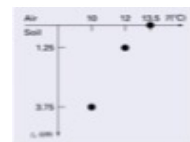
1. The position of a moving bicycle is given by the equation $x = 0.7t^3 - 3t^2 + 5t$, where x is measured in meters and t is measured in seconds.
    a. Find the velocity and the acceleration at t = 2sec using analytical method.
    b. Obtain the velocity and the acceleration at t = 2sec using first forward difference approximation and first central difference approximation.
    c. Compare the results obtained in b with a.
2. The temperature measurements according to the depth of the ground in a certain region are as follows:

depth (cm)	0.00	1.25	3.75
temperature (°C)	13.5	12	10



Heat flux can be calculated by Fourier's Law using the following equation:

$q = -k\,\frac{dT}{dz}$ (q=heat flux[W/m^2], k=thermal conductivity[W/m·K], for soil k=0.5)

Using the numerical differentiation method, calculate the temperature gradient at the boundary (z=0) between the air and the ground and use it to calculate the thermal flux from the air to the ground.

3. The second forward difference approximation equation with accuracy ($O(\Delta x^2)$) is given below:

$$f'_{high}(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2\Delta x}$$

Derive the above high-accuracy differential equation by referring to the low-accuracy ($O(\Delta x)$) first-order and second-order differentials as follows:

$f'_{low}(x_i) = \frac{f(x_{i+1})-f(x_i)}{\Delta x}$ $f''_{low}(x_i) = \frac{f(x_{i+2})-2f(x_{i+1})+f(x_i)}{\Delta x^2}$

(Hint: Replace the low-accuracy differential equations in the expansion of $f(x_{i+1})$.)

4. The second order differential equation for a potential distribution in one-dimensional space when there is no electric charge is called the Laplace equation and is as follows:

$$\nabla^2\Phi(x) = \left(\frac{d}{dx}\right)^2 \Phi(x) = 0$$

4.1. For a given vector $y = [\,f(x_0), f(x_1), f(x_2) \dots f(x_n)]$, where $x_0 = 0$, $x_n = 2$, $\Delta x = x_n - x_{n-1} = 0.02$. Create a numpy array for the matrix L, where L@y = y''. L should be created using the equation: $f''(x_i) = \frac{f(x_{i+1})-2f(x_i)+f(x_{i-1})}{\Delta x^2}$

4.2. Given the boundary conditions of the potential $\Phi(x_0) = 0$ and $\Phi(x_n) = -5$. Modify the previously created L matrix appropriately to obtain the matrix A and vector b in the form of a linear equation A@y=b from the Laplace equation. Solve the linear equation to find the potential $\Phi(x)$.