1 Introduction

In this report, we will perform an analysis of the physics relevant in the launch of the Saturn V rocket with the intention of sending it to the Moon. In section 2, we will calculate and analyze the gravitational potential field of the Earth-Moon system. In section 3, we will calculate and analyze the gravitational force field of the Earth-Moon. In section 4, we will compute various parameters relevant to the launch performance of Stage 1 of the Saturn V rocket, and in section 5, we will summarize the approximations made and the limitations of the analysis done in this report.

2 The Gravitational Potential of the Earth-Moon System

The gravitational potential a distance R from a point source with mass M is given by

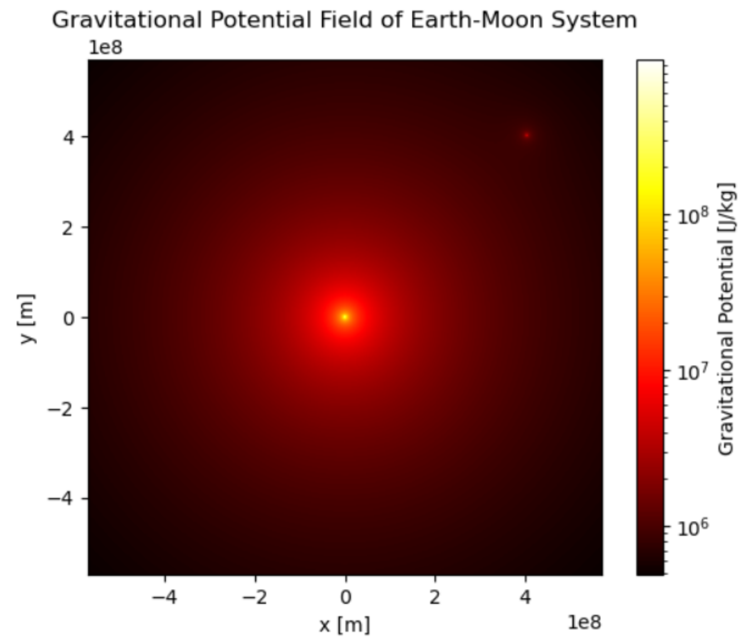
$$\Phi(R) = -\frac{GM}{R} \tag{1}$$

In this report, we approximate the Earth and the Moon as point sources. The total gravitational potential at a point is, by the principle of superposition, the sum of the potentials from each mass, therefore the gravitational potential of a point r in the Earth-Moon system is given by

$$\Phi_{tot}(r) = \Phi_E(r) + \Phi_M(r) \tag{2}$$

where Φ_E is the potential of the Earth, and Φ_M is the potential of the Moon.

The calculations of these potentials were made, assuming the Earth to be at the origin and the Moon to be at $(\frac{d_{E\to M}}{\sqrt{2}}, \frac{d_{E\to M}}{\sqrt{2}})$, and the magnitude of the potential was plotted on a 2D plane using the numpy.pcolormesh command, giving Figure 1.



Figur 1: 2D Colormesh plot of the Gravitational Potential of the Earth-Moon System

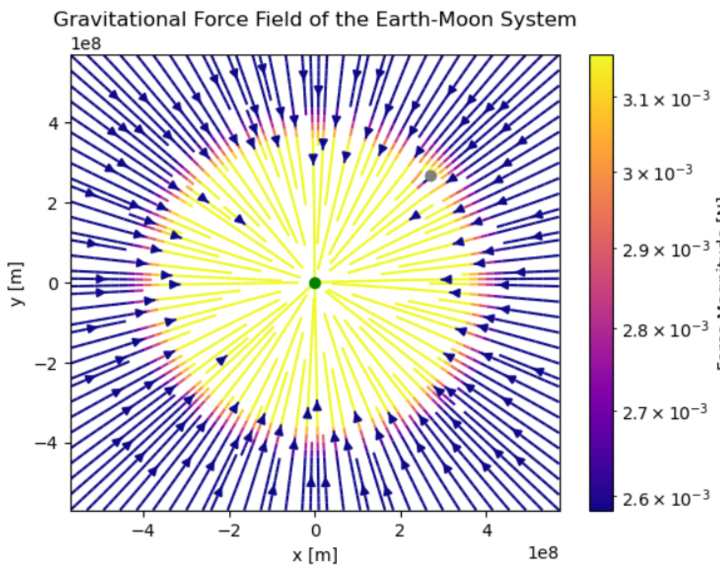
3 The Gravitational Force of the Earth-Moon System

