1 Introduction

In this task we are interested in Diffusion Limited Aggregation (DLA). The main concept is to form structures by sedimentation of randomly walking particles. Arising structure has properties of self similar objects, e.g. its dimension is not equal to dimension in which it is formed. By slight modification of basic algorithms very challenging and complex effects can be simulated - e.g. lightning, electric discharge and many others. I followed the theory included in [1].

2 Algorithm Description

There is a variety of ways how to implement DLA, I chose implementation on a square lattice. I chose a seed in the middle of the lattice. To save computational time I did not release walkers from the brim of lattice but I chose square region of side $2r_{min}$ with the seed in the middle and randomly picked the initial position of walker on the perimeter of this region. When the sedimentation occurs or walker reaches the brim of region surrounding the seed having the side $2r_{max}$, new walker is generated. I chose $r_{min} = 10$ and when maximal distance of aggregated particles from the seed overreached $0.7r_{min}$, the generation region was pushed further of 10 sites. r_{max} was always either equal to $2r_{min}$ or $\sim N/2$.

So I created lattice of zeros and set a seed (I put number one to corresponding site). Then walkers were aggregating and to obtain time information about aggregation I changed after every 1000 aggregations number that was written in the newly occupied site (I started with ones, after 1000 aggregations I switched to twos, then threes,...).

When suppressing noise, by every potential aggregation I generated random number deciding whether aggregation would be accepted.

To obtain fractal dimension I implemented sand-box method and then fitted linear part of the log-log plot of mass vs. radius with slope being fractal dimension. I counted number of sites in square regions centred in the seed with radius step of 5.

For the more detailed information about the topic see [1].

3 Results

In fig.1 the resulting cluster for lattice $500x500$ is shown, in figures fig.2-3 two different clusters with deposition-time shown. We can observe that newly occupied are almost always sites at the outer brim of cluster as random walkers usually hit these first sooner than they could get more "inside" the cluster. This is different in the Eden model where all perimeter sites are occupied with equal probabilities.

Results of noise suppression is shown in fig.4-6 for various values of acceptance probability, specifically $(0.2, 0.1, 0.05)$. It can be seen that the square symmetry of lattice is reflected in the cluster.

Sandbox method plot can be seen in fig.7. Fitted function was of form $m = r^{d_f}$ where m is mass (number of occupied sites) and r is radius of the region. For five clusters I obtained following data:

$$
\begin{aligned}
d_{f1} &= 1.88 \pm 0.01 \\
d_{f2} &= 1.88 \pm 0.02 \\
d_{f3} &= 1.90 \pm 0.01 \\
d_{f4} &= 1.90 \pm 0.01 \\
d_{f5} &= 1.88 \pm 0.02
\end{aligned}
$$

