

*Solve 1, 2ab, 3ab first (the rest is for practice).*

**Problem 1** (Useless symbols). Answer the following questions. Justify your answer.

- (a) Find an example of a grammar in which there is a generating variable only reachable via nongenerating variables.
- (b) When reducing a grammar, which variables do we need to remove first: nongenerating or unreachable?
- (c) Is it possible for a reachable generating variable to become nongenerating after the removal of unreachable variables?

**Problem 2** (Convert to ChNF). Convert the following context-free grammar to Chomsky normal form:

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|---|--|
| <p>(a)</p> $G = (\{S, A, B\}, \{0, 1\}, S, \mathcal{P})$ $\mathcal{P} = \{S \rightarrow 0AB,$ $A \rightarrow 0A0 \mid 11,$ $B \rightarrow 0\}$  | <p>(c)</p> $G = (\{S, A, B\}, \{0, 1\}, S, \mathcal{P})$ $\mathcal{P} = \{S \rightarrow 0A10B10,$ $A \rightarrow 1A0 \mid \lambda,$ $B \rightarrow 1B00 \mid \lambda\}$  |
| <p>(b)</p> $G = (\{S, A, B\}, \{0, 1\}, S, \mathcal{P})$ $\mathcal{P} = \{S \rightarrow A \mid 0SA \mid \lambda,$ $A \rightarrow 1A \mid 1 \mid B1,$ $B \rightarrow 0B \mid 0 \mid \lambda\}$ | <p>(d)</p> $G = (\{S, E, F\}, \{(\cdot), *, +, , 1\}, S, \mathcal{P})$ $\mathcal{P} = \{S \rightarrow (E),$ $E \rightarrow F + F \mid F * F,$ $F \rightarrow S \mid 1\}$ |

**Problem 3** (The CYK algorithm). Using the CYK algorithm determine if  $w \in L(G)$ .

- (a)  $w = 0110$ ,  $G = (\{S, A, B\}, \{0, 1\}, S, \mathcal{P})$ ,
- $$\mathcal{P} = \{S \rightarrow 0 \mid AB,$$
- $$A \rightarrow 1 \mid SA \mid SB,$$
- $$B \rightarrow AS \mid BA \mid 0\}$$
- (b)  $w = abcb$ ,  $G = (\{S, A, B, C\}, \{a, b, c\}, S, \mathcal{P})$ ,
- $$\mathcal{P} = \{S \rightarrow CA \mid CB,$$
- $$B \rightarrow CBA \mid CB \mid BA \mid BB,$$
- $$C \rightarrow ABC \mid BC,$$
- $$A \rightarrow a, B \rightarrow b, C \rightarrow c\}$$

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$$\begin{aligned} \text{(c) } w = abcb, G = (\{S, A, B, C\}, \{a, b, c\}, S, \mathcal{P}), \\ \mathcal{P} = \{S \rightarrow CA \mid CB, \\ B \rightarrow CBA \mid CB \mid BA \mid BB, \\ C \rightarrow ABC \mid BC, \\ A \rightarrow a, B \rightarrow b, C \rightarrow c\} \end{aligned}$$