The gravitational force felt by a mass m_2 at a distance r from a point source with mass M_1 is given by

$$\vec{F} = -\frac{GMm}{|r|^2}\hat{r}_{21} \tag{3}$$

where $\hat{r}_{21}$ is the unit vector pointing from m_2 to M_1. Similar to the potential, the gravitational force follows the principle of superposition, so the total force is the sum of the force from each mass in the system, i.e. the Earth and the Moon.

To compute the force contribution each mass, a function was created that calculated the magnitude of the force vector, and projected along the $\hat{r}_{21}$ direction. This function returned the x and y components of the vector field, so that the contribution from the Earth and the Moon and be summed component-wise to construct the total force vector field, that was then plotted using the numpy.streamplot command, yielding Figure 2.



Figur 2: Gravitational Force Vector Field of the Earth-Moon System with the Earth's (green dot) and the Moon's (grey dot) positions marked (Markers not to scale).

As shown in Figure 2, the force vectors point towards the Earth, increasing in magnitude as the distance gets smaller, with a small perturbation near the location of the Moon. The magnitudes were computed using $m = 5500\,kg$ as the mass of the Saturn V rocket.

4 Projected Performance of the Saturn V Stage 1

The total burnout time of the Saturn V rocket is given by

$$T = \frac{m_0 - m_f}{\dot{m}} \tag{4}$$

where $m_0 = 2.8 \times 10^6\ kg$ is the wet mass, $m_f = 7.5 \times 10^5\ kg$ is the dry mass, and $\dot{m} = 1.3 \times 10^4\ \frac{kg}{s}$ is the burn rate of the rocket. The total burnout time is then $T = 157.69\ s$.

The change in velocity at a time t due to the ejection of spent fuel is given by the Tsiolkovsky rocket equation,

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m_0 - \dot{m}t}\right) - gt \tag{5}$$

where $v_e = 2.4 \times 10^3\ \frac{m}{s}$ is the exhaust velocity and $g = 9.81\ \frac{m}{s^2}$ is the acceleration due to gravity.

The altitude of the rocket after the burnout time is then

$$h = \int_0^T \Delta v(t) dt \tag{6}$$

This indefinite integral was calculated using the scipy.integrate.quad function giving an altitude of $h = 74.093\ km$.

5 Discussion and Future Work

In performing these calculations for the expected performance of the Saturn V rocket, a number of physical approximations were made. Namely, the effect of external forces such as air resistance, and the effects of wind. Additionally, the Tsiolkovsky rocket equation assumes that all of the momentum from the expelled fuel propels the rocket in the vertically, however not all of the material is expelled straight downward, so there would be a non-zero change in momentum parallel to the ground, causing the true change in momentum in the vertical direction to be lower than expected.

To make the calculations more accurate, a different method would have to be used to calculate the motion of the rocket. Factors such as air resistance and other environmental conditions would have to be considered, as well as a more robust representation of the momentum transfer present in the thrust mechanism.

The test done demonstrates that the calculation of $T = 157.69\ s$ was a slight underestimate, and $h = 74.093\ km$ was an overestimate. As mentioned prior, the calculation for h neglects factor such as air resistance, which would cause it to be an overestimate. The difference in burn time likely comes from the assumption that the burn rate $\dot{m}$ is a constant. Realistically, you would expect that the burn rate would be variable to some degree, directly affecting all of the calculations done in this report.