2023 전산물리 10 주차 과제

1. Using the Romberg method, we can calculate an integral with a maximum accuracy of O(h^8) using the following R_{ij} values.

$$\begin{matrix} R_{1,1} & & & \\ R_{2,1} & R_{2,2} & & \\ R_{3,1} & R_{3.2} & R_{3,3} & \\ R_{4,1} & R_{4,2} & R_{4,3} & R_{4,4} \end{matrix}$$

Calculate the R_{ij} values to obtain $I = \int_0^{\pi/4} ln(1 + tan\, x)dx$.

2. Calculate the integral of a function $\int_0^{\pi} f(x)\, dx$ using the Trapezoidal rule, Simpson's rule, and Romberg method for the given data. Explain which method has the highest accuracy.

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
$f(x)$	1.0000	0.3431	0.2500	0.3431	1.0000

3. Use the recursive trapezoidal rule to evaluate $\int_0^{\pi/4} \ln(1 + \tan(x))dx$. Explain the results.

4. Use Gauss-Legendre quadrature with two or four nodes to calculate the following integral, and compare against a sufficiently converged integral value

$$\int_1^{\pi} \frac{\ln(x)}{x^2 - 2x + 2} dx$$

5. The integral of the Gaussian function can be expressed as follows:

$$\int_{-\infty}^{+\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

Using the recursive trapezoidal method, let's increase the number of panels and calculate the error and integral values for $\int_{-10}^{+10} e^{-x^2}\, dx$ until the error with respect to $\sqrt{\pi}$ is less than 1e-10.

6. Calculate the integral value using a mixture of Simpson's 1/3 rule and the 3/8 rule for the following table.

x	0.00	0.16	0.32	0.48	0.64	0.80
f(x)	0.20	1.2969	1.7434	3.1860	3.1819	0.232

7. The work required to lift an object with mass m from the Earth's surface to a height H above the surface, where the radius of the Earth is R, is given by the following expression:

$$W = -\Delta U = \int_{R}^{R+H} G\frac{Mm}{r^2}dr$$

$$(G = 6.673 \times 10^{-11}[J \cdot m/kg^2], M = 5.972 \times 10^{24}[kg], R = 6371[km])$$

The space development company, SpaceX, plans to launch and place 1,600, 2,800, and 7,500 satellites at altitudes of 550 km, 1,150 km, and 340 km, respectively. Each satellite has a mass of 200 kg, and the effects of centrifugal force and kinetic energy due to rotation are neglected. The work required to place all the satellites at their respective altitudes using only the potential energy gained from their ascent should be calculated using either the Romberg or Simpson 1/3 integration routine.