NTIN071 A&G: Tutorial 9 – Pumping lemma for context-free languages

Solve 1, 2a-j, 3 first (the rest is for practice).

Problem 1 (Pumping lemma)**.** Recall the statement and proof of the Pumping lemma for context-free languages. Compare with the version for regular languages.

Problem 2 (Proving non-context-freeness)**.** Decide if the following languages are context-free. Prove that your answer is correct.

(a) $L = \{0^i 1^i \mid i \geq 0\}$

(b) $L = \{0^i 1^j 0^i \mid i, j \geq 0\}$

(c) $L = \{0^i 1^j 0^i \mid 0 \leq i \leq j\}$

(d) $L = \{0^i 1^j 0^i \mid 0 \leq j \leq i\}$

(e) $L = \{0^i 1^i 2^i \mid i \geq 0\}$

(f) $L = \{0^{2i} 1^{3i} 0^i \mid i \geq 0\}$

(g) $L = \{0^i 1^j 2^k \mid 0 \leq i \leq j \leq k\}$

(h) $L = \{ww \mid w \in \{0,1\}^*\}$

(i) $L = \{ww^R \mid w \in \{0,1\}^*\}$

(j) $L = \{ww^R \mid w \in \{0,1\}^*, |w|_0 = |w|_1\}$

(k) $L = \{1^{n^2} \mid n \geq 0\}$

(l) $L = \{1^{n^2+n+1} \mid n \geq 0\}$

(m) $L = \{1^p \mid p \text{ is a prime}\}$

(n) $L = \{0^i 1^j \mid 0 \leq i \leq j^2\}$

Problem 3 (Pumping linear languages)**.** Recall that a grammar is *linear*, if it only contains production rules of the form $A \to uBw$ and $A \to w$, where $A, B \in V$ and $u, w \in T^*$.

(a) Formulate a Pumping lemma for linear languages.

(b) Proof the statement using derivations from a (reduced) linear grammar.

(c) How is the constant n from the lemma related to a linear grammar for the given language?

(d) Show that the language $L = \{w \in \{0,1\}^* \mid |w|_0 = |w|_1\}$ is not linear.

(e) Where does L lie within the Chomsky hierarchy?