

NTIN071 A&G: TUTORIAL 4 – MORE ON CLOSURE PROPERTIES, REGULAR EXPRESSIONS

Solve 1, 2a-f, 3a-e, 4abc, 5a (the rest is for practice).

Problem 1 (Delete). Let L be a regular language over the alphabet $\Sigma = a, b$. Describe the following languages in set notation. Are these languages (necessarily) also regular? Prove or disprove.

Language consisting of all words obtained from words of the language L by...

- (a) ...deleting all occurrences of the letter a .
- (b) ...deleting the initial letter and writing this letter at the end of the word.
- (c) ...deleting the longest contiguous sequence of a 's from the beginning of the word.

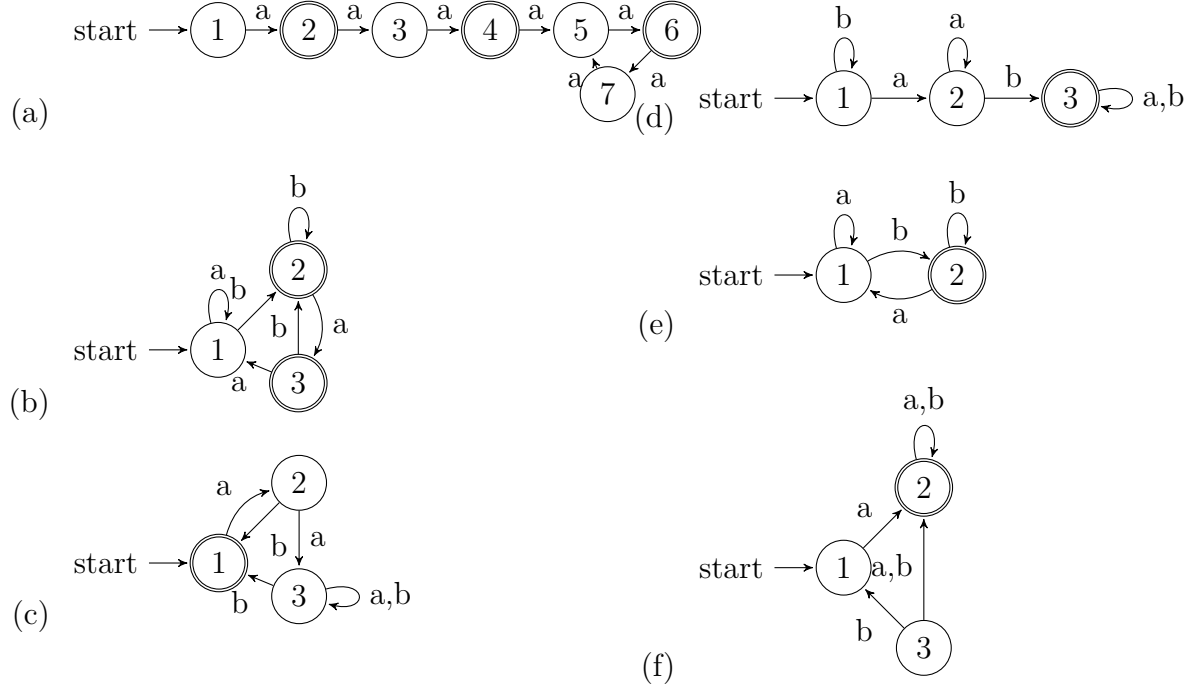
Problem 2 (Constructing regular expressions). Find regular expressions representing the following languages over $\Sigma = \{a, b\}$ consisting of words that:

- (a) start with 'abba',
- (b) end with 'abba',
- (c) start with 'ab' and end with 'ba',
- (d) contain 'abba' or 'bab' as a subword,
- (e) do not contain 'aa' as a subword,
- (f) contain an even number of a 's,
- (g) have at least 2 letters and the first letter is the same as the last letter,
- (h) the first two letters are the same as the last two letters.

Problem 3 (Regex to automaton). Construct finite automata accepting languages described by the following regular expressions:

- | | |
|----------------------|----------------------------------|
| (a) $ab + ba$ | (e) $((ab + c) + a(bc)^* + b)^*$ |
| (b) $a^2 + b^2 + ab$ | (f) $((ab + c)^*a(bc)^* + b)^*$ |
| (c) $a + b^*$ | (g) $(01^* + 101)^*0^*1$ |
| (d) $(ab + c)^*$ | (h) $(01)^*11(01)^*(0 + 1)^*00$ |

Problem 4 (Automaton to regex). Construct regular expressions for languages accepted by the following automata.



Problem 5 (Testing equivalence of regular expressions).

- (a) Describe an algorithm to test equivalence of two regular expressions.
- (b) Apply it to the following pair of regular expressions:

$$(a + b)(a + b)^* \quad \text{and} \quad a(a + b)^* + b(a + b)^*$$

Problem 6 (Are regular expressions regular?). Fix a finite alphabet Σ . Is the language consisting of all regular expressions over Σ a regular language?