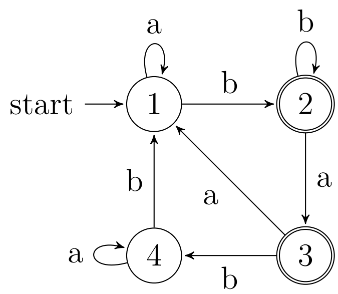
NTIN071 A&G: Tutorial 7 – Formal grammars, regular and context-free grammars, bonus: two-way automata

Solve 1a-d, 2, 3, 4ab first (the rest is for practice, the bonus section on 2-way automata won't be tested)

Problem 1 (Constructing grammars)**.** Design grammars (of the highest possible type) which generate the following languages (the alphabet is $\Sigma = \{a, b\}$ unless specified otherwise).

(a) $L = \Sigma^*$
(b) $L = \{w \mid |w|_b \text{ is even}\}$
(c) $L = \{ww^R \mid w \in \Sigma^*\}$
(d) $L = \{a^{2i}b^j \mid i \leq j\}$
(e) $L = \{w \mid |w|_a = 2|w|_b\}$
(f) $L = \{a^ib^jc^k \mid i = j \text{ or } j = k\}$

Problem 2 (FA to grammar)**.** For the following automaton, find an equivalent grammar. Which class of the Chomsky hierarchy does it belong to?



Problem 3 (Type 3 grammar to FA)**.** For the following right linear grammar, construct an equivalent finite automaton: $G = (\{S, A, B, C\}, \{a, b\}, \mathcal{P}, S)$ where $\mathcal{P}$ consists of the following production rules:

$$
\begin{aligned}
S &\to abS \mid babA \mid \epsilon \\
A &\to abA \mid aB \mid bC \\
B &\to abS \mid B \mid bC \mid \epsilon \\
C &\to aab \mid A \mid aA \mid \epsilon
\end{aligned}
$$

Problem 4 (Testing properties of context-free languages)**.** Design an (efficient) algorithm which decides in a given CFG satisfies the given property:

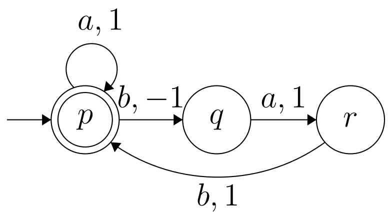
(a) $L(G) \neq \emptyset$
(b) $\epsilon \in L(G)$
(c) $L(G)$ is a finite language

Problem 5 (Small grammars generating large (finite) languages)**.** Find a sequence of CFGs $G_1, G_2, G_3, \ldots$ (over a given alphabet Σ) such that G_n generates exactly all words of length $\leq 2^n$ (and no other words), and the size of G_n (for simplicity, say the number of symbols in bodies of production rules) is in $O(n)$.

Bonus: Two-way automata

Problem 6 (Inspect and convert a 2-way automaton)**.** Consider the following two-way automaton.

	a	b
$\to *p$	$p, 1$	$q, -1$
q	$r, 1$	
r		$p, 1$



(a) Determine the language accepted by this automaton.
(b) Determine the functions f_u and the congruence $\sim$ for all words of length at most 4.
(c) Convert it to an equivalent one-way automaton.

Problem 7 (Without 2-way automata this is hard)**.** Given a DFA A, design a nondeterministic finite automaton accepting the language $L' = \{\#w\# \mid ww^R \in L(A)\}$. Do not use two-way automata.

Problem 8 (Constructing 2-way automata)**.** Let L be a regular language over the alphabet Σ accepted by a finite automaton A and $\# \notin \Sigma$. Construct a two-way finite automaton accepting the given language:
(a) $L' = \{\#w\# \mid ww^R \in L\}$
(b) $L' = \{\#w\# \mid (\exists u \in \Sigma^*)(wu \in L \ \wedge \ |w| = |u|)\}$