

Photo album multiobjective QAP

Master's Degree first year project

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1.1. QAP definitions

Origin

QAP was introduced by Koopmans and Beckmann in 1957

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- NP-Hard problem

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Explanation

- NP-Hard problem
- Assign a set of facilities to a set of locations
- Minimize the total assignment cost

1.1. QAP definitions

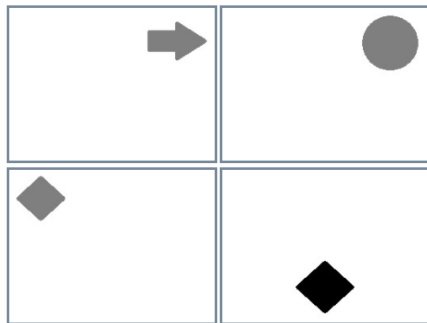


Figure: Page 1

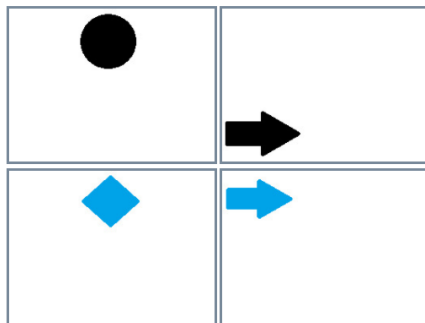


Figure: Page 2

Example

$$p = \{7, 3, 1, 2, 8, 5, 6, 4\}$$

1.1. QAP definitions

Solution definitions

- $N = \{1, 2, \dots, n\}$, the solution representation

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Matrix of QAP

- $S = (s_{ij})$ is an $n \times n$ matrix where s_{ij} is the computed similarity distance between photos i and j .
- $D = (d_{ij})$ is an $n \times n$ matrix where d_{ij} is the euclidean distance between photos i and j .

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Single objective function to minimize

$$\min_{\phi \in S_n} \sum_{i=1}^n \sum_{j=1}^n s_{ij} \cdot d_{\phi(i)\phi(j)}$$

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1.2. Photo album mQAP

Multiobjective function to minimize with $k \in [1, 2]$

$$\min f_1(\phi) = \sum_{i=1}^n \sum_{j=1}^n s_{ij}^1 \cdot d_{\phi(i)\phi(j)}$$

$$\min f_2(\phi) = \sum_{i=1}^n \sum_{j=1}^n s_{ij}^2 \cdot d_{\phi(i)\phi(j)}$$

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Domination definition

$\phi \prec \phi'$, if $f_k(\phi') \leq f_k(\phi)$ for all $k \in [1, 2]$

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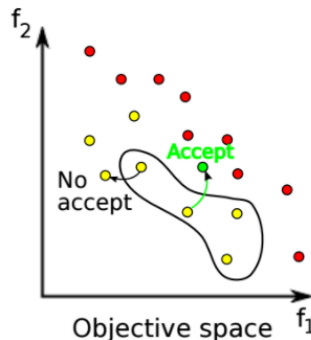
2.1. Random walk

Algorithm 1: Random walk

Input: **nbEval** evaluation stopping
criteria

Output: A

```
1  $A := \theta$ ;  
2 evaluation := 0;  
3 repeat  
4    $s := \text{select randomly a solution}$ ;  
5    $A := A + s$ ;  
6    $A := \text{getNonDominated}(A)$ ;  
7   evaluation := evaluation + 1;  
8 until  $\text{evaluation} \geq \text{nbEval}$ ;
```



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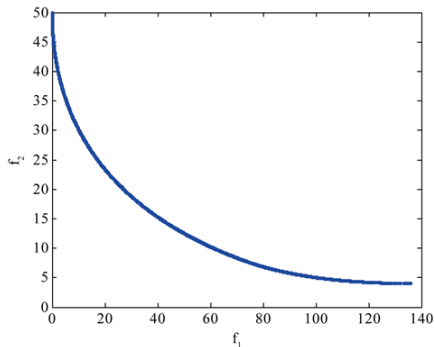
2.2. Pareto Local Search

Algorithm 2: Pareto Local Search

Input: A_0 an initial set of non dominated solutions,
nbEval evaluation stopping criteria

Output: A

```
1  $A := A_0$ ;  
2 explored :=  $A_0$ ;  
3 evaluation := 0;  
4 repeat  
5    $s :=$  select randomly a solution  $\notin A$ ;  
6   foreach  $s' \in V(s)$  do  
7     if  $s' \notin \text{explored}$  then  
8        $A := A + s'$ ;  
9        $A := \text{getNonDominated}(A)$ ;  
10      evaluation := evaluation + 1;  
11    end  
12    explored := explored +  $s'$ ;  
13  end  
14 until evaluation  $\geq$  nbEval;
```



