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Homework 5

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GEOS597 Homework #5: Seafloor subsidence due to cooling

Due: 10/17/2016

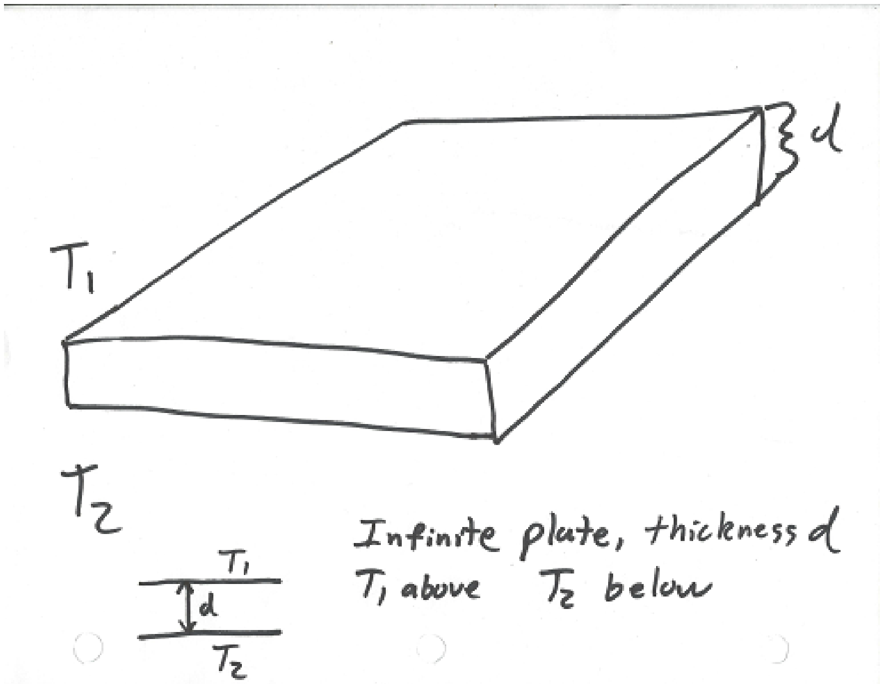
```
close all;
clear all;
clc;
```

Part 1: Conductive heat flow

1.1: A model

Here is a drawing of our simple plate model.

```
[V,map] = imread('scan0002.png', 'png');
imshow(V,map)
```



1.2: Heat flow

$$Q = -k\frac{T_2 - T_1}{d}$$

In this equation, the heat flow *down* (Q) is negative, which means that positive Q(heat) actually flows up, from higher temperature to lower temperature, as we would expect. Negative flow down ==> positive Q up.

1.3: Thermal conductivities

- **Thermal Conductivity (Watts/m*C):** *
- **Silver: 406** *
- **Magnesium: 156** #
- **Glass: 0.8** *
- **Rock: 2-7** #
- **Wood: 0.04-0.12** *

***from Young, Hugh D., University Physics, 7th Ed. Table 15-5**

 # from engineeringtoolbox.com

1.4: The heat transport equation

The heat transport equation will become:

$$Q = -k\frac{T(z+dz)-T(z)}{dz}$$

The derivative term on the right hand side =

$$\frac{\partial T}{\partial z}$$

So the heat equation becomes:

$$Q(T,z) = -k * \frac{\partial T}{\partial z}$$

or

$$Q(T,z) = -k * \nabla T$$

1.5: The conservation equation

Compute derivative of Q:

$$c_p\rho * \frac{\partial T}{\partial t} = A + k\frac{\partial^2 T}{\partial z^2}$$

Heat generation A = 0 gives us:

$$c_p\rho * \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2}$$

We can condense all the constants into a new value κ, which will be equal to:

