Problem 2

For case (a), because the speed is in the earth frame, it won't be affected by the rotation. So everything is simple.

$$^{E}V = \begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix}$$

$$\theta(t) = \begin{pmatrix} 0 \\ 0 \\ t \cdot r \end{pmatrix}$$

As

$$^{E}V = \begin{pmatrix} \cos(rt) & -\sin(rt) & 0 \\ \sin(rt) & \cos(rt) & 0 \\ 0 & 0 & 1 \end{pmatrix} {}^{B}V$$

we have

$$^{B}V = \begin{pmatrix} V\cos(rt) \\ -V\sin(rt) \\ 0 \end{pmatrix}$$

For case (b), the speed always aligns with the body X-axis, i.e.

$$^{\mathrm{B}}V = \begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix}$$

Therefore, using the above rotation matrix

$$^{E}V = \begin{pmatrix} \cos(rt) & -\sin(rt) & 0 \\ \sin(rt) & \cos(rt) & 0 \\ 0 & 0 & 1 \end{pmatrix} {}^{B}V$$

We have

$$^{E}V = \begin{pmatrix} V\cos(rt) \\ V\sin(rt) \\ 0 \end{pmatrix}$$

Case (a) is in the static force equilibrium (total forces sum to zero) because it's velocity doesn't change, while case (b) is not (velocity changes direction).

Both case (a) & (b) are in the static moment equilibrium (total moments sum to zero). This is because they are all rotating at constant angular velocity. We don't need to apply any moments to them to keep rotating.

Problem 3

$$[A] = \begin{bmatrix} \left(\frac{D}{L}\right)_o \frac{2g}{V_o} & -g \\ \frac{2g}{V_o^2} & 0 \end{bmatrix} \text{ and } [B] = \begin{bmatrix} \frac{1}{m} & 2Kg \\ 0 & \frac{g}{V_o C_{Lo}} \end{bmatrix}$$

The eigenvalues of A are computed using

$$\det(\lambda I - A) = 0$$

Just by expanding this equation and we'll got

$$\lambda^2 + 2\frac{g}{V_o}\left(\frac{D}{L}\right)_o \lambda + 2\left(\frac{g}{V_o}\right)^2 = 0$$

Note that as mentioned on Ed, the correct matrices should be

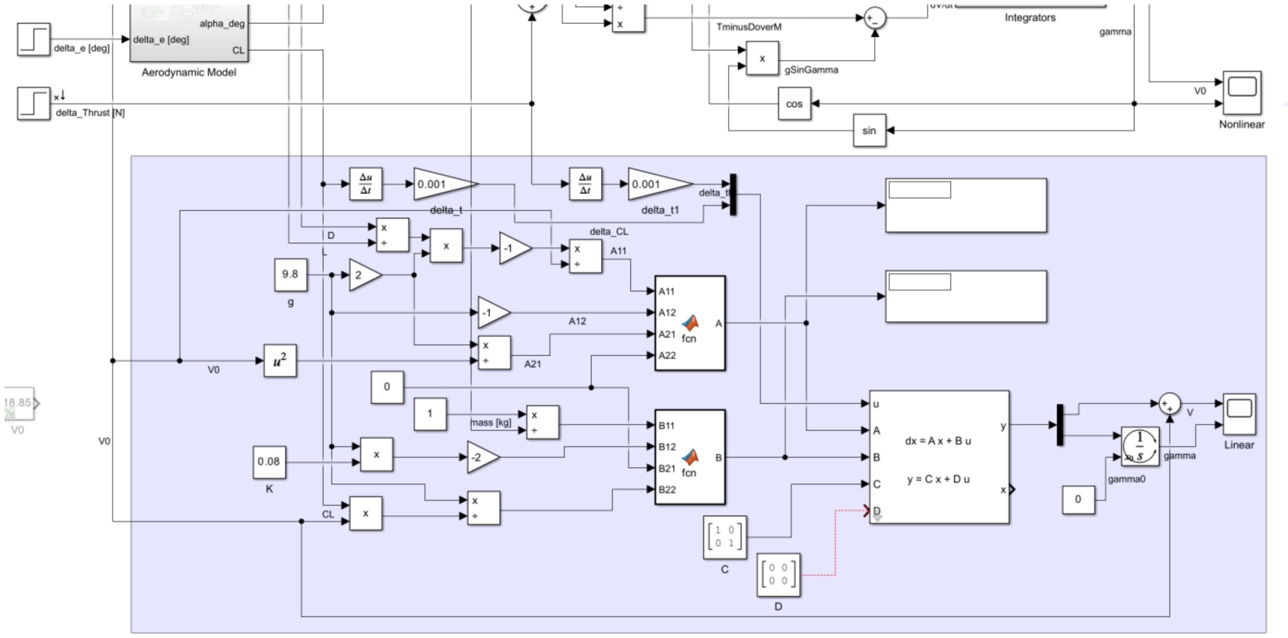
$$[A] = \begin{bmatrix} -\left(\frac{D}{L}\right)_o \frac{2g}{V_o} & -g \\ \frac{2g}{V_o^2} & 0 \end{bmatrix}$$

$$[B] = \begin{bmatrix} \frac{1}{m} & -2Kg \\ 0 & \frac{g}{V_o C_{Lo}} \end{bmatrix}$$

