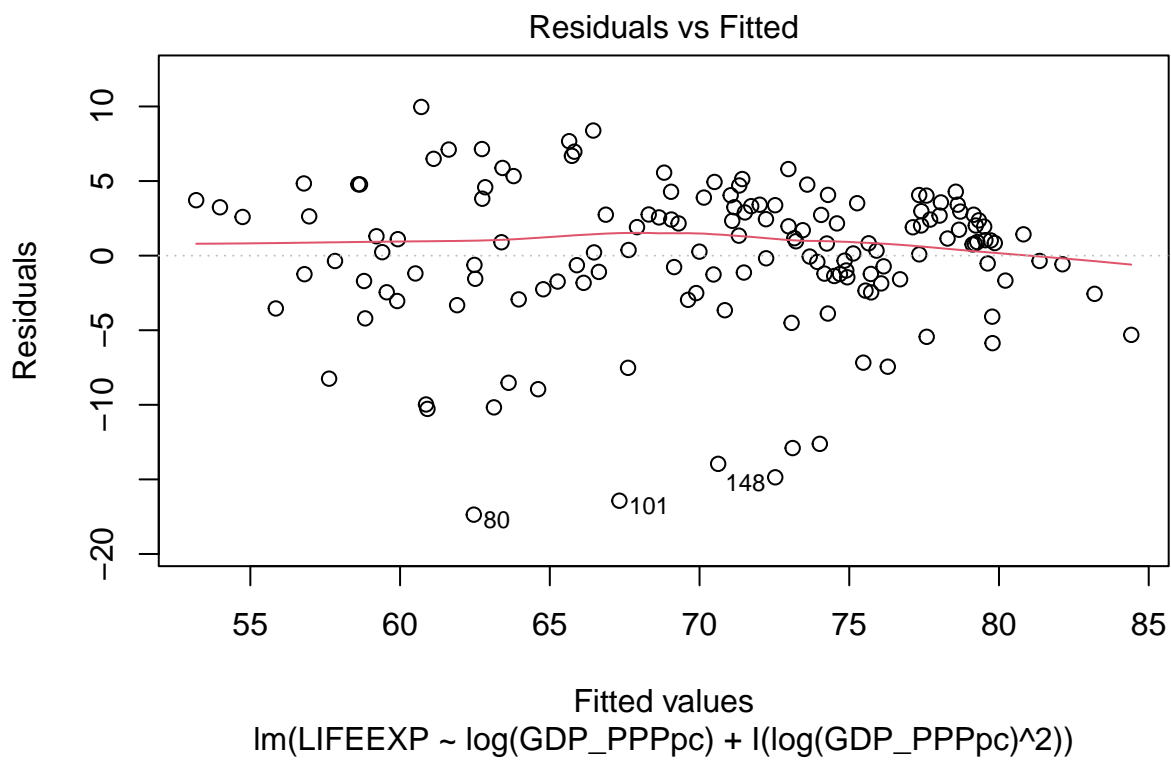
```
plot(model3)
```

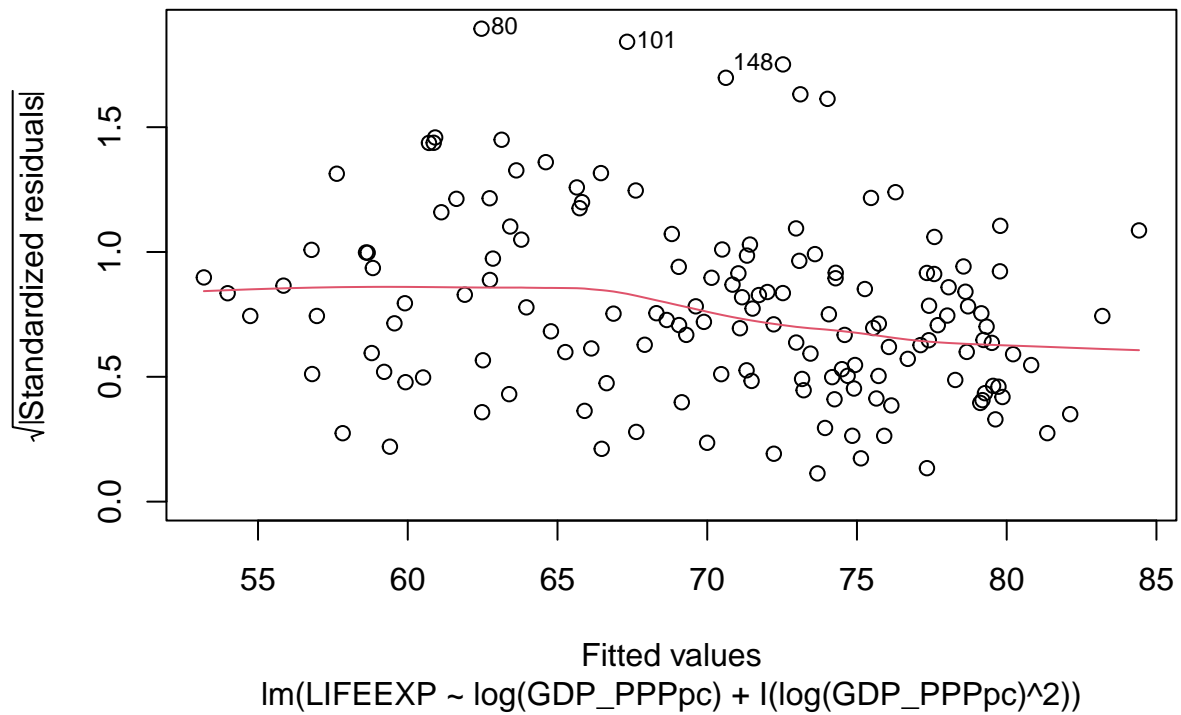
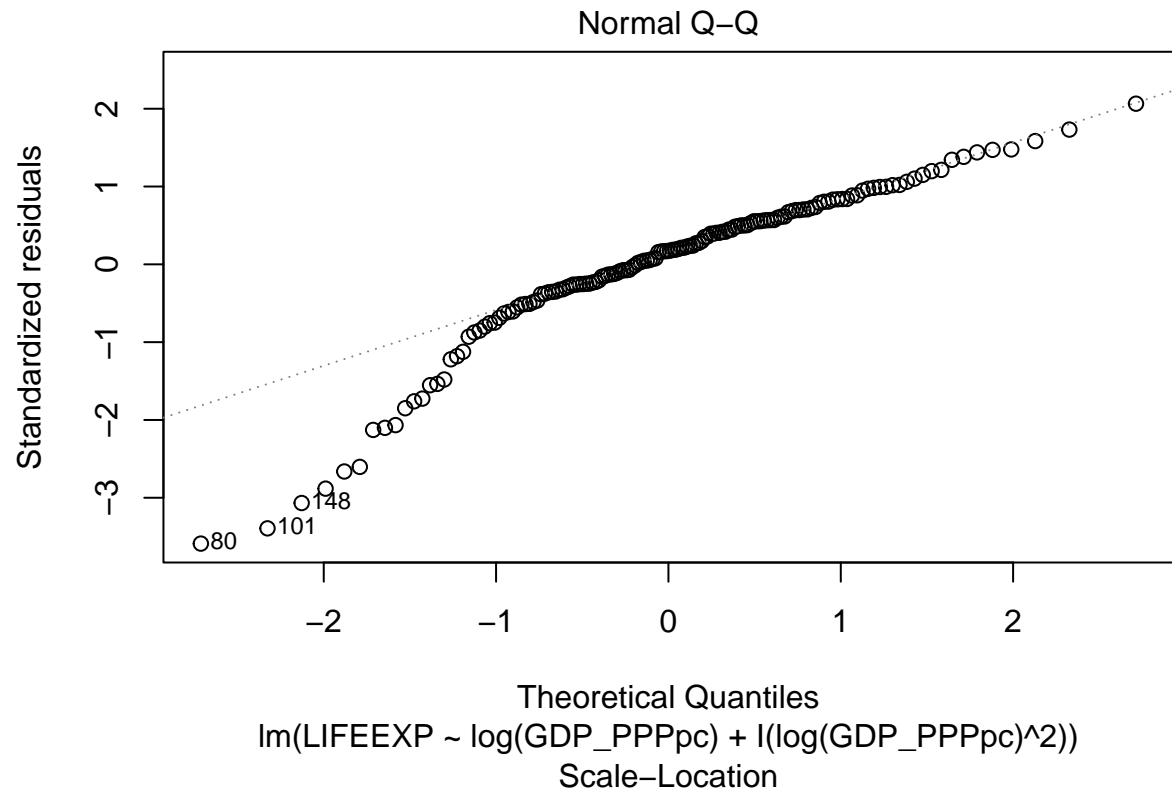


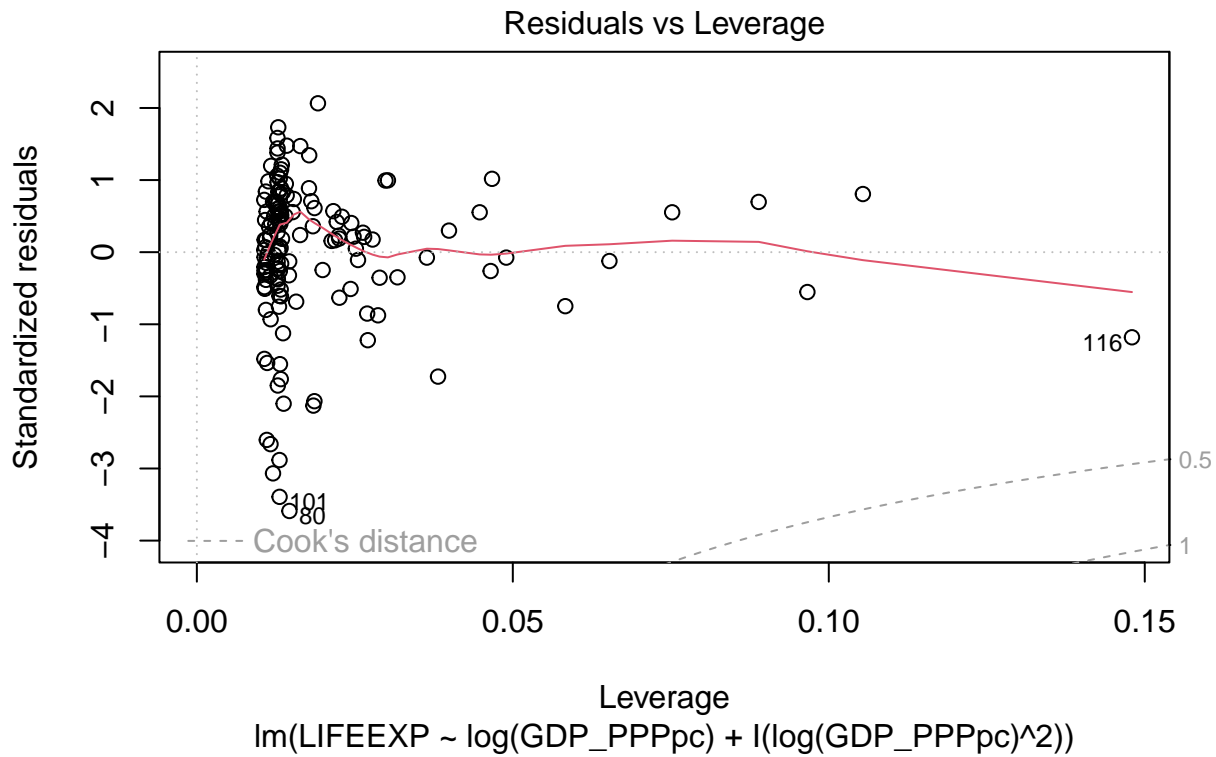




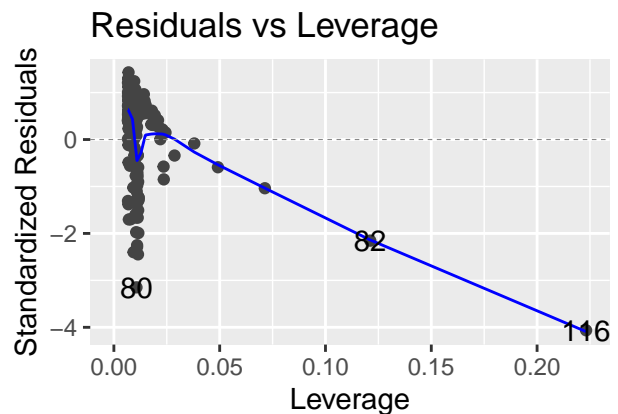
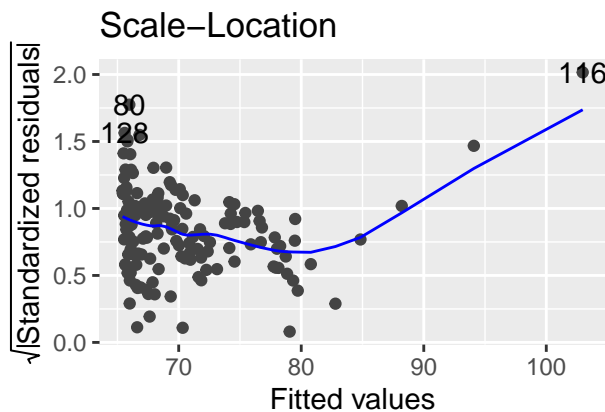
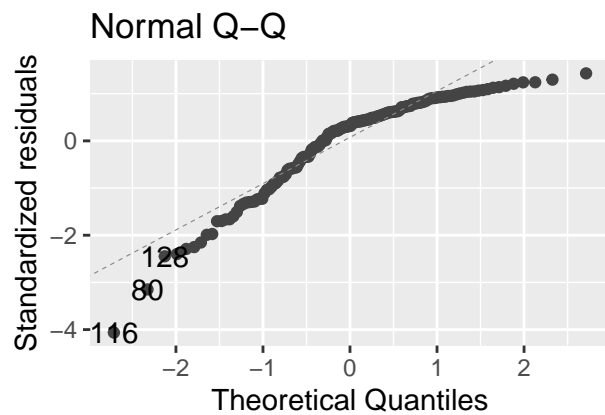
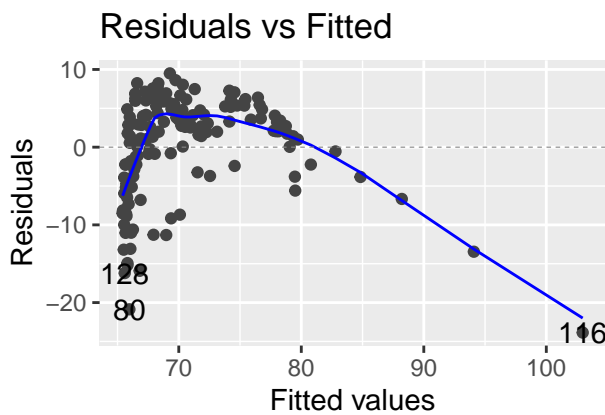
```
plot(model4)
```







```
library(ggfortify)
autoplot(model1)
```



Test auf Heteroskedastie ##### Breusch-Pagan Test - H_0 = Homoskedastie

```
library(lmtest)
bptest(model1)
```

```
##
## studentized Breusch-Pagan test
##
## data: model1
## BP = 3.4002, df = 1, p-value = 0.06519
```

```
bptest(model2)
```

```
##
## studentized Breusch-Pagan test
##
## data: model2
## BP = 4.7918, df = 1, p-value = 0.0286
```

```
bptest(model3)
```

```
##
## studentized Breusch-Pagan test
##
## data: model3
## BP = 7.1428, df = 1, p-value = 0.007527
```

```
bptest(model4)
```

```
##
## studentized Breusch-Pagan test
##
## data: model4
## BP = 7.3991, df = 2, p-value = 0.02474
```

Goldfeld-Quandt Test

- H_0 = Homoskedastie

```
gqtest(model1, alternative = "less")
```

```
##
## Goldfeld-Quandt test
##
## data: model1
## GQ = 2.088, df1 = 73, df2 = 73, p-value = 0.999
## alternative hypothesis: variance decreases from segment 1 to 2
```

```
gqtest(model1, alternative = "greater") # Default
```

```
##  
## Goldfeld-Quandt test  
##  
## data: model1  
## GQ = 2.088, df1 = 73, df2 = 73, p-value = 0.0009676  
## alternative hypothesis: variance increases from segment 1 to 2
```

```
gqtest(model1, alternative = "two.sided")
```

```
##  
## Goldfeld-Quandt test  
##  
## data: model1  
## GQ = 2.088, df1 = 73, df2 = 73, p-value = 0.001935  
## alternative hypothesis: variance changes from segment 1 to 2
```

```
gqtest(model2)
```

```
##  
## Goldfeld-Quandt test  
##  
## data: model2  
## GQ = 1.8364, df1 = 73, df2 = 73, p-value = 0.005111  
## alternative hypothesis: variance increases from segment 1 to 2
```

```
gqtest(model3)
```

```
##  
## Goldfeld-Quandt test  
##  
## data: model3  
## GQ = 2.1784, df1 = 73, df2 = 73, p-value = 0.0005293  
## alternative hypothesis: variance increases from segment 1 to 2
```

```
gqtest(model4)
```

```
##  
## Goldfeld-Quandt test  
##  
## data: model4  
## GQ = 1.7924, df1 = 72, df2 = 72, p-value = 0.007141  
## alternative hypothesis: variance increases from segment 1 to 2
```

Reaktion auf Heteroskedastie Siehe Skript!

```
var_covar_matrix <- sandwich::vcovHC(model2, type = "HC1")
lmtest::coeftest(model2, vcov. = var_covar_matrix)

##
## t test of coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   15.11241    2.95323   5.1172 9.48e-07 ***
## log(GDP_PPPpc) 6.07485    0.30545  19.8879 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(model2)
```

```
##
## Call:
## lm(formula = LIFEEXP ~ log(GDP_PPPpc), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.3420  -1.7187   0.6502   3.0746   9.7919
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    15.1124     3.0494   4.956 1.95e-06 ***
## log(GDP_PPPpc)  6.0748     0.3322  18.287 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.876 on 148 degrees of freedom
## Multiple R-squared:  0.6932, Adjusted R-squared:  0.6911
## F-statistic: 334.4 on 1 and 148 DF, p-value: < 2.2e-16
```

Übungsaufgabe: Regression Lebenserwartung und Gini

Experimentiere mit unterschiedlichen Regressionsmodellen.