$$\kappa = \frac{k}{c_p\rho}$$

Part 2: Oceanic lithosphere cooling

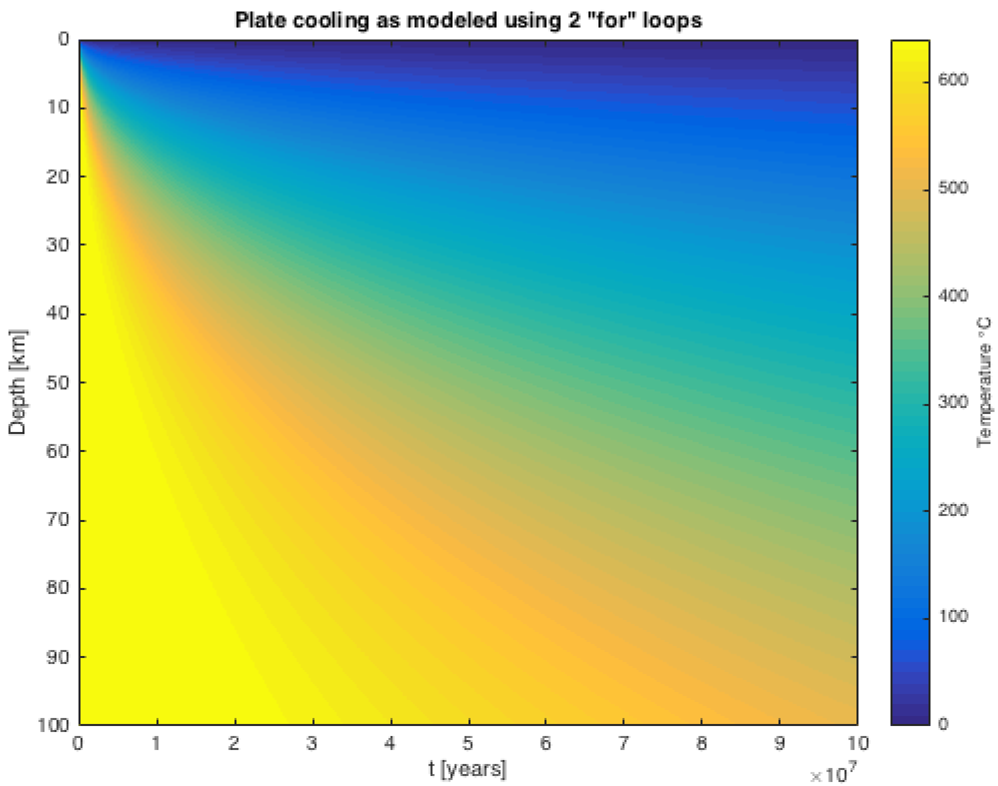
2.1: Set up the model domain and compute

```
z = 0:0.1:100;                % depth [km]
dz = z(2)-z(1);               % depth step size [km]
t = 0:1E5:1E8;                % time [yr]
dt = t(2)-t(1);               % time step [yr]
kappa = (1E-6);               % thermal diffusivity [m^2/s]
To = 640;                     % initial temp. at ridge [deg. C]
tic;
```

```
T = zeros(length(z),length(t));
for j = 1:length(t);
    for i = 1:length(z);
        T(i,j) = To * erf (1000*z(i)./(2*sqrt(kappa*3.154e7*t(j))));
    end
end
runTime = toc;
```

