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Powell's Conjugate Direction Method Report

1) Program Description:

Powell's Conjugate Direction Method (PCDM) is an unconstrained multi-dimensional method for finding the minimum of a quadratic function that is strictly convex. PCDM also works for general functions because "For a general function f one simply repeats the iteration until a point sufficiently close to a minimum has been reached. Because f is approximately quadratic near the minimum" (1). PCDM starts with n direction vectors u which represent the column vectors of the identity matrix and n is the dimensions of the objective function. Then we perform a line search to update our initial point to get a better approximation to the minimizer. At the end of every iteration we discard the first direction vector by letting each direction vector u_i equal to u_{i+1}. After k iterations all directions vectors are conjugate based on Theorem 3.4 by Richard Brent (2).

General Algorithm (2):

1) For i = 1, … , n, compute t to minimize $f(x_{i-1} + tu_i)$ and define $x_i = x_{i-1} + tu_i$ (Note: Perform line search using 1 dimensional Newton method to calculate t)
2) For i = 1, ..., n-1, replace u_i by u_{i+1}.
3) Replace u_n by $x_n - x_0$
4) Compute t to minimize $f(x_0 + tu_n)$ and define $x_0 = x_0 + tu_n$

Stopping Criteria is similar to other methods:

1) After k iterations, where k > K and K is a determined max number of iterations.
2) The difference between x_0 after k-1 iterations and after k iterations is less than some small value epsilon

My program performs Powell's Conjugate Direction Method for any general function of n dimensions.

2) Program Effectiveness

PCDM has quadratic convergence. Comparing the number of iterations taken on function 1 with methods we've seen in class:

PCDM: 6 iterations, {0.955 145 782, 0.022 427 109, 1.836 281 938}

Polak-Ribiere Conjugate Gradient: 9 iterations, (0.955 145 782, 0.022 427 109, 1.836 281 938).

Steepest Descent: 12 iterations, (0.968 108 959, 0.015 991 593, 1.889 312 940).

Limited Memory BFGS: 8 iterations, (0.955 145 782, 0.022 427 109, 1.836 281 938)

Newton's Method w/ Line Search: 3 iterations, {0.955 145 782, 0.022 427 109, 1.836 281 938}

PCDM performs relatively well in regard to speed (# of iterations to reach minimizer) and accuracy.

References

1. Brent, Richard P. (1973). Algorithms for Minimization Without Derivatives, 126-127.

2. Buckley, A. G. (1976). Constrained minimization using Powell's conjugacy approach. SIAM Journal on Numerical Analysis, 13(4), 522-523.