NTIN071 A&G: Tutorial 4 – Closure properties of regular languages

Teaching goals: The student is able to

- formally describe a construction of an automaton based on other automata
- decide whether regular languages are closed under various set and string operations, including more complex ones, and prove or disprove it

In-class problems

Problem 1 (Closure under set and string operations)**.** Given DFAs A, B, construct an automaton C recognizing the given language. (Give a formal description of the automaton.)

(a) $L(A) - L(B)$

(b) $L(A).L(B)$

(c) $L(A)^+$

(d) $L(A)^*$

(e) $L(A)^R$

A	a	b
$\rightarrow 0$	1	2
$* 1$	3	0
2	4	5
3	0	2
4	2	5
5	0	3

B	a	b
$\rightarrow 0$	0	5
$* 1$	1	3
2	2	5
3	3	2
$* 4$	6	1
5	5	1
$* 6$	4	2

Problem 2 (Delete)**.** Let L be a regular language over the alphabet $\Sigma = \{a, b\}$. Desribe the following languages in set notation. Decide if they are (necessarily) also regular, prove or disprove. The language of all words obtained from words of the language L by...

(a) ... deleting all occurrences of the letter a.

(b) ... deleting the initial letter and writing this letter at the end of the word.

(c) ... deleting the longest contiguous sequence of a's from the beginning of the word.

Extra Practice and Thinking

Problem 3 (Shift)**.** Given a regular language L over an alphabet Σ, define the language L' as follows. Is the language L' necessarily regular?

$$L' = \{uv \mid u, v \in \Sigma^*, vu \in L\}$$

Problem 4 (Cut)**.** Consider two regular languages L and M over an alphabet Σ, and define the language K as follows. Is the language K necessarily regular?

$$K = \{uw \mid u, w \in \Sigma^*, (\exists v \in M)\, uvw \in L\}$$

Problem 5 (Switch final and nonfinal states)**.** If we switch accepting and nonaccepting states in a given NFA, will the language recognized by the resulting automaton be the complement of the language recognized by the original NFA? Justify your answer.