So we get the average

$$\bar{d_f} = 1.89 \pm 0.01$$

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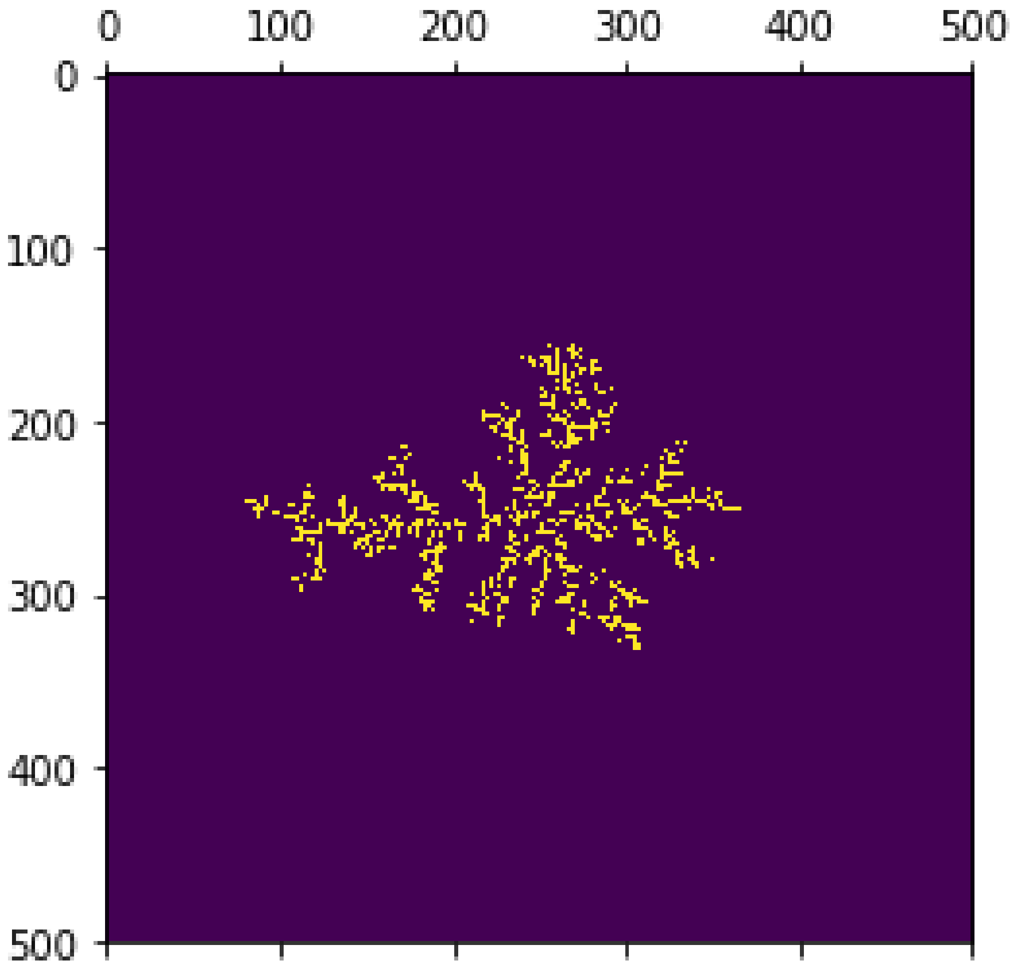

Figure 1: DLA cluster for lattice $N = 500$ with no noise reduction, sites numbering is denoted on the axes

4 Discussion

From coloured figures we can see that aggregation region is different from e.g. Eden model and random walkers usually occupy outer parts of cluster. After noise reduction procedure we should observe square lattice reflection in the clusters. Actually, it can be seen to some extent also in the previous figures 2 and 3. It could be caused by small distance of r_{min} from the edge of cluster. Namely in fig.5 and 6 the vertically-horizontal growth of cluster can be observed.

Using sandbox method, we obtained very similar results yielding to fractal dimension 1.89 what is more than theoretical value of 1.7. This can be due to small lattice or non-ideal randomness of the system. Other way how to obtain better results is to create DLA in continuous space, algorithm requires slight modifications that I do not do it this task.

References

[1] Hermann, H.: *http://www.ifb.ethz.ch/education/bsc-courses/bsc-intro-comphys.html*, 11.11.2018

