Think Like A Physicist

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Foreword

First Principles Thinking is a teachable and incredibly powerful algorithm for efficiently solving real world problems and building the future.

It is the best algorithm I know to make a brain useful.

These notes are compiled from the International Physics Olympiad Australian training program held in Canberra c. 2005. I have found, over the intervening 18 years, that the skills and knowledge taught during this intensive two week program have formed the basis of my ability to apply physics to everyday problems.

I have assembled these notes with the intention to create the most singularly terse summary of undergraduate physics ever written, accessible to motivated professionals who know how to Google. Necessarily, they do not cover the lab or test-taking aspects of the Olympiad training program.

I hope you find thinking like a physicist to be as rewarding as I do.

— Casey Handmer, 25 November, 2023.

Part I

Foundations

1 How to Think Like a Physicist

1.1 SI Units

Table 1.1: SI Base Units

Base Quantity	Base Unit Name	Symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

The five fundamental units are length (meter, m), time (second, s), mass (kilogram, kg), charge (coulomb, C), and temperature (kelvin, K). Ampere (A), mole (mol), and candela (cd) are derived from these base units.

All other units are derived from combinations of these fundamental units.

Table 1.2: Derived Units

Concept	Unit	Derivation
Force	Newton (N)	$kg \cdot m \cdot s^{-2}$
Energy	Joule (J)	$kg \cdot m^2 \cdot s^{-2}$
Power	Watt (W)	$kg \cdot m^2 \cdot s^{-3}$
Current	Ampere (A)	$C \cdot s^{-1}$

1.2 Dimensional Analysis and Sanity Checking

Vital tool for both checking (always!!) and fudging. It is part of a consistency check — units on both sides of the equation must be the same. A successful dimension check is followed by a :).

Similarly, equations with vectors must have the same vector shape on each side.

Math and formulas are a tool, not the final authority. Never formula fit. Understand what is going on and check that the answers are sensible.

1.3 Solving Problems with the 7 Ds and the little s

Using this basic algorithm to approach problems from first principles is the single biggest update you are likely to make to your thinking, ever.[1]

1. **Diagrams** — Big, 2/3 page, as many as you need. Load the problem into your GPU.
2. **Directions** — Mark it (negative/positive).
3. **Definitions and Given Data** — Put it on the page (all of them).
4. **Diagnosis** — How? What type of problem is this? E.g. conservation principles, force laws, angular momentum.
5. **Derivation** — Write down the fundamental equations, the diagnosis expressed as symbols. You will need as many equations as variables. Add to the diagram if necessary. Check dimensions.
6. **Determination** (or D'algebra) — Math manipulations to get the answer. Box it!
7. **Dimensions** — **Check dimensions and limiting cases/sanity check**. $LHS = RHS$ then :).
8. **substitutions** — Only if necessary, do rough calculation by hand and check units, include an error term.

Example Problem 1

A body, mass m, slides without friction from rest at height H to lower height h, colliding with a horizontal spring of strength k. By how much does it compress?

> ⚠️ TODO: Diagram
>
> Diagram showing: (1) Initial state — mass m at height H on a frictionless ramp. (2) Final state — mass at height h compressing a horizontal spring by distance x. Include direction arrows for gravity and coordinate axes.

Definitions & Data

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[1]A more thorough exploration of this algorithm can be found at Casey Handmer's blog post In Space, No One Can Reason By Analogy.

Variable	Description
m	Mass of object
H	Initial height
h	Final height
k	Spring constant
g	Acceleration due to gravity $(9.81\,\mathrm{m/s}^2)$
x	Compression of spring
U_{gpe}	Gravitational potential energy
U_E	Elastic potential energy
E	Energy

Diagnosis

Conservation of Energy (E is conserved).

$$U_{gpe} \rightarrow U_E$$

$$mgH = mgh + \frac{1}{2}kx^2$$

Derivation

$$F = -kx$$

$$U = -\int F\,\mathrm{d}x$$
$$= -\int_{x_i}^{x_f} kx\,\mathrm{d}x$$
$$= \frac{1}{2}kx_f^2$$

Determination

$$mg\left(H - h\right) = \frac{1}{2}kx^2$$

$$\frac{2mg\left(H - h\right)}{k} = x^2$$

$$\boxed{x = \sqrt{\frac{2mg(H-h)}{k}}}$$

Dimensions

$$L = \sqrt{\frac{MLT^{-2}\,(L)}{MT^{-2}}}$$
$$= \sqrt{L^2} = L \quad \checkmark$$

Limiting Cases

- $H \to h$: $H - h = 0 \implies x = 0$ — makes sense!
- $m \to 0 \implies x \to 0$ — makes sense!
- $k \to \infty \implies x \to 0$ — makes sense!
- $k \to 0 \implies x \to \infty$ — makes sense!

Substitution

Not needed.

Example Problem 2

A man (mass m) is pulling a piano (mass M) up a hill (angle θ to horizontal), using a pulley. The coefficient of kinetic friction is μ_k.

> ⚠️ TODO: Diagram
>
> Diagram showing: (1) Inclined plane at angle θ. (2) Piano (mass M) on the incline connected by string over pulley to man (mass m) hanging vertically. Include direction arrows and coordinate axes.

> ⚠️ TODO: Free Body Diagrams
>
> (1) Man: weight mg down, tension T up. (2) Piano: weight Mg decomposed into components along/perpendicular to incline, normal force N, friction $\mu_k N$ opposing motion, tension T up the incline.

