Module in

NMTL01E – Numerical Methods

SESSION TOPIC : GAUSS ELIMINATION METHOD

LEARNING OBJECTIVES:

At the end of the session, the students will:

1. Recognize the types of matrices
2. Explain the row echelon form and row elementaryoperations
3. Find the values of the variables in linear equations using Gauss Elimination Method (Row Echelon Form)

KEY TERMS

Square matrix	Identity matrix	Scalar matrix	Diagonal matrix
Upper triangular matrix	swap	pivot	Row echelon

CORE CONTENT

A. Types of Matrices

1. **Square** –matrix has the same number of rows as columns.
   Ex:

$$Square\ Matrix\ M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

2. **Identity Matrix** – has 1s on the main diagonal and 0s everywhere else:
   Ex:

$$Identity\ matrix\ I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. **Diagonal Matrix** – has zero anywhere not on the main diagonal:
   Ex:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4. **Scalar Matrix –** has all main diagonal entries the same, with zero everywhere else:
   Ex:

$$A = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

5. **Lower triangular** is when all entries above the main diagonal are zero:
   Ex:

Lower Triangular Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 6 & 5 & 1 \end{bmatrix}$$

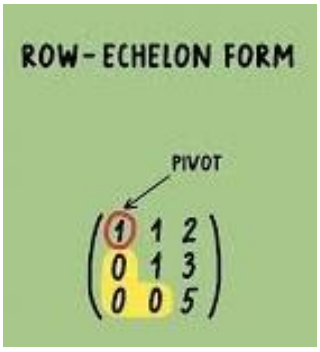
6. **Upper triangular** is when all entries below the main diagonal are zero:
   Ex:

Upper triangular matrix: U

$$\begin{bmatrix} 1 & 1/2 & 3 & 0 \\ 0 & 5 & 0 & 1 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

B. **Gaussian Elimination Method**

Gaussian elimination is the process of using valid row operations on a matrix until it is in reduced row echelon form.



Echelon form

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

Reduced echelon form

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A matrix is in Row Echelon form if it has the following properties:

- Any row consisting entirely of zeros occurs at the bottom of the matrix.
- For each row that does not contain entirely zeros, the first non-zero entry is 1 (called a leading 1).
- For two successive (non-zero) rows, the leading 1 in the higher row is further left than the leading one in the lower row.

Row Elementary Operations

1. Swap
2. Scale
3. Pivot

Sample 1.

Given the linear equations below, find the values of x,y,z using Gauss Elimination Method

x + y -z=7

x-y+2z=3

2x+y+z=9

Sol'n: Augmented Matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 7 \\ 1 & -1 & 2 & 3 \end{array}\right] \quad R_1-R_2 \rightarrow R_2$$

2 1 1 9

1 1 -1 7

0 2 -3 4

2 1 1 9 $-2R_1 + R_3 \rightarrow R_3$

1 1 -1 7

0 2 -3 4 $R_2(1/2) \rightarrow R_2$

0 -1 3 -5

1 1 -1 7

0 1 -3/2 2 $R_2 + R_3 \rightarrow R_3$

0 -1 3 -5

1 1 -1 7

0 1 -3/2 2

0 0 3/2 -3 $R_3(2/3) \rightarrow R_3$

1 1 -1 7

0 1 -3/2 2

0 0 1 -2 (This is already in row echelon form. Apply back substitution (z=-2))

Solve for y:

y-3/2z=2

y-3/2(-2)=2

y+3=2

y=-1

1 1 -1 7

x+y-z=7

x-1+2=7

x=6

check: x=6 y=-1 z=-2

x + y -z=7

x-y+2z=3

2x+y+z=9

Using 1st equation:

x + y -z=7

6-1+2=7

7=7

Using 2nd equation:

x-y+2z=3

6+1-4=3

3=3

Using 3rd equation:

2x+y+z=9

2(6)-1-2=9

9=9

IN-TEXT ACTIVITY

- ✓ Video
  https://www.youtube.com/watch?v=2GKESu5atVQ -Gauss Elim. Mathod
- ✓ Pdf copy to be uploaded in myLPU

SELF ASSESSMENT

QUIZ

REFERENCES

https://www.mathsisfun.com/algebra/matrix-types.html