NTIN071 A&G: Tutorial 1 – Deterministic finite automaton, recognized language, regular languages

Teaching goals: After this tutorial the student is able to

- use basic terminology and notation from formal languages and automata theory
- explain the formal definition of a DFA, and the language it recognizes
- describe a language recognized by a given DFA, in set notation
- construct (and describe formally) a DFA recognizing a given language
- prove the closure of regular languages under basic set operations

In-class problems (think-pair-share)

Problem 1 (Constructing a DFA for a given language)**.** Construct a DFA recognizing the given language.

(a) $L = \{w \in \{a,b\}^* \mid |w|_a \text{ is not divisible by } 3\}$

(b) $L = \{w \in \{a,b\}^* \mid 2 \text{ divides } |w|_a \text{ or } 3 \text{ divides } |w|_b\}$

(c) $L = \{w \in \{a,b\}^* \mid 2 \text{ divides } |w|_a \text{ and } 3 \text{ divides } |w|_b\}$

(d) $L = \{w \in \{0,1\}^* \mid w \text{ is a binary encoding of a nonnegative integer divisible by } 3\}$

Problem 2 (DFA given by a table)**.** Draw a state diagram and describe the recognized language in set notation.

(a)

	0	1
→ p	q	p
∗ q	r	q
∗ r	p	r

(b)

	0	1
→ p	p	q
∗ q	r	q
∗ r	p	q

Problem 3 (Describing a language and constructing a DFA for a given property)**.** Construct a DFA accepting exactly those words over the alphabet $\Sigma = \{a,b\}$ that satisfy the given property. Describe the language in set notation.

(a) starts 'abba' (b) ends 'abba' (c) contains 'abba' or 'bab' as a subword

Problem 4 (Regular languages and set operations)**.** Let L, L' be regular languages over the same alphabet. Show that the following is true:

(a) $\Sigma^* \setminus L$ is a regular language

(b) $L \cup L'$ is a regular language

(c) $L \cap L'$ is a regular language

Extra Practice and Thinking

Problem 5. Construct a DFA recognizing the given language.

(a) $L = \{w \in \{a,b\}^* \mid |w|_a \text{ is even}\}$

(b) $L = \{w \in \{a,b\}^* \mid |w|_b \text{ is divisible by } 3\}$

(c) $L = \{w \in \{a,b\}^* \mid 2 \text{ or } 3 \text{ divides } |w|_a\}$

(d) $L = \{w \in \{a,b\}^* \mid 2 \text{ and } 3 \text{ divides } |w|_a\}$

(e) $L = \{w \in \{0,1\}^* \mid w \text{ is a binary encoding of a nonnegative integer divisible by } 5\}$

Problem 6. Draw a state diagram and describe the recognized language in set notation.

(a)

	0	1
$\rightarrow *$ p	q	p
q	r	q
r	p	r

(b)

	0	1
$\rightarrow$ p	p	q
q	p	r
$*$ r	p	r

Problem 7. Construct a DFA recognizing the language of all words over $\Sigma = \{a,b\}$ satisfying the property:

(a) has at least 2 letters and the first letter is the same as the last letter

(b) has at least 2 letters and the first two letters are the same as the last two letters

Problem 8. What if L, L' are regular languages over different (but not necessarily disjoint) alphabets? Are the languages $\Sigma^* \setminus L$, $L \cup L'$, $L \cap L'$ necessarily regular?

Problem 9. Would you be able to show that L^R (i.e., words from L written in reverse) is regular whenever L is regular?