# NTIN071 A&G: Tutorial 10 – Properties of context-free languages, DPDA

**Teaching goals:** The student is able to

- decide whether context-free languages (CFL) are closed under various operations
- give a formal definition of a deterministic pushdown automaton (DPDA)
- explain the difference between accepting by final state and by empty stack for DPDA, i.e., the classes of languages $\mathrm{L_{DPDA}}$ and $\mathrm{N_{DPDA}}$
- explain and illustrate the relationship between $\mathrm{L_{DPDA}}$ and $\mathrm{N_{DPDA}}$ with other classes
- decide whether $\mathrm{L_{DPDA}}$ and $\mathrm{N_{DPDA}}$ are closed under various operations

## In-class problems

**Problem 1** (Non-palindromes)**.** Consider the language $L = \{w \in \{a,b\}^* \mid w = w^R\}$.

(a) Is $L$ context-free?
(b) Is its complement $\overline{L}$ context-free?

**Problem 2** (Closure Properties)**.** Are (I) CFL, (II) $\mathrm{L_{DPDA}}$, (III) $\mathrm{N_{DPDA}}$ closed under the following operations? Prove or disprove.

(a) Intersection
(b) Complement
(c) Intersection with a regular language
(d) Homomorphism

**Problem 3.** Show that:

(a) $\{a^n b^{n+1} \mid n \geq 0\} \in \mathrm{N_{DPDA}}$
(b) $\{a^n b^{n+k} \mid n \geq 0, k \in \{0,1\}\} \in \mathrm{L_{DPDA}} \setminus \mathrm{N_{DPDA}}$
(c) $\{a^n b^{n \cdot 2} \mid n \geq 0\} \in \mathrm{L_{DPDA}}$
(d) $\{a^n b^{n \cdot k} \mid n \geq 0, k \in \{1,2\}\} \in \mathrm{CFL} \setminus \mathrm{L_{DPDA}}$

## Extra Practice and Thinking

**Problem 4** (Closure Properties)**.** Are (I) CFL, (II) $\mathrm{L_{DPDA}}$, (III) $\mathrm{N_{DPDA}}$ closed under the following operations? Prove or disprove.

(a) Union
(b) Concatenation
(c) Iteration
(d) Reversal
(e) Substitution of a regular language
(f) Inverse homomorphism
(g) Right/left derivation
(h) Union with a regular language

**Problem 5** (Bonus: Context-sensitive Grammar)**.** Let $G = (\{S,A,B,C\}, \{a,b,c\}, \mathcal{P}, S)$, where:

$$\mathcal{P} = \{S \to aSBC \mid aBC, B \to BBC, C \to CC, CB \to BC,$$
$$aB \to ab, bB \to bb, bC \to bc, cC \to cc\}$$

What language does it generate? Is the grammar $G$ context-sensitive? If not, find an equivalent context-free grammar.