NTIN071 A&G: Tutorial 8 – Pumping lemma for context-free languages

Teaching goals: The student is able to

- give a formal statement of the Pumping lemma for context-free languages
- explain the proof of the Pumping lemma for context-free languages
- apply the Pumping lemma to prove that a given language is not context-free

In-class problems

Problem 1 (Pumping lemma: statement and proof)**.** (a) Formulate the Pumping Lemma for context-free languages (without consulting your notes).

(b) Compare the statement to the version for regular languages.

(c) Explain the idea behind its proof.

(d) Demonstrate pumping on the language $L = \{ww^R \mid w \in \{a, b\}^*\}$.

Problem 2 (Pumping lemma: application)**.** Decide if the following languages are context-free. Prove that your answer is correct.

(a) $L = \{0^i 1^j 0^i \mid i, j \geq 0\}$

(b) $L = \{0^i 1^j 0^i \mid 0 \leq i \leq j\}$

(c) $L = \{0^i 1^j 2^k \mid 0 \leq i \leq j \leq k\}$

(d) $L = \{ww \mid w \in \{0, 1\}^*\}$

(e) $L = \{ww^R \mid w \in \{0, 1\}^*, |w|_0 = |w|_1\}$

(f) $L = \{1^{n^2+n+1} \mid n \geq 0\}$

Extra Practice and Thinking

Problem 3 (Pumping and right-linear grammars)**.** Give an alternative proof of the Pumping lemma for regular languages that is based on derivations from a right-linear grammar.

Problem 4 (Pumping linear languages)**.** Recall that a grammar is *linear*, if it only contains production rules of the form $A \to uBw$ and $A \to w$, where $A, B \in V$ and $u, w \in T^*$.

(a) Formulate a Pumping lemma for linear languages.

(b) Proof the statement using derivations from a (reduced) linear grammar.

(c) How does the pumping constant n from the lemma relate to a linear grammar for L?

(d) Show that the language $L = \{w \in \{0, 1\}^* \mid |w|_0 = |w|_1\}$ is not linear.

(e) Where does L lie within the Chomsky hierarchy?

Problem 5 (Pumping lemma: application)**.** Decide if the following languages are context-free. Prove that your answer is correct.

(a) $L = \{0^i 1^i \mid i \geq 0\}$

(b) $L = \{0^i 1^j 0^i \mid 0 \leq j \leq i\}$

(c) $L = \{0^i 1^i 2^i \mid i \geq 0\}$

(d) $L = \{0^{2i} 1^{3i} 0^i \mid i \geq 0\}$

(e) $L = \{ww^R \mid w \in \{0,1\}^*\}$

(f) $L = \{1^{n^2} \mid n \geq 0\}$

(g) $L = \{1^p \mid p \text{ is a prime}\}$

(h) $L = \{0^i 1^j \mid 0 \leq i \leq j^2\}$