Modelle

```
model5 <- lm(LIFEEXP ~ GINI_post, lifeexp_data)
summary(model5)

##
## Call:
## lm(formula = LIFEEXP ~ GINI_post, data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -20.853 -5.839 1.694 5.828 12.863
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  91.5636      3.2164  28.468 < 2e-16 ***
## GINI_post    -0.5569      0.0830  -6.709 3.88e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.709 on 148 degrees of freedom
## Multiple R-squared:  0.2332, Adjusted R-squared:  0.228
## F-statistic: 45.01 on 1 and 148 DF, p-value: 3.88e-10
```

```
model6 <- lm(LIFEEXP ~ log(GINI_post), lifeexp_data)
summary(model6)
```

```
##
## Call:
## lm(formula = LIFEEXP ~ log(GINI_post), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -20.429  -5.474   1.779   5.902  12.793
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  146.921      11.294  13.009 < 2e-16 ***
## log(GINI_post) -21.152       3.117  -6.786 2.59e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.689 on 148 degrees of freedom
## Multiple R-squared:  0.2373, Adjusted R-squared:  0.2322
## F-statistic: 46.05 on 1 and 148 DF, p-value: 2.591e-10
```

```
model7 <- lm(log(LIFEEXP) ~ GINI_post, lifeexp_data)
summary(model7)
```

```
##
## Call:
## lm(formula = log(LIFEEXP) ~ GINI_post, data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.34381 -0.07779  0.03034  0.08779  0.18807
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.559278   0.048782  93.463 < 2e-16 ***
## GINI_post    -0.008248   0.001259  -6.552 8.84e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.1169 on 148 degrees of freedom
## Multiple R-squared: 0.2248, Adjusted R-squared: 0.2196
## F-statistic: 42.92 on 1 and 148 DF, p-value: 8.842e-10
```

```
model8 <- lm(log(LIFEEXP) ~ log(GINI_post), lifeexp_data)
summary(model8)
```

```
##
## Call:
## lm(formula = log(LIFEEXP) ~ log(GINI_post), data = lifeexp_data)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
##	-0.33757	-0.07700	0.03081	0.09048	0.18680

```
##
## Coefficients:
```

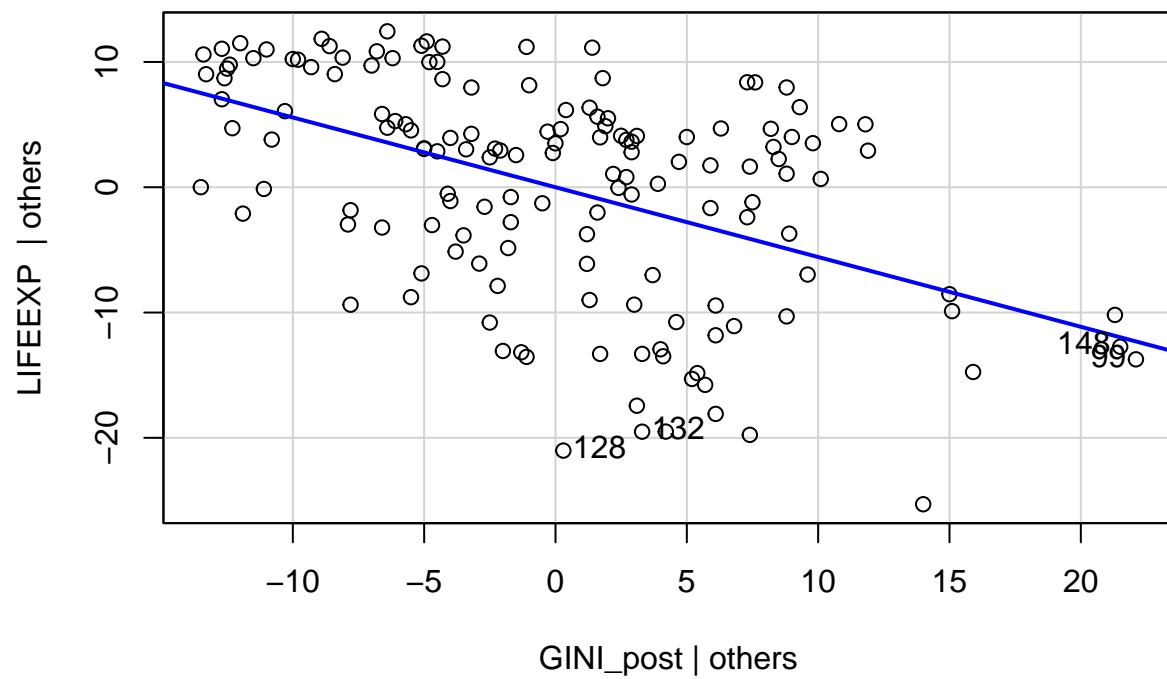
	Estimate	Std. Error	t value	Pr(> t)
## (Intercept)	5.37547	0.17147	31.349	< 2e-16 ***
## log(GINI_post)	-0.31226	0.04733	-6.598	6.95e-10 ***

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1167 on 148 degrees of freedom
## Multiple R-squared: 0.2273, Adjusted R-squared: 0.2221
## F-statistic: 43.53 on 1 and 148 DF, p-value: 6.948e-10
```

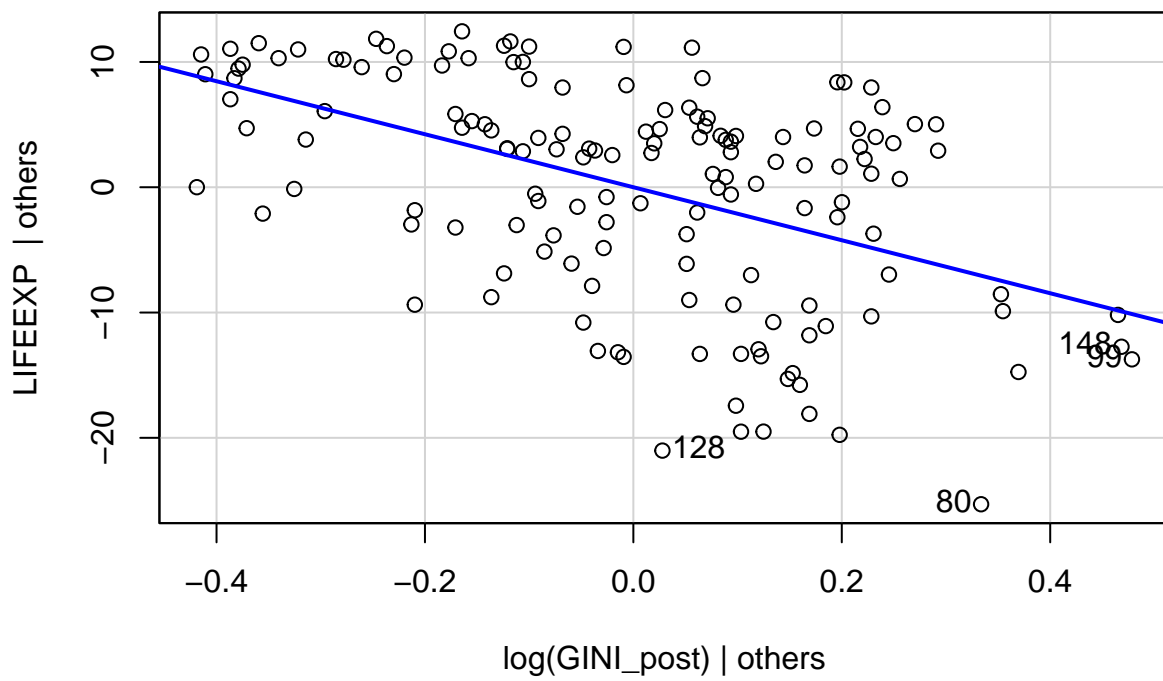
Regressionsdiagnostik

Plots

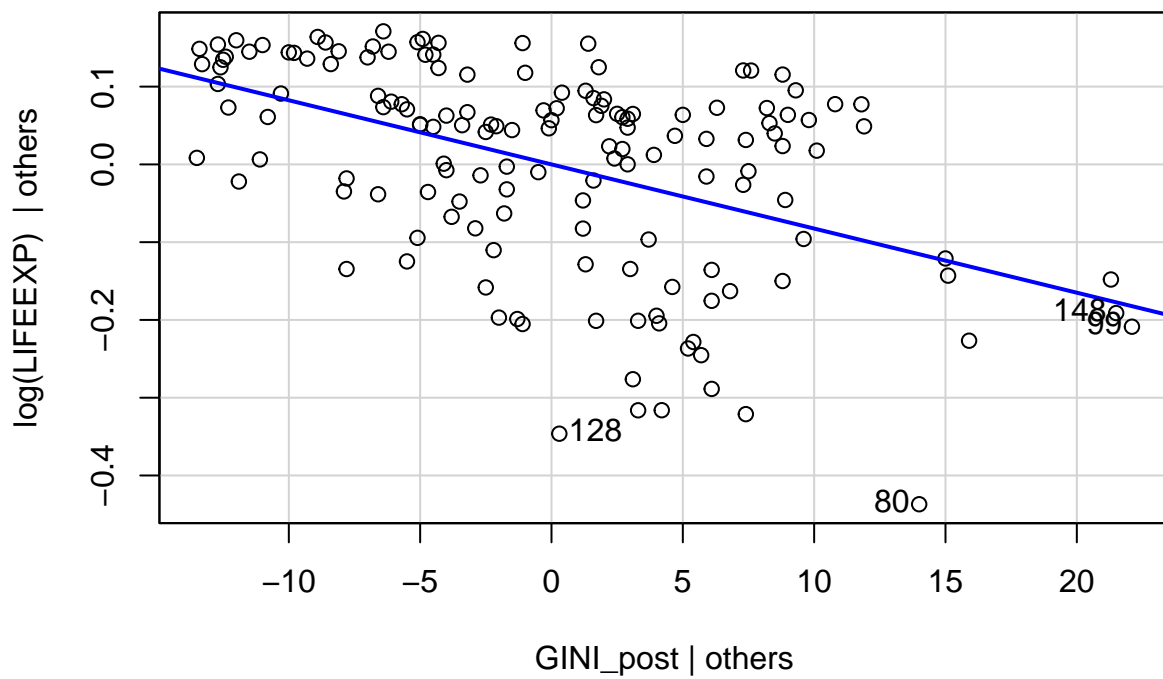
```
avPlots(model5)
```



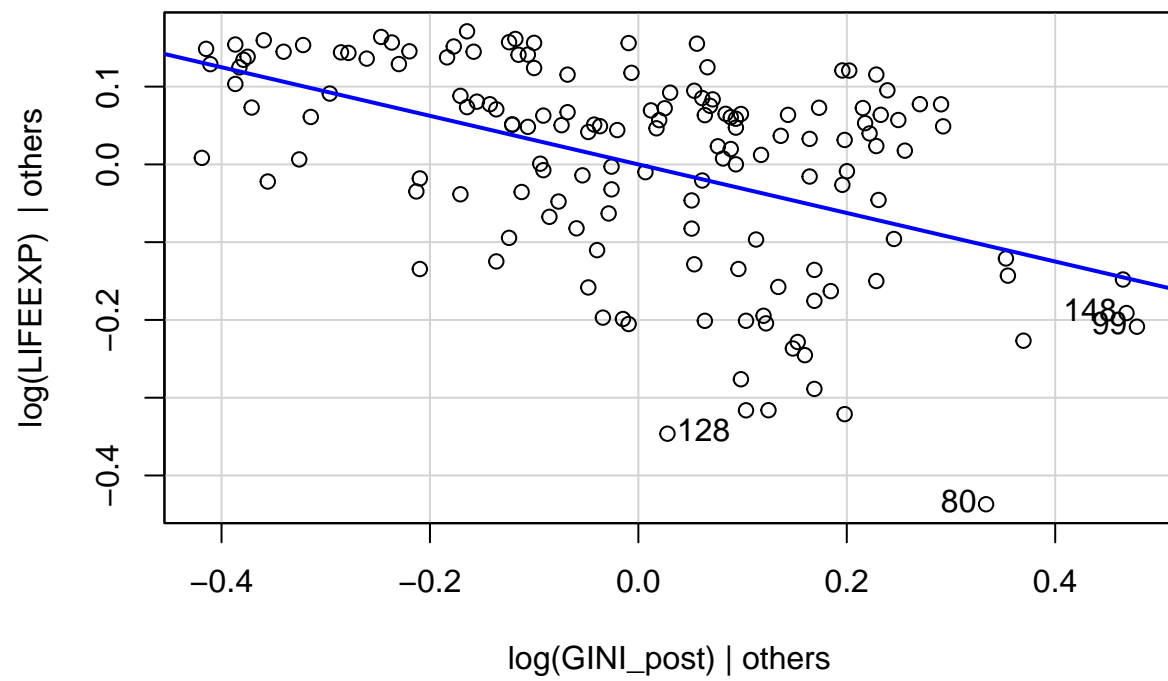
```
avPlots(model6)
```



```
avPlots(model7)
```

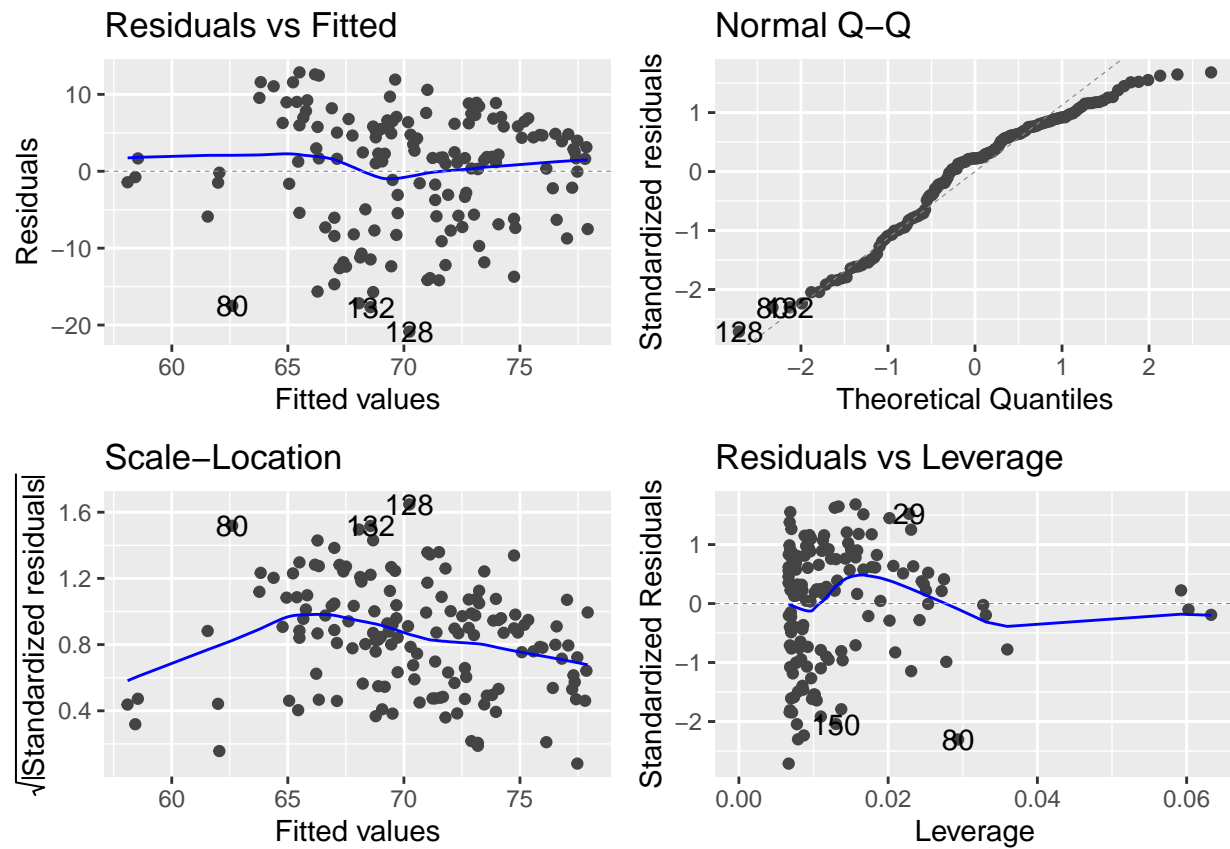



```
avPlots(model8)
```

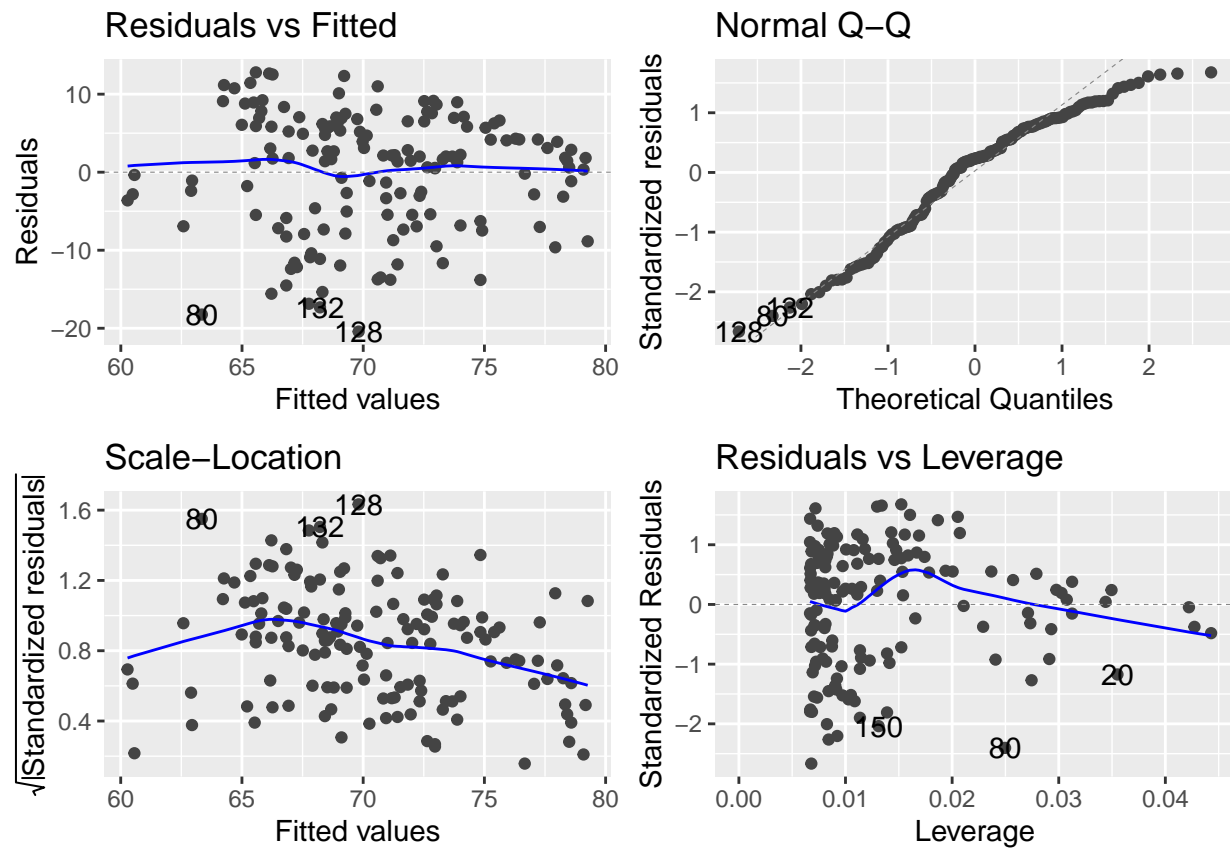


AV-Plot, QQ-Plot, etc.