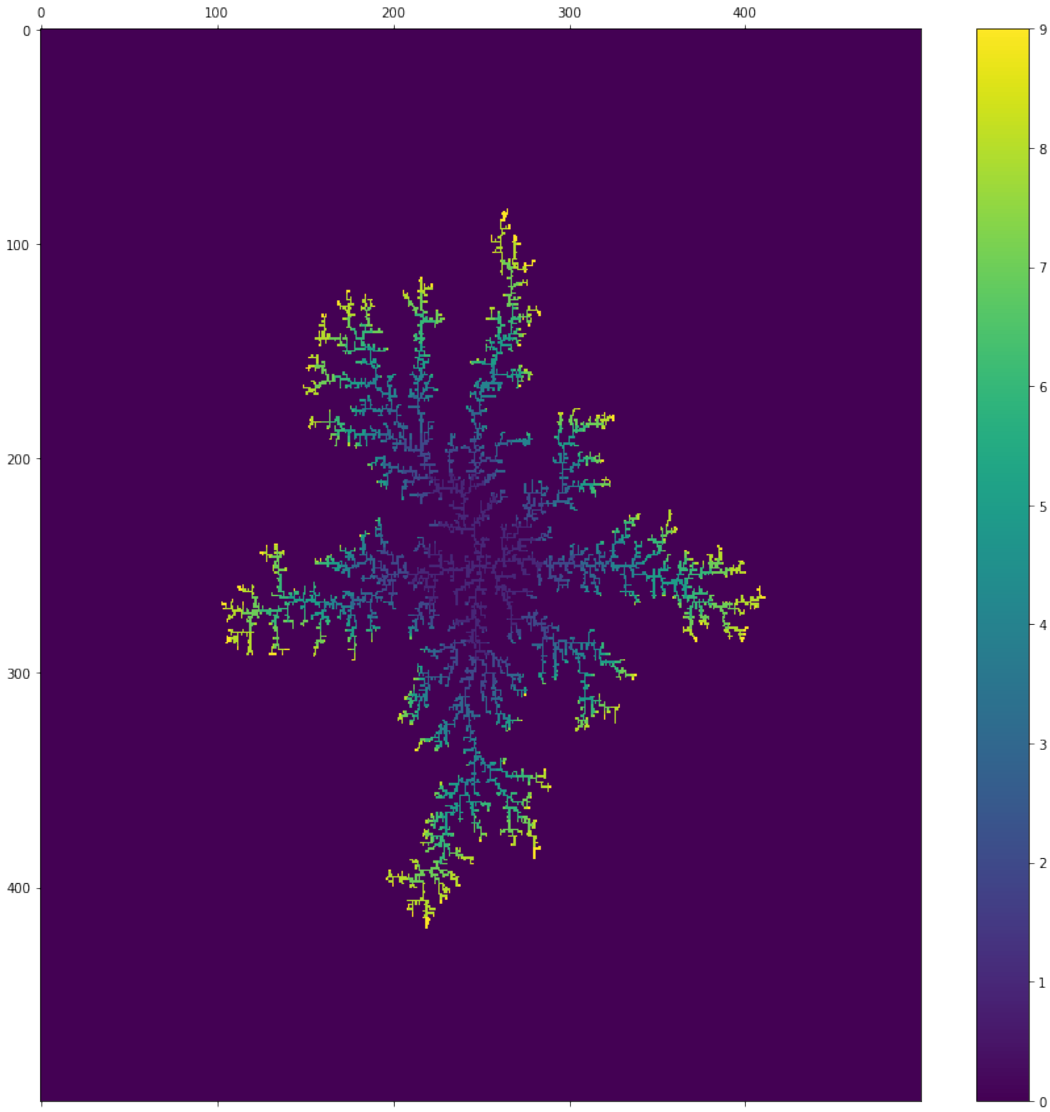

Figure 2: DLA cluster for lattice $N = 500$ with no noise reduction, sites numbering is denoted on the axes, color denoting the aggregation time per one thousand particles, 8 correspond to time 8000 (8000 aggregated particles)