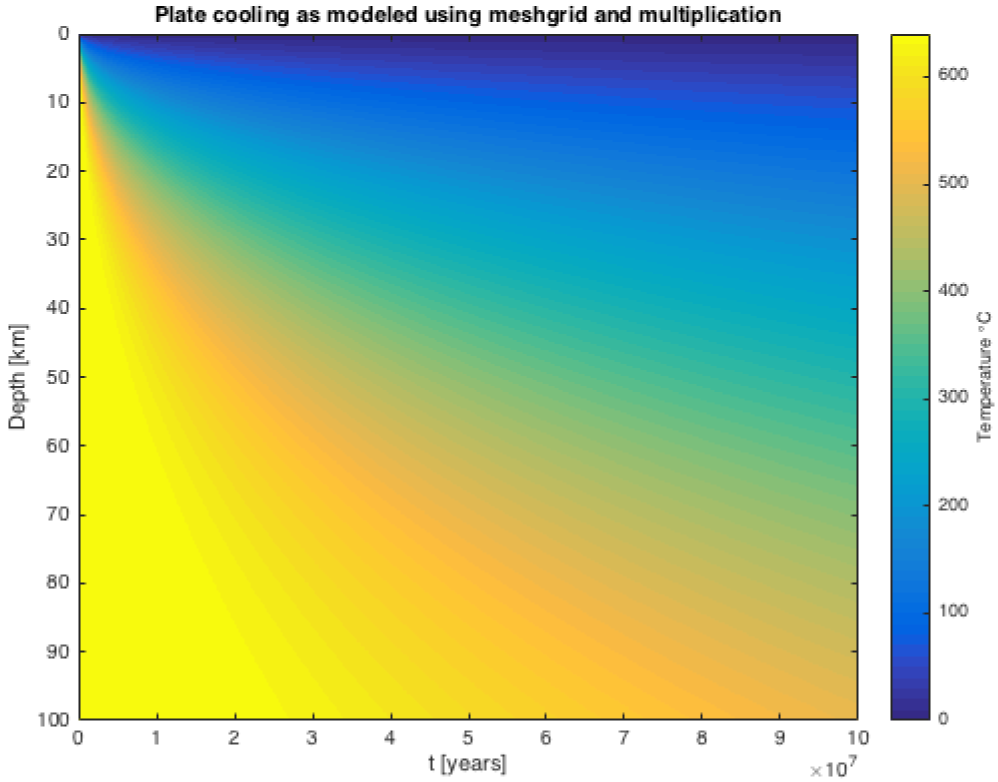
EXTRA CREDIT: runTime >>> runTime2. Using the piecewise matrix multiplication of two matrices rather than using two for loops is much less computationally expensive.

```
h = figure;
imagesc(t, z, T);
c = colorbar;
c.Label.String = 'Temperature \circC';
xlabel ('t [years]')
ylabel ('Depth [km]')
title('Plate cooling as modeled using 2 "for" loops')
```




```
tic
[TIME, DEPTH] = meshgrid (t,z);
T2 = To * erf (1000*DEPTH./(2*sqrt(3.154e7*kappa*TIME)));
runTime2 = toc;
h = figure;
imagesc(t, z, T2);
c = colorbar;
c.Label.String = 'Temperature \circC';
```

```
xlabel ('t [years]')
ylabel ('Depth [km]')
title('Plate cooling as modeled using meshgrid and multiplication')
```



2.2: Analyze model output

- This model makes sense given our boundary conditions. Newly formed parts of the plate are near the boundary condition value of 640C = To, while as time goes on (and depth increases) more of the plate is cooled by the ocean water due to conduction, and consequently more of the plate cools toward the ocean temperature(which we assumed, incorrectly, to be zero). The result is the temperature gradient we see in our figure.

- Many factors control the rate of temperature decay:

1. Ocean Temperature

2. Specific heat of the water and plate

3. Density of the plate

4. Ocean currents/convection

5. The relative thermal conductivity(s)

- We could relate this model with distance from the ridge axis if we know the plate velocity. We could multiply the age by the plate velocity, and we we would get back the distance from the ridge.

- We know from physics that the density of water is greatest at 4 degrees C, We also found published values(Wikipedia) suggesting 2-3 degrees at the bottom of the ocean. A more appropriate boundary condition for T(z=0) would be 4 degrees C, or the 2-3 degrees figure, because we know that the temperature at a "deep enough" point in the ocean is a constant 4 degrees, which has to do with the thermal expansion of water. Either of these would be a better boundary condition than t = 0.

- 640C seems like an appropriate temperature for a mid ocean ridge, since, essentially, magma is being cooled into solid rock at the ridge. Liquid magma (according to Wikipedia) ranges from approximately 700-1300C, taking a value at the lower end of this scale seems appropriate, since the magma will be on the verge of solidifying as the plate forms.

