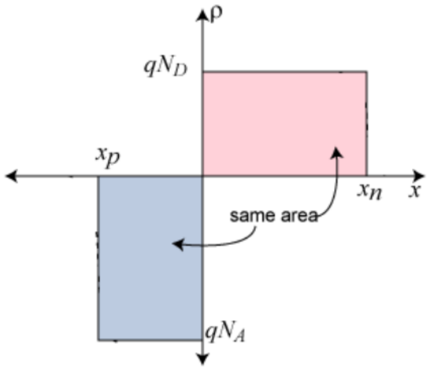
2023 전산물리 과제

1. When charges are distributed in one-dimensional space, the potential is represented by the Poisson's equation as follows:

$$\frac{d^2\Phi(x)}{dx^2} = \rho(x)$$



Given the charge distribution in the depletion region of a p-n junction as shown in the right figure, calculate and plot the potential.

Use the values X_p = *-10*, X_n = *20*, N_D = *1*, N_A = *2*, and q = *-1*, and consider the range of x from -30 to 40 to determine the potential.

2. The Lennard-Jones potential, which represents the general interaction between two particles, is given by the following equation:

$$U(r) = 4\epsilon[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}]$$

Solve the equation for the distance r of a particle as a function of time $r(t)$,

$$\frac{d^2r(t)}{dt^2} = -\frac{1}{m}\frac{dU(r)}{dr}$$

from $t = 0$ to $t = 3$, and plot it. Assume the initial conditions $r(t = 0) = 1.05$ and $r'(t = 0) = 0.0$, with $\varepsilon = \sigma = m = 1$. Use the numerical differentiation method ($\frac{dU(r)}{dr}$, previously learned) to solve the problem.

3. The SIR model, which represents the trend of disease spread, is as follows:

$$\frac{dS}{dt} = -\frac{\beta}{N} \cdot S \cdot I$$

$$\frac{dI}{dt} = \frac{\beta}{N} \cdot S \cdot I - \gamma \cdot I$$

$$\frac{dR}{dt} = \gamma \cdot I$$

In this model, S represents the susceptible population [in individuals], I represents the infected population [in individuals], and R represents the recovered population (or deceased) [in individuals]. It is assumed that the recovered population does not get infected again. Additionally, β represents the daily transmission rate [day^-1], and γ represents the daily recovery rate [day^-1]. The total population N is assumed to be 50 million, and at t=0 [day], there are 10 infected individuals. Given γ = 0.05, let's plot the graphs of I and R over 365 days based on different levels of social distancing measures, represented by β values of 0.2, 0.1, and 0.05. Furthermore, let's find out how long it takes for the healthcare system to collapse when the number of infected individuals exceeds 100,000 for each transmission rate.

4. Integrate the following initial value problem from $x = 0$ to $x = 1$ with an interval of $h = 1/3$.

$$y' = -2y^2 \ , \qquad y(0) = 1.5$$

(a) Find the solutions at each x-step using Euler method by hand. (Hint: check example 7.1).

(b) Repeat the resolution by hand using the second order Modified Euler's Runge-Kutta method. (Hint: check example 7.3).

$$\mathbf{K}_0 = h\mathbf{F}(x, \mathbf{y})$$

$$\mathbf{K}_1 = h\mathbf{F}\left(x + \frac{h}{2}, \mathbf{y} + \frac{1}{2}\mathbf{K}_0\right)$$

$$\mathbf{y}(x + h) = \mathbf{y}(x) + \mathbf{K}_1$$