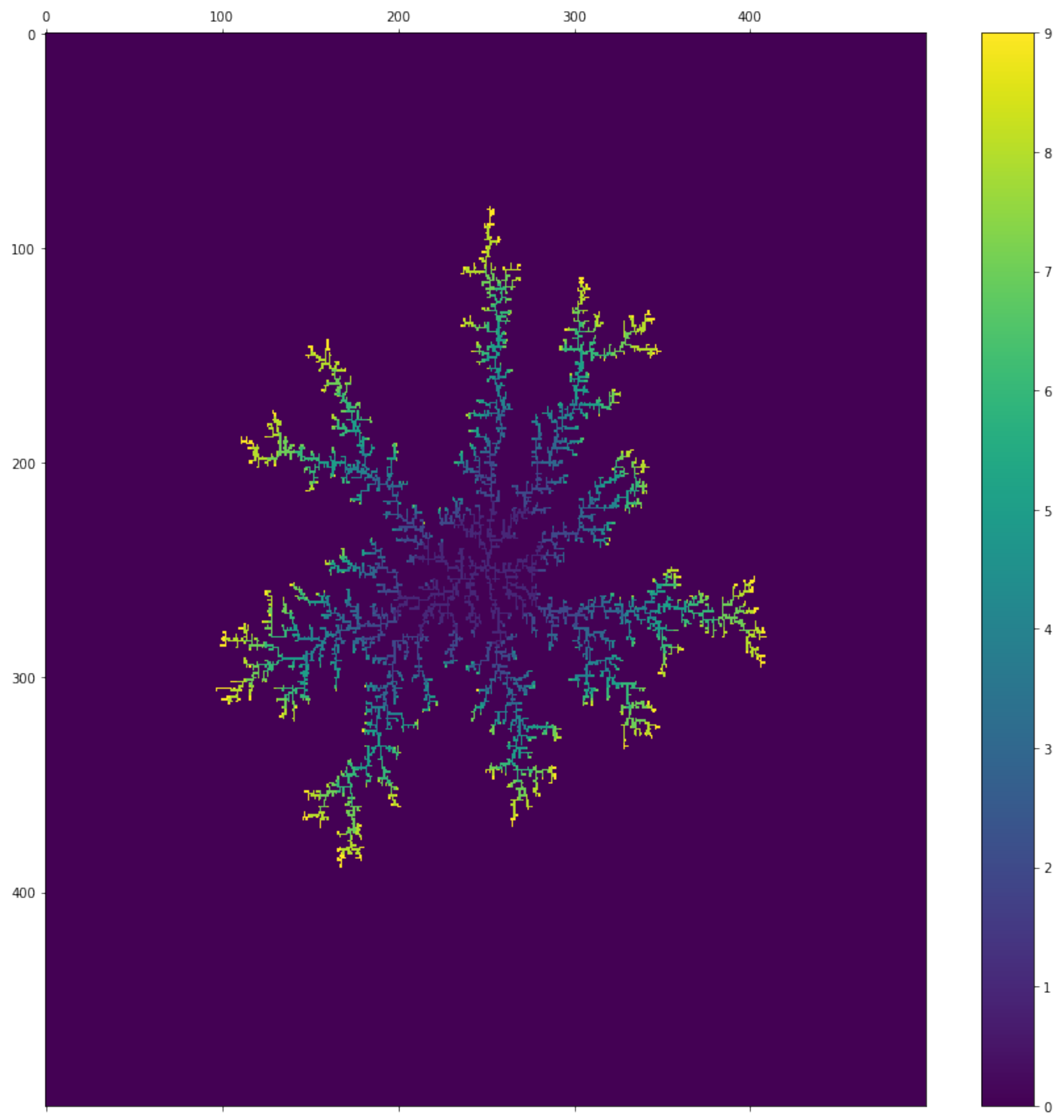

Figure 3: DLA cluster for lattice $N = 500$ with no noise reduction, sites numbering is denoted on the axes, color denoting the aggregation time