Part 3: Plate Velocity and the depth of oceans

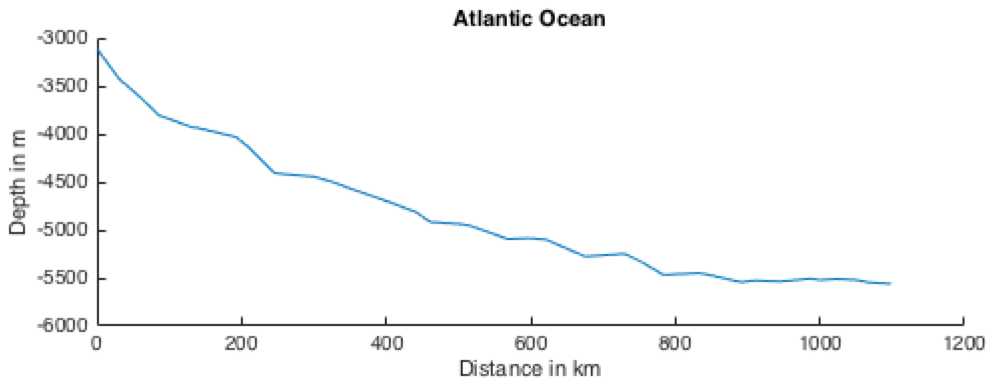
3.1 Load and plot sea-floor depth data

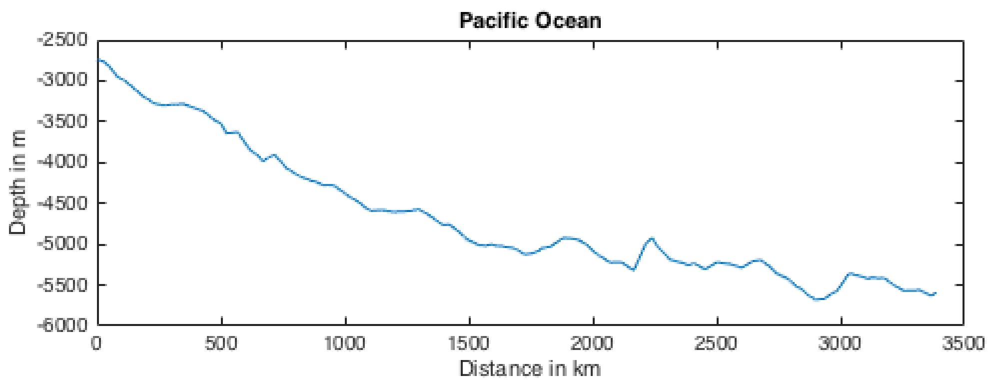
First we load our data:

```
load('spreadingData.mat');
```

Now we want a plot depth(z) vs distance from the ridge(x)

```
h=figure;
subplot(2,1,1);
hold on
plot(Bath.atlanticx,Bath.atlanticz);
title('Atlantic Ocean');
xlabel('Distance in km');
ylabel('Depth in m');
subplot(2,1,2);
plot(Bath.pacificx,Bath.pacificz);
title('Pacific Ocean');
xlabel('Distance in km');
ylabel('Depth in m');
```





3.2 A half-space model

For this step we will attempt to find a value for plate velocity that closely matches the depth data that we have.

```
velocity = 45;  % in [Km/Ma]
dpacific = zeros(1,numel(Bath.pacificx));
for ii = 1:numel(Bath.pacificx);
    dpacific(ii) = -(2.65 + 0.345*(Bath.pacificx(ii)/velocity)^(1/2));

end
dpacific = dpacific*1E03;
```

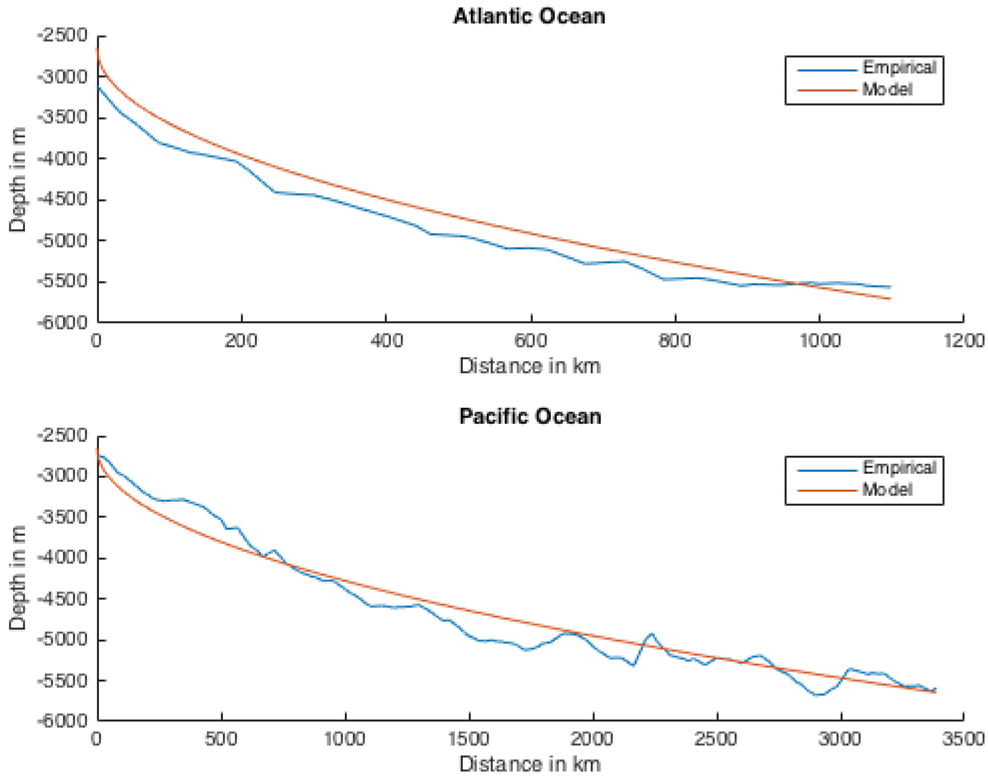
Now for the Atlantic side

```
velocity = 14;  % in [Km/Ma]
datlantic = zeros(1,numel(Bath.atlanticx));
for ii = 1:numel(Bath.atlanticx);
    datlantic(ii) = -(2.65 + 0.345*(Bath.atlanticx(ii)/
velocity)^(1/2));

end
datlantic = datlantic*1E03;
```

