NTIN071 A&G: Tutorial 4 – Closure properties of regular languages

Teaching goals: The student is able to

- formally describe a construction of an automaton based on other automata
- decide whether regular languages are closed under various set and string operations, including more complex ones, and prove or disprove it

In-class problems

Problem 1 (Closure under set and string operations)**.** Given DFAs A, B, construct an automaton C recognizing the given language. (Give a formal description of the automaton.)

(a) $L(A) - L(B)$

(b) $L(A).L(B)$

(c) $L(A)^+$

(d) $L(A)^*$

(e) $L(A)^R$

A	a	b
$\to 0$	1	2
$* 1$	3	0
2	4	5
3	0	2
4	2	5
5	0	3

B	a	b
$\to 0$	0	5
$* 1$	1	3
2	2	5
3	3	2
$* 4$	6	1
5	5	1
$* 6$	4	2

Problem 2 (Delete)**.** Let L be a regular language over the alphabet $\Sigma = \{a, b\}$. Desribe the following languages in set notation. Decide if they are (necessarily) also regular, prove or disprove. The language of all words obtained from words of the language L by. . .

(a) . . . deleting all occurrences of the letter a.

(b) . . . deleting the initial letter and writing this letter at the end of the word.

(c) . . . deleting the longest contiguous sequence of a's from the beginning of the word.

Extra Practice and Thinking

Problem 3 (Prefixes)**.** Are regular languages closed under the following operations? Prove or disprove. (In the following, L is a regular language over an alphabet Σ.)

(a) $\text{init}(L) = \{w \in \Sigma^* \mid \text{there is } u \in \Sigma^* \text{ such that } wu \in L\}$

(b) $\text{min}(L) = \{w \in L \mid \text{there is no } u \in L, v \in \Sigma^+ \text{ such that } w = uv\}$

(c) $\text{max}(L) = \{w \in L \mid \text{there is no } u \in \Sigma^+ \text{ such that } wu \in L\}$

Problem 4 (Shift)**.** Given a regular language L over an alphabet Σ, define the language L' as follows. Is the language L' necessarily regular?

$$L' = \{uv \mid u, v \in \Sigma^*, vu \in L\}$$

Problem 5 (Cut)**.** Consider two regular languages L and M over an alphabet Σ, and define the language K as follows. Is the language K necessarily regular?

$$K = \{uw \mid u, w \in \Sigma^*, (\exists v \in M)\, uvw \in L\}$$

Problem 6 (Switch final and nonfinal states)**.** If we switch accepting and nonaccepting states in a given NFA, will the language recognized by the resulting automaton be the complement of the language recognized by the original NFA? Justify your answer.

Problem 7 (Iterations of unary languages)**.** Show that for any language L over the alphabet $\Sigma = \{a\}$, the language L^* is regular.