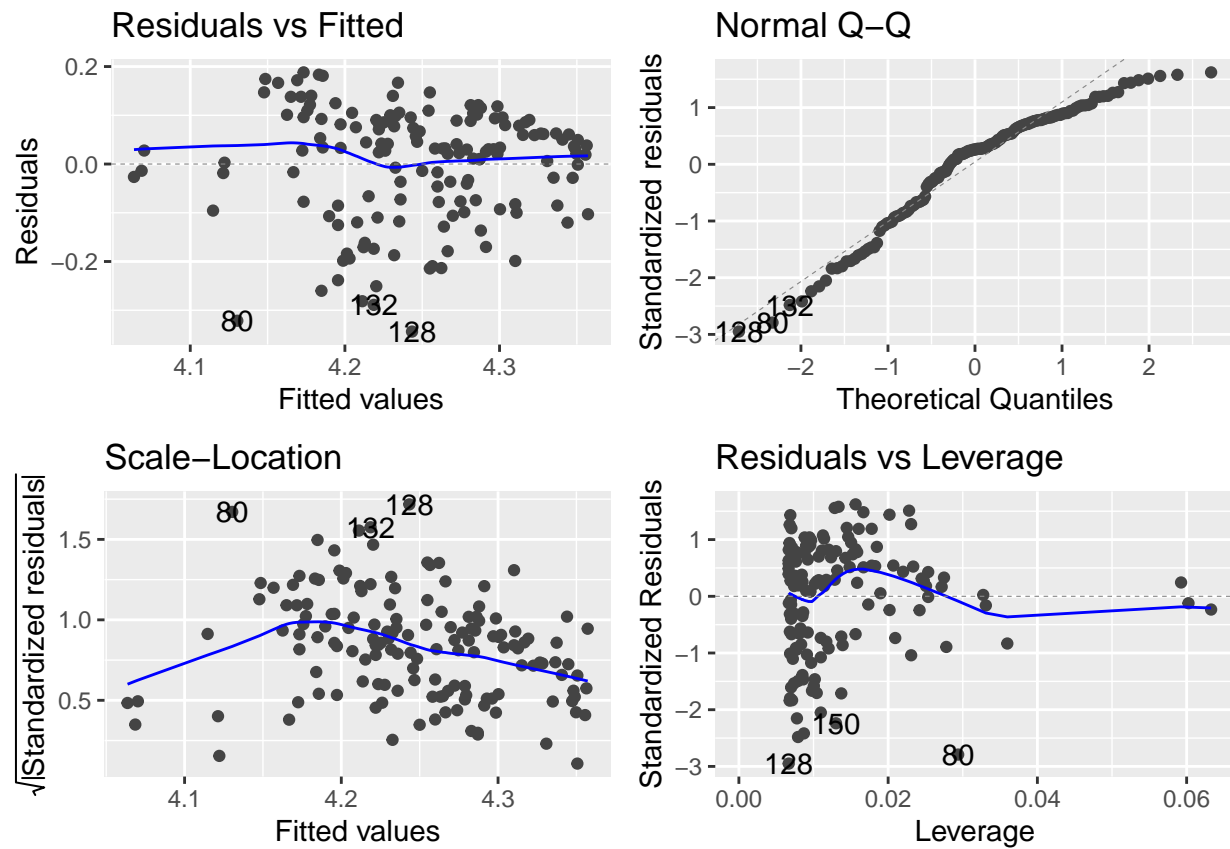
```
autoplot(model5)
```



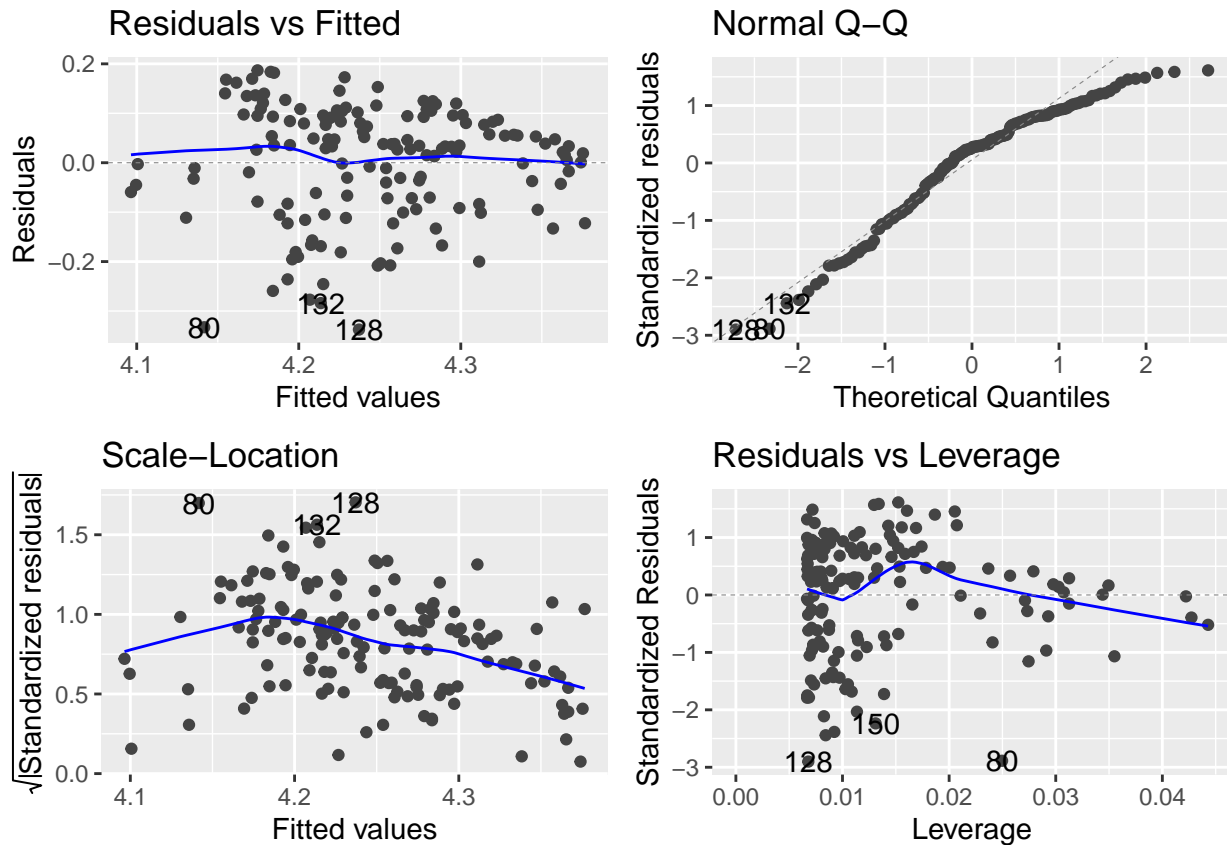
```
autoplot(model16)
```



```
autoplot(model17)
```



```
autoplot(model18)
```



Regressionen: Lebenserwartung und Mordrate

Modelle

```
model9 <- lm(LIFEEXP ~ MORTRATE, lifeexp_data)
summary(model9)
```

```
##
## Call:
## lm(formula = LIFEEXP ~ MORTRATE, data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.7510  -1.8849  -0.0202   2.7103  24.0910
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  77.660903   0.451427  172.03  <2e-16 ***
## MORTRATE     -0.197703   0.008393  -23.56  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.04 on 148 degrees of freedom
```

```
## Multiple R-squared:  0.7894, Adjusted R-squared:  0.788
## F-statistic: 554.9 on 1 and 148 DF,  p-value: < 2.2e-16
```

```
model10 <- lm(LIFEEXP ~ log(MORTRATE), lifeexp_data)
summary(model10)
```

```
##
## Call:
## lm(formula = LIFEEXP ~ log(MORTRATE), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.2422  -2.4686   0.6188   2.6438   7.2300
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   90.9512     0.8155  111.53  <2e-16 ***
## log(MORTRATE)  -6.8608     0.2538  -27.03  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.613 on 148 degrees of freedom
## Multiple R-squared:  0.8316, Adjusted R-squared:  0.8304
## F-statistic: 730.7 on 1 and 148 DF,  p-value: < 2.2e-16
```

```
model11 <- lm(log(LIFEEXP) ~ MORTRATE, lifeexp_data)
summary(model11)
```

```
##
## Call:
## lm(formula = log(LIFEEXP) ~ MORTRATE, data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.24680 -0.02441  0.00357  0.03596  0.37191
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.3558780  0.0066897  651.13  <2e-16 ***
## MORTRATE     -0.0029960  0.0001244  -24.09  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05987 on 148 degrees of freedom
## Multiple R-squared:  0.7968, Adjusted R-squared:  0.7954
## F-statistic: 580.3 on 1 and 148 DF,  p-value: < 2.2e-16
```

```
model12 <- lm(log(LIFEEXP) ~ log(MORTRATE), lifeexp_data)
summary(model12)
```

```
##
## Call:
```

```
## lm(formula = log(LIFEEXP) ~ log(MORTRATE), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.273439 -0.035280  0.006534  0.045288  0.111400
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.54976    0.01343  338.81  <2e-16 ***
## log(MORTRATE) -0.10146    0.00418  -24.27  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0595 on 148 degrees of freedom
## Multiple R-squared:  0.7993, Adjusted R-squared:  0.7979
## F-statistic: 589.3 on 1 and 148 DF,  p-value: < 2.2e-16
```

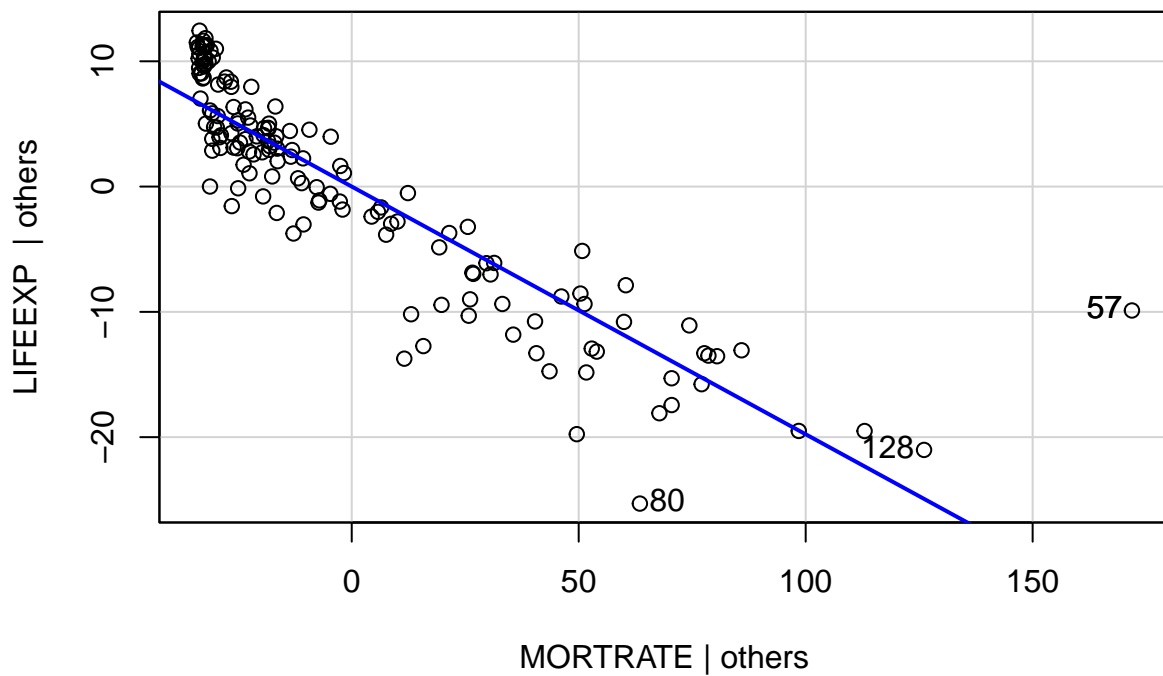
```
model13 <- lm(LIFEEXP ~ log(MORTRATE) + I(log(MORTRATE)**2), lifeexp_data)
summary(model13)
```

```
##
## Call:
## lm(formula = LIFEEXP ~ log(MORTRATE) + I(log(MORTRATE)^2), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.5721  -1.9768   0.7967   2.0409  11.8451
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    80.3630     1.7784  45.188 < 2e-16 ***
## log(MORTRATE)     1.4221     1.2914   1.101  0.273
## I(log(MORTRATE)^2) -1.3777     0.2115  -6.513 1.1e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.194 on 147 degrees of freedom
## Multiple R-squared:  0.8693, Adjusted R-squared:  0.8675
## F-statistic: 488.8 on 2 and 147 DF,  p-value: < 2.2e-16
```

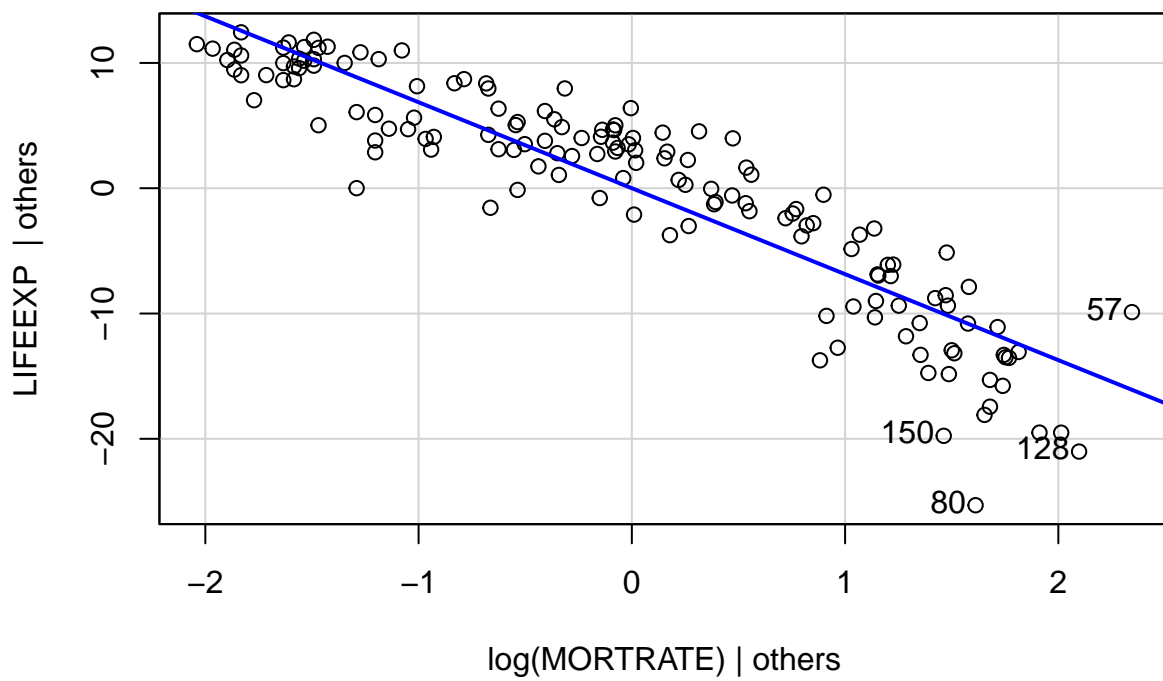
Regressionsdiagnostik

Scatter Plots

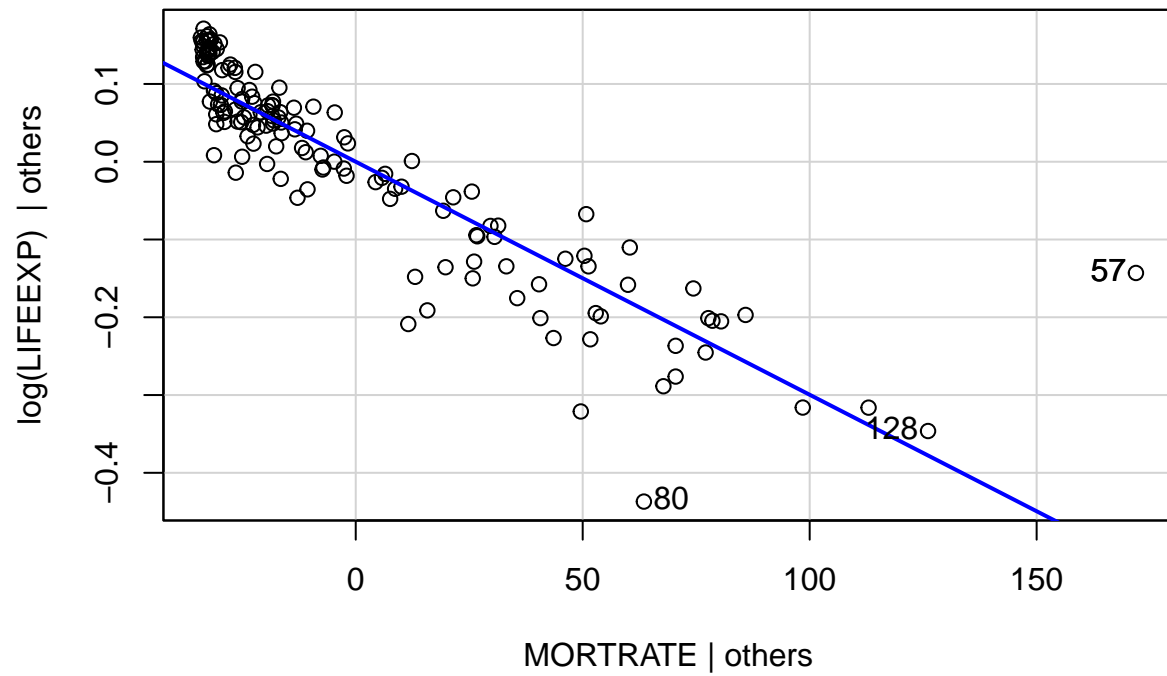
```
avPlots(model9)
```

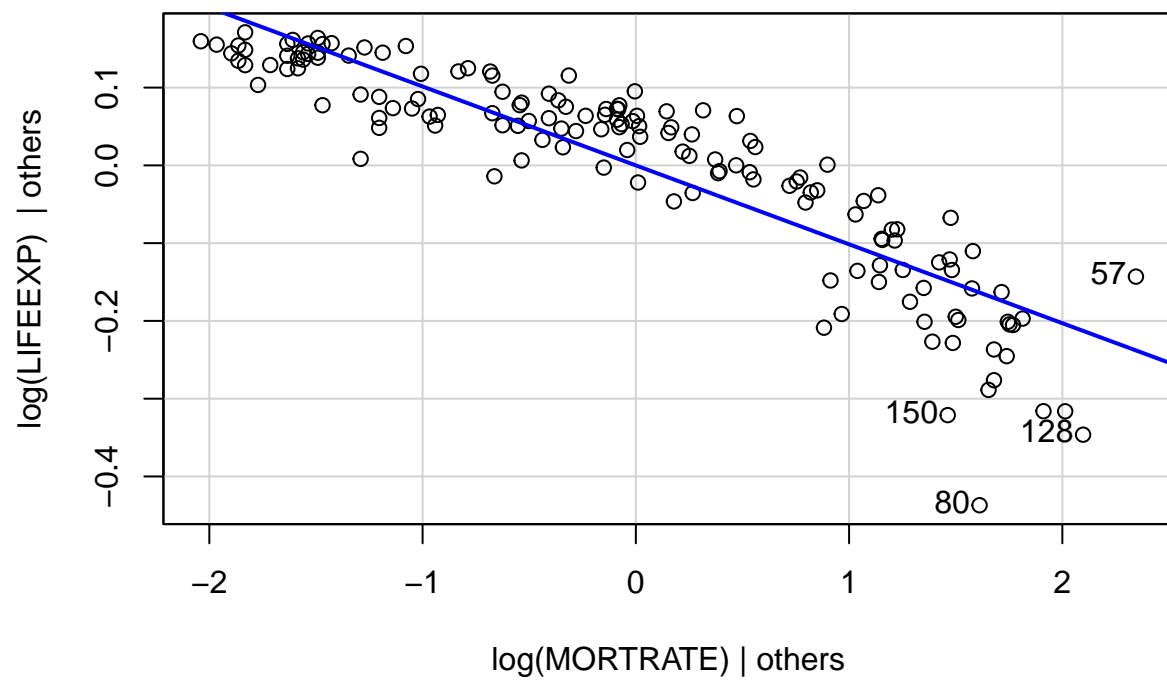
```
avPlots(model10)
```



```
avPlots(model11)
```