Now we re-plot the previous data with the depths predicted by our simplified model

```
h=figure;
```

```
subplot(2,1,1);
hold on
plot(Bath.atlanticx,Bath.atlanticz);
plot(Bath.atlanticx,datlantic);
legend toggle;
legend('Empirical','Model');
title('Atlantic Ocean');
xlabel('Distance in km');
ylabel('Depth in m');
subplot(2,1,2);
hold on
plot(Bath.pacificx,Bath.pacificz);
plot(Bath.pacificx,dpacific);

legend toggle;
legend('Empirical','Model');
title('Pacific Ocean');
xlabel('Distance in km');
ylabel('Depth in m');
```



The value 2.65 represents the depth at t=0, or the depth at the mid-ocean ridge in this model.

The best velocities we found to fit the data, without actually doing statistical mechanics, are as follows:

1. Pacific: 45 km/Ma

2. Atlantic: 14 km/Ma

This works out to 4.5 cm/yr for the Pacific and 1.4 cm/yr for the Atlantic

Published value for the Atlantic spreading: ~2.5cm/yr Pacific: ~10cm/yr

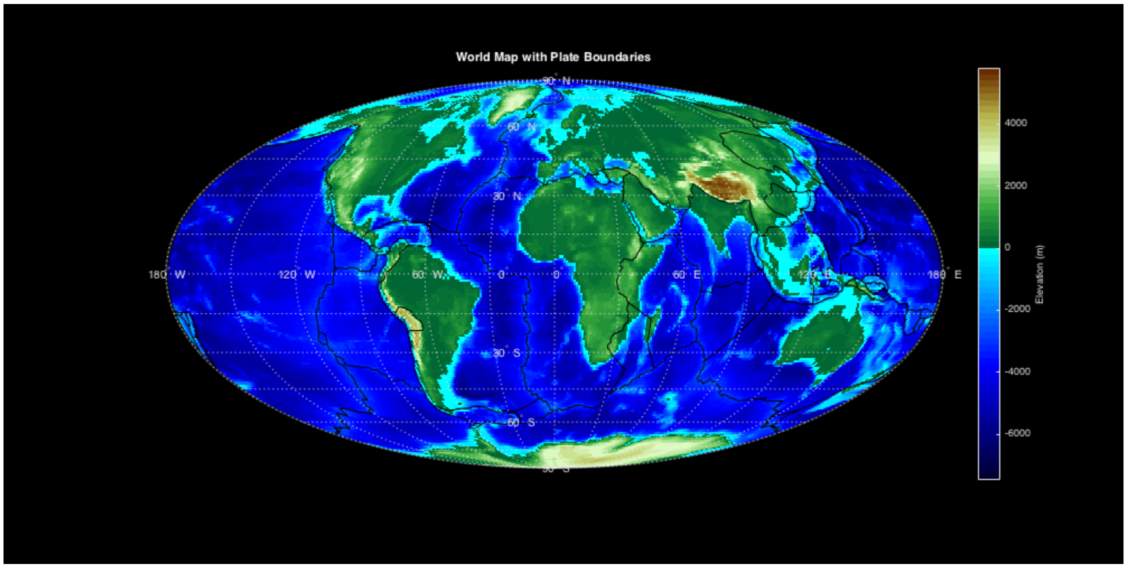
These values do not match our predictions exactly, but they are fairly reasonable considering our very simplified model.

