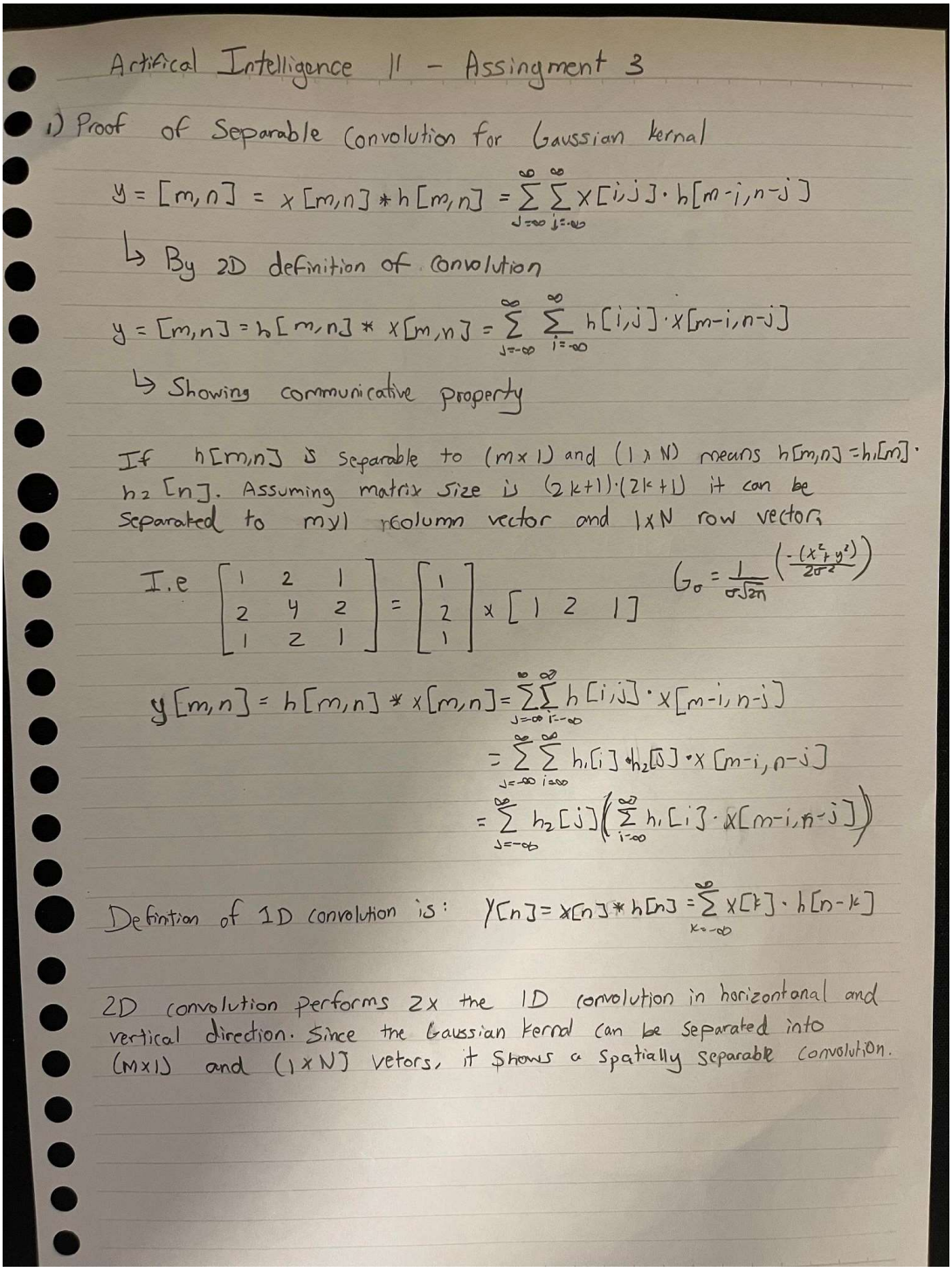
CS4442 - Assignment 3 Report

Problem 1) Separable Convolution



Artifical Intelligence II - Assingment 3

i) Proof of Separable Convolution for Gaussian kernal

$$y = [m,n] = x[m,n] * h[m,n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} x[i,j] \cdot h[m-i, n-j]$$

