NTIN071 A&G: Tutorial 2 – Pumping Lemma, Myhill–Nerode theorem

Teaching goals: The student is able to

- state and prove the Pumping Lemma
- apply the Pumping Lemma to prove nonregularity of a given language
- state and prove the Myhill-Nerode theorem
- apply the Myhill-Nerode theorem to prove regularity, to construct a DFA
- apply the Myhill-Nerode theorem to prove nonregularity

In-class problems

Problem 1 (Pumping Lemma: statement)**.** (a) Formulate the Pumping Lemma for regular languages (without consulting your notes).

(b) How is the number n form its statement related to a recognizing automaton?

(c) Prove it (without consulting your notes).

(d) Demonstrate pumping on $L = \{w \in \{a,b\}^* \mid w \text{ contains } abba \text{ as a subword}\}$.

Problem 2 (Pumping Lemma: application)**.** Use the Pumping Lemma to prove that the following languages are not regular. (The alphabet is $\Sigma = \{a, b\}$.)

(a) $L = \{a^i b^j \mid i \geq j\}$

(b) $L = \{a^{i^2} \mid i \geq 0\}$

(c) $L = \{a^i b^{i+j} a^j \mid i, j \geq 0\}$

(d) $L = \{ww^R \mid w \in \Sigma^*\}$, where w^R is w reversed

Problem 3 (Myhill–Nerode theorem: statement)**.** (a) Formulate the Myhill–Nerode theorem and recall the idea of its proof (without consulting your notes).

(b) Show that if we forget any of the conditions on the equivalence $\sim$, the resulting statement is not true.

Problem 4 (Myhill–Nerode theorem: application)**.** Prove or disprove using the Myhill–Nerode theorem that the following languages are regular.

(a) $L = \{aa, ab, ba\}$

(b) $L = \{a^i b^j \mid i \geq j\}$

(c) $L = \{a^{i^2} \mid i \geq 0\}$

(d) $L = \{ww^R \mid w \in \Sigma^*\}$, where w^R is w reversed

(e) $L = \{a^i b^{i+j} a^j \mid i, j \geq 0\}$

Extra Practice and Thinking

Problem 5. Use the Pumping Lemma to prove that the following languages are not regular. (The alphabet is $\Sigma = \{a, b\}$.)
(a) $L = \{a^i b^j \mid i \leq j\}$

(b) $L = \{a^{2^i} \mid i \geq 0\}$

(c) $L = \{ww \mid w \in \Sigma^*\}$

Problem 6. Prove or disprove using the Myhill–Nerode theorem that the following languages are regular.
(a) $L = \{a^i b^j \mid i \leq j\}$

(b) $L_k = \{a^i b^j \mid i \leq j \leq k\}$ for a fixed $k \in \mathbb{N}$

(c) $L = \{a^{2^i} \mid i \geq 0\}$

(d) $L = \{ww \mid w \in \Sigma^*\}$

Problem 7 (Pumping Lemma: generalization)**.** (a) Can we change the condition $|uv| \leq n$ with $|vw| \leq n$, that is, *iterate near the end*? Prove or disprove.

(b) Can we iterate near a chosen position in the word? How to formulate (and prove) such a generalization?

Problem 8 (Equivalences on words)**.** Give an example of an equivalence relation $\sim$ on Σ^* which:

(a) is a right and a left congruence

(b) is a right but not a left congruence

(c) is of finite index