Part 4: Global Oceanic plate ages

4.1 Load topo data and plot sea-floor depths

```
load ('topo.mat');

h=figure;

h.InvertHardcopy='off'; %  Ensure that the colors of the saved figure
 match the colors on the display
h.Color='k'; % changes background color
h.Position=[100 100 1000 500]; % sets image position and size(coords
 of lower left and upper right)
h.PaperPositionMode='auto'; % saves image position for print and
 figure projection

%%setup map axes
ax=axesm('Mollweid','Frame','on','Grid','on'); % sets projection type
 and turns frame and gridding on
setm(ax,'MLabelLocation',60); % sets position for labels of the
 longitude lines every 60 degrees
setm(ax,'PLabelLocation',30); % sets position for labels for latitude
 lines every 30 degrees
mlabel('MLabelParallel',0); %labels longitude lines along the equator
plabel('PLabelMeridian',-25); % labels latitude lines along the chosen
 meridian
axis('off'); % turns normal square axis off
setm(ax,'FontColor',[0.9 0.9 0.9]); % sets color of labels to work on
 black background
setm(ax,'GColor',[0.9 0.9 0.9]); % sets color of grid to work with
 black background
title('\color{white}World Map with Plate Boundaries')
c=colorbar('color', [0.9 0.9 0.9]); % adds colorbar and sets font
 color
c.Label.String= 'Elevation (m)'; % adds label to colorbar

% These commands add coastlines and plot elevation
load('coastlines'); % loads built-in MATLAB data called coastlines
plotm(coastlat, coastlon);

LAT = topolatlim(1):topolatlim(2);
LON =  topolonlim(1):topolonlim(2);


[lon, lat] = meshgrid(LON,LAT); % compute the lat/lon of every grid
 point in topo
```

```
pcolorm(lat,lon,topo); % plot the matrix of elevations on the map
demcmap(topo); % give it a better colormap

% load the plate boundary data and plot the plate boundaries

[platelat, platelon] = importPlates('All_boundaries.txt');
plotm(platelat,platelon,'k');
```



4.2 Kill the topography and get the units right

Set elevation above sea level to zero

```
for j = 1:numel(topo);
    if topo(j) >= 0;
        topo(j) = 0;
    end
end
```

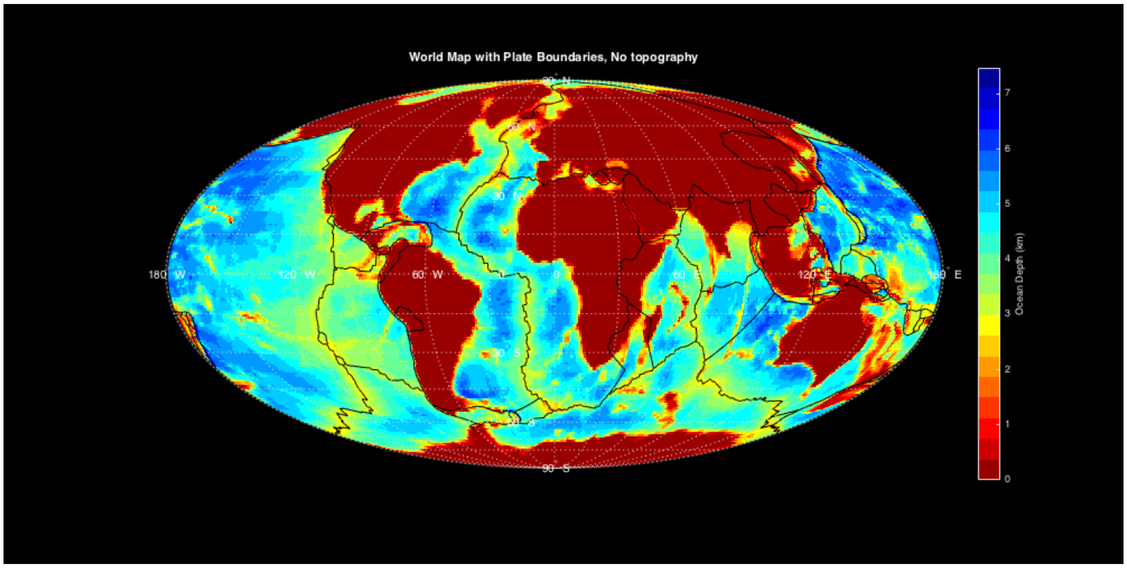
Now I'll switch the values to be positive, and change the units to km

```
depth = -topo./1000;
```

