NTIN071 A&G: Tutorial 11 – Turing Machines

Teaching goals: The student is able to

- explain the formal definition of deterministic and nondeterministic Turing machines
- describe computation graph, define the recognized language, the computed function
- perform the computation of a given Turing machine on a given input
- determine the language recognized by a given Turing machine
- construct a Turing machine recognizing given language, computing given function
- analyze various variants & modifications of the Turing machine computation model

In-class problems

Problem 1 (A Turing machine)**.** Consider the following TM.

	B	a	b	c
$\to q_0$	(q_1, B, L)	(q_0, a, R)	(q_0, c, R)	(q_0, c, R)
q_1	(q_2, B, R)	(q_1, c, L)		(q_1, b, L)
$*q_2$				

(a) Draw the state diagram.
(b) Describe the computation (by a sequence of configurations) for $w = aabca$.
(c) What language does the machine recognize? What function does it compute?

Problem 2 (Erase all 1s)**.** Design a TM over the alphabet $\{0, 1\}$ which will erase all 1's from the input and then return to the beginning (e.g. if it starts in the configuration $q_0 0011010$, then it will halt in the configuration $q_F 0000$ for some $q_F \in F$).

Problem 3 (Predecessor)**.** Construct a Turing machine T that for a given input natural number $x > 0$ in binary encoding outputs its predecessor, i.e., $x - 1$ (in binary encoding as well) and returns the head to the beginning of the output.

(a) Draw the state diagram of T.
(b) Write a sequence of *configurations* that the machine goes through during some accepting computation for the input word $w = 10100$.

Construct a deterministic, single-tape, single-track machine. (If you want e.g. a two-track machine, program it yourself.) A number in binary encoding must not start with 0, unless it is equal to 0. Examples of input and output configurations:

- from the configuration $q_0 1$ the machine should finish in $f0$ for some $f \in F$,
- from the configuration $q_0 1001$ the machine should finish in $f1000$ for some $f \in F$,
- from the configuration $q_0 100$ the machine should finish in $f11$ for some $f \in F$.

Problem 4 (One-way infinite tape)**.** Describe how to convert a Turing machine with a (single) two-way infinite tape to a Turing machine whose tape is only infinite in one direction, to the right. (You can assume that the second TM's tape contains a special delimiter $\triangleright$ in its first field.)

Problem 5 (Nondeterministic test of non-primeness)**.** Design a nondeterministic TM which will recognize the language $L = \{1^n \mid n \text{ is not a prime number}\}$.

Extra Practice and Thinking

Problem 6 (Programming TMs)**.** Design a TM which will accept the language L. Write down the sequence of configurations that shows that the given word w is accepted.

(a) $L = \{0^n1^n2^n \mid n \geq 0\}$, $w = 001122$
(b) $L = \{w \in \{0,1\}^* \mid |w|_0 = |w|_1\}$, $w = 100110$
(c) $L = \{ucu^R \mid u \in \{0,1\}^*\}$, $w = 10c01$
(d) $L = \{ucu \mid u \in \{0,1\}^*\}$, $w = 10c10$
(e) $L = \{uu \mid u \in \{0,1\}^*\}$, $w = 110110$

Problem 7 (Reverse)**.** Design a TM which will create the reverse of the input word.

Problem 8 (Memory blocks)**.** Design a TM which will switch the contents of two memory blocks. Specifically, if it starts in the configuration $q_0u\#v\#w\#x\#y$ (where $u, v, w, x, y \in \Sigma \setminus \{\#\}$), then it halts in the configuration $fu\#x\#w\#v\#y$ for some $f \in F$.

Problem 9 (Head moves)**.** Consider modifications of Turing machines in which the allowed moves of the head are the following. What class of languages they recognize?

(a) left (L) and right (R),
(b) stay (N) and right (R),
(c) stay (N) and left (L),
(d) left (L), right (R), and stay (N).

Problem 10 (Only two actions at once)**.** Show that any single-tape Turing machine M can be converted to a Turing machine M' which is allowed to execute only two of the three actions at one step, that is, any instruction either

- changes state and head position, or
- changes state and tape symbol, or
- changes head position and tape symbol,

but no instruction can perform all three of these actions.

Problem 11 (Right or restart)**.** Consider a Turing machine model where the tape is only one-way infinite (to the right) and the head can only perform two types of movement: right (R) or RESTART (that is, return to the first field of the tape). Show how to convert a single-tape Turing machine to a Turing machine of this kind.

Problem 12 (Rewrite at most once)**.** Consider a single-tape Turing machine which is allowed to change any field (i.e., it can rewrite the symbol with a different symbol) on the tape at most once. Show that this model is equivalent to a regular single-tape TM.

Problem 13 (Don't rewrite input)**.** Explain why if a single-tape Turing machine is forbidden to modify the fields containing the input, it is equivalent to a finite automaton. (And therefore such TMs only recognize regular languages. It is enough to give the main idea, not a detailed construction.)

Problem 14 (Closure properties)**.** Show that *recursive* languages and *recursively enumerable* languages are closed under (a) *union*, (b) *intersection*, (c) *concatenation*, (d) *iteration*.

Moreover, show that (e) *recursive languages are closed under complementation*, but (f) *recursively enumerable languages are not.*