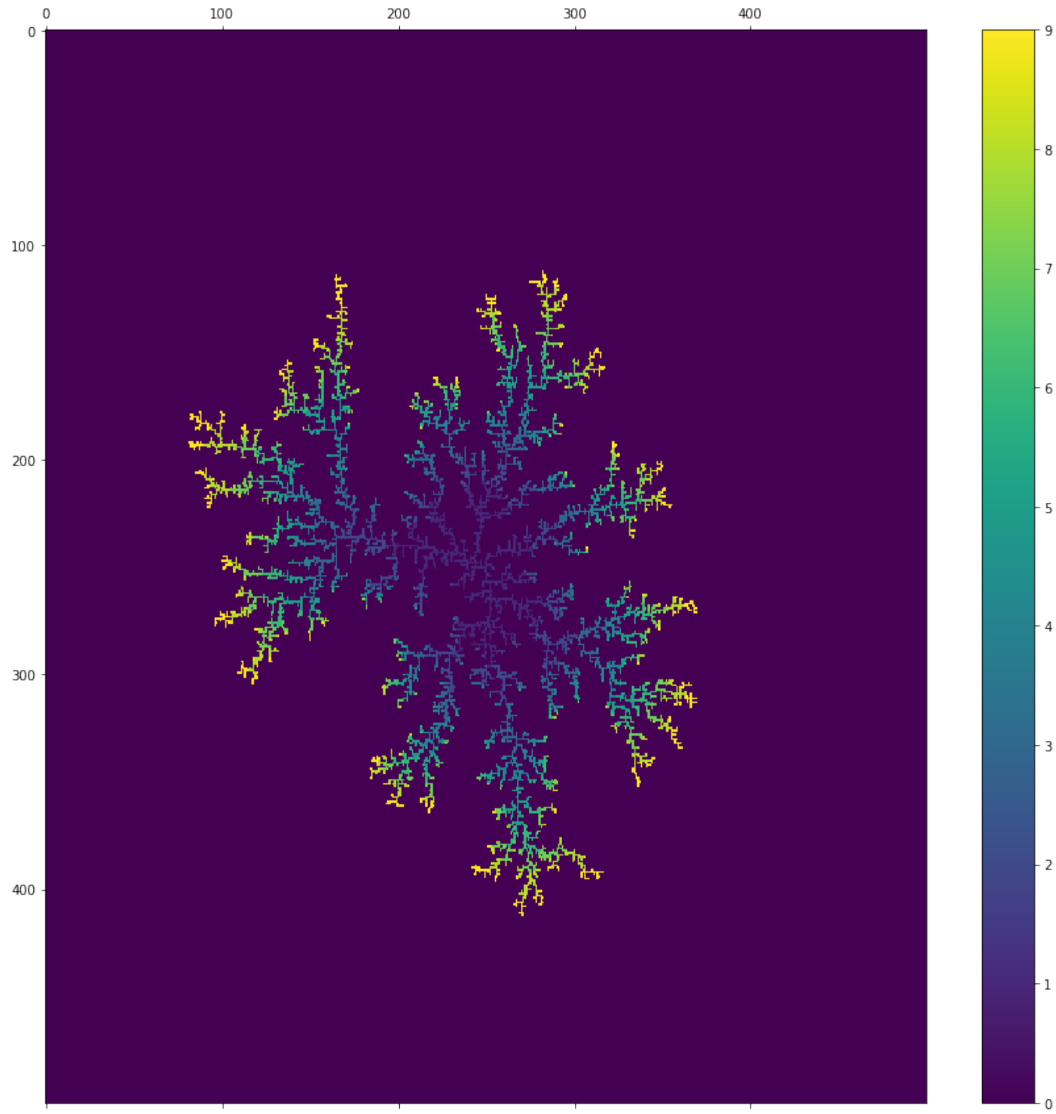

Figure 4: DLA cluster for lattice $N = 500$ with noise reduction, every fifth particle was accepted, sites numbering is denoted on the axes, color denoting the aggregation time

