NTIN071 A&G: Tutorial 3 – Equivalent and minimal representations, automata homomorphism, nondeterminism, closure properties

Solve 1abc only for A&B, 2, 3a (the rest is for practice).

Problem 1 (Equivalent and minimal representations)**.** For the automata below:

(a) Find and remove all unreachable states.

(b) Determine the state equivalence (indistinguishability) relations. (Moreover, for any distinguishable pair of states find all minimal-length distinguishing words.)

(c) Construct their reducts.

(d) Are any two of the automata equivalent? Use the algorithm from the lecture.

A:

	a	b
$\to *0$	1	2
1	3	0
2	4	5
3	0	2
4	2	5
5	0	3

B:

	a	b
$\to *0$	0	5
1	1	3
2	2	7
3	3	2
$*4$	6	1
5	5	1
$*6$	4	2
7	7	0

C:

	a	b
$\to 1$	2	3
2	2	4
$*3$	3	5
4	2	7
$*5$	6	3
$*6$	6	6
7	7	4
8	2	3
9	9	4

Problem 2 (Automata homomorphism)**.** Find DFA A, B such that:

(a) Both are reduced and they are not isomorphic.

(b) A is homomorphic onto B but they are not isomorphic.

(c) They are equivalent but not isomorphic.

(d) They are both homomorphic onto C but not isomorphic with it. Moreover, A is not homomorphic onto B and B is not homomorphic onto A.

$$C = (\{p, q\}, \{0, 1\}, \{((p, 0), q), ((p, 1), p), ((q, 0), p), ((q, 1), q)\}, p, \{q\})$$

Problem 3 (Subset construction)**.** Given a nondeterministic finite automaton with λ-transitions, construct an equivalent reduced DFA.

(a)

	a	b	λ
$*A$	$\{A, C\}$	$\{B\}$	$\emptyset$
B	$\{B, D\}$	$\emptyset$	$\emptyset$
$*C$	$\{E\}$	$\{D\}$	$\emptyset$
D	$\{A\}$	$\{C, D\}$	$\emptyset$
$\to *E$	$\emptyset$	$\emptyset$	$\{A, C\}$

(b)

	a	b	λ
$\rightarrow A$	$\{E\}$	$\{B\}$	$\emptyset$
B	$\emptyset$	$\{C\}$	$\{D\}$
$\rightarrow C$	$\emptyset$	$\{D\}$	$\emptyset$
$*D$	$\emptyset$	$\emptyset$	$\emptyset$
E	$\{F\}$	$\emptyset$	$\{B, C\}$
F	$\{D\}$	$\emptyset$	$\emptyset$

Problem 4 (Closure properties)**.** Construct an automaton accepting the given language.

(a) $L(A) - L(B)$
(b) $L(A).L(B)$
(c) $L(A)^+$
(d) $L(A)^*$
(e) $L(A)^R$

A	a	b
$\rightarrow *\ 0$	1	2
1	3	0
2	4	5
3	0	2
4	2	5
5	0	3

B	a	b
$\rightarrow *\ 0$	0	5
1	1	3
2	2	7
3	3	2
$*\ 4$	6	1
5	5	1
$*\ 6$	4	2
7	7	0