Now I'll plot this new depth matrix using the same figure commands as before:

```
h=figure;

h.InvertHardcopy='off'; %  Ensure that the colors of the saved figure
 match the colors on the display
h.Color='k'; % changes background color
h.Position=[100 100 1000 500]; % sets image position and size(coords
 of lower left and upper right)
h.PaperPositionMode='auto'; % saves image position for print and
 figure projection

%%setup map axes
```

```
ax=axesm('Mollweid','Frame','on','Grid','on'); % sets projection type
 and turns frame and gridding on
setm(ax,'MLabelLocation',60); % sets position for labels of the
 longitude lines every 60 degrees
setm(ax,'PLabelLocation',30); % sets position for labels for latitude
 lines every 30 degrees
mlabel('MLabelParallel',0); %labels longitude lines along the equator
plabel('PLabelMeridian',-25); % labels latitude lines along the chosen
 meridian
axis('off'); % turns normal square axis off
setm(ax,'FontColor',[1 1 1]); % sets color of labels to work on black
 background
setm(ax,'GColor',[0.9 0.9 0.9]); % sets color of grid to work with
 black background
title('\color{white}World Map with Plate Boundaries, No topography')
c=colorbar('color', [0.9 0.9 0.9]); % adds colorbar and sets font
 color
c.Label.String= 'Ocean Depth (km)'; % adds label to colorbar

% These commands add coastlines and plot elevation
load('coastlines'); % loads built-in MATLAB data called coastlines
plotm(coastlat, coastlon);

LAT = topolatlim(1):topolatlim(2);
LON =  topolonlim(1):topolonlim(2);


[lon, lat] = meshgrid(LON,LAT); % compute the lat/lon of every grid
 point in topo
pcolorm(lat,lon,depth); % plot the matrix of elevations on the map
demcmap(depth); % give it a better colormap

% load the plate boundary data and plot the plate boundaries

[platelat, platelon] = importPlates('All_boundaries.txt');
plotm(platelat,platelon,'k');

% Use colormap suggested in assignment:

colormap( flipud( jet(20) ) );
% This command sets the color properties of the figure to a matrix
% of values, the flipud command flips the matrix upside-down, and
 jet(20) is the matrix of
% RGB values for the colors in the colormap, and 20 is the number of
% colors included.
```



4.3 Compute sea-floor age

Rearranging equation (3) for t gives:

$$t = (\frac{d - 2.65}{0.345})^2$$

4.4 Plot oceanic lithosphere age map

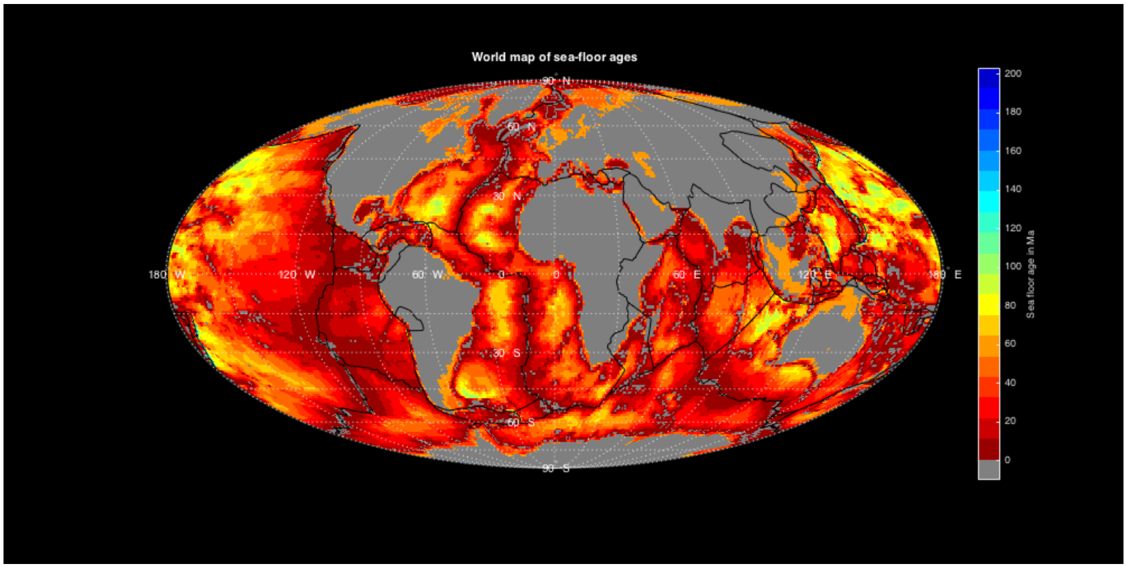
First we need a matrix with the ages in Ma as the entries, we will accomplish this using the above equation, then I'll set the values at the continent to -10 Ma as suggested.

```
ageArray = zeros(180,360);

for ii = 1:numel(depth);

    ageArray(ii) = ((depth(ii)-2.65)/(0.345))^2;

    if ageArray(ii) == ((0-2.65)/0.345)^2;
        ageArray(ii) = -10;
    end

end
```

