NTIN071 A&G: Tutorial 2 – Pumping lemma, Myhill–Nerode theorem

Solve 1, 2a, 3a-f, 4, 5, 6a-f first (the rest is for practice).

Problem 1 (Pumping lemma: statement)**.** (a) Formulate the Pumping lemma for regular languages (without consulting your notes).

(b) How is the number n form its statement related to a recognizing automaton?

(c) Prove it (without consulting your notes).

Problem 2 (Pumping lemma: generalization)**.** (a) Can we change the condition $|uv| \leq n$ with $|vw| \leq n$, that is, *iterate near the end*? Prove or disprove.

(b) Can we iterate near a chosen position in the word? How to formulate (and prove) such a generalization?

Problem 3 (Pumping lemma: application)**.** Determine which of the following languages are nonregular and show that using the Pumping lemma. (The alphabet is $\Sigma = \{a, b\}$.)

(a) $L = \{aa, ab, ba\}$

(b) $L = \{a^i b^j \mid i \leq j\}$

(c) $L = \{a^i b^j \mid i \geq j\}$

(d) $L_k = \{a^i b^j \mid i \leq j \leq k\}$ for a given $k \in \mathbb{N}$

(e) $L = \{a^{2^i} \mid i \geq 0\}$

(f) $L = \{ww^R \mid w \in \Sigma^*\}$, where w^R denotes the word w written in reverse

(g) $L = \{a^i b^{i+j} a^j \mid i, j \geq 0\}$

(h) $L = \{ww \mid w \in \Sigma^*\}$

Problem 4 (Equivalences on words)**.** Give an example of an equivalence relation $\sim$ on Σ^* which:

(a) is a right and a left congruence

(b) is a right but not a left congruence

(c) is of finite index

Problem 5 (Myhill–Nerode theorem: statement)**.** Formulate the Myhill–Nerode theorem and recall the idea of its proof (without consulting your notes).

Problem 6 (Myhill–Nerode theorem: application)**.** Prove or disprove using the Myhill–Nerode theorem that the following languages are regular.

(a) $L = \{aa, ab, ba\}$

(b) $L = \{a^i b^j \mid i \leq j\}$

(c) $L = \{a^i b^j \mid i \geq j\}$

(d) $L_k = \{a^i b^j \mid i \leq j \leq k\}$ for a fixed $k \in \mathbb{N}$

(e) $L = \{a^{2^i} \mid i \geq 0\}$

(f) $L = \{ww^R \mid w \in \Sigma^*\}$, where w^R is w reversed

(g) $L = \{a^i b^{i+j} a^j \mid i, j \geq 0\}$

(h) $L = \{ww \mid w \in \Sigma^*\}$