

GAUSS SEIDEL

- an iterative method and an improved form of Jacobi method
- also known as the *successive displacement method*.
- named after Carl Friedrich Gauss (Apr. 1777–Feb. 1855) and Philipp Ludwig von Seidel (Oct. 1821–Aug. 1896).
- the starting values are $u_2 = u_3 = u_4 = 0$
- The difference between the Gauss–Seidel and Jacobi methods is that the Jacobi method uses the values obtained from the previous step while the Gauss–Seidel method always applies the latest updated values during the iterative procedures

Given:

$$5x_1 - x_2 + 2x_3 = 12$$

$$3x_1 + 8x_2 - 2x_3 = -25$$

$$x_1 + x_2 + 4x_3 = 6$$

Solution:

Find the equations for x_1, x_2, x_3

$$x_1 = (12 + x_2 - 2x_3) / 5$$

$$x_2 = (-25 - 3x_1 + 2x_3) / 8$$

$$x_3 = (6 - x_1 - x_2) / 4$$

	1	2	3	4	5
x_1	0	$=(12+0-2(0))/5$ $=12/5$ $=2.4$	$=(12-4.025-2(1.90625))/5$ $=0.8325$	$=(12-2.961-2(2.03))/5$ $=0.9958$ (1)	
x_2	0	$=(-25-3(2.4)+2(0))/8$ $=-4.025$	$=(-25-3(0.8325)+2(1.90625))/8$ $=-2.961$	$=(-25-3(0.9958)+2(2.03))/8$ $=-2.991$ (-3)	
x_3	0	$=(6-2.4-(-4.025))/4$ $=1.90625$	$=(6-0.8325-(-2.961))/4$ $=2.03$	$=(6-0.9958-(-2.991))/4$ $=1.9988$ (2)	

Check: $x_1=1$ $x_2=-3$ $x_3=2$

$$5x_1 - x_2 + 2x_3 = 12$$

$$5 \cdot 1 - (-3) + 2 \cdot 2 = 12$$

$$3x_1 + 8x_2 - 2x_3 = -25$$

$$3 \cdot 1 + 8(-3) - 2(2) = -25$$

$$-25 = -25$$

$$x_1 + x_2 + 4x_3 = 6$$

$$1 - 3 + 4(2) = 6$$

$$6 = 6$$

<https://www.sciencedirect.com/topics/engineering/gauss-seidel-method>