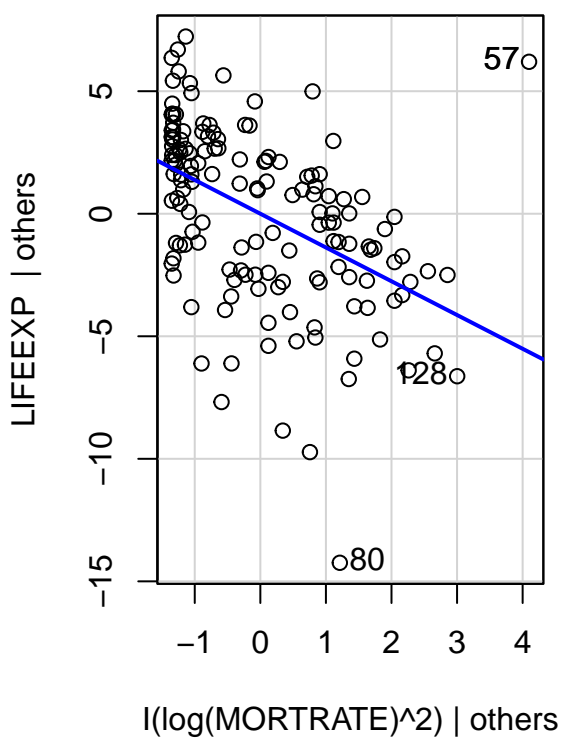
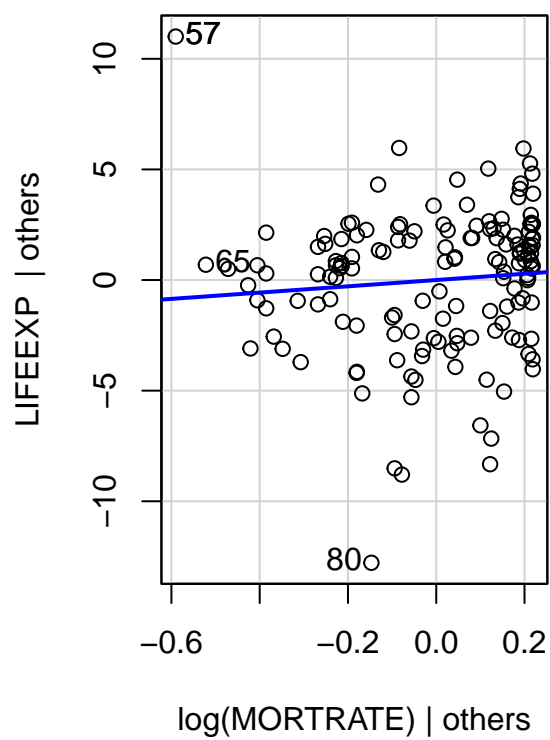


```
avPlots(model12)
```

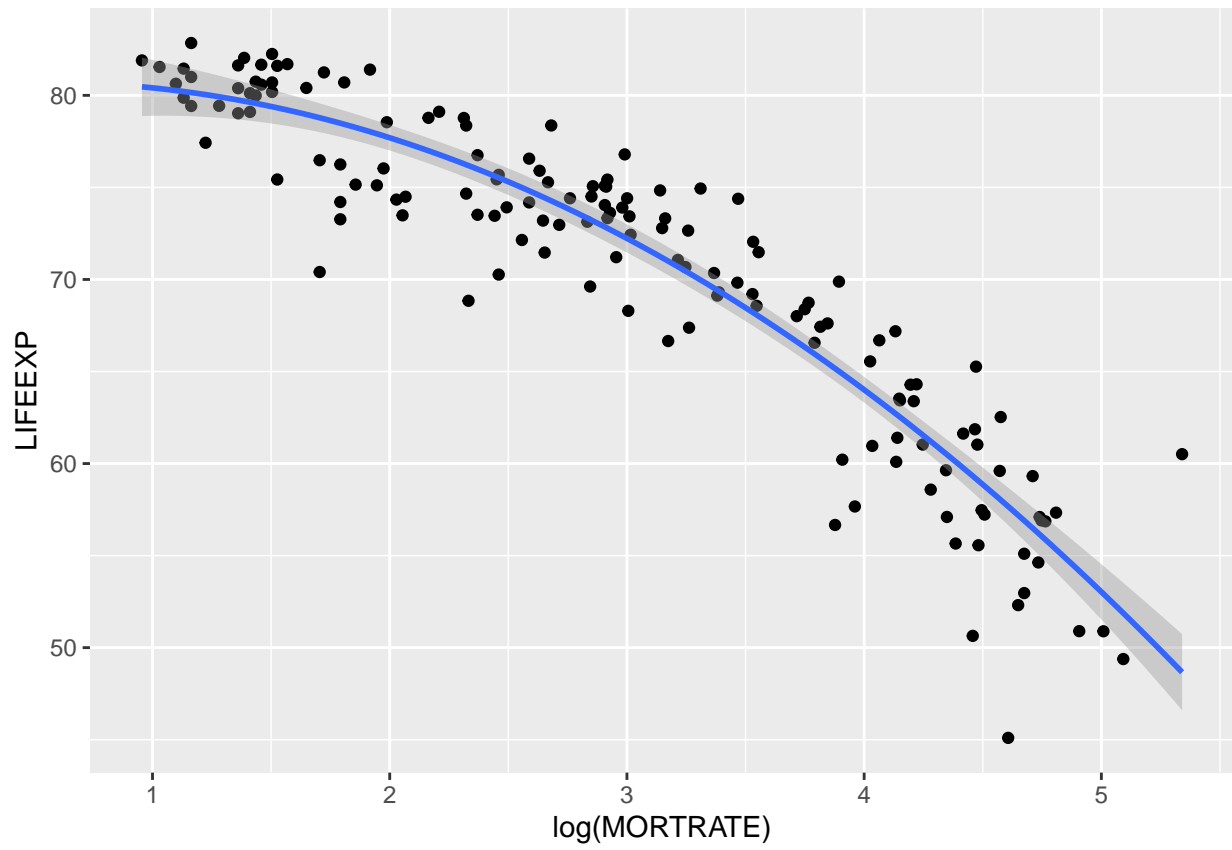


```
avPlots(model13)
```

Added-Variable Plots

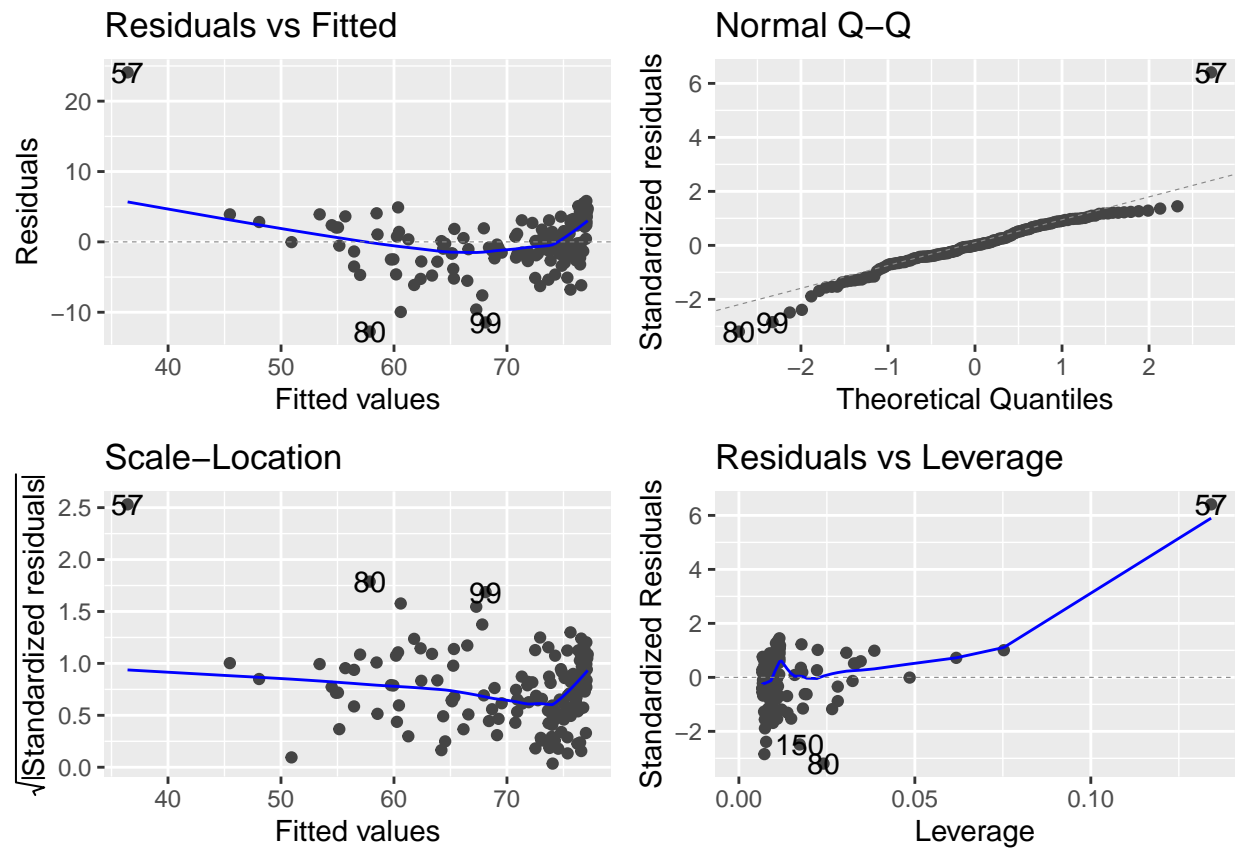


```
ggplot(lifeexp_data, aes(x = log(MORTRATE), y = LIFEEXP))+
  geom_point()+
  geom_smooth(method = "lm", formula = y ~ x + I(x**2))
```

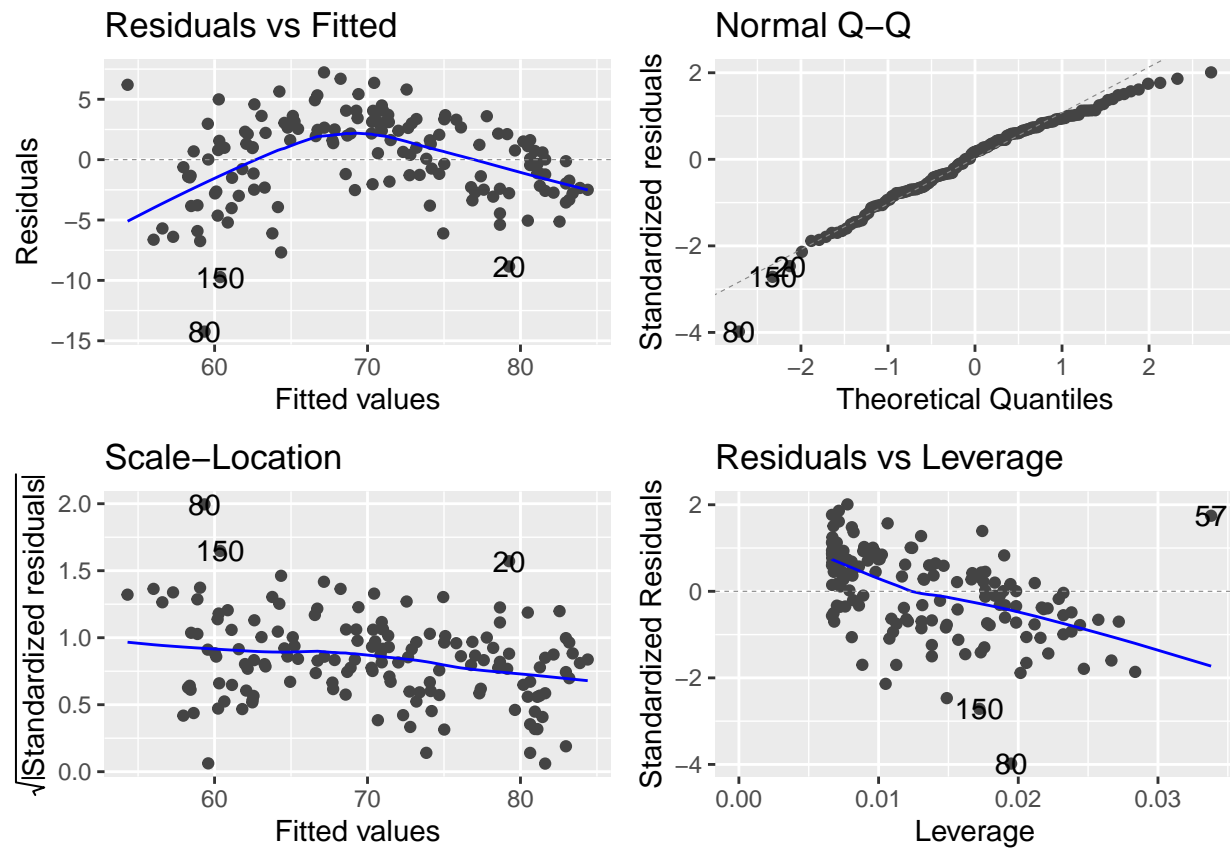


TA-, QQ-Plots, etc.

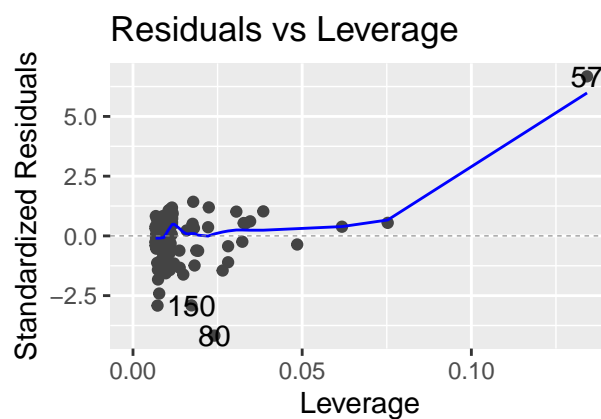
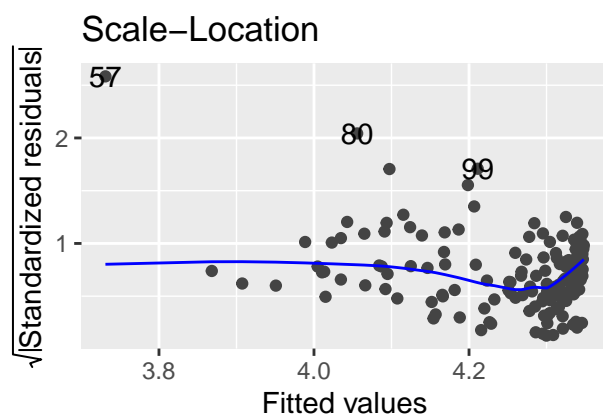
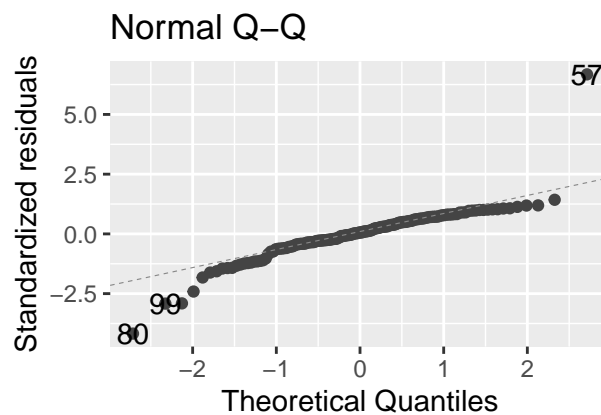
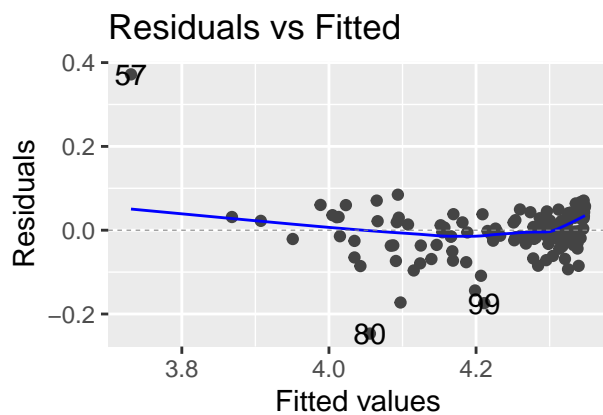
```
autoplot(model9)
```



