Physics Thinking

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Preface

Consider this as an option for developing (and publishing?) the book.

To learn more about Quarto books visit https://quarto.org/docs/books.

1 Introduction

This is a book created from markdown and executable code.

Table 1.1: SI units

Base Quantity	Base Unit	Symbol
length	meter	m
time	second	s
mass	kilogram	kg
electric current	ampere	A
Thermodynamic temperature	Kelvin	K
Amount of substance	mole	mol
Luminous Intensity	candela	cd

Table 1.2: Fundamental Units

length	time	mass	charge	temperature
m	s	kg	C	K

Table 1.3: Combinations

Concept	Units
Force	$kg\ m\ s^{-2} = N$
Energy	$kg\ m^2\ s^{-2} = N\ m = J$
Power	$kg\ m^2\ s^{-3} = J\ s^{-1} = W$
Current	$C\ s^{-1}$

Dimensional analysis: always checking and fudging (?).

Same units go to the same side of the equation!

Vectors vs scalars

Math is a tool, not the be all and end all – don't simply formula fit.

Sensible answers! Check!

1.1 Solving Problems with the 7D's and the little S

- Diagram: Big! (2/3 of a page) and as many as you need. Graphs
- Directions: Mark it (negative/positive)
- Definitions & Data: Put it on the page (all of them)
- Diagnosis: Type (how) conservation principles, force laws, angular momentum.
- Derivation: Equations (diagnosis in symbols) as many equations as variables add to diagram dimensions
- Determination - D'algebra - box the answer.
- Dimensions - Check and limiting cases – if makes sense then possibly right (if not prob wrong). LHS = RHS then :grinning:.
- Substitution: if necessary - do rough calc by hand an dunits include error!

2 Mechanics

2.1 Example Problem

Diagram

TODO

Directions

TODO

Definitions & Data

Variable	Description
m	Mass of object
H	Initial height
h	Final height
k	Spring constant
g	Acceleration due to gravity
x	Compression of spring
U_{gpe}	gravitational potential energy
U_E	elastic potential energy
E	energy

Diagnosis

Conservation of Energy (E is conserved).

$$U_{gpe} \to U_E$$

$$mgH = mgh + \frac{1}{2}kx^2$$

Derivation

$$F = -kx$$

$$\begin{aligned} U &= -\int F \, \mathrm{d}x \\ &= -\int_{x_i}^{x_f} kx \, \mathrm{d}x \\ &= \frac{1}{2} k x_f^2 \end{aligned}$$

Determination

$$mg\,(H-h) = \frac{1}{2}kx^2$$

$$\frac{2mg\,(H-h)}{k} = x^2$$

$$\boxed{x = \sqrt{\frac{2mg(H-h)}{k}}}$$

Dimensions

$$\begin{aligned} L &= \sqrt{\frac{MLT^{-2}\,(L)}{MT^{-2}}} \\ &= \sqrt{L^2} \end{aligned}$$

Limiting Cases

$$H \to h$$

$$H - h = 0 \implies x = 0$$

makes sense!

$$m \to 0 \implies x \to 0$$

makes sense!

$$k \to \infty \implies x \to 0$$

makes sense!

$$k \to 0 \implies x \to \infty$$

makes sense!

Substitution

Not needed.

2.2 Example Problem

Title: Acceleration and Tension in a Two-Mass Pulley System on an Incline

A system consists of two masses, a man m and piano M, where m is connected by a string over the pulley to the piano M, which is positioned on an inclined plane with an angle θ from the horizontal. The mass M experiences kinetic friction characterized by a coefficient μ_k with the inclined plane. The piano M is heavy enough that its sliding down the ramp and pulling the man m up. Determine the acceleration a of the system.

Diagram

TODO: Create a diagram showing the two masses, m and M, the inclined plane with angle θ, and the direction of the frictional force opposing the motion of M up the incline.

Directions

TODO gravity points down

- Direction of a for man is vertically upwards.
- Direction of a for piano downward along the incline.

Definitions & Data

- m: Mass hanging from the pulley (kg)
- M: Mass on the inclined plane (kg)
- θ: Angle of the inclined plane from the horizontal (radians)
- μ_k: Coefficient of kinetic friction
- g: Acceleration due to gravity

Diagnosis

Consider the forces on each mass and draw a free body diagram:

TODO FBD

- Mass m: Gravitational force mg and tension T.
- Mass M: Gravitational force component along the incline, normal force, frictional force $f_k = \mu_k N$, and tension T.
- Conservation of String: string is the same length, so when pulled the forces are transmitted without diminishing.

Derivation

1. Write down the force equations for each mass.
   - For mass m: $mg - T = ma$ (Downward force - tension = mass times acceleration).
   - For mass M: $Mg\sin(\theta) - f_k - T = Ma$ (Component of gravity along incline - friction - tension = mass times acceleration).

$$|F| = (m + M) * |a|$$

$$N = Mg\cos\theta$$

$$T = T$$

$$\therefore \sum F = Mg\sin\theta + \mu_k N - mg$$

D'algebra

$$|F| = (m+M) * |a| = Mg\sin\theta + \mu_k N - mg$$

$$\begin{aligned} |a| &= \frac{Mg\sin\theta + \mu_k N - mg}{(m+M)} \\ &= \frac{Mg\sin\theta + \mu_k Mg\cos\theta - mg}{(m+M)} \end{aligned}$$

Determination

$$\boxed{|a| = g\left(\frac{M\sin\theta + \mu_k M\cos\theta - m}{(m+M)}\right)}$$

Dimensions

$$LHS = LT^{-2}$$

$$RHS = LT^{-2}\left(\frac{M+M-m}{M}\right)$$

Looks right!

Limiting Cases

Consider extreme cases like:

- What happens when $\theta = 0$?
  - acceleration is difference in weight of the man from the frictional force. Makes sense.
- What happens when $\theta = 90^\circ$?
  - acceleration is the difference in masses divided by the sum of the masses times g. Makes sense.
- What happens when $\mu_k = 0$ or is very large?
  - Larger $\mu_k \implies$ smaller a.
- What happens if $M \gg m$?

- The piano will slide as though nothing is attached (only forces of friction and force due to gravity matter). Makes sense.

- What happens if $m \gg M$?
  - The weight of the man will pull the piano up the incline. Makes sense.

Substitution

TODO

3 Summary

A work in progress.

References