

## Problem 1

```
In [ ]: import matplotlib.pyplot as plt

def p(x) :
    if (-10 <= x) and (x < 0) :
        return -2
    elif (0 < x) and (x <= 20) :
        return 1
    return 0

psi_init = 0
psi_dif_init = 0
x_init = -40
h = 0.05

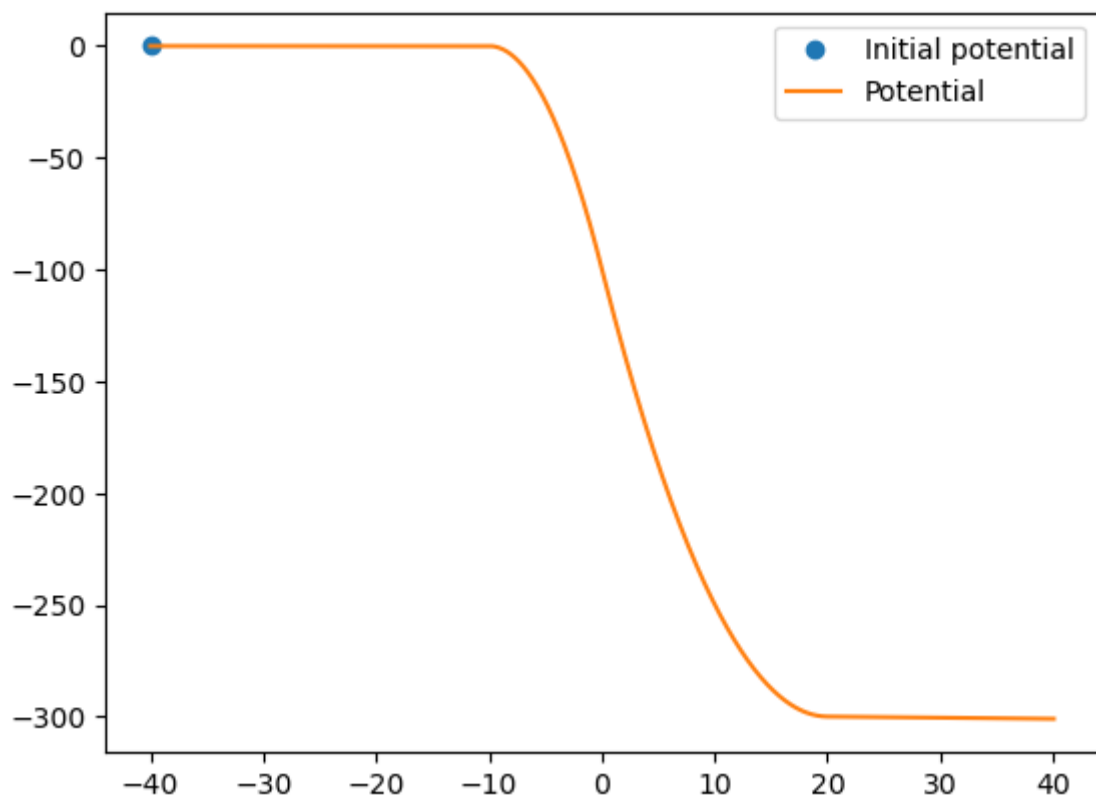
plt.plot(x_init, psi_init, 'o', label="Initial potential")

x_list = [x_init]
psi_list = [psi_init]
while x_init <= 40 and x_init >= -40:

    psi_dif_dif = p(x_init)
    psi_dif_init += psi_dif_dif * h
    psi_init += psi_dif_init * h
    x_init += h

    x_list.append(x_init)
    psi_list.append(psi_init)

plt.plot(x_list, psi_list, label="Potential")
plt.legend()
plt.show()
```



