

# NTIN071 A&G: TUTORIAL 4 – CLOSURE PROPERTIES OF REGULAR LANGUAGES

**Teaching goals:** The student is able to

- formally describe a construction of an automaton based on other automata
- decide whether regular languages are closed under various set and string operations, including more complex ones, and prove or disprove it

## IN-CLASS PROBLEMS

**Problem 1** (Closure under set and string operations). Given DFAs  $A, B$ , construct an automaton  $C$  recognizing the given language. (Give a formal description of the automaton.)

(a) $L(A) - L(B)$	A	a	b	B	a	b
(b) $L(A).L(B)$	$\rightarrow 0$	1	2	$\rightarrow 0$	0	5
(c) $L(A)^+$	* 1	3	0	* 1	1	3
(d) $L(A)^*$	2	4	5	2	2	5
(e) $L(A)^R$	3	0	2	3	3	2
	4	2	5	* 4	6	1
	5	0	3	5	5	1
				* 6	4	2

**Problem 2** (Delete). Let  $L$  be a regular language over the alphabet  $\Sigma = \{a, b\}$ . Describe the following languages in set notation. Decide if they are (necessarily) also regular, prove or disprove. The language of all words obtained from words of the language  $L$  by...

- ...deleting all occurrences of the letter  $a$ .
- ...deleting the initial letter and writing this letter at the end of the word.
- ...deleting the longest contiguous sequence of  $a$ 's from the beginning of the word.

## EXTRA PRACTICE AND THINKING

**Problem 3** (Prefixes). Are regular languages closed under the following operations? Prove or disprove. (In the following,  $L$  is a regular language over an alphabet  $\Sigma$ .)

- $\text{init}(L) = \{w \in \Sigma^* \mid \text{there is } u \in \Sigma^* \text{ such that } wu \in L\}$
- $\text{min}(L) = \{w \in L \mid \text{there is no } u \in L, v \in \Sigma^+ \text{ such that } w = uv\}$
- $\text{max}(L) = \{w \in L \mid \text{there is no } u \in \Sigma^+ \text{ such that } wu \in L\}$

**Problem 4** (Shift). Given a regular language  $L$  over an alphabet  $\Sigma$ , define the language  $L'$  as follows. Is the language  $L'$  necessarily regular?

$$L' = \{uv \mid u, v \in \Sigma^*, vu \in L\}$$

**Problem 5** (Cut). Consider two regular languages  $L$  and  $M$  over an alphabet  $\Sigma$ , and define the language  $K$  as follows. Is the language  $K$  necessarily regular?

$$K = \{uw \mid u, w \in \Sigma^*, (\exists v \in M) uvw \in L\}$$

**Problem 6** (Switch final and nonfinal states). If we switch accepting and nonaccepting states in a given NFA, will the language recognized by the resulting automaton be the complement of the language recognized by the original NFA? Justify your answer.