

NTIN071 A&G: TUTORIAL 1 – DETERMINISTIC FINITE AUTOMATON, RECOGNIZED
LANGUAGE, REGULAR LANGUAGES

Solve 1abcd, 2ab, 3abcd, 4ab first (the rest is for practice).

Problem 1 (Constructing a DFA for a given language). Construct a DFA recognizing the given language.

- (a) $L = \{w \in \{a, b\}^* \mid |w|_a \text{ is even}\}$
- (b) $L = \{w \in \{a, b\}^* \mid |w|_a \text{ is not divisible by 3}\}$
- (c) $L = \{w \in \{0, 1\}^* \mid w \text{ is a binary encoding of a nonnegative integer divisible by 5}\}$
- (d) $L = \{w \in \{a, b\}^* \mid 2 \text{ divides } |w|_a \text{ or } 3 \text{ divides } |w|_b\}$
- (e) $L = \{w \in \{a, b\}^* \mid 2 \text{ or } 3 \text{ divides } |w|_a\}$
- (f) $L = \{w \in \{a, b\}^* \mid |w|_b \text{ is divisible by 3}\}$
- (g) $L = \{w \in \{a, b\}^* \mid 2 \text{ divides } |w|_a \text{ and } 3 \text{ divides } |w|_b\}$

Problem 2 (DFA given by a table). Draw a state diagram and describe the recognized language in set notation.

<p>(a)</p> <table style="display: inline-table; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">$\rightarrow * p$</td> <td style="padding: 5px;">q</td> <td style="padding: 5px;">p</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">q</td> <td style="padding: 5px;">r</td> <td style="padding: 5px;">q</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">r</td> <td style="padding: 5px;">p</td> <td style="padding: 5px;">r</td> </tr> </table>		0	1	$\rightarrow * p$	q	p	q	r	q	r	p	r	<p>(b)</p> <table style="display: inline-table; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">$\rightarrow p$</td> <td style="padding: 5px;">p</td> <td style="padding: 5px;">q</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">q</td> <td style="padding: 5px;">p</td> <td style="padding: 5px;">r</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">* r</td> <td style="padding: 5px;">p</td> <td style="padding: 5px;">r</td> </tr> </table>		0	1	$\rightarrow p$	p	q	q	p	r	* r	p	r	<p>(c)</p> <table style="display: inline-table; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">$\rightarrow p$</td> <td style="padding: 5px;">q</td> <td style="padding: 5px;">p</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">* q</td> <td style="padding: 5px;">r</td> <td style="padding: 5px;">q</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">* r</td> <td style="padding: 5px;">p</td> <td style="padding: 5px;">r</td> </tr> </table>		0	1	$\rightarrow p$	q	p	* q	r	q	* r	p	r	<p>(d)</p> <table style="display: inline-table; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"></td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">$\rightarrow p$</td> <td style="padding: 5px;">p</td> <td style="padding: 5px;">q</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">* q</td> <td style="padding: 5px;">r</td> <td style="padding: 5px;">q</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">* r</td> <td style="padding: 5px;">p</td> <td style="padding: 5px;">q</td> </tr> </table>		0	1	$\rightarrow p$	p	q	* q	r	q	* r	p	q
	0	1																																																	
$\rightarrow * p$	q	p																																																	
q	r	q																																																	
r	p	r																																																	
	0	1																																																	
$\rightarrow p$	p	q																																																	
q	p	r																																																	
* r	p	r																																																	
	0	1																																																	
$\rightarrow p$	q	p																																																	
* q	r	q																																																	
* r	p	r																																																	
	0	1																																																	
$\rightarrow p$	p	q																																																	
* q	r	q																																																	
* r	p	q																																																	

Problem 3 (Describing a language and constructing a DFA for a given property). Construct a DFA accepting exactly those words over the alphabet $\Sigma = \{a, b\}$ that satisfy the given property. Describe the language in set notation.

- (a) starts ‘abba’
- (b) ends ‘abba’
- (c) contains ‘abba’ or ‘bab’ as a subword
- (d) has at least 2 letters and the first letter is the same as the last letter
- (e) the first two letters are the same as the last two letters

Problem 4 (Regular languages and set operations). Let L, L' be regular languages over the same alphabet. Show that the following is true:

- (a) $\Sigma^* \setminus L$ is a regular language
- (b) $L \cup L'$ is a regular language
- (c) $L \cap L'$ is a regular language
- (d) What if the alphabets of L and L' are different (but possibly sharing some symbols)?
- (e) Would you be able to show that L^R (i.e., words from L written in reverse) is also regular?