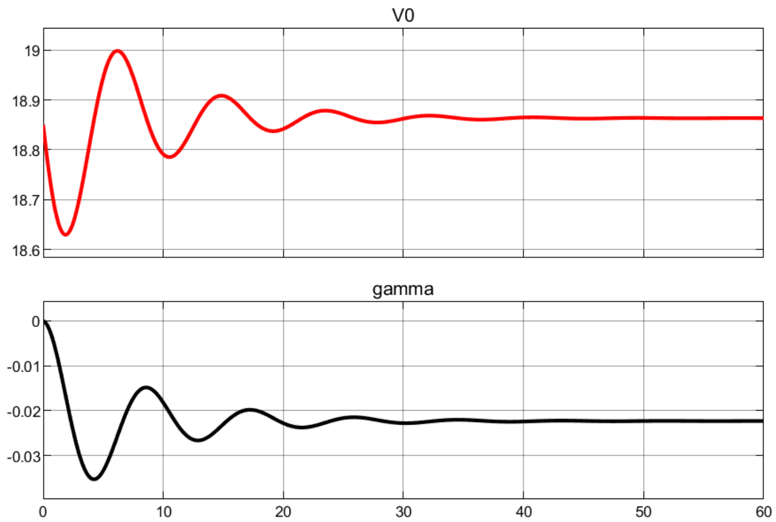
The eigenvalues are

$$\lambda = -\frac{gD}{V_0 L} \pm \frac{g}{V_0 L}\sqrt{D^2 - 2L^2}$$

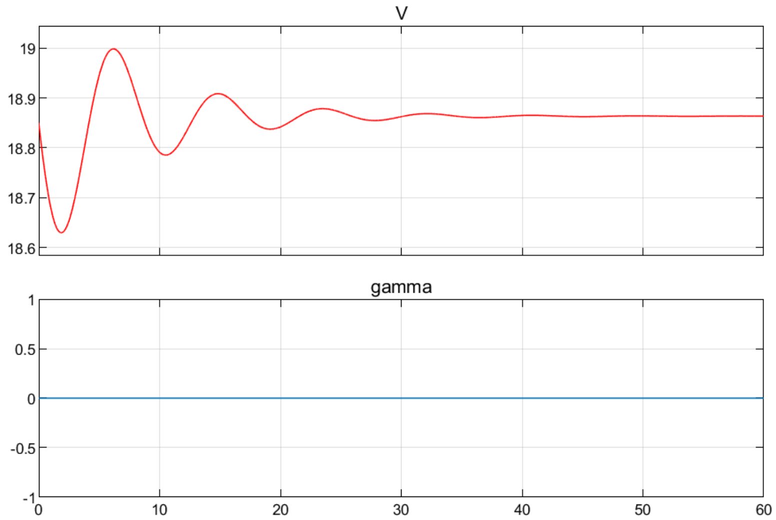
My Simulink model is as below. The linear part I implemented is marked in the blue area. I left the calculation of matrices outside of the state-space model because they need to be updated every iteration using the states provided by the nonlinear output in the previous iteration. The input block parameters are: thrust 1.55N, delta angle 6 degrees.



The output of the nonlinear system is



And the linearized system is



We can see that the velocity outputs of the two systems are very close. Their oscillation periods are about 4.8 seconds. And the theoretical period is given by

$$T \approx \sqrt{2}\frac{\pi V_0}{g} \approx 4.52s$$

where I chose V_0 as the median velocity in the graph $10\mathrm{m/s}$. The output γ seems to be always zero. This is a problem but I haven't identify the reason.

Then let's take a look back at the two eigenvalues

$$\lambda = -\frac{gD}{V_0 L} \pm \frac{g}{V_0 L}\sqrt{D^2 - 2L^2}$$

As drag is usually much smaller than lift, the sqrare root term should be

$$\lambda = -\frac{gD}{V_0 L} \pm i\frac{g}{V_0 L}\sqrt{2L^2 - D^2} \approx -\frac{gD}{V_0 L} \pm i\frac{\sqrt{2}g}{V_0}$$

This means the eigenvalue will always be at the left half plane, indicting stabilty for the linearized system. However, the term $-\frac{gD}{V_0 L}$ can still be very small in absolute value, which means the system is not robust. And as the eigenvalues go closer to the y-axis, it will become less stable and would oscillate longer. The period can be calculated using

$$T = \frac{\sqrt{2}g}{V_0}\pi$$

We can see this by increasing the thrust from 1.55N to 3.0N (let the aircraft fly at a higher speed), and the damping rate of oscillation reduces obviously, as shown below.