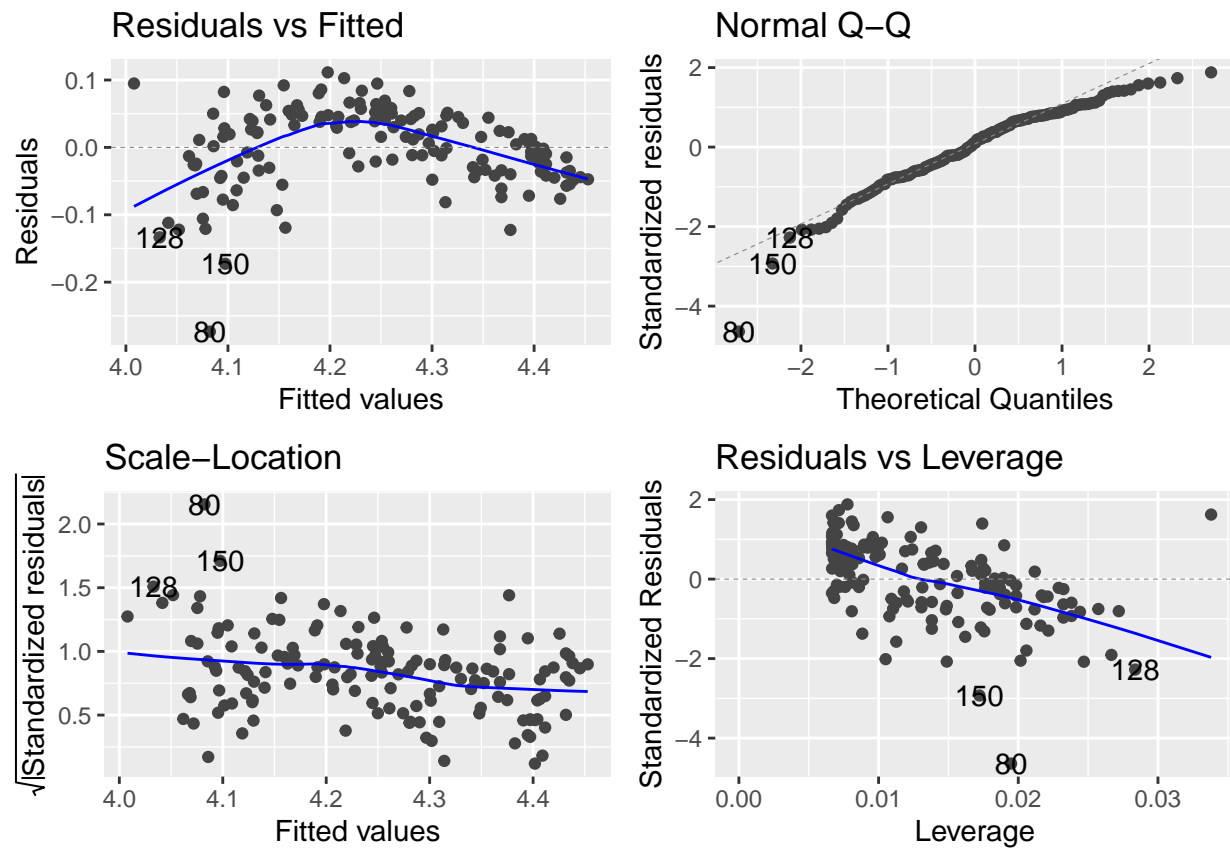
```
autoplot(model110)
```



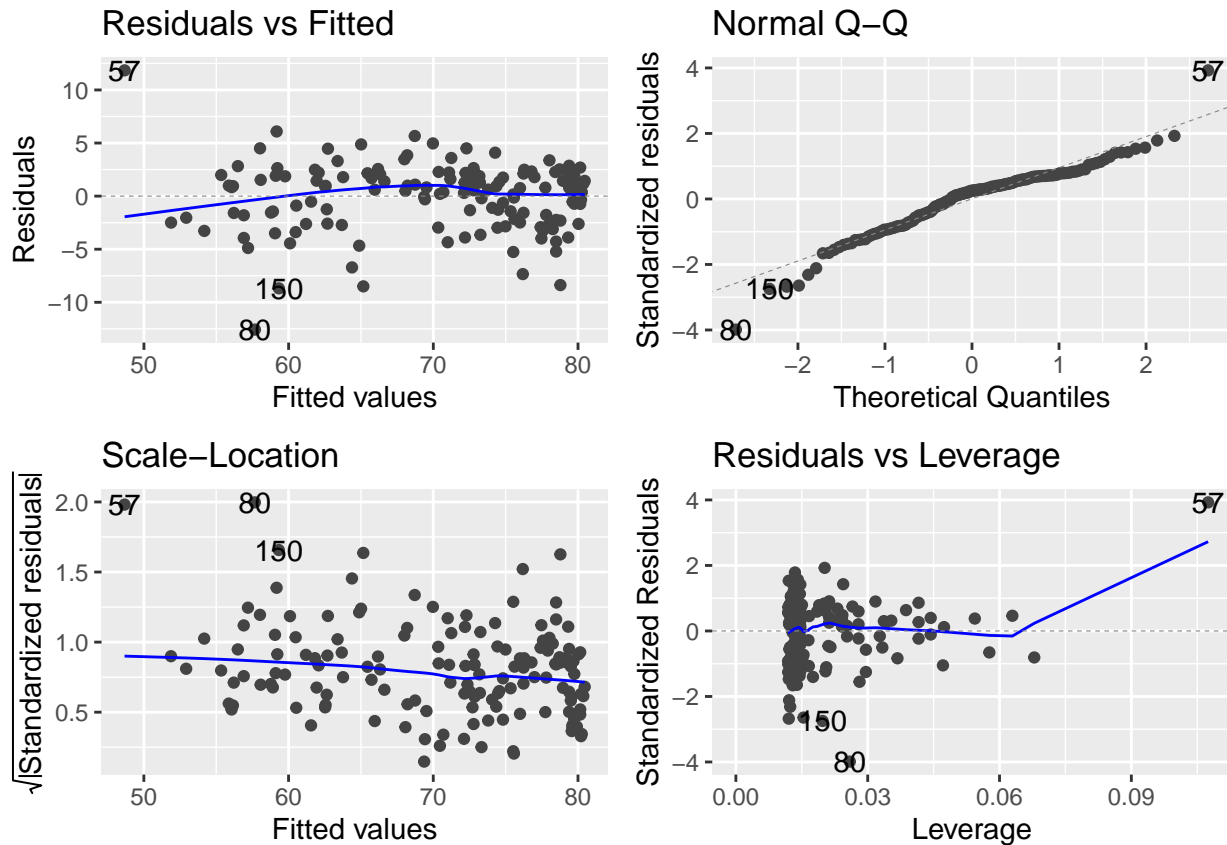
```
autoplot(model111)
```



```
autoplot(model112)
```



```
autoplot(model113)
```

Multivariate Regression

Modelle und Diagnostik

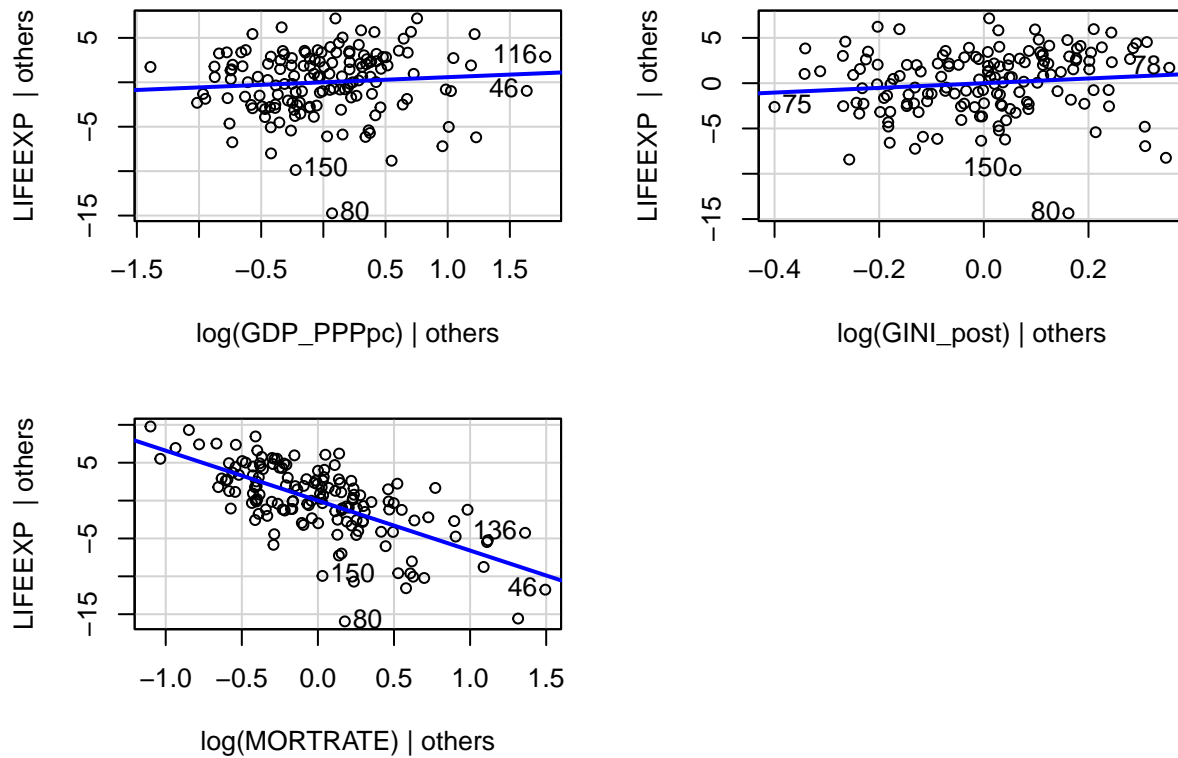
```
model14 <- lm(LIFEEXP ~ log(GDP_PPPpc) + log(GINI_post) + log(MORTRATE), lifeexp_data)
summary(model14)
```

```
##
## Call:
## lm(formula = LIFEEXP ~ log(GDP_PPPpc) + log(GINI_post) + log(MORTRATE),
##     data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.7835  -2.2006   0.5298   2.4959   7.1272
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    75.5242     8.0831   9.343  <2e-16 ***
## log(GDP_PPPpc)  0.5763     0.5473   1.053   0.294
## log(GINI_post)  2.5931     1.8210   1.424   0.157
## log(MORTRATE)  -6.5934     0.6189 -10.654 <2e-16 ***
## ---
```

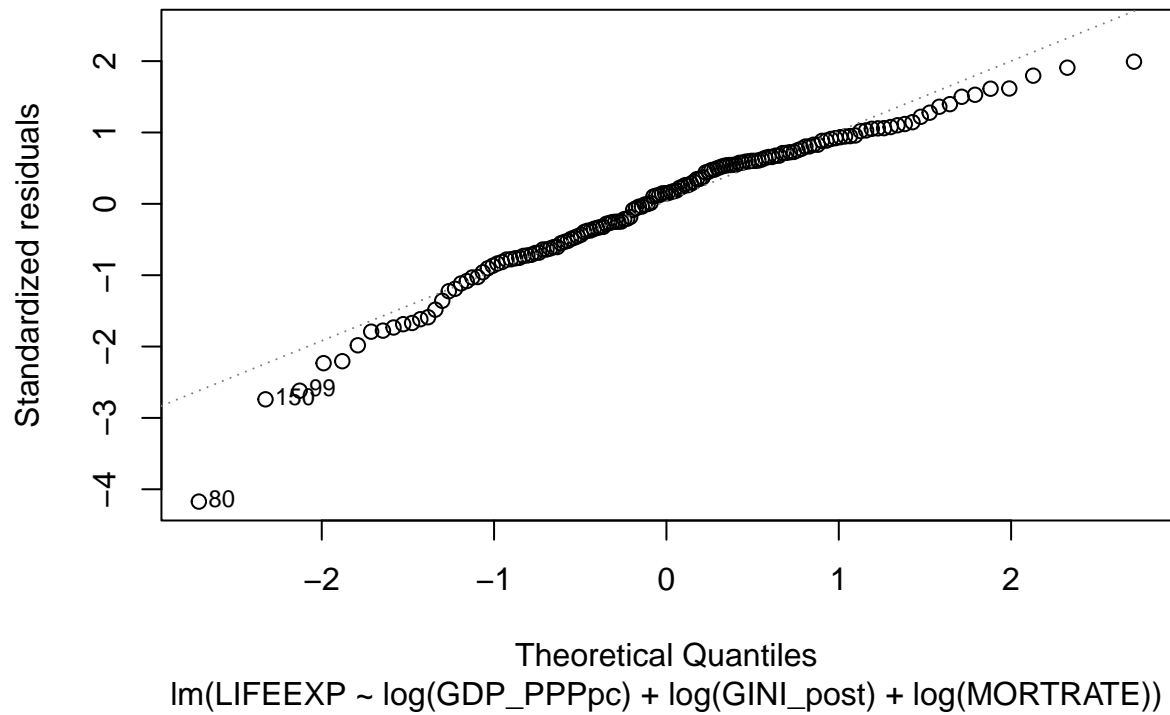
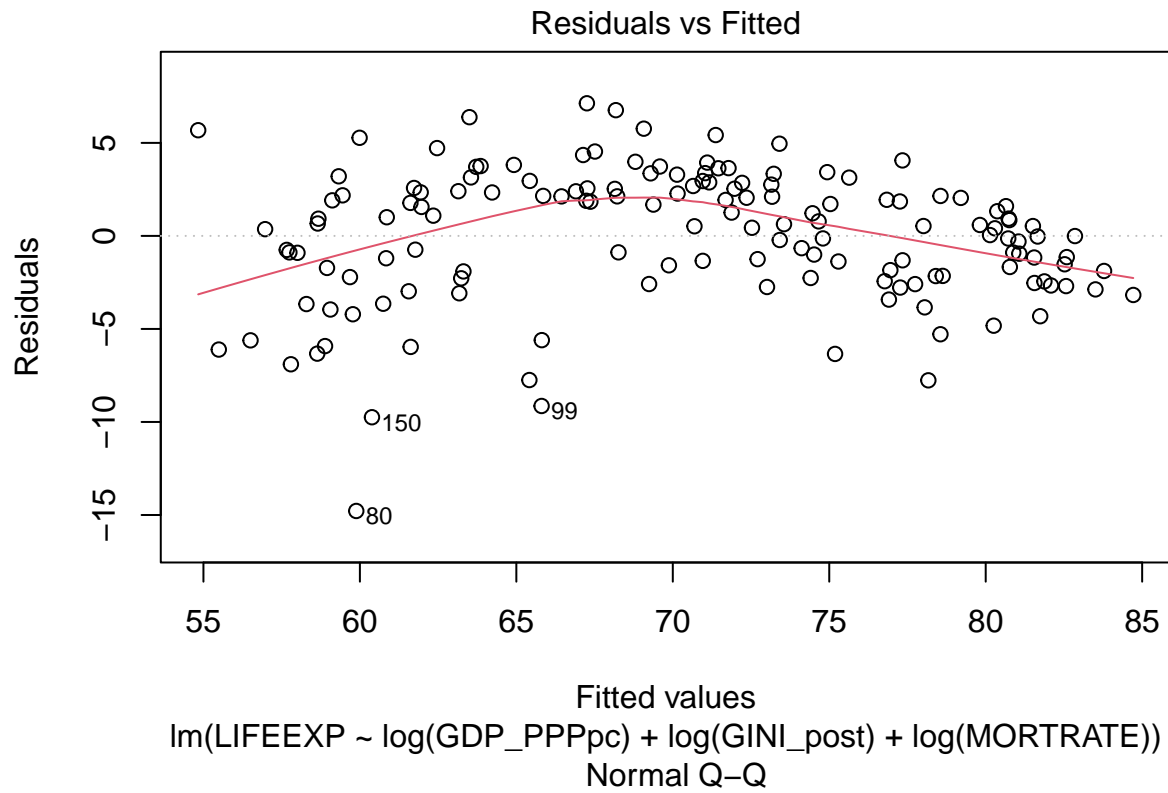
```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.592 on 146 degrees of freedom
## Multiple R-squared:  0.8358, Adjusted R-squared:  0.8324
## F-statistic: 247.8 on 3 and 146 DF,  p-value: < 2.2e-16
```

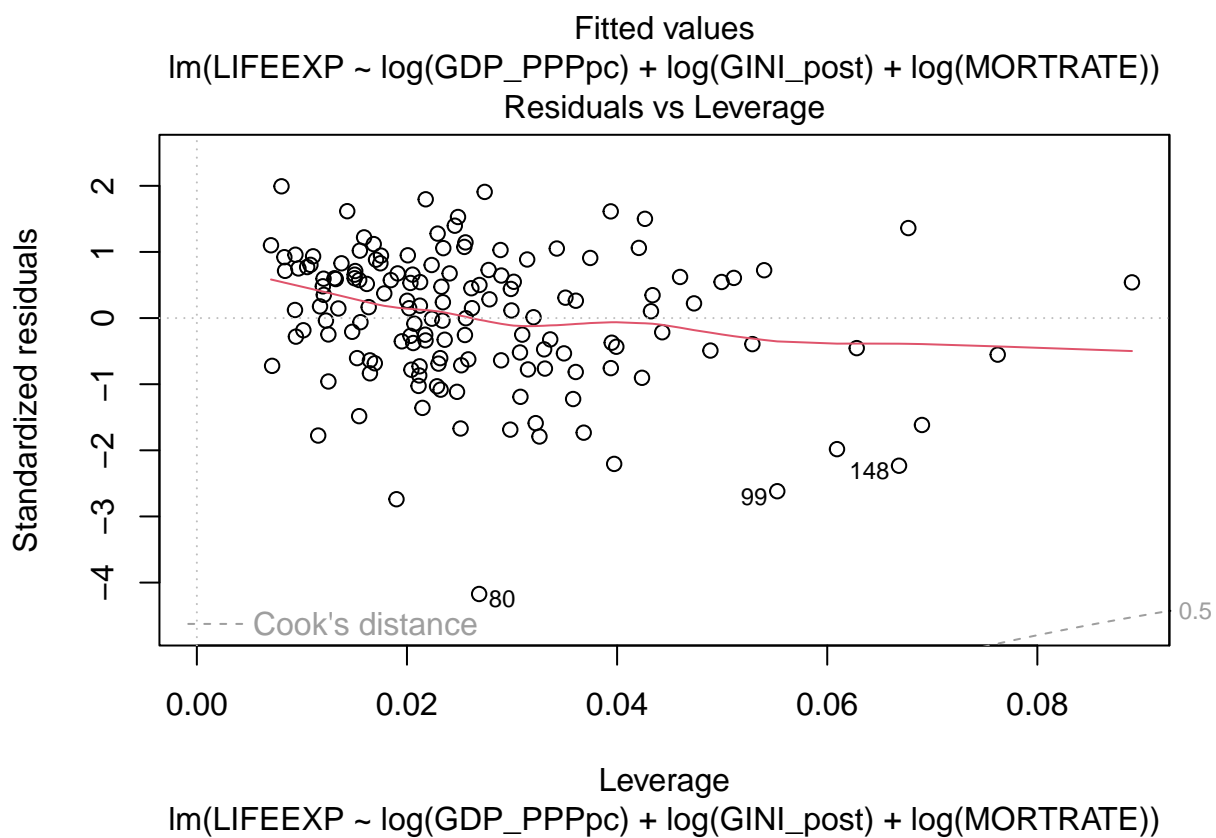
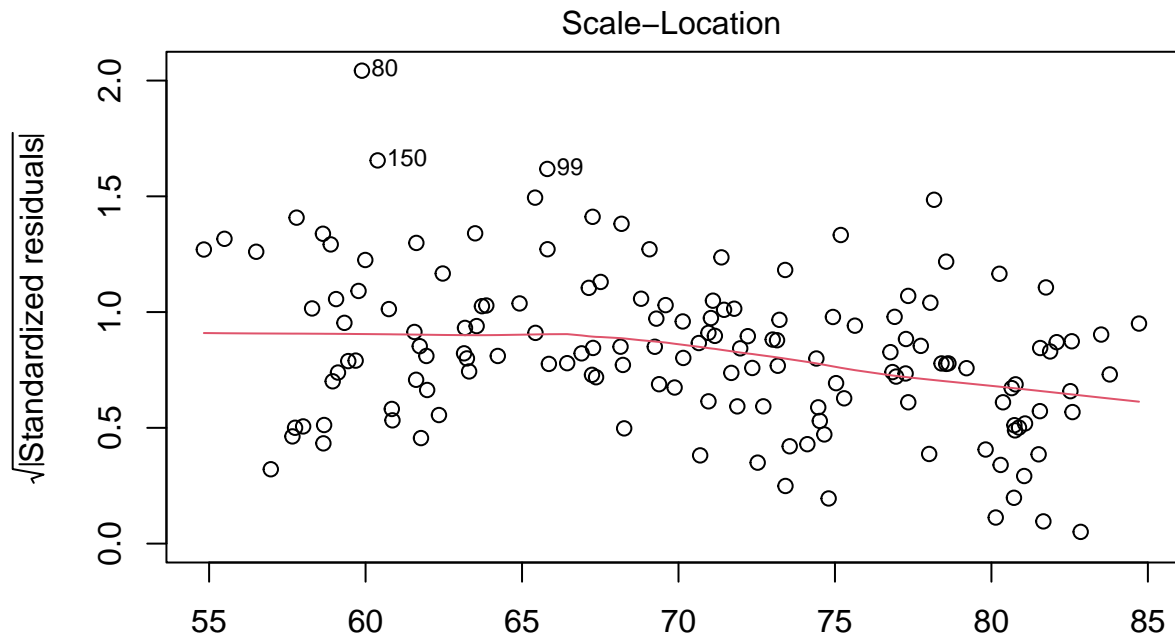
```
avPlots(model14)
```

