NTIN071 A&G: Tutorial 9 – Pushdown automata

Teaching goals: The student is able to

- state the formal definition of a PDA, acceptance by empty stack and by final state
- construct a pushdown automaton for a given language
- convert between acceptance by empty stack and acceptance by final state
- convert a context-free grammar into a pushdown automaton
- convert a pushdown automaton into a context-free grammar

In-class problems

Problem 1 (PDA Construction)**.** Design a pushdown automaton recognizing the given language. For (a), (b), (c), accept by empty stack; for (d), (e), (f) by final state.

(a) $L = \{ww^R \mid w \in \{0,1\}^*\}$
(b) $L = \{w \in \{(,)\}^* \mid w \text{ is correctly parenthesized}\}$
(c) $L = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$
(d) $L = \{a^{2n} b^{3n} \mid n \geq 0\}$
(e) $L = \{w \in \{0,1\}^* \mid |w|_0 = |w|_1\}$
(f) $L = \{u2v \mid u, v \in \{0,1\}^* \text{ and } |u| \neq |v|\}$

Problem 2 (Final State vs. Empty Stack)**.** Convert selected pushdown automata constructed in the previous problem from final state acceptance to empty stack acceptance, and vice versa. (Try both conversions.)

Problem 3 (CFG to PDA Conversion)**.** For a given grammar G, construct a PDA A such that $L(G) = N(A)$. Additionally, for a given word $w \in L(G)$, find the leftmost derivation from G and simulate the computation of A (write the accepting sequence of configurations).

(a) $G = (\{S, T, X\}, \{a, b\}, \mathcal{P}, S)$ with the following rules, $w = aaaabbb$

$$\begin{aligned}\mathcal{P} = \{&S \to aTXb,\\ &T \to XTS \mid \epsilon,\\ &X \to a \mid b\}\end{aligned}$$

(b) $G = (\{S, T, X\}, \{(,), *, +, 1\}, P, S)$ with the following rules, $w = 1 + 1 * (1 + 1)$

$$\begin{aligned}P = \{&S \to S + T \mid T,\\ &T \to T * X \mid X,\\ &X \to 1 \mid (S)\}\end{aligned}$$

Problem 4 (PDA to CFG Conversion)**.** Convert the pushdown automata from Problem 1 (a), (b) into context-free grammars. For a reasonably long word w accepted by the given automaton, find the leftmost derivation of this word in the constructed grammar.

Extra Practice and Thinking

Problem 5 (Bonus: Context-Sensitive Grammar)**.** Consider $G = (\{S, A, B, C\}, \{a, b, c\}, S, P)$, where:

$$P = \{S \to aSBC \mid aBC, B \to BBC, C \to CC, CB \to BC,$$
$$aB \to ab, bB \to bb, bC \to bc, cC \to cc\}$$

What language does it generate? Is grammar G context-sensitive? If not, find an equivalent context-sensitive grammar.

Problem 6 (PDA Construction)**.** Design pushdown automata for the following languages. (They may accept by final state or empty stack; for some, construct both; for others, try converting between these two acceptance methods.)

(a) $L = \{w \mid w \in \{0, 1\}^*, |w|_1 \geq 3\}$

(b) $L = \{w \in \{0, 1\}^* \mid w = w^R\}$

(c) $L = \{a^i b^j c^k \mid i + j = k\}$

(d) $L = \{w \in \{(,), [,]\}^* \mid w \text{ is correctly parenthesized}\}$