

Teaching goals: The student is able to

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IN-CLASS PROBLEMS

Problem 1 (Useless symbols). Recall the process of converting a context-free grammar to Chomsky Normal Form. Then answer the following questions. Justify.

- (a) Find an example of a grammar in which there is a generating variable only reachable via nongenerating variables.
- (b) When reducing a grammar, which variables do we need to remove first: nongenerating or unreachable?
- (c) Is it possible for a reachable generating variable to become nongenerating after the removal of unreachable variables?

Problem 2 (Convert to ChNF). Convert the following context-free grammars to Chomsky normal form:

(a)

$$\begin{aligned} G_1 &= (\{S, A, B\}, \{0, 1\}, S, \mathcal{P}) \\ \mathcal{P} &= \{S \rightarrow 0AB, \\ &\quad A \rightarrow 0A0 \mid 11, \\ &\quad B \rightarrow 0\} \end{aligned}$$

(b)

$$\begin{aligned} G_2 &= (\{S, A, B\}, \{0, 1\}, S, \mathcal{P}) \\ \mathcal{P} &= \{S \rightarrow 0A10B10, \\ &\quad A \rightarrow 1A0 \mid \epsilon, \\ &\quad B \rightarrow 1B00 \mid \epsilon\} \end{aligned}$$

Problem 3 (The CYK algorithm). Using the CYK algorithm determine if $w \in L(G)$.
 $w = 0110$, $G = (\{S, A, B\}, \{0, 1\}, S, \mathcal{P})$,

$$\begin{aligned} \mathcal{P} &= \{S \rightarrow 0 \mid AB, \\ &\quad A \rightarrow 1 \mid SA \mid SB, \\ &\quad B \rightarrow AS \mid BA \mid 0\} \end{aligned}$$

EXTRA PRACTICE AND THINKING

Problem 4 (Convert to ChNF). Convert the following context-free grammar to Chomsky normal form:

(a)

$$\begin{aligned} G &= (\{S, A, B\}, \{0, 1\}, S, \mathcal{P}) \\ \mathcal{P} &= \{S \rightarrow A \mid 0SA \mid \epsilon, \\ &\quad A \rightarrow 1A \mid 1 \mid B1, \\ &\quad B \rightarrow 0B \mid 0 \mid \epsilon\} \end{aligned}$$

(b)

$$G = (\{S, E, F\}, \{(\cdot), *, +, , 1\}, S, \mathcal{P})$$

$$\begin{aligned}\mathcal{P} = \{ & S \rightarrow (E), \\ & E \rightarrow F + F \mid F * F, \\ & F \rightarrow S \mid 1\}\end{aligned}$$

Problem 5 (The CYK algorithm). Using the CYK algorithm determine if $w \in L(G)$.

(a) $w = abcbb$, $G = (\{S, A, B, C\}, \{a, b, c\}, S, \mathcal{P})$,

$$\begin{aligned}\mathcal{P} = \{ & S \rightarrow CA \mid CB, \\ & B \rightarrow CBA \mid CB \mid BA \mid BB, \\ & C \rightarrow ABC \mid BC, \\ & A \rightarrow a, B \rightarrow b, C \rightarrow c\}\end{aligned}$$

(b) $w = abcbb$, $G = (\{S, A, B, C\}, \{a, b, c\}, S, \mathcal{P})$,

$$\begin{aligned}\mathcal{P} = \{ & S \rightarrow CA \mid CB, \\ & B \rightarrow CBA \mid CB \mid BA \mid BB, \\ & C \rightarrow ABC \mid BC, \\ & A \rightarrow a, B \rightarrow b, C \rightarrow c\}\end{aligned}$$