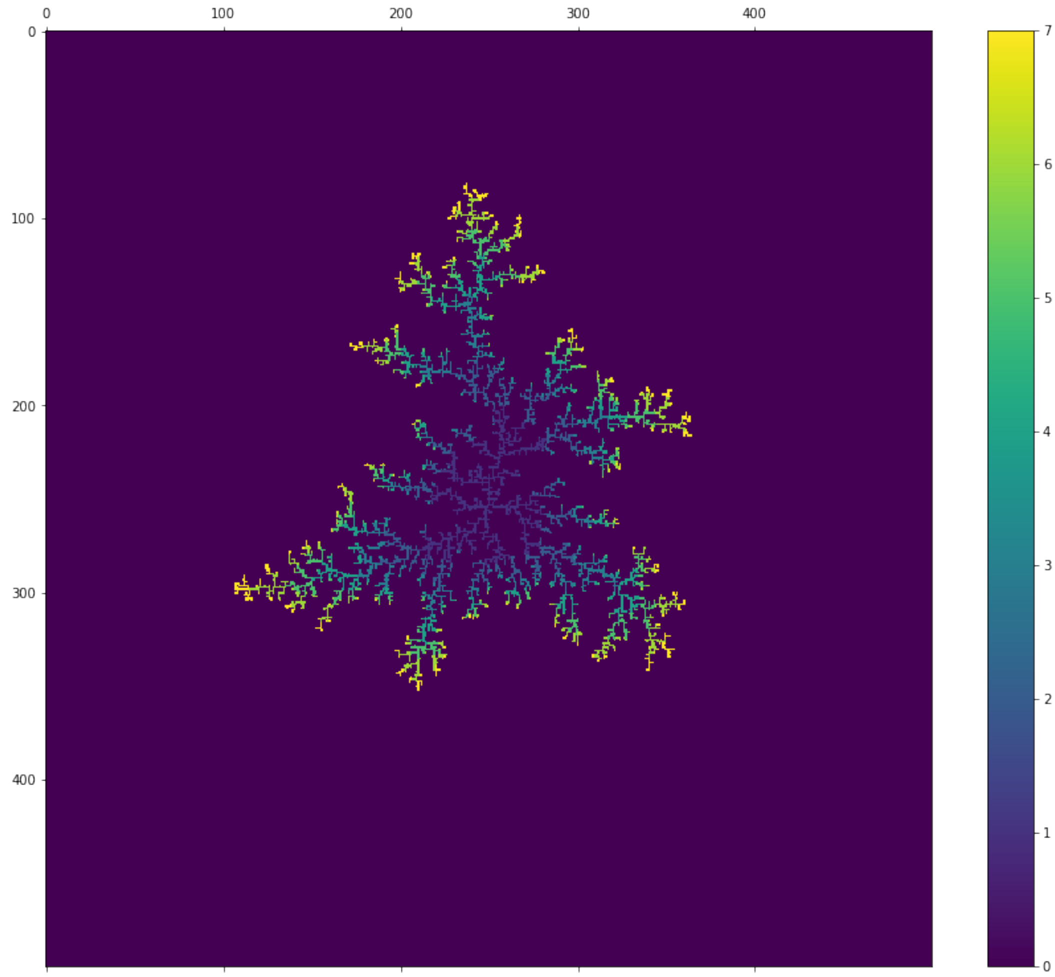

Figure 5: DLA cluster for lattice $N = 500$ with noise reduction, every tenth particle was accepted, sites numbering is denoted on the axes, color denoting the aggregation time