Now we want to plot the resulting figure.

```
h=figure;

h.InvertHardcopy='off'; %  Ensure that the colors of the saved figure
 match the colors on the display
h.Color='k'; % changes background color
h.Position=[100 100 1000 500]; % sets image position and size(coords
 of lower left and upper right)
h.PaperPositionMode='auto'; % saves image position for print and
 figure projection
```

```
%%setup map axes
ax=axesm('Mollweid','Frame','on','Grid','on'); % sets projection type
 and turns frame and gridding on
setm(ax,'MLabelLocation',60); % sets position for labels of the
 longitude lines every 60 degrees
setm(ax,'PLabelLocation',30); % sets position for labels for latitude
 lines every 30 degrees
mlabel('MLabelParallel',0); %labels longitude lines along the equator
plabel('PLabelMeridian',-25); % labels latitude lines along the chosen
 meridian
axis('off'); % turns normal square axis off
setm(ax,'FontColor',[1 1 1]); % sets color of labels to work on black
 background
setm(ax,'GColor',[0.9 0.9 0.9]); % sets color of grid to work with
 black background
title('\color{white}World map of sea-floor ages')
c=colorbar('color', [0.9 0.9 0.9]); % adds colorbar and sets font
 color
c.Label.String= 'Sea floor age in Ma'; % adds label to colorbar

% These commands add coastlines and plot elevation
load('coastlines'); % loads built-in MATLAB data called coastlines
plotm(coastlat, coastlon);

LAT = topolatlim(1):topolatlim(2);
LON =  topolonlim(1):topolonlim(2);


[lon, lat] = meshgrid(LON,LAT); % compute the lat/lon of every grid
 point in topo
pcolorm(lat,lon,ageArray); % plot the matrix of elevations on the map
demcmap(ageArray); % give it a better colormap

% load the plate boundary data and plot the plate boundaries

[platelat, platelon] = importPlates('All_boundaries.txt');
plotm(platelat,platelon,'k');


% Then we want the colormap information from the assignment:

cmap = flipud( jet(20) );  % This creates a 20x3 matrix of RGB values
 called cmap,
% that is inverted to go from red to blue instead of blue to red.
cmap = [0.5 0.5 0.5; cmap]; % this adds gray as the first row of cmap
cmap(end,:) = []; % removes the last row of cmap so it has 20 rows
 again
colormap(cmap); % Sets the colormap properties to use the matrix we
 created(cmap)
```



4.5 Discuss your results

- This map makes sense because the youngest sea-floors are generally found near spreading centers, and the oldest are found in the middle of plates.

- To find the max age, we'll use the max() function

```
p = max(max(ageArray));
```

The oldest age on the map is 195.4323 Ma

- find location of oldest age

```
[I,J] = ind2sub(size(ageArray),find(ageArray == p));
```

This gives us a coordinate of (135,152), which, with the way our data is oriented, is located in the north west pacific at 45N and 152E. We know that this area is home to the oldest sea-floors on Earth, so this result makes sense.

Now we want the youngest age:

```
b = min(min(abs(ageArray))); % takes care of imposed negative ages of
 continents.
```

```
[I2,J2] = ind2sub(size(ageArray),find(ageArray == b));
```

This command gives us three points where the sea floor age = 0!

1. (114,65) ==> 24N,65E

2. (107,89) ==> 17N,89E

3. (171,164) ==> 81N,164E

All these places appear to be at plate boundaries of spreading centers, which makes sense to us.

- There are many assumptions in this model:

1. Have not accounted for sediment transport
2. We used our half space model which assumes all physical parameters to be constant.
3. This model assumes all spreading centers occur at a depth of 2.65km.
4. We are assuming that depth is solely a function of time, which we know is not true.
5. There are difinetly more

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