

*Solve 1, 2a, 3a-f, 4, 5a-f first (the rest is for practice).*

**Problem 1** (Pumping lemma: statement). (a) Formulate the Pumping lemma for regular languages (without consulting your notes).

(b) How is the number  $n$  from its statement related to a recognizing automaton?

(c) Prove it (without consulting your notes).

**Problem 2** (Pumping lemma: generalization). (a) Can we change the condition  $|uv| \leq n$  with  $|vw| \leq n$ , that is, *iterate near the end*? Prove or disprove.

(b) Can we iterate near a chosen position in the word? How to formulate (and prove) such a generalization?

**Problem 3** (Pumping lemma: application). Determine which of the following languages are nonregular and show that using the Pumping lemma. (The alphabet is always  $\Sigma = \{a, b\}$ .)

(a)  $L = \{aa, ab, ba\}$

(b)  $L = \{a^i b^j \mid i \leq j\}$

(c)  $L = \{a^i b^j \mid i \geq j\}$

(d)  $L_k = \{a^i b^j \mid i \leq j \leq k\}$  for a given  $k \in \mathbb{N}$

(e)  $L = \{a^{2^i} \mid i \geq 0\}$

(f)  $L = \{ww^R \mid w \in \Sigma^*\}$ , where  $w^R$  denotes the word  $w$  written in reverse

(g)  $L = \{a^i b^{i+j} a^j \mid i, j \geq 0\}$

(h)  $L = \{ww \mid w \in \Sigma^*\}$

**Problem 4** (Equivalences on words). Give an example of an equivalence relation on  $\Sigma^*$  which:

(a) is a right and a left congruence

(b) is a right but not a left congruence

(c) is of finite index

**Problem 5** (Myhill-Nerode theorem: application). Prove or disprove using the Myhill-Nerode theorem that the following languages are regular.

- (a)  $L = \{aa, ab, ba\}$
- (b)  $L = \{a^i b^j \mid i \leq j\}$
- (c)  $L = \{a^i b^j \mid i \geq j\}$
- (d)  $L_k = \{a^i b^j \mid i \leq j \leq k\}$  for a fixed  $k \in \mathbb{N}$
- (e)  $L = \{a^{2^i} \mid i \geq 0\}$
- (f)  $L = \{ww^R \mid w \in \Sigma^*\}$ , where  $w^R$  is  $w$  reversed
- (g)  $L = \{a^i b^{i+j} a^j \mid i, j \geq 0\}$
- (h)  $L = \{ww \mid w \in \Sigma^*\}$