↳ By 2D definition of convolution

$$y = [m,n] = h[m,n] * x[m,n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} h[i,j] \cdot x[m-i, n-j]$$

↳ Showing communicative property

If $h[m,n]$ is separable to $(m \times 1)$ and $(1 \times N)$ means $h[m,n] = h_1[m] \cdot h_2[n]$. Assuming matrix size is $(2k+1) \cdot (2k+1)$ it can be separated to myl rcolumn vector and $1 \times N$ row vector:

I.e $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$

$G_\sigma = \frac{1}{\sigma\sqrt{2\pi}} \left(\frac{-(x^2+y^2)}{2\sigma^2} \right)$

$$y[m,n] = h[m,n] * x[m,n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} h[i,j] \cdot x[m-i, n-j]$$

$$= \sum_{j=-\infty}^{\infty} \sum_{i=\infty}^{\infty} h_1[i] \cdot h_2[0] \cdot x[m-i, n-j]$$

$$= \sum_{j=-\infty}^{\infty} h_2[j] \left(\sum_{i=\infty}^{\infty} h_1[i] \cdot x[m-i, n-j] \right)$$

Definition of 1D convolution is: $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$

2D convolution performs 2x the 1D convolution in horizontal and vertical direction. Since the Gaussian kernal can be separated into $(M \times 1)$ and $(1 \times N)$ vetors, it shows a spatially separable convolution.

Is Sobel kernal spitially Separable.

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

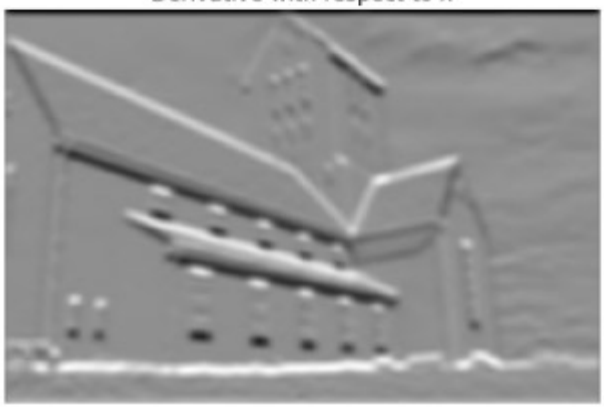
∴ The Sobel kernal is spatially separable since it can be put into a $(M \times 1)$ and $(1 \times N)$ vectors.