1 Problem presentation

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- Random walk
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- **MOEA/D**
- Two-Phase Local Search

3 Algorithms comparisons

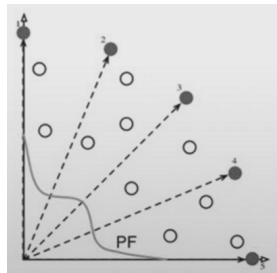
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2.3. MOEA/D - Weighted sum

Multiobjective Evolutionary Algorithm Based on Decomposition

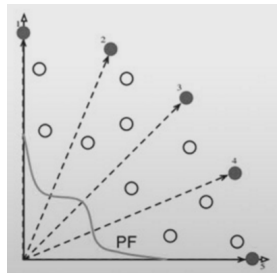
Method which decomposes multiobjective problems into a number of scalar sub problems and optimizes them simultaneously.



2.3. MOEA/D - Weighted sum

Multiobjective Evolutionary Algorithm Based on Decomposition

Method which decomposes multiobjective problems into a number of scalar sub problems and optimizes them simultaneously.



Weighted sum single objective scalarizing method

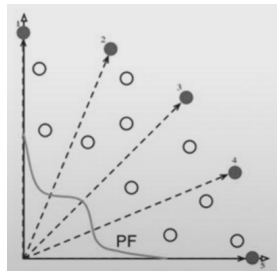
$$g_{\lambda}(x) = \lambda_1 \cdot f_1(x) + \lambda_2 \cdot f_2(x)$$

where $x \in S_n$ is a candidate solution, and $\lambda = (\lambda_1, \lambda_2)$ is a weighting coefficient vector.

2.3. MOEA/D - Tchebycheff

Multiobjective Evolutionary Algorithm Based on Decomposition

Method which decomposes multiobjective problems into a number of scalar sub problems and optimizes them simultaneously.



Tchebycheff single objective scalarizing method

$$g_{\lambda}(x) = \min \left\{ \lambda_1 * |f_1(x) - r_1|, \lambda_2 * |f_2(x) - r_2| \right\}$$

where r is a reference point in the objective space, as example $r(0,0)$.

2.3. MOEA/D

Algorithm 3: Multiobjective Evolutionary Algorithm Based on Decomposition

Input: **N** the number of sub problem, **T** the number of the weight vectors in the neighborhood of each weight vector, **g** the single objective scalarizing approach, **nbEval** evaluation stopping criteria

Output: EP

$$1 \quad EP := \theta;$$

```
2  $\lambda := \text{computeWeightVectors}(N)$ ;
```

3 $B :=$ generating with $B(i) = \{i_1, \dots, i_T\}$ where $\lambda_{i_1}, \dots, \lambda_{i_T}$ are the closest weight vectors to λ_i ;

4 $P :=$ initial population x_1, \dots, x_N of each sub problem set randomly;

5 $FV :=$ matrix which contains objective values of each P solution where FV_i is the F -Value of x_i represented as

$$FV_i = F(x_i);$$

6 $z :=$ reference point generating with min value of each objective found so far into FV ;

```
7 evaluation := 0;
```

8 repeat

```

9 | for  $i := 0$  to  $N$  do

```

 $k, l := \text{random indexes from } B(i);$

$s :=$ new solution from $\{x_k, x_l\}$ using genetic operators;

$s' :=$ new solution produce from s using improvement heuristic;

$z :=$ set min value of each objective found so far into FV to update reference point z :

for $j := 0$ to T do

if $g(s') < g(P(j))$ then

$$P(j) := s';$$
$$FV_i := F(s'_i);$$
$$EP := EP + P(j);$$
$$EP := \text{getNonDominated}(EP);$$

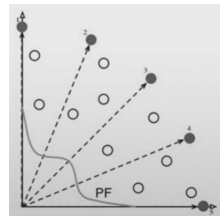
end

```
evaluation := evaluation + 1;
```

end

end

```
24 until evaluation ≥ nbEval:
```



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2.4. Two-Phase Local Search

Algorithm 4: Two-phase Local Search

Input: **N** the number of sub problem, **T** the number of the weight vectors in the neighborhood of each weight vector, **g** the single objective scalarizing approach, **nbEvalMOEAD** MOEAD evaluation stopping criteria, **nbEvalPLS** PLS evaluation stopping criteria

Output: **A**

- 1 $A := \text{MOEAD_Algo}(\text{nbEvalMOEAD}, N, T, g);$
 - 2 $A := \text{PLS_Algo}(\text{nbEvalPLS}, A);$
-

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3.1. Test context

Language

All algorithms source code is in Scala multi paradigm language. Scala has been selected to get benefit of its functional paradigm for this mQAP.

