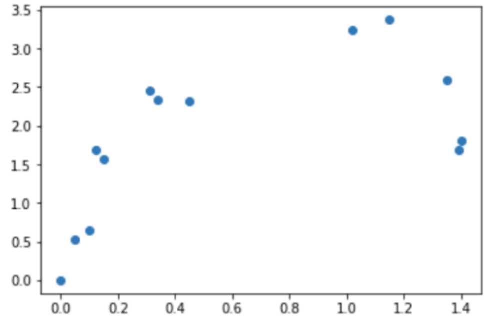
HW6, regression and roots.

1. Polynomial Regression Analysis
The file 'quiz5-1.txt' contains the results of measuring the height of a ball over time after being thrown vertically multiple times at the same speed. The first column of the file (the x-axis of the graph) represents the time [sec] elapsed after the ball was released from the hand, and the second column (y-axis) represents the height [m].



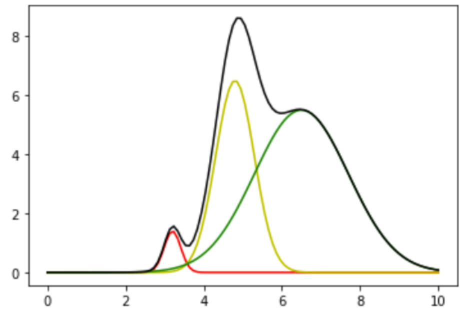
1-1. First, let's use the '*genfromtxt*' function from numpy to load data from a file. Then, let's fit the data with 1st, 2nd, and 4th degree functions using the polyfit function, and plot the fitting results along with the data using the poly1d function.

1-2. Which function among the 1st, 2nd, and 4th degree functions fitted the data most accurately, and if additional experiments were conducted to measure more data, write your own thoughts on which function's accuracy would improve.

1-3. In vertical free-falling motion, the height is given by $y = y_0 + v_0 t - \frac{1}{2} g t^2$. Let's calculate the initial velocity and gravitational acceleration from the fitting coefficients obtained earlier.

2. Curvefit Regression Analysis
We want to fit the data "quiz5-1.txt" used in problem 1 with a 2nd degree function ($y = a\,t^2 + b\,t$). with $y_0 = 0$. Let's use the *scipy.optimize.curve_fit* function to fit the data with $y = a\,t^2 + b\,t$, and calculate the initial velocity and gravitational acceleration.

3. Complex Curvefit Regression Analysis
The file "quiz5-3.txt" is the result of the sum of three Gaussian functions.



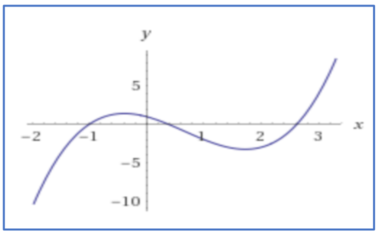
A Gaussian function is a bell-shaped curve represented by $a\,e^{-\frac{(x-b)^2}{c^2}}$,where , a is the height of the curve's peak, b is the position of the center of the peak, and c determines the width of the bell. Using the *scipy.optimize.curve_fit* function, find the nine parameters (a1, b1, c1, a2, b2, c2, a3, b3, c3) that make up the three Gaussian functions.

(Hint: If the fitting does not work well, try putting initial prediction values in the p0 option of the curve_fit function.)

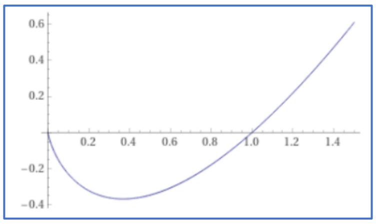
4. Find the local minimum and roots

If the sign of $f''(x)$ is positive, then solution x^* that minimizes $f(x)$ has the same value as the root of $f'(x) = 0$

2−1 We want to find the critical point x^* and the minimum value $f(x^*)$ of the equation $f(x) = x^3 - 2x^2 - 2x + 1$ using the Bisection method and the Newton method, each with an error of less than five decimal places. Let's compare the computation time after finding a critical point with each method.



2−2 We want to find the critical point $x*$ and the minimum value $f(x^*)$ of the equation $f(x) = x\,lnx$ using the Bisection method and the Newton method, each with an error of less than five decimal places. Let's compare the computation time after finding a critical point with each method.
(Note that it may not be possible to calculate the solution with some methods, depending on the attempts made.)



5. Find the roots

The *scipy.special.zeta* function provides the Riemann zeta function, which is defined as follows:

$$\zeta(x) = \sum_{k=1}^{\infty} \frac{1}{k^x}$$

Let's solve the following problem using any convenient method.

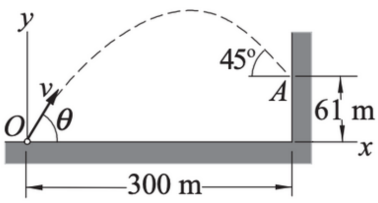
3-1 Find all x such that $\zeta(x) = 0$ within the range of [-11, -1].

3-2 Find all x in the range [1, 10] for which $\zeta(x) = \frac{\pi^2}{6}$.

6. A projectile is launched like in the following figure. Assume that the motion is subject to linear in velocity friction with the following equations of motion:

$$x = \frac{v_{0x}}{\gamma}\left(1 - e^{-\gamma t}\right)$$

$$y = \left(\frac{v_{0y}}{\gamma} + \frac{g}{\gamma^2}\right)\left(1 - e^{-\gamma t}\right) - \frac{g}{\gamma}t$$



Take the gravitational acceleration to be $g = 9.81\ m/s^2$ and $\gamma = 0.01\ v_0$.

Find the velocity v_0, θ and time of flight t if the projectile is to hit the target A at an angle of 45^o.