Added-Variable Plots



```
plot(model14)
```





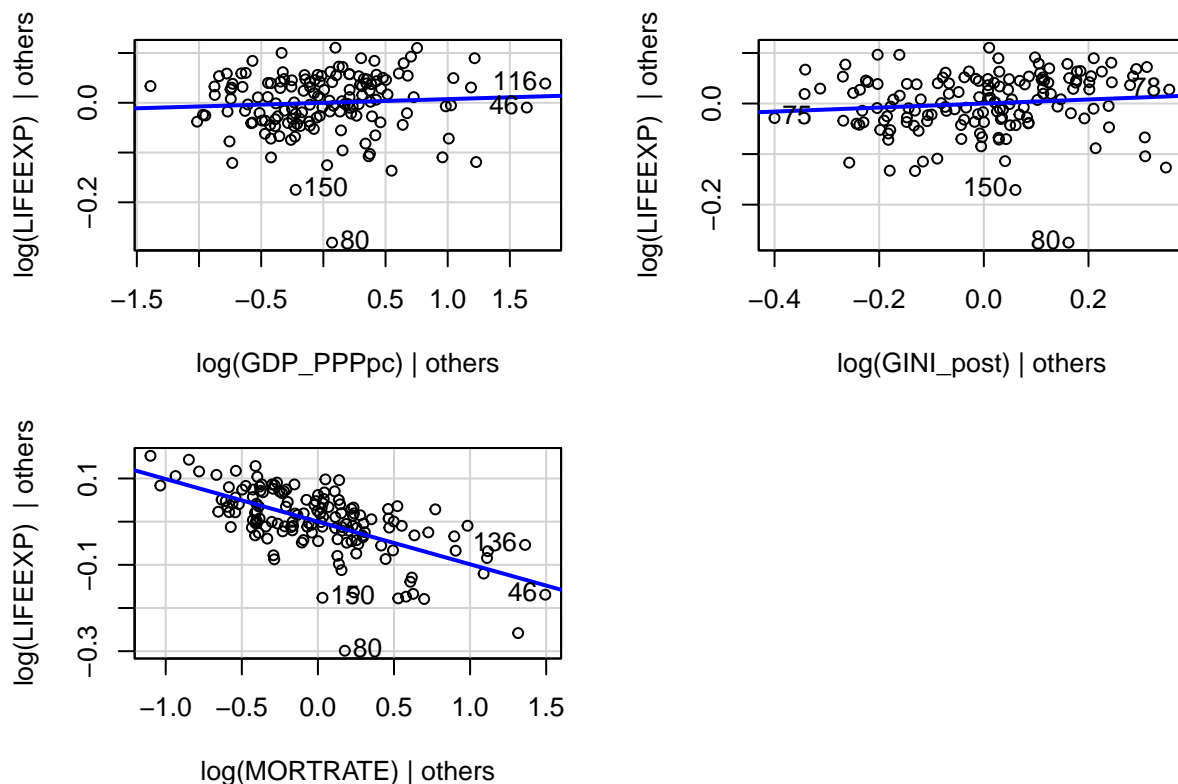
```
model115 <- lm(log(LIFEEXP) ~ log(GDP_PPPpc) + log(GINI_post) + log(MORTRATE), lifeexp_data)
summary(model115)
```

```
##
## Call:
```

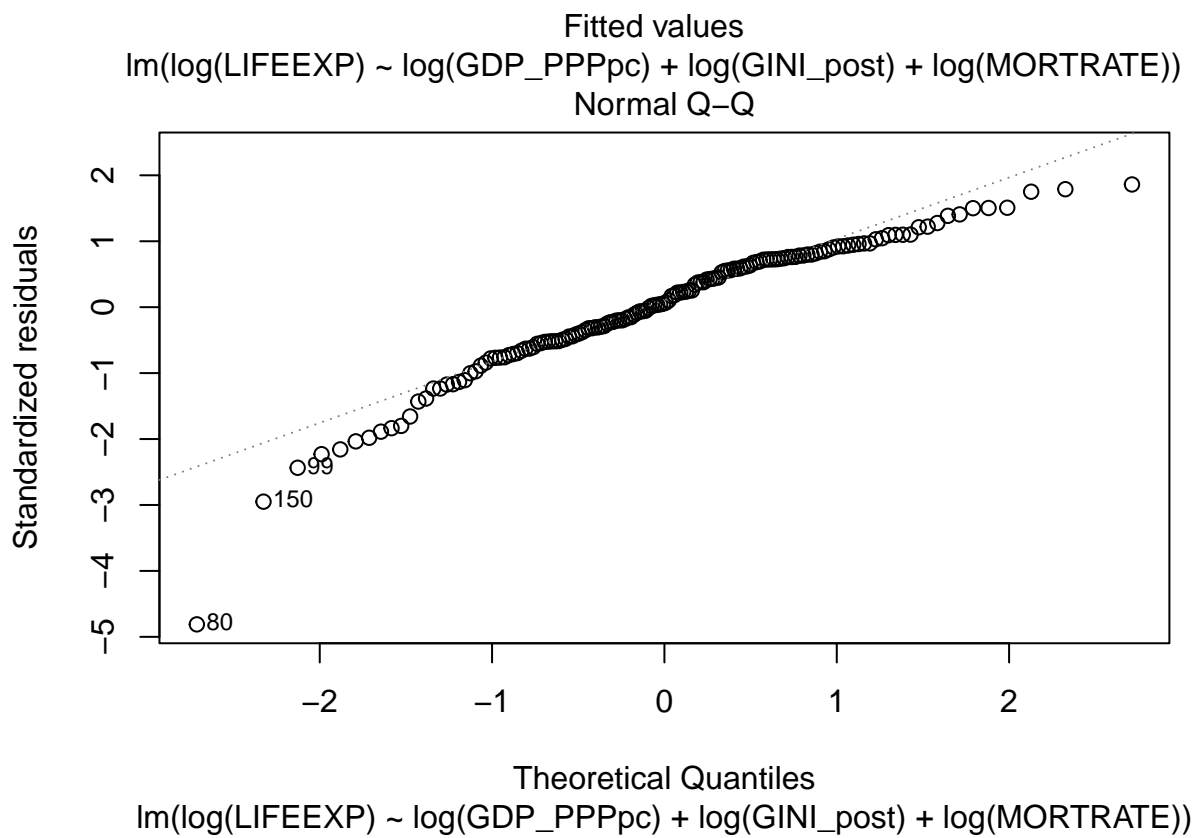
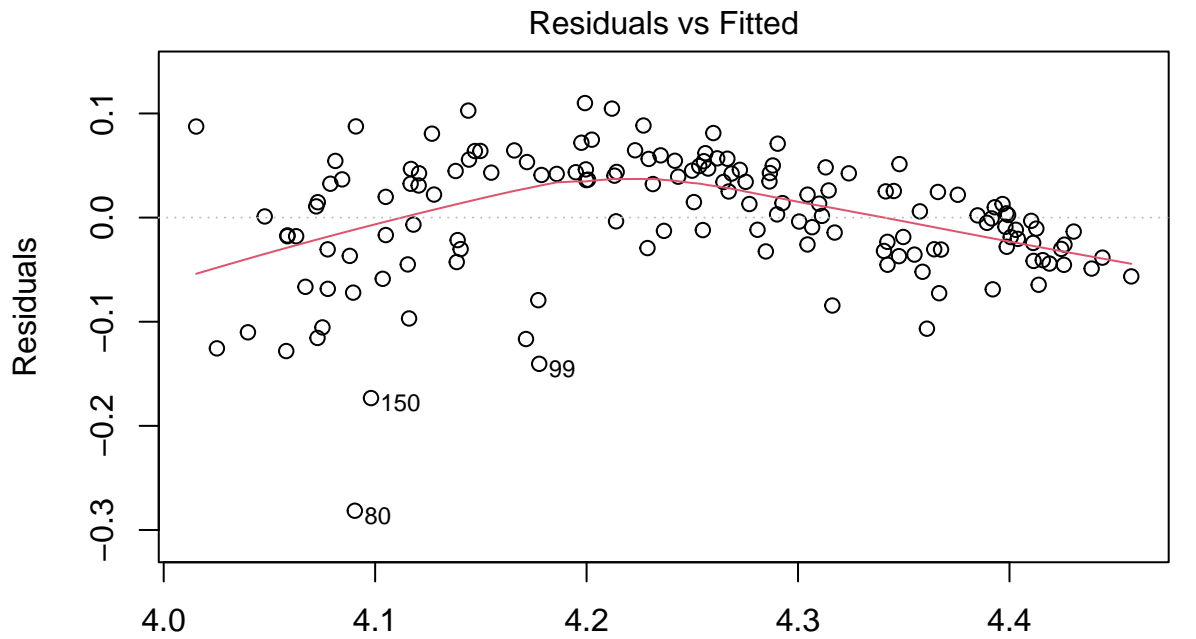
```
## lm(formula = log(LIFEEXP) ~ log(GDP_PPPpc) + log(GINI_post) +
##     log(MORTRATE), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.281505 -0.030490  0.003568  0.043083  0.109980
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.330392   0.133503  32.437  <2e-16 ***
## log(GDP_PPPpc) 0.007357   0.009039   0.814   0.417
## log(GINI_post) 0.039869   0.030076   1.326   0.187
## log(MORTRATE) -0.098729   0.010221  -9.659  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.05932 on 146 degrees of freedom
## Multiple R-squared:  0.8032, Adjusted R-squared:  0.7991
## F-statistic: 198.6 on 3 and 146 DF,  p-value: < 2.2e-16
```

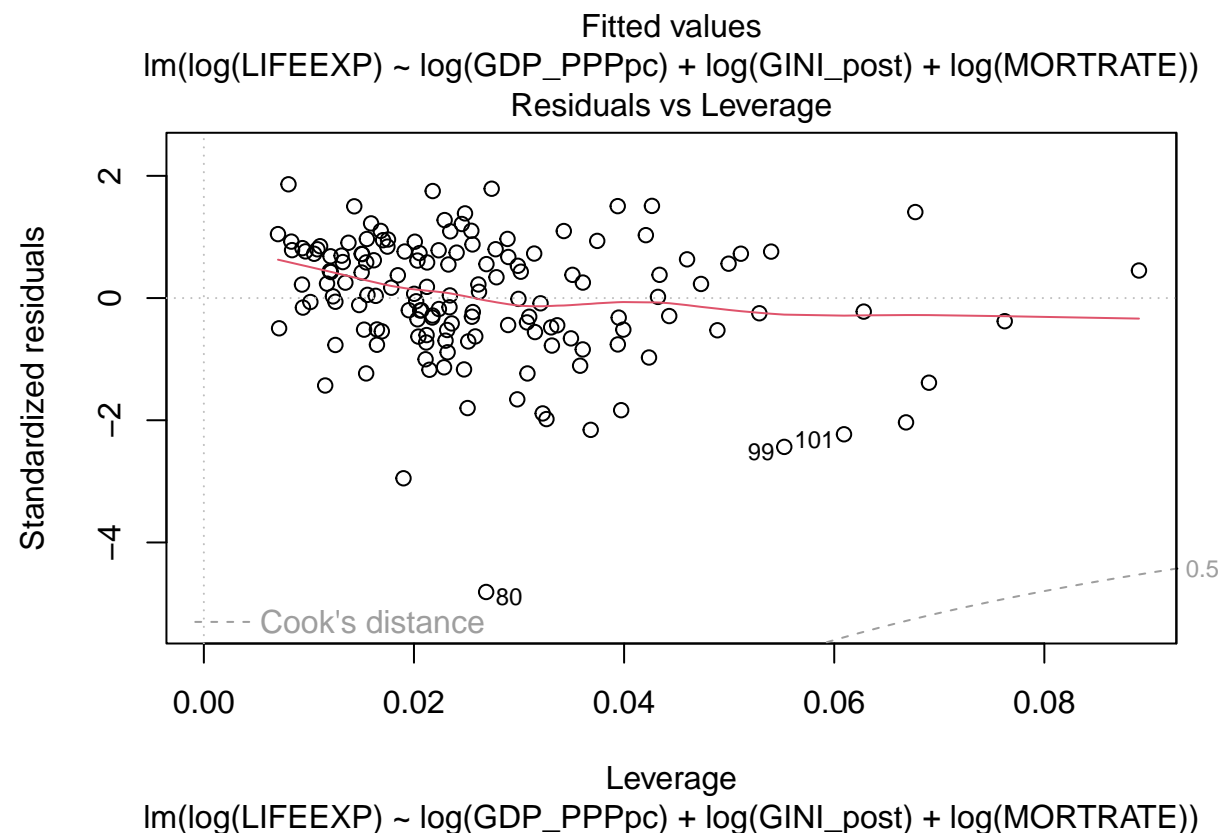
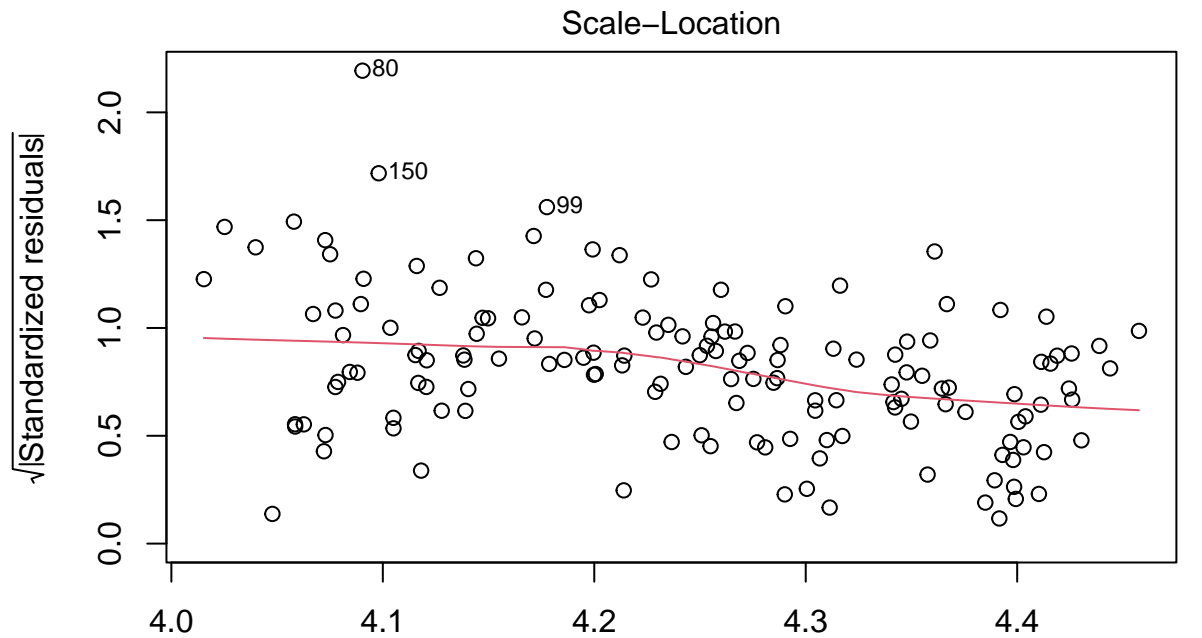
```
avPlots(model15)
```

Added-Variable Plots



```
plot(model15)
```





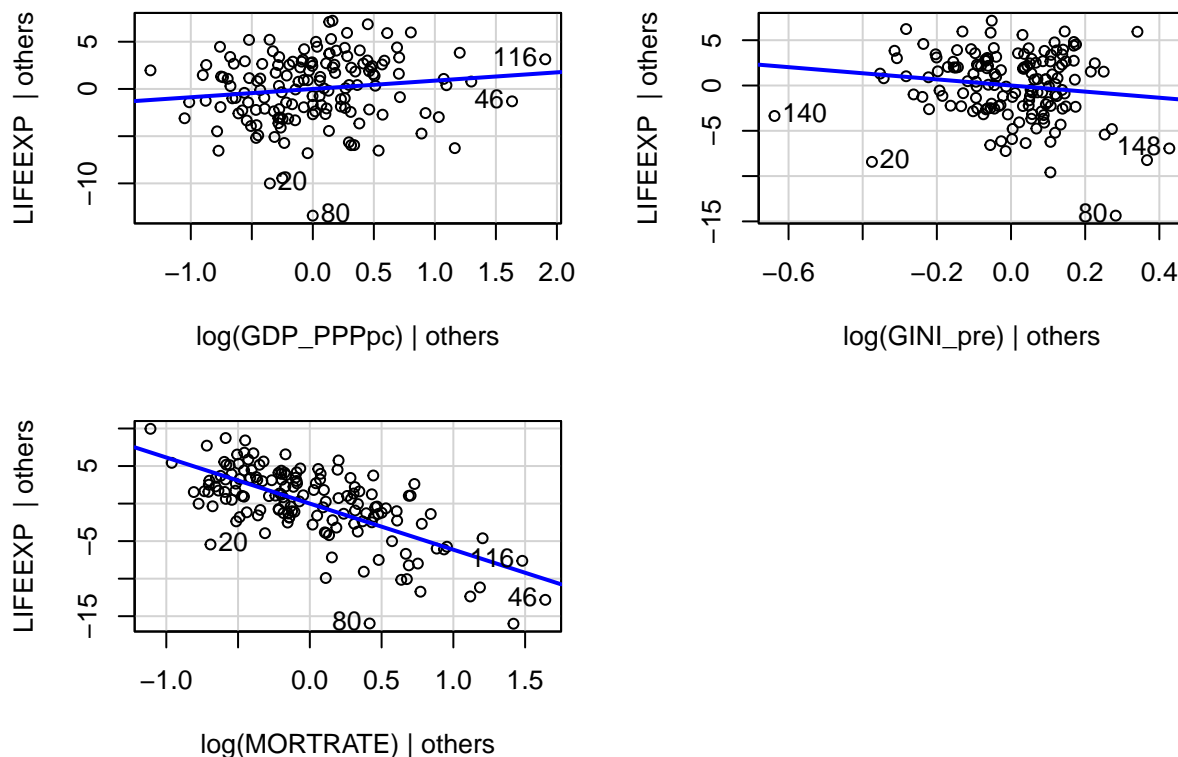
```
model116 <- lm(LIFEEXP ~ log(GDP_PPPpc) + log(GINI_pre) + log(MORTRATE), lifeexp_data)
summary(model116)
```

```
##
## Call:
```