## Problem 2

Use numerical differentiation method : Finite difference approximation

$$f(x)' \approx \frac{f(x+h) - f(x)}{h} + O(h)$$

```
In [ ]: import numpy as np
import matplotlib.pyplot as plt

def U(r) :
    return 4 * (r**(-12) - r**(-6))

def dU_dr(r, h) :
    return (U(r + h) - U(r)) / h

t_list = []
r_list = []

t_init = 0
r_init = 1.05
r_dif_init = 0

F = np.zeros(shape=(2), dtype=float)
F[0] = r_dif_init
F[1] = -1 * dU_dr(r_init, 0.01)

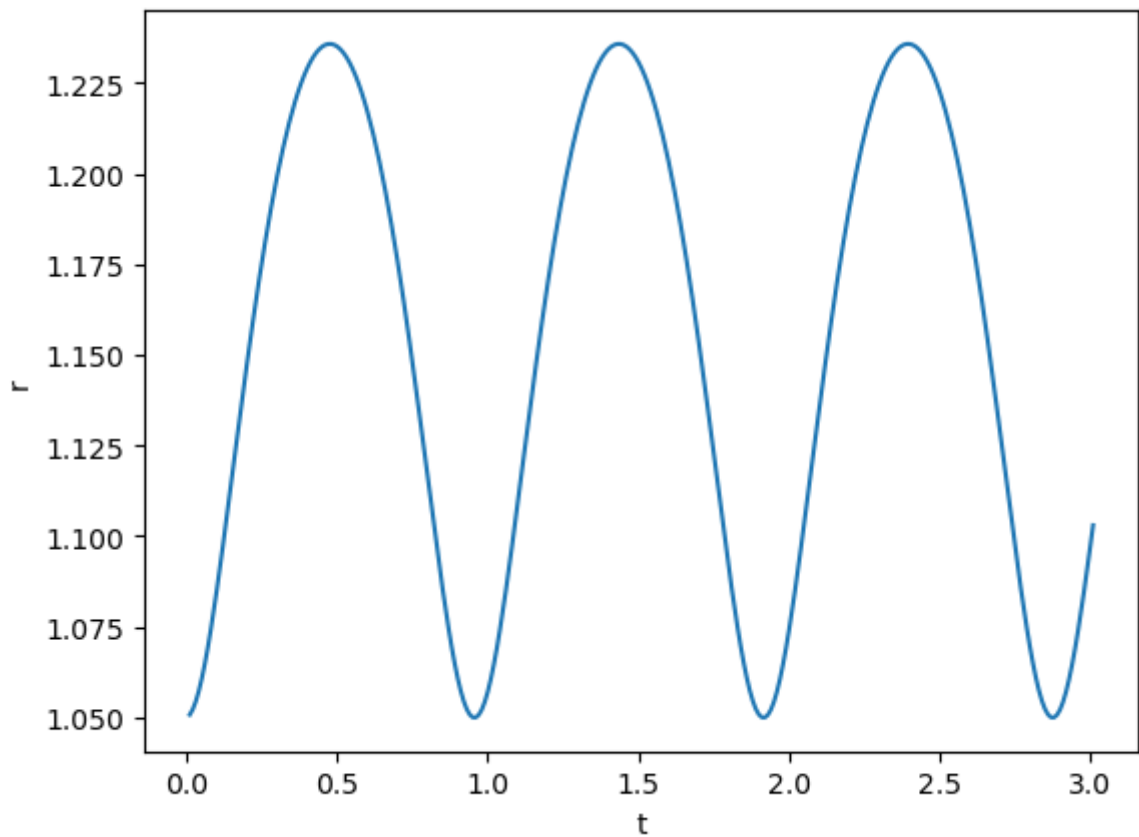
t_step = 0.01
while t_init <= 3 :

    r_dif_init += F[1] * t_step
    r_init += r_dif_init * t_step
    t_init += t_step

    F[0] = r_dif_init
    F[1] = -1 * dU_dr(r_init, 0.01)

    t_list.append(t_init)
    r_list.append(r_init)

plt.plot(t_list, r_list)
plt.xlabel("t")
plt.ylabel("r")
plt.show()
```



