2018550025 물리학과 정준상

1. $x = [0, 1, 2]$

$y = [1, 4, 3]$

(a) degree of polynomial = 2, $P_2(x) = y_0 l_0(x) + y_1 l_1(x) + y_2 l_2(x)$

$$l_0 = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} = \frac{1}{2}(x-1)(x-2) = \frac{1}{2}x^2 - \frac{3}{2}x + 1$$

$$l_1 = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} = -x(x-2) = -x^2 + 2x$$

$$l_2 = \frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1} = \frac{1}{2}x(x-1) = \frac{1}{2}x^2 - \frac{1}{2}x$$

$$\Rightarrow P_2(x) = l_0 + 4l_1 + 3l_2 = -2x^2 + 5x + 1$$

(b) by the equation of curvature,

$$(x_0 - x_1)k_0 + 2(x_0 - x_2)k_1 + (x_1 - x_2)k_2 = 6\left(\frac{y_0 - y_1}{x_0 - x_1} - \frac{y_1 - y_2}{x_1 - x_2}\right)$$

$$-k_0 - 4k_1 - k_2 = 6 \cdot 4$$

$\longrightarrow$ since $k_0 = k_2 = 0$, $k_1 = 24/-4 = -6$.

Applying to cubic spline equation,

$$f_{0,1}(x) = -x^3 + 4x + 1$$

$$f_{1,2}(x) = x^3 - 6x^2 + 10x - 1$$

```python
import sympy
from IPython.display import display

X = sympy.Symbol("x")
k = [0, -6, 0]
x = [0, 1, 2]
y = [1, 4, 3]

for i in range(0, 2) :

    expression = k[i] / 6 * (((X - x[i + 1])**3) / (x[i] - x[i + 1]) - (X - x[i + 1]) * (x[i] - x[i + 1]))
    expression -= k[i + 1] / 6 * (((X - x[i])**3) / (x[i] - x[i + 1]) - (X - x[i]) * (x[i] - x[i + 1]))
    expression += (y[i] * (X - x[i + 1]) - y[i + 1] * (X - x[i])) / (x[i] - x[i + 1])

    expand = sympy.expand(expression)

    print("f_({},{}) : ".format(i, i + 1))
    display(expand)

```

[13] ✓ 0.0s Python

f_(0,1) :

$-1.0x^3 + 4.0x + 1$

f_(1,2) :

$1.0x^3 - 6.0x^2 + 10.0x - 1.0$

(c)

since $-k_0 - 4k_1 - k_2 = 6 \cdot 4$ and $k_0 = 0$, $k_1 = -5$,

$k_2 = -24 + 20 = -4$

Applying to cubic spline equation,

$$f_{0,1}(x) = -\frac{5}{6}x^3 + \frac{23}{6}x + 1$$

$$f_{1,2}(x) = \frac{x^3}{6} - 3x^2 + \frac{41}{6}x$$

```python
import sympy
from IPython.display import display

X = sympy.Symbol("x")
k = [0, -5, -4]
x = [0, 1, 2]
y = [1, 4, 3]

for i in range(0, 2):
    expression = k[i] / 6 * (((X - x[i + 1])**3) / (x[i] - x[i + 1]) - (X - x[i + 1]) * (x[i] - x[i + 1]))
    expression -= k[i + 1] / 6 * (((X - x[i])**3) / (x[i] - x[i + 1]) - (X - x[i]) * (x[i] - x[i + 1]))
    expression += (y[i] * (X - x[i + 1]) - y[i + 1] * (X - x[i])) / (x[i] - x[i + 1])

    expand = sympy.expand(expression)
    numerator, denominator = sympy.fraction(expand)
    fraction = numerator / denominator

    display(sympy.Eq(sympy.Symbol(f"f_({i},{i+1})(x)"), sympy.nsimplify(fraction, rational=True, tolerance=1.0e-12)))
```

[27] ✓ 0.0s Python

$$f_{(0,1)(x)} = -\frac{5x^3}{6} + \frac{23x}{6} + 1$$

$$f_{(1,2)(x)} = \frac{x^3}{6} - 3x^2 + \frac{41x}{6}$$