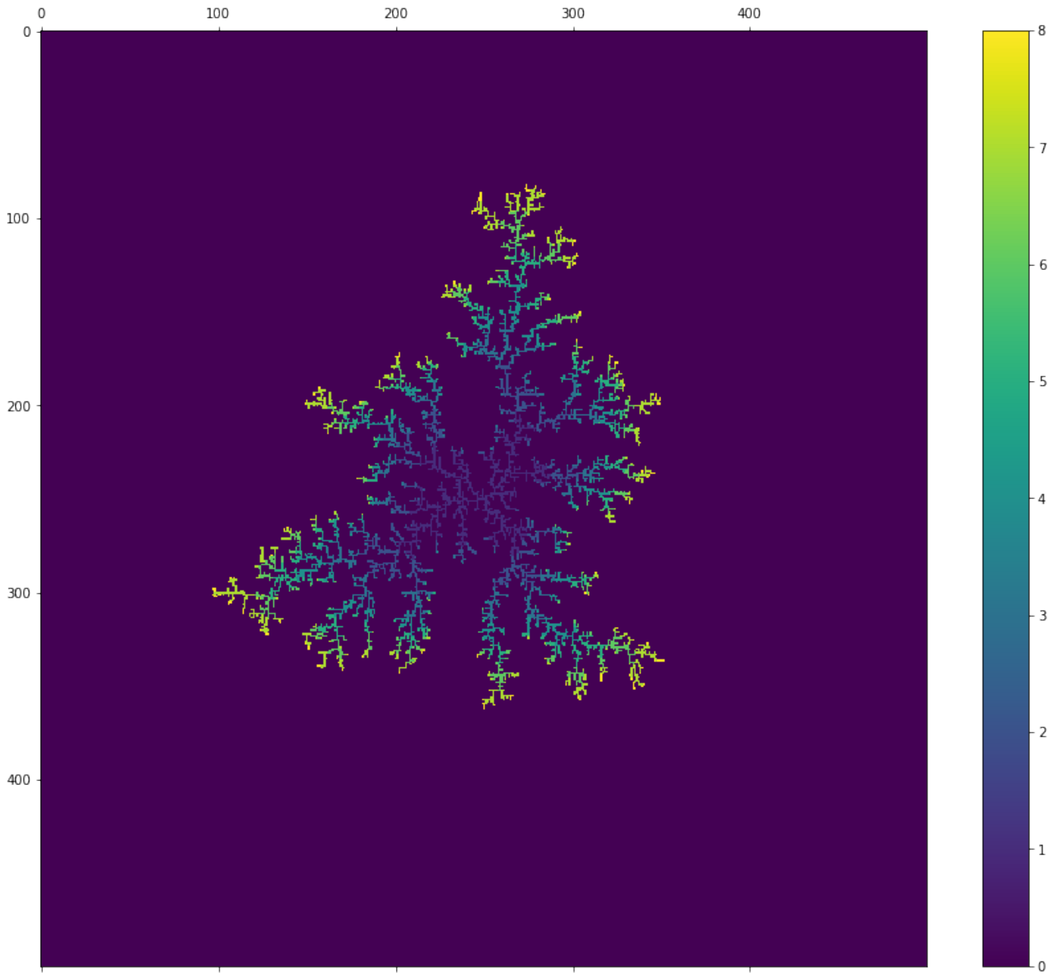

Figure 6: DLA cluster for lattice $N = 500$ with noise reduction, every twentieth particle was accepted, sites numbering is denoted on the axes, color denoting the aggregation time

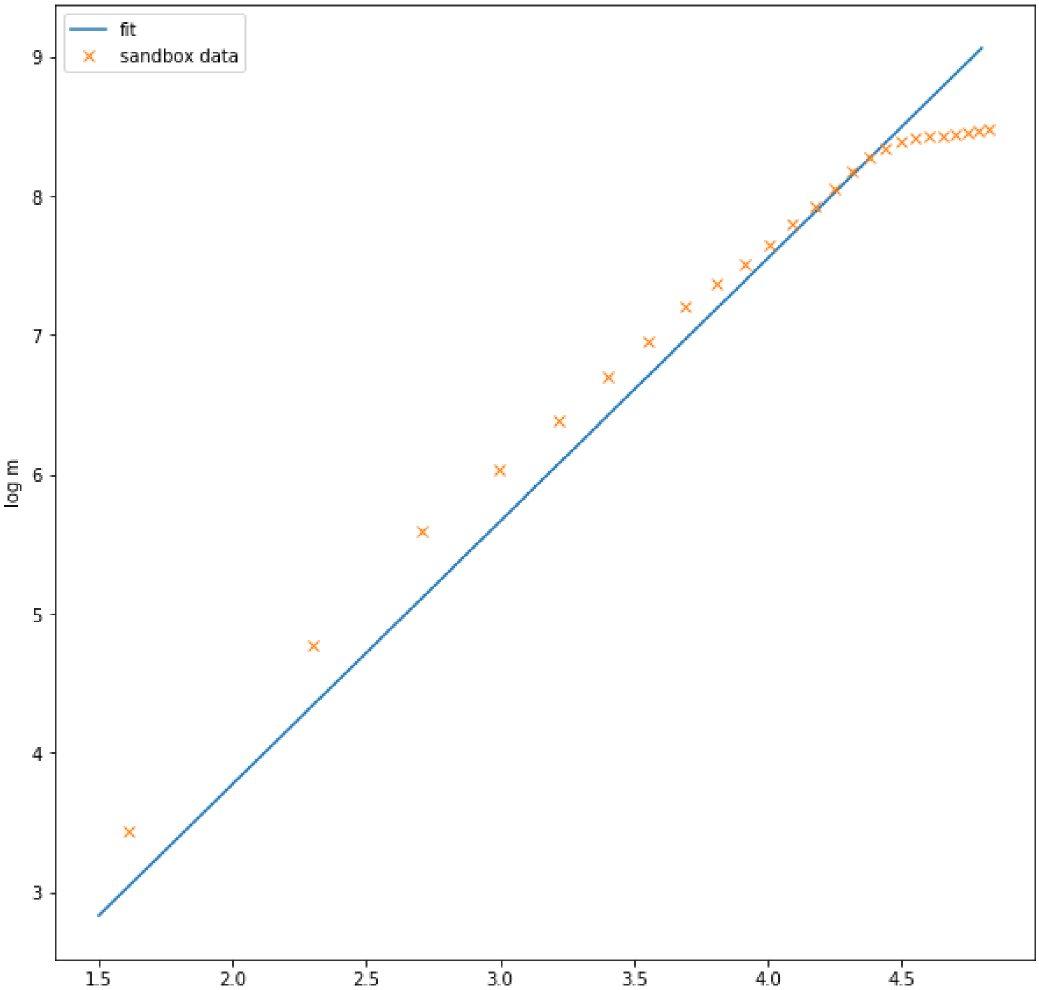

Figure 7: Log mass vs. log radius data and their fit in the linear region