### Problem 3

```
In [ ]: import matplotlib.pyplot as plt

def dS_dt(S, beta, N, I) :
    return -1 * beta * S * I / N

def dI_dt(I, beta, gamma, N, S) :
    return beta * S * I / N - 1 * gamma * I

def dR_dt(gamma, I) :
    return gamma * I

gamma = 0.05
N = 50000000
beta_list = [0.2, 0.1, 0.05]

fig, axes = plt.subplots(ncols=3, figsize=(15, 3))

for beta, ax in zip(beta_list, axes) :
    t_init = 0
    S_init = N
    I_init = 10
    R_init = 0

    t_list = [t_init]
    S_list = [S_init]
    I_list = [I_init]
    R_list = [R_init]
    valid = True
    while t_init <= 365 :
        S_current = S_init
        I_current = I_init
        R_current = R_init
```

```

S_init += dS_dt(S_current, beta, N, I_current)
I_init += dI_dt(I_current, beta, gamma, N, S_current)
R_init += dR_dt(gamma, I_current)
t_init += 1

t_list.append(t_init)
S_list.append(S_init)
I_list.append(I_init)
R_list.append(R_init)

if I_init > 100000 and valid :
    print("Beta [{}] has exceed 100000 at day = {}".format(beta, t_init))
    valid = False

ax.plot(t_list, S_list, label="S")
ax.plot(t_list, I_list, label="I")
ax.plot(t_list, R_list, label="R")
ax.set_title("Beta : {}".format(beta))
ax.legend()

if valid :
    print("Beta [{}] didn't exceed 100000".format(beta))
del t_list, S_list, I_list, R_list

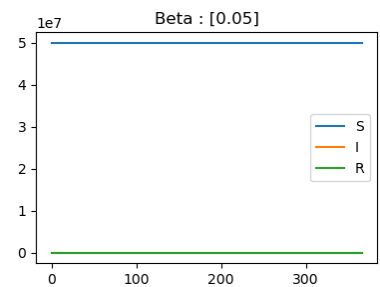
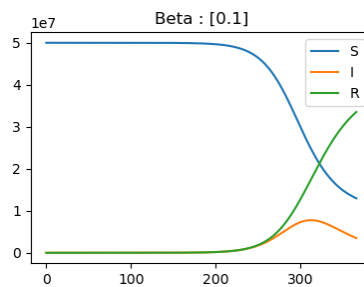
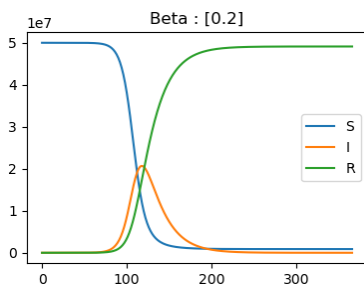
plt.show()

```

Beta [0.2] has exceed 100000 at day = [66]

Beta [0.1] has exceed 100000 at day = [189]

Beta [0.05] didn't exceed 100000



## Problem 4

$$4. \quad g' = -2g^2, \quad g(0) = 1.5, \quad h = 1/3$$

$$(a) \quad g'(0) = -2 \cdot (1.5)^2, \quad g(0+h) = g(0) + g'(0) \cdot h = 0.0$$

$$g'(h) = -2 \cdot 0^2, \quad g(h+h) = g(h) + g'(h) \cdot h = 0.0$$

$$g'(2h) = -2 \cdot 0^2, \quad g(2h+h) = g(2h) + g'(2h) \cdot h = 0.0$$

$$(b) \quad f(x, g) = -2g^2 = f(g)$$

$$k_0 = hf(g) = -\frac{2}{3}g^2, \quad k_1 = hf\left(g + \frac{1}{2}k_0\right) = -\frac{2}{3}\left(g - \frac{1}{3}g^2\right)^2$$

$$= -\frac{2}{27}g^2(3-g)^2$$

$$g(0+h) = g(0) + k_1(g=1.5) \quad g(h+h) = g(h) + k_1(g=1.125)$$

$$= 1.5 - 0.375$$

$$= 1.125 - 0.32959$$

$$= 1.125$$

$$= 0.79541$$

$$g(2h+h) = g(2h) + k_1(g=0.79541)$$

$$= 0.79541 - 0.22778$$

$$= 0.56764$$