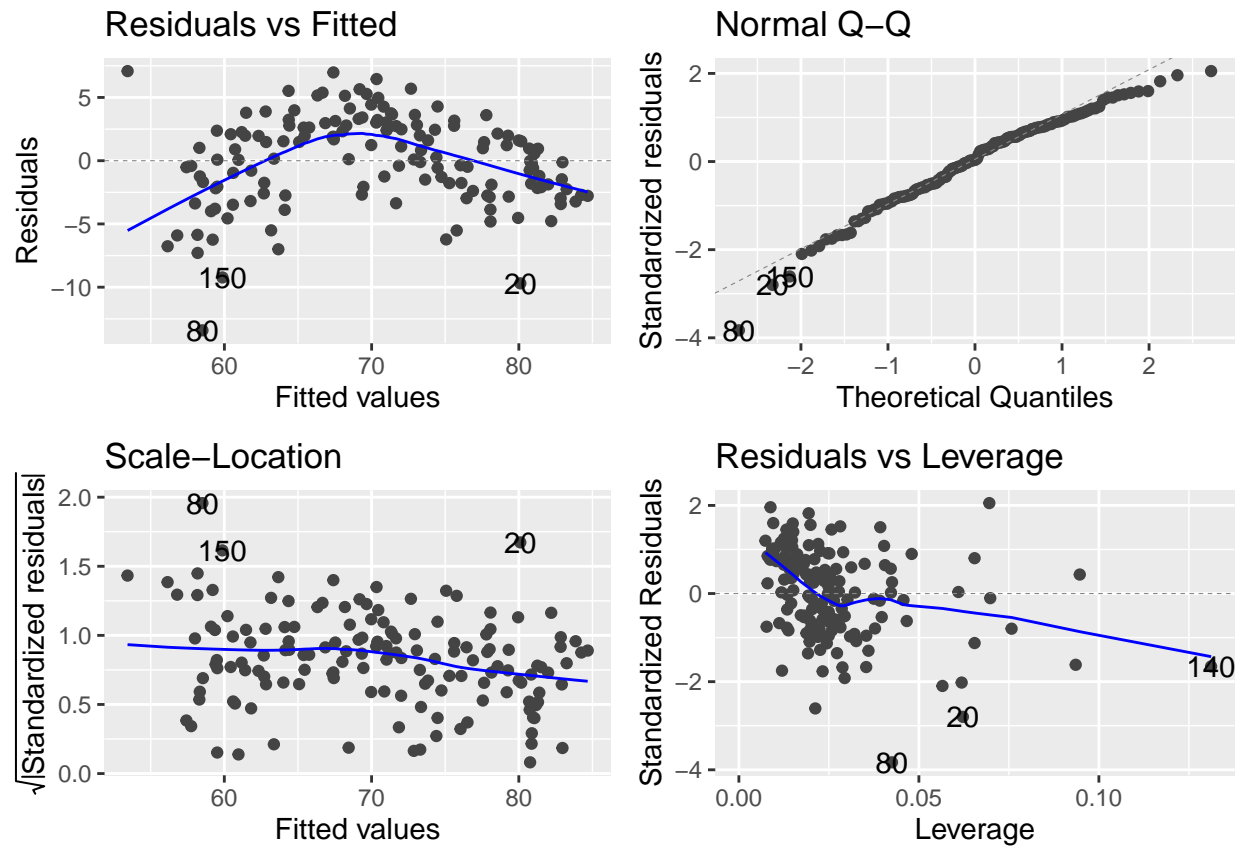
```
## lm(formula = LIFEEXP ~ log(GDP_PPPpc) + log(GINI_pre) + log(MORTRATE),
##     data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.4107  -2.2339   0.1399   2.6224   7.0845
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    93.5269     8.9175  10.488 <2e-16 ***
## log(GDP_PPPpc)   0.8783     0.5445   1.613  0.109
## log(GINI_pre)   -3.3764     1.8956  -1.781  0.077 .
## log(MORTRATE)   -6.1423     0.5545 -11.078 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.578 on 146 degrees of freedom
## Multiple R-squared:  0.8371, Adjusted R-squared:  0.8337
## F-statistic: 250 on 3 and 146 DF, p-value: < 2.2e-16
```

```
avPlots(model16)
```

Added-Variable Plots




```
autoplot(model16)
```

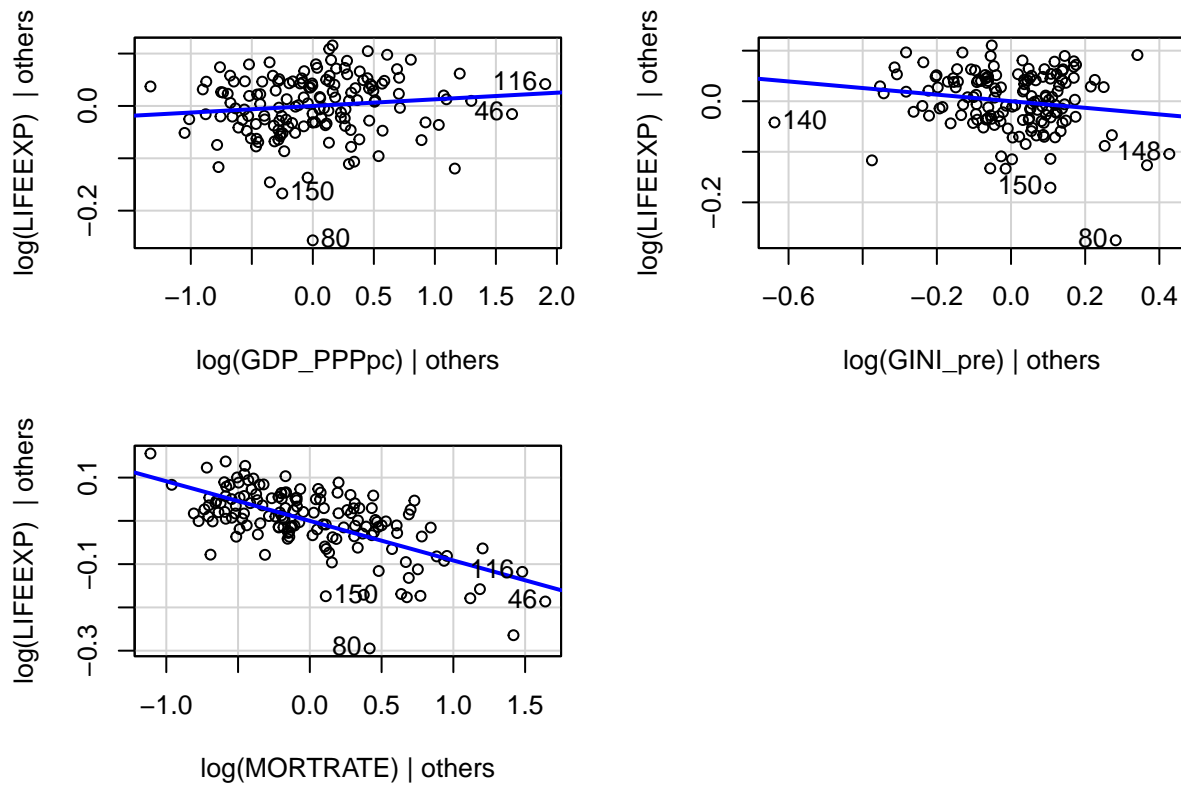


```
model17 <- lm(log(LIFEEXP) ~ log(GDP_PPPpc) + log(GINI_pre) + log(MORTRATE), lifeexp_data)
summary(model17)
```

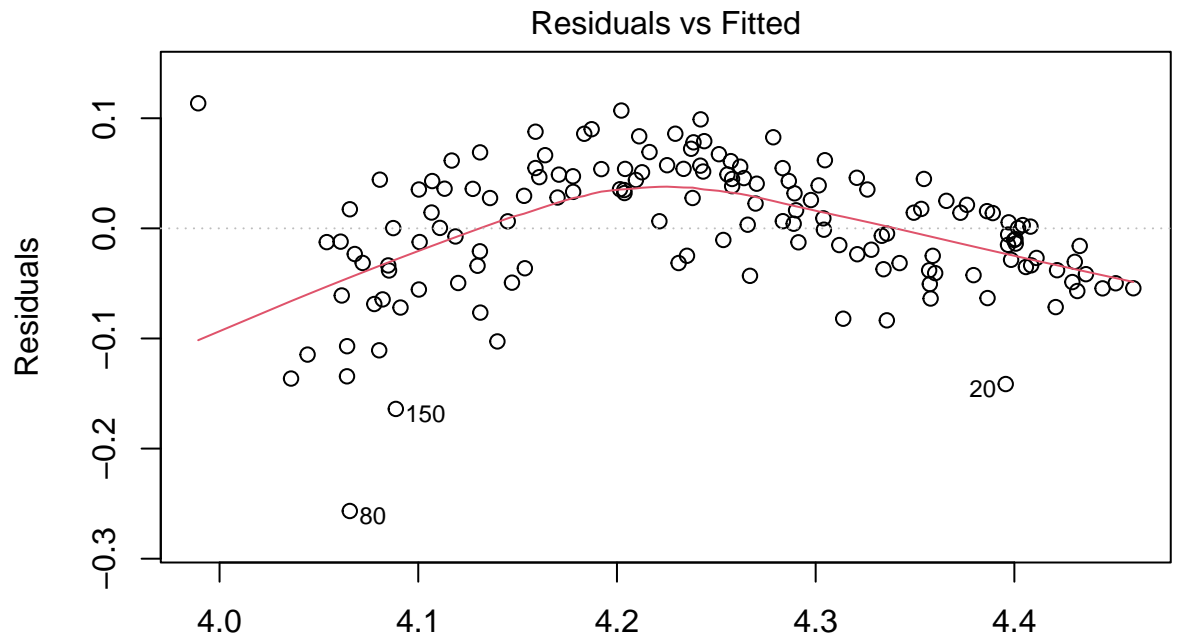
```
##
## Call:
## lm(formula = log(LIFEEXP) ~ log(GDP_PPPpc) + log(GINI_pre) +
##     log(MORTRATE), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.256659 -0.034799  0.003779  0.044156  0.113580
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.650472   0.146561  31.731  <2e-16 ***
## log(GDP_PPPpc) 0.012645   0.008949   1.413   0.1598
## log(GINI_pre) -0.065151   0.031155  -2.091   0.0382 *
## log(MORTRATE) -0.091548   0.009113 -10.046  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0588 on 146 degrees of freedom
## Multiple R-squared:  0.8066, Adjusted R-squared:  0.8026
## F-statistic: 202.9 on 3 and 146 DF, p-value: < 2.2e-16
```

```
avPlots(model17)
```

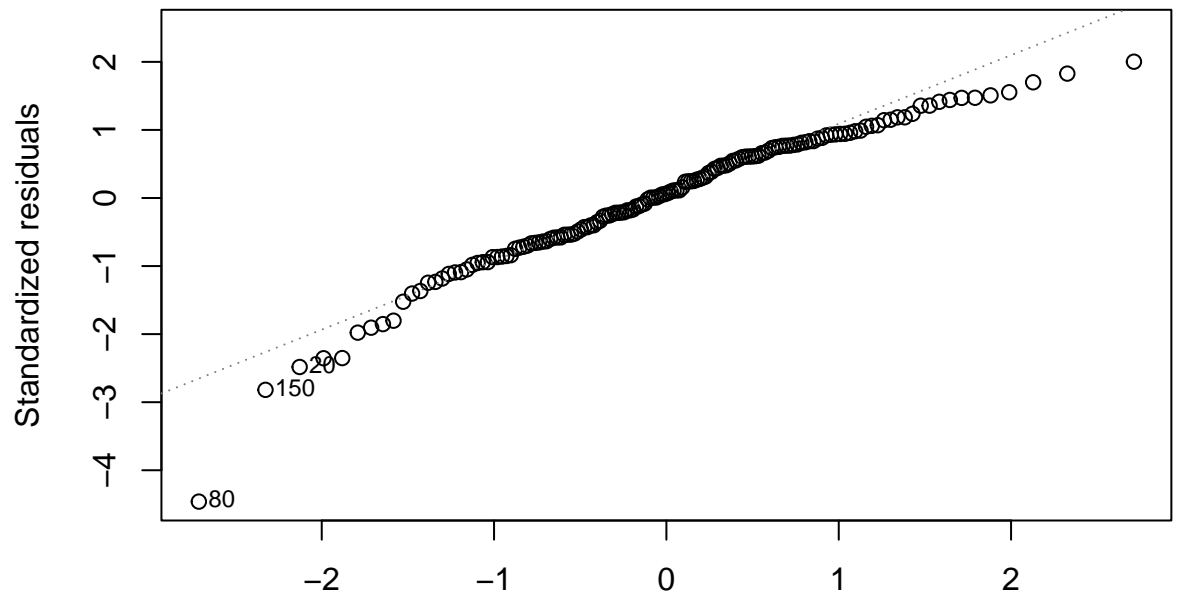
Added-Variable Plots



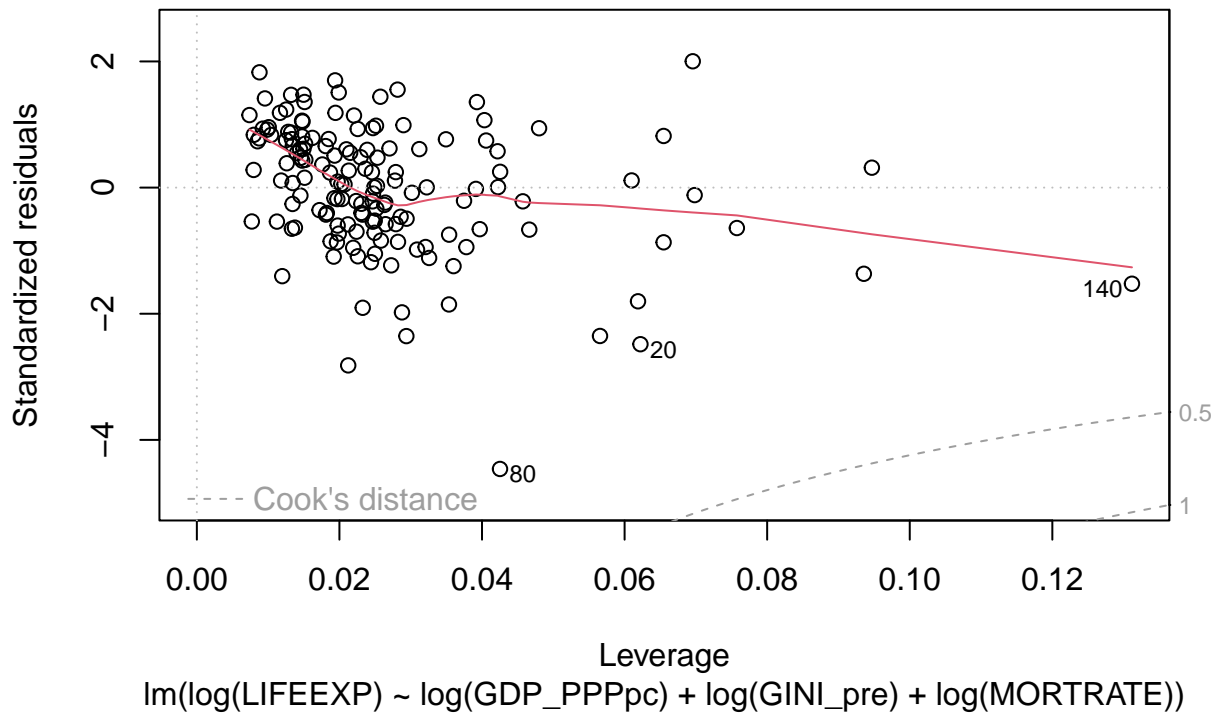
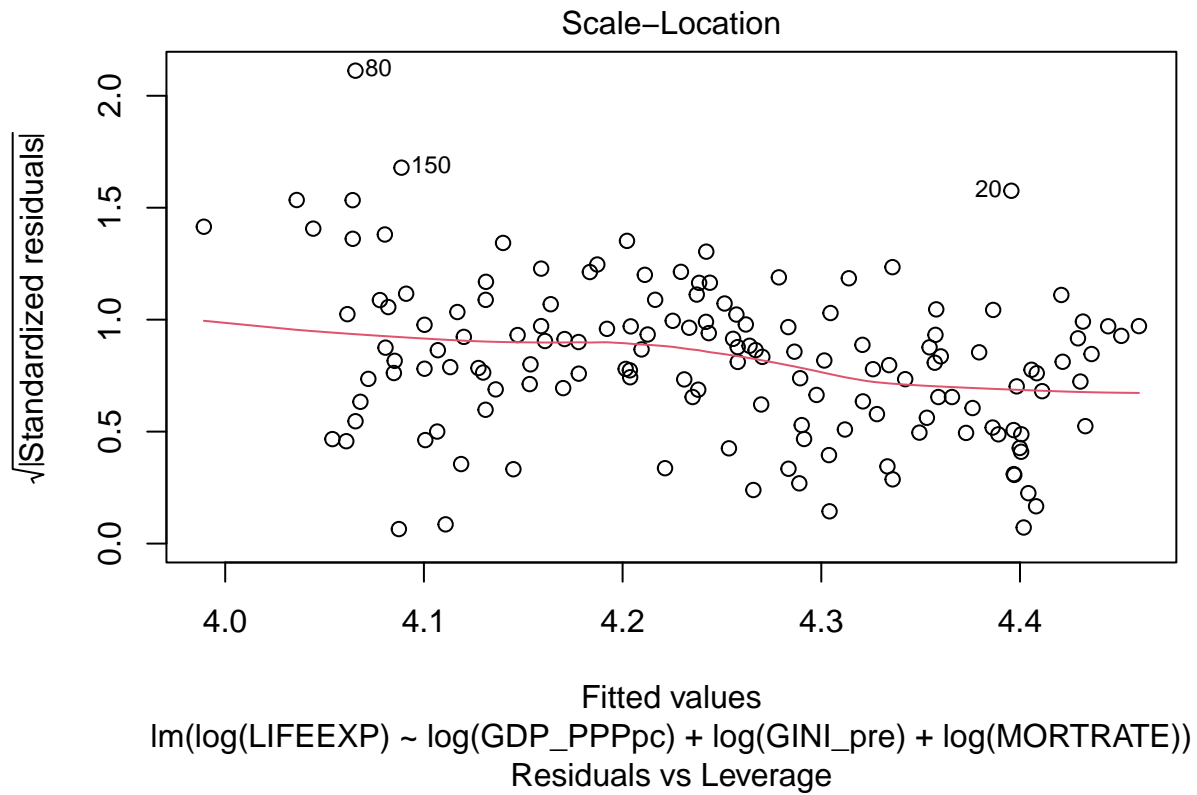
```
plot(model17)
```