Test platform

The platform used for test suites is a Cloud platform solution with 1 vCPU and 1.7 GB of RAM

3.1. Test context

Album photo size and disposition

- $N = \{1, 2, \dots, 16\}$

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- 4 pages which each contains a 2 per 2 photos matrix.

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Criteria choice

- $f_1 \rightarrow$ Grey AVG

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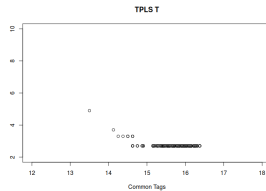
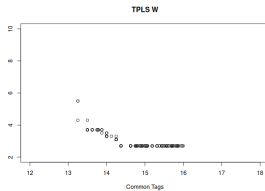
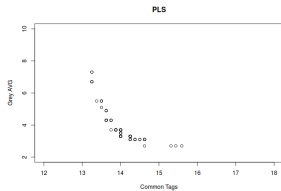
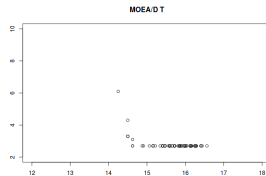
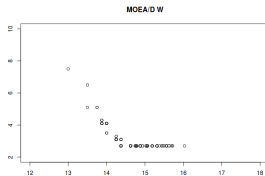
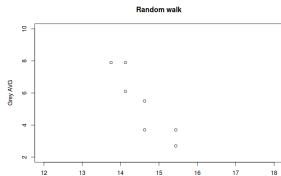
$$\min f_2(\phi) = \sum_{i=1}^n \sum_{j=1}^n s_{ij}^2 \cdot d_{\phi(i)\phi(j)}$$

Criteria choice

- $f_1 \rightarrow$ Grey AVG
- $f_2 \rightarrow$ Common Tags

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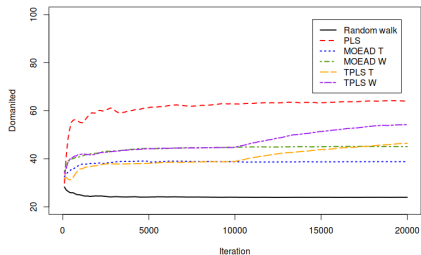
3.2. Landscapes



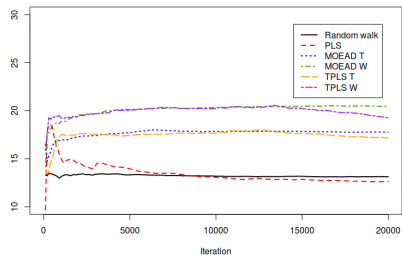
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3.3. Features - Dominated feature

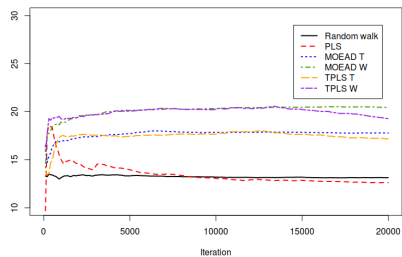
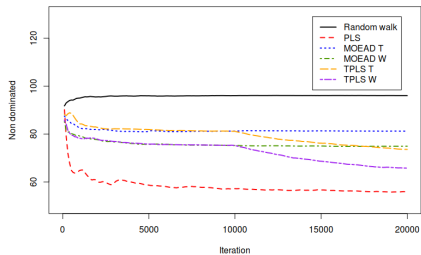
Mean



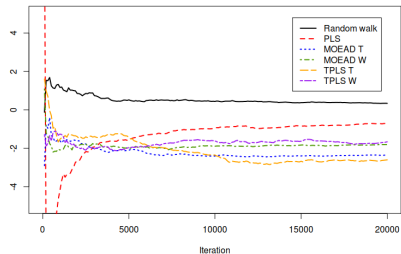
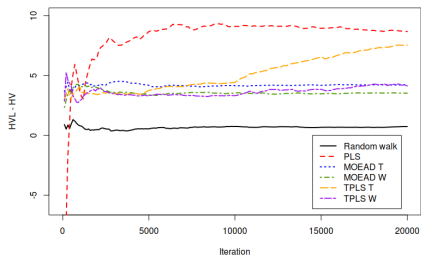
Standard deviation



3.3. Features - Non dominated



3.3. Features - (HVL - HV)



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4. Web platform



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5. Conclusion

- Two-phase Local Search a good compromised
- Algorithm complexity
- Other programming language
- Client customization