NTIN071 A&G: Homework assignment

Each problem is worth 5 points, totaling 20 points. The solution must be entirely your own work. Describe all steps in your solution, the objects you construct, and proofs in sufficient detail and using the formalism from the lecture.

Problem 1. Solve the following sample test:

(a) Construct a context-free grammar generating the language $L = \{a^n b^k a^{3n} \mid n, k \geq 0\}$. Write down a derivation for the word $w = abbaaa$.

(b) Convert the grammar from the previous problem to Chomsky normal form.

(c) Prove that the language $L = \{a^{n^5} \mid n \geq 0\}$ is not regular.

(d) Construct a pushdown automaton accepting, by empty stack, the language $L = \{w \in \{0,1\}^* \mid |w|_0 \geq |w|_1 + 1\}$. Write down a sequence of configurations for the word $w = 10001$.

(e) Prove that the language $L = \{0^i 1^j 2^k 3^\ell \mid i = j = k \text{ or } \ell = 0\}$ is not context-free.

(f) Construct a deterministic finite automaton that accepts exactly those words over the alphabet $\{0,1\}$ which end with the sequence 010.

Problem 2. Let us have a pair of regular languages L, M over the alphabet $\{0,1\}$. Assume that we have DFAs A, B such that $L = L(A)$ and $M = L(B)$. Define the language K as follows:

$$K = \{uvw \mid uw \in L, v \in M\}$$

(a) Construct a ϵ-NFA C recognizing language K. (The construction must be formally described but you do not have to prove that C accepts the language K.)

Demonstrate your construction also on the following example: Let $L = \{w \in \{0,1\}^* \mid |w| \text{ is divisible by } 3\}$ and $M = \{0^{2n+1}11 \mid n \geq 0\}$.

(b) Draw the state diagrams of some DFAs A, B accepting the languages L, M (respectively).

(c) Draw the state diagram of the ϵ-NFA C obtained by your construction.

(d) Write a sequence of states that the automaton C goes through during some accepting computation for the input word $w = 001100$.

Problem 3. Consider the following language over the alphabet $\{0, 1, \#\}$:

$$L = \{w\#s^R \mid w, s \in \{0,1\}^* \text{ and } s \text{ is a substring of } w\}$$

(Note: s^R denotes the word s written backwards; a substring is a contiguous subsequence, including empty and the entire word.)

(a) Prove that L is not regular.

(b) Construct a context-free grammar G generating the language L.

(c) Write down the derivation of the word 1001#00.

(d) Convert the grammar G from part (a) into a pushdown automaton accepting by empty stack.

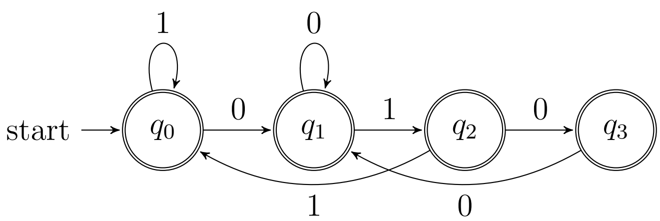
(e) Write a sequence of configurations for the word 1001#00.

Problem 4. Consider the following languages over the alphabet $\Sigma = \{0, 1\}$:

- L_1 is the language generated by the context-free grammar $G = (\{S\}, \Sigma, \mathcal{P}, S)$ with production rules $\mathcal{P} = \{\, S \to SS \mid 0S1 \mid \epsilon \,\}$ (where ϵ denotes the empty word),
- L_2 is the language recognized by the deterministic finite automaton

$$A = (\{q_0, q_1, q_2, q_3\}, \Sigma, \delta_A, q_0, \{q_0, q_1, q_2, q_3\})$$

whose transition function δ_A si given by the following state diagram:



Construct a pushdown automaton recognizing the intersection $L = L_1 \cap L_2$ of the lanugages L_1 and L_2.