NTIN071 A&G: Tutorial 3 – Equivalent and minimal representations, testing properties, nondeterminism, subset construction

Teaching goals: After this tutorial the student is able to

- define reachable states, state equivalence, reduced automaton, automata homomorphism, NFA, ϵ-NFA
- apply the reachable states and state equivalence algorithms to reduce a DFA
- design efficient algorithms to test basic properties of automata
- apply the subset construction to convert an NFA or ϵ-NFA to a DFA

In-class problems

Problem 1 (Equivalent and minimal representations)**.** For the automata below:

(a) Find and remove all unreachable states.

(b) Determine the state equivalence (indistinguishability) relations. (Moreover, for any distinguishable pair of states find all minimal-length distinguishing words.)

(c) Construct their reducts.

A	a	b
$\rightarrow *$ 0	1	2
1	3	0
2	4	5
3	0	2
4	2	5
5	0	3

B	a	b
$\rightarrow *$ 0	0	5
1	1	3
2	2	7
3	3	2
$*$ 4	6	1
5	5	1
$*$ 6	4	2
7	7	0

Problem 2 (Testing properties)**.** Consider finite automata A, B. Design an algorithm that decides the following properties. (Can you estimate the time complexity?)

(a) $L(A) = \emptyset$, (b) $L(A) = L(B)$, (c) $L(A) \subseteq L(B)$, (d) $L(A)$ is finite.

Apply the algorithm to the automata A, B from the previous problem.

Problem 3 (Subset construction)**.** Given a nondeterministic finite automaton with ϵ-transitions, construct an equivalent reduced DFA.

	a	b	ϵ
$\rightarrow A$	$\{E\}$	$\{B\}$	$\emptyset$
B	$\emptyset$	$\{C\}$	$\{D\}$
$\rightarrow C$	$\emptyset$	$\{D\}$	$\emptyset$
$*D$	$\emptyset$	$\emptyset$	$\emptyset$
E	$\{F\}$	$\emptyset$	$\{B, C\}$
F	$\{D\}$	$\emptyset$	$\emptyset$

Extra Practice and Thinking

Problem 4 (Reducing a DFA)**.** Reduce the following DFA:

C	a	b
$\rightarrow 1$	2	3
2	2	4
$* 3$	3	5
4	2	7
$* 5$	6	3
$* 6$	6	6
7	7	4
8	2	3
9	9	4

Problem 5 (Subset construction)**.** Construct an equivalent reduced DFA.

	a	b	ϵ
$*A$	$\{A, C\}$	$\{B\}$	$\emptyset$
B	$\{B, D\}$	$\emptyset$	$\emptyset$
$*C$	$\{E\}$	$\{D\}$	$\emptyset$
D	$\{A\}$	$\{C, D\}$	$\emptyset$
$\rightarrow *E$	$\emptyset$	$\emptyset$	$\{A, C\}$

Problem 6 (Automata homomorphism)**.** Find DFA A, B such that:

(a) Both are reduced and they are not isomorphic.

(b) A is homomorphic onto B but they are not isomorphic.

(c) They are equivalent but not isomorphic.

(d) They are both homomorphic onto C but not isomorphic with it. Moreover, A is not homomorphic onto B and B is not homomorphic onto A.

$$C = (\{p, q\}, \{0, 1\}, \{((p, 0), q), ((p, 1), p), ((q, 0), p), ((q, 1), q)\}, p, \{q\})$$

Problem 7 (Regular? Reduce)**.** Let L be a language over the alphabet $\{a, b\}$ consisting of words that do not contain three consecutive occurrences of the same letter. Decide if L is regular. If it is, find a reduced DFA accepting L.

Problem 8 (Subset construction reduced)**.** Is the result of the subset construction (where we only generate reachable states) always a reduced DFA? Prove or disprove.