NTIN071 A&G: Tutorial 3 — Equivalent and minimal representations, testing properties, nondeterminism, closure under operations

Solve 1abc only for A&B, 2, 3a, 4 (the rest is for practice).

Problem 1 (Equivalent and minimal representations). For the automata below:

- (a) Find and remove all unreachable states.
- (b) Determine the state equivalence (indistinguishability) relations. (Moreover, for any distinguishable pair of states find all minimal-length distinguishing words.)
- (c) Construct their reducts.
- (d) Are any two of the automata equivalent? Use the algorithm from the lecture.

A	a	b	$B \mid a \mid b$	\mathbf{C}	a	b
$\rightarrow * 0$	1	2	$\rightarrow * 0 0 5$	$\rightarrow 1$	2	3
1	3	0	$1 \mid 1 \mid 3$	2	2	4
2	4	5	$2 \mid 2 \mid 7$	* 3	3	5
3	0	2	$3 \mid 3 \mid 2$	4	2	7
4	2	5	* 4 6 1	* 5	6	3
5	0	3	5 5 1	* 6	6	6
	1		* 6 4 2	7	7	4
			$7 \mid 7 \mid 0$	8	2	3
			'	9	9	4

Problem 2 (Testing properties). Consider finite automata A, B. Design an algorithm that decides the following properties. (Can you estimate the time complexity?)

(a)
$$L(A) = \emptyset$$
,

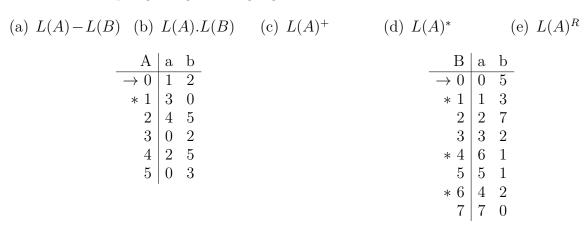
(c)
$$L(A) \subseteq L(B)$$
,

(b)
$$L(A) = L(B)$$
,

(d)
$$L(A)$$
 is finite.

Problem 3 (Subset construction). Given a nondeterministic finite automaton with λ -transitions, construct an equivalent reduced DFA.

Problem 4 (Closure under operations). For the automata A, B given below, construct an automaton accepting the given language.



Problem 5 (Automata homomorphism). Find DFA A, B such that:

- (a) Both are reduced and they are not isomorphic.
- (b) A is homomorphic onto B but they are not isomorphic.
- (c) They are equivalent but not isomorphic.
- (d) They are both homomorphic onto C but not isomorphic with it. Moreover, A is not homomorphic onto B and B is not homomorphic onto A.

$$C = (\{p,q\},\{0,1\},\{((p,0),q),((p,1),p),((q,0),p),((q,1),q)\},p,\{q\})$$

Problem 6 (Regular? Reduce). Let L be a language over the alphabet $\{a, b\}$ consisting of words that do not contain three consecutive occurrences of the same letter. Decide if L is regular. If it is, find a reduced DFA accepting L.

Problem 7 (Switch final and nonfinal states). If we switch accepting and nonaccepting states in a given NFA, will the language accepted by the resulting automaton be the complement of the language accepted by the original NFA? Justify your answer.