Problem 2) Edge Detection

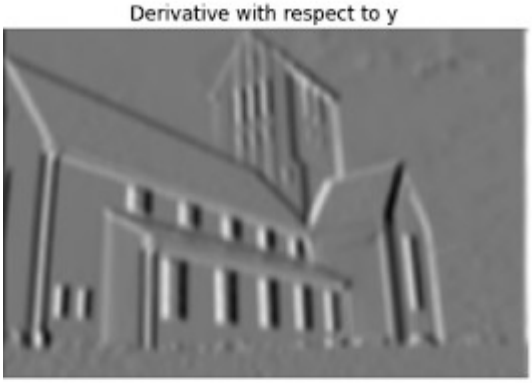
The Original Image

Gaussian Smoothing sigma is 2

Derivative with respect to x

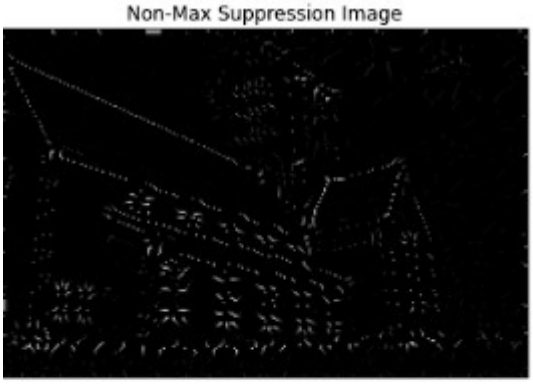


Derivative with respect to y

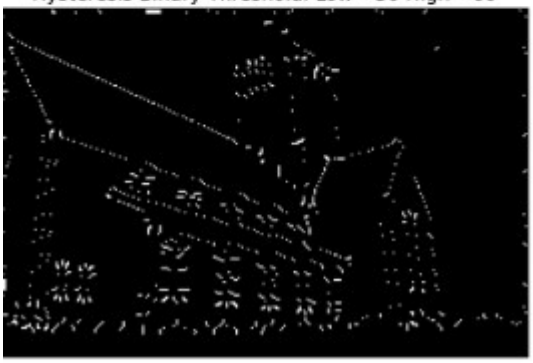


Gradient Intensity Image

Non-Max Suppression Image



Hysteresis Binary Threshold: Low= 30 High= 60



Problem 3) Corner Detection

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(-0.5, 275.5, 181.5, -0.5)
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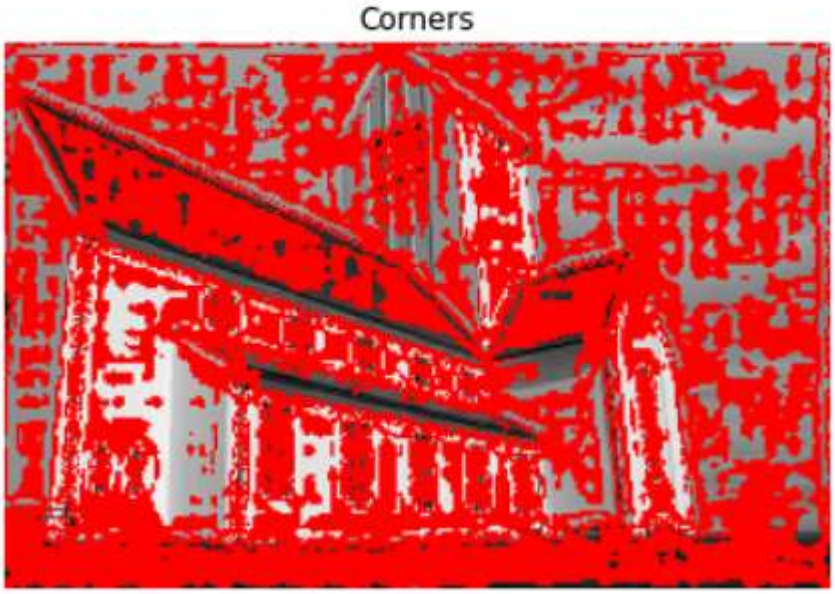


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[<matplotlib.lines.Line2D at 0x1881ce14370>]
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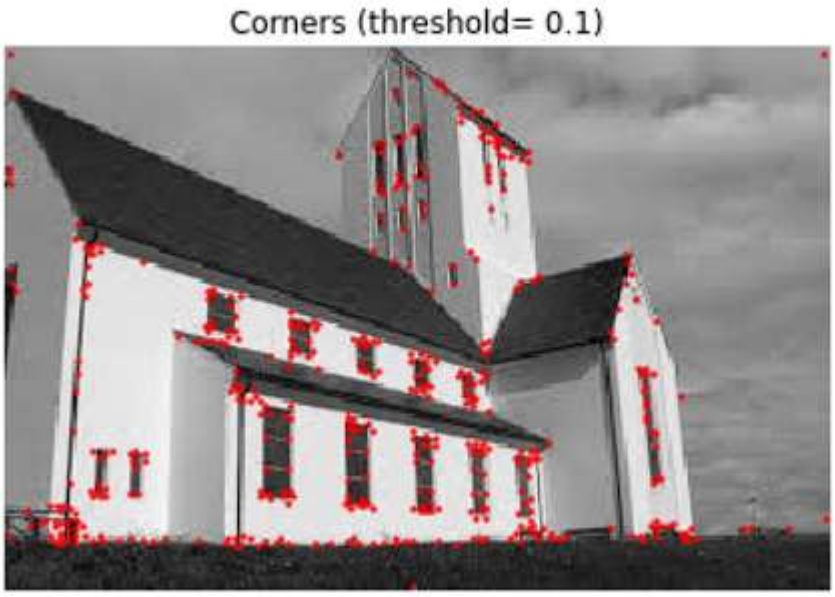

References:

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https://iq.opengenus.org/separable-convolution/

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