Fitted values
 $\ln(\log(\text{LIFEEXP}) \sim \log(\text{GDP_PPPpc}) + \log(\text{GINI_pre}) + \log(\text{MORTRATE}))$
 Normal Q-Q



Theoretical Quantiles
 $\ln(\log(\text{LIFEEXP}) \sim \log(\text{GDP_PPPpc}) + \log(\text{GINI_pre}) + \log(\text{MORTRATE}))$



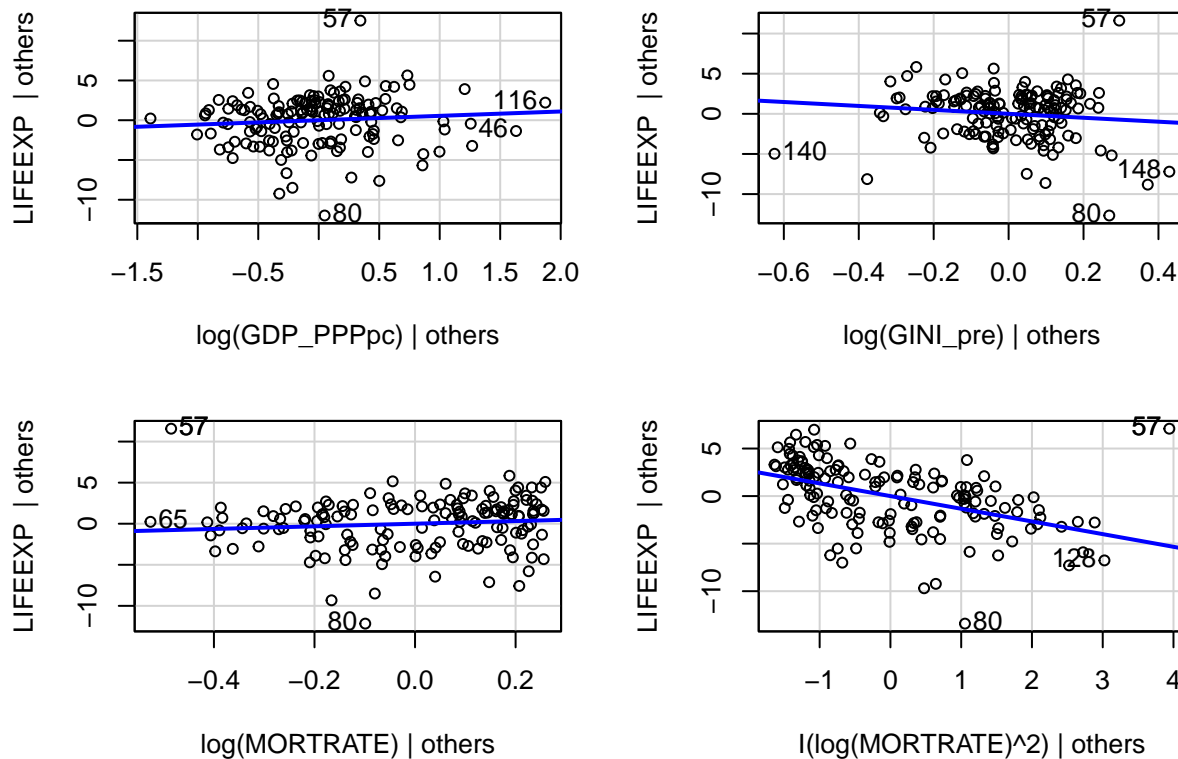
```
model118 <- lm(LIFEEXP ~ log(GDP_PPPpc) + log(GINI_pre) + log(MORTRATE) + I(log(MORTRATE)**2), lifeexp_d)
summary(model118)
```

```
##
## Call:
```

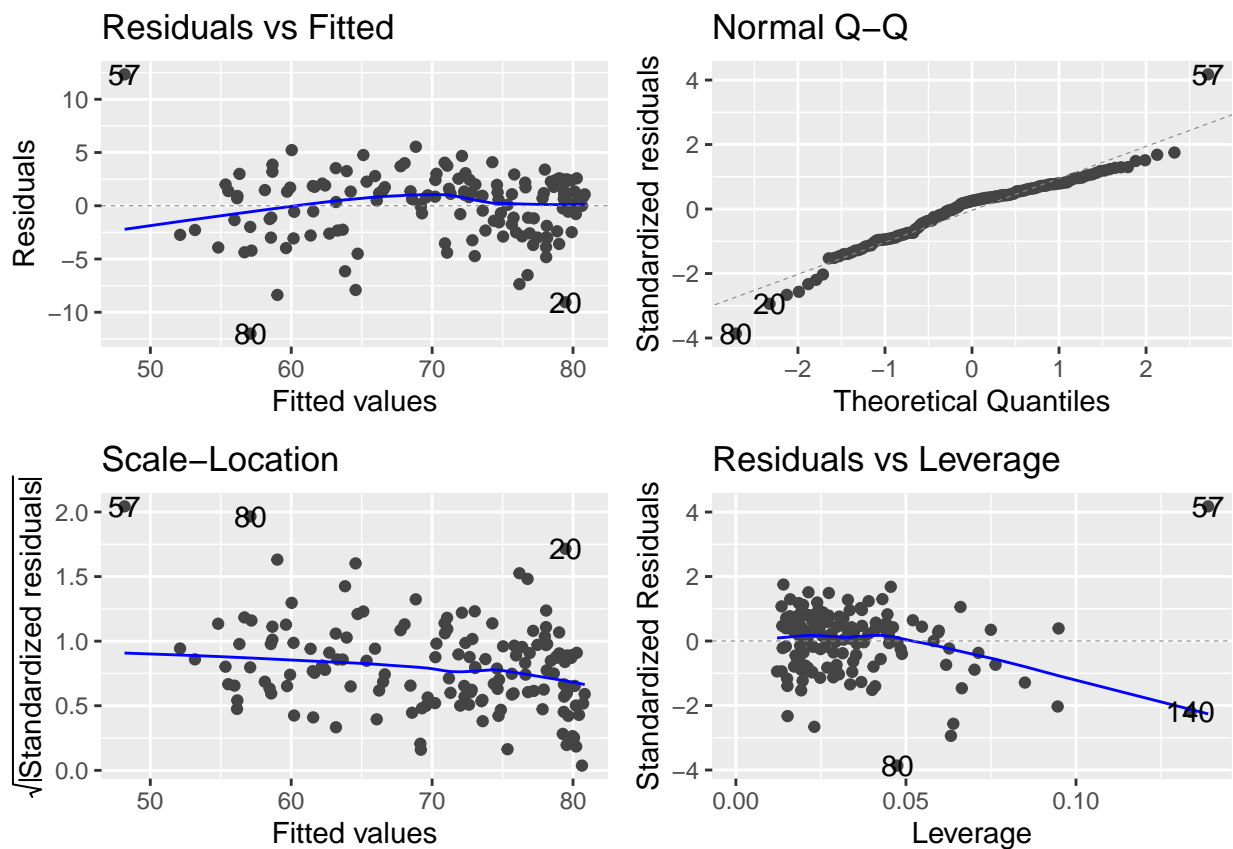
```
## lm(formula = LIFEEXP ~ log(GDP_PPPpc) + log(GINI_pre) + log(MORTRATE) +
##     I(log(MORTRATE)^2), data = lifeexp_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -12.0064  -2.1861   0.7992   1.9784  12.3395
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    83.6320     8.0921  10.335 < 2e-16 ***
## log(GDP_PPPpc)    0.5493     0.4874   1.127   0.262
## log(GINI_pre)   -2.4538     1.6936  -1.449   0.150
## log(MORTRATE)    1.5955     1.3291   1.200   0.232
## I(log(MORTRATE)^2) -1.3333     0.2127  -6.270 3.91e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.184 on 145 degrees of freedom
## Multiple R-squared:  0.8718, Adjusted R-squared:  0.8683
## F-statistic: 246.6 on 4 and 145 DF,  p-value: < 2.2e-16
```

```
avPlots(model18)
```

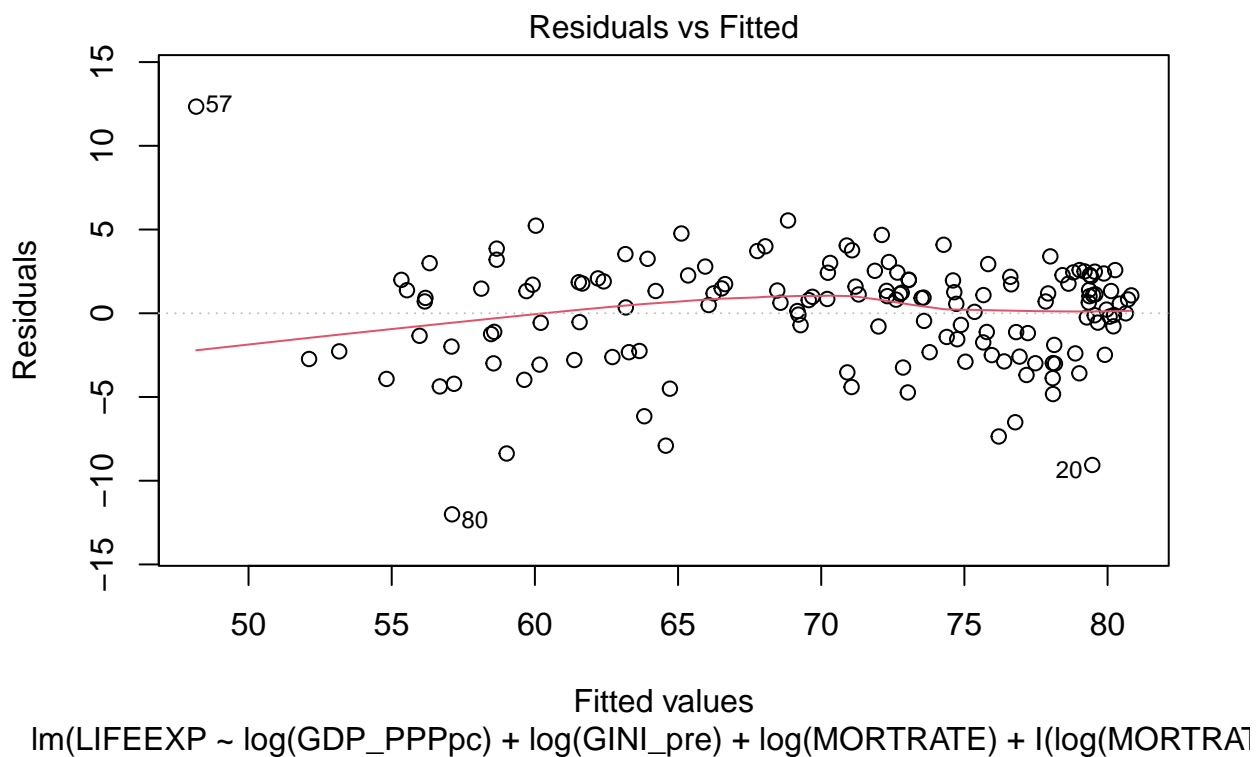
Added-Variable Plots

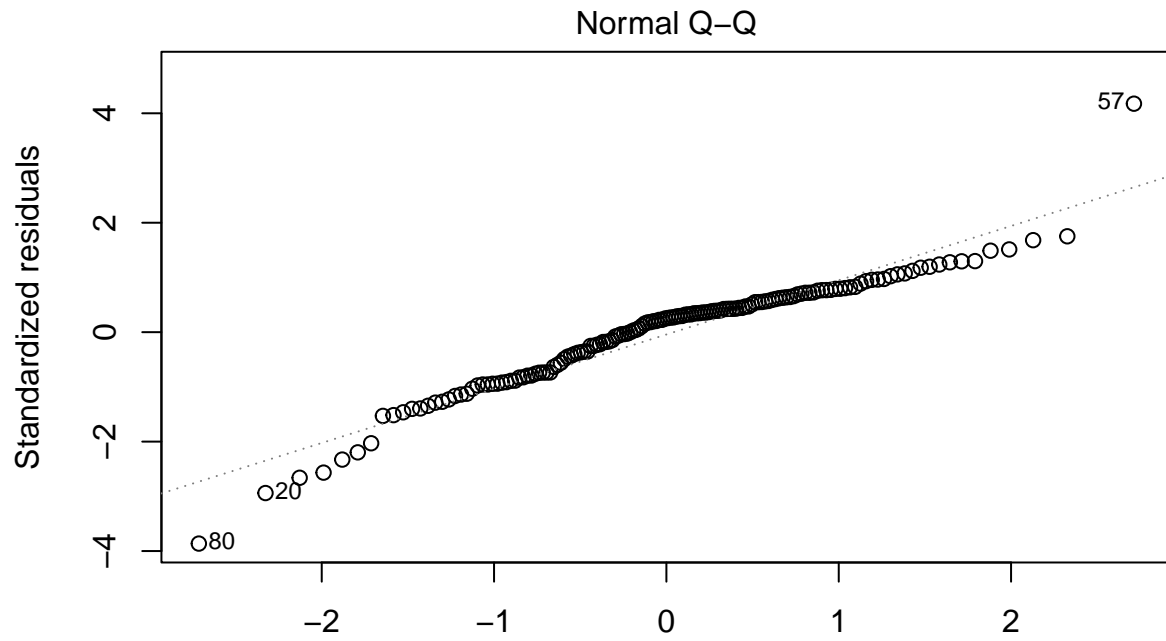


```
autoplot(model18)
```

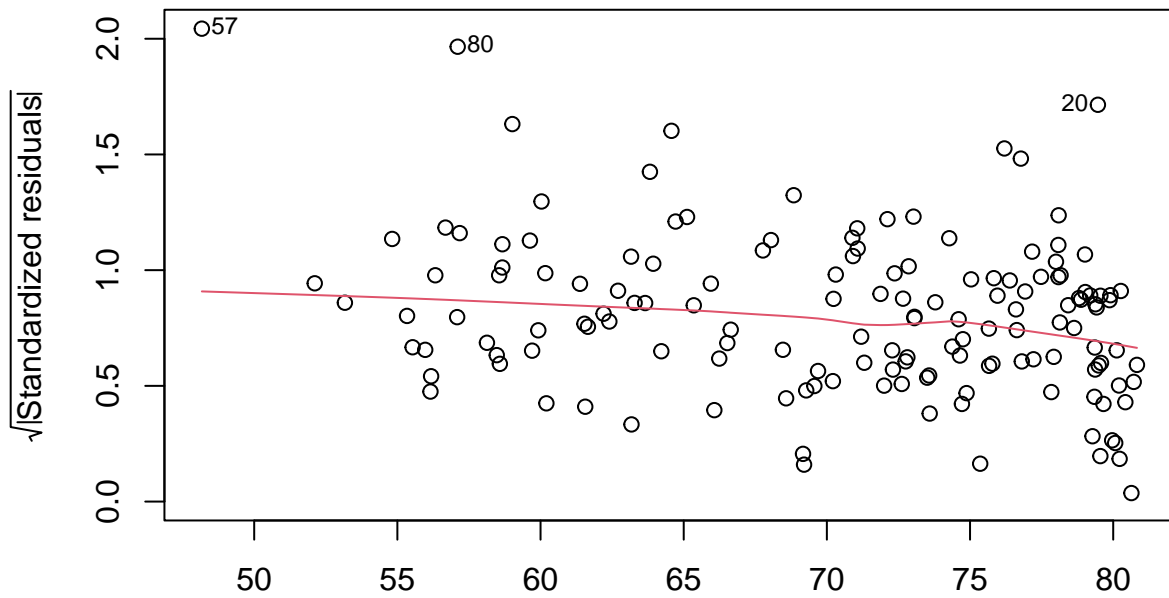


```
plot(model18)
```



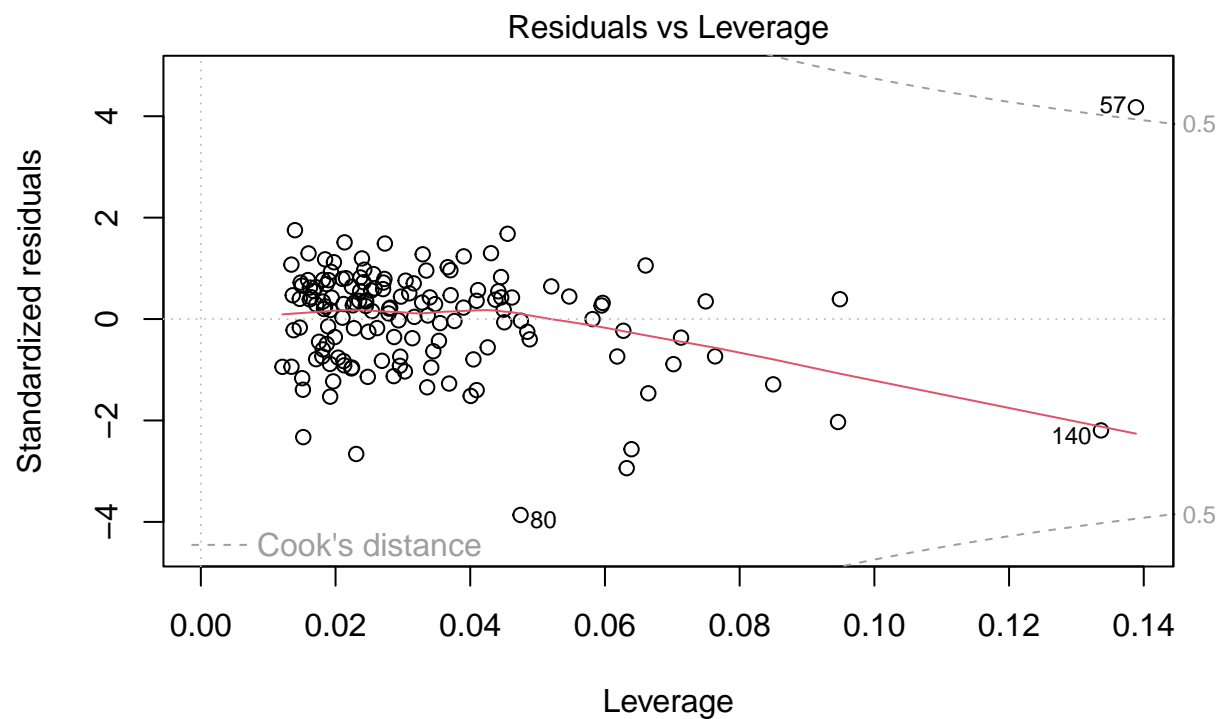


Im(LIFEEXP ~ log(GDP_PPPpc) + log(GINI_pre) + log(MORTRATE) + I(log(MORTRATE)^2))



Fitted values

Im(LIFEEXP ~ log(GDP_PPPpc) + log(GINI_pre) + log(MORTRATE) + I(log(MORTRATE)^2))



$\text{lm}(\text{LIFEEXP} \sim \log(\text{GDP_PPPpc}) + \log(\text{GINI_pre}) + \log(\text{MORTRATE}) + \text{I}(\log(\text{MORTRATE}))$