Definitions & Data

- m: Mass of man (kg)
- M: Mass of piano (kg)
- θ: Angle of incline from horizontal
- μ_k: Coefficient of kinetic friction
- g: Acceleration due to gravity
- T: Tension in string
- N: Normal force
- a: Acceleration of the system

Diagnosis

Forces: free body diagram. Conservation of string.

- Man: $F = ma$
- Piano: component of gravity along incline, normal force, friction $f_k = \mu_k N$, tension T.

Derivation

For the man:
$$ma = mg - T$$
$$T = m(g - a)$$

For the piano:
$$N = Mg\cos\theta$$
$$\sum F = Mg\sin\theta + \mu_k N - mg$$

Determination

$$\boxed{a = g\left(\frac{M\sin\theta + \mu_k M\cos\theta - m}{M + m}\right)}$$

Dimensions

$$LHS = LT^{-2}, \quad RHS = LT^{-2}\left(\frac{M + M - M}{M}\right) \quad \checkmark$$

Limiting Cases

- If $M \gg m$: acceleration to the right (piano dominates). Makes sense.
- If $m \gg M$: acceleration negative (man pulls piano up). Makes sense.
- $\theta = 0$: acceleration depends on friction vs man's weight. Makes sense.
- $\theta = 90°$: reduces to Atwood machine. Makes sense.

This working includes an error caught during the limiting cases check, and left in to show how this works.

2 Math Toolkit

2.1 Differentiation — Chain Rule

a)

$$\frac{d}{dr}\left(6r^3(\cos 2r + 5r)^7\right) = 6r^2(\cos 2r + 5r)^6(3\cos 2r + 2r(25 - 7\sin 2r))$$

b)

$$x = (5te^{2t+1} + 2)^8$$

$$v = \frac{dx}{dt} = 40e^{2t+1}(1 + 2t)(5te^{2t+1} + 2)^7$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 40e^{2t+1}(5te^{2t+1} + 2)^6\left(8(1 + t) + (7 + 32t(1 + t))5e^{2t+1}\right)$$

2.2 Partial Derivatives

Simply take derivatives with respect to each variable, but consider extreme points if finding maxima or minima.

$$f(x, y) = 5xy + ye^x$$

$$\frac{\partial}{\partial x}f(x, y) = 5y + ye^x$$

$$\frac{\partial}{\partial y}f(x, y) = 5x + e^x$$

2.3 Integration

Substitution

$$\int_{1/2}^{1} \sqrt{1 - x^2}\, dx \quad \text{Let } x = \cos u, \quad dx = -\sin u\, du$$

$$\Rightarrow I = \int_{\pi/2}^{\pi/6} \sqrt{1 - \cos^2 u} \cdot (-\sin u)\, du = \int_{\pi/6}^{\pi/2} \sin^2 u\, du$$

Using $\cos 2u = 1 - 2\sin^2 u$:

$$I = \frac{1}{2} \int_{\pi/6}^{\pi/2} (1 - \cos 2u)\, du = \frac{1}{2} \left[-\frac{\sin 2u}{2} - u \right]_{\pi/6}^{\pi/2} = \frac{\sqrt{3}}{8} - \frac{\pi}{6}$$

Integration by Parts

Basically the inverse of the product rule:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\int u\, dv = uv - \int v\, du$$

Example:

$$\int \tan^{-1} x\, dx$$

Let $u = \tan^{-1} x$, $dv = dx$.

$$x = \tan u, \quad \frac{dx}{du} = 1 + \tan^2 u = 1 + x^2$$

$$du = \frac{1}{1 + x^2}\, dx, \quad v = x$$

$$\Rightarrow \int \tan^{-1} x\, dx = x\tan^{-1} x - \int \frac{x}{1 + x^2}\, dx = x\tan^{-1} x - \frac{1}{2}\ln(1 + x^2) + C$$

2.4 Kinematic Equations

$$v = \frac{dx}{dt}, \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Set a constant and $v(t = 0) = v_0 = u$:

$$dv = a\,dt, \quad \int_u^v dv = \int_0^t a\,dt, \quad \boxed{v = u + at}$$

$$dx = (u + at)\,dt, \quad \int_{x_0}^x dx = \int_0^t (u + at)\,dt, \quad \boxed{x = x_0 + ut + \frac{1}{2}at^2}$$

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$$

$$a\,dx = v\,dv, \quad \int_0^x a\,dx = \int_u^v v\,dv, \quad \boxed{v^2 = u^2 + 2ax}$$

2.5 Differential Equations

There are infinite varieties of DEs, but nearly all physically relevant ones can be solved quickly by inspection. Here is a very incomplete summary of linear equations.

First Order: Rearrange and integrate.

Second Order: Simple Harmonic Motion!

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Guess the answer of the form:

$$x = A\cos\omega t + B\sin\omega t$$

or

$$x = Ae^{i\omega t + \phi} = Ae^{i\omega t} + Be^{-i\omega t}$$

For example, springs. Nearly everything in nature is a spring of one form or another.

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

So by inspection, $\omega^2 = k/m$.

Another example: Solve $v = ktx$.

$$\frac{dx}{dt} = ktx, \quad \frac{dx}{x} = kt\,dt, \quad \int_{x_0}^{x} \frac{dx}{x} = k\int_0^t t\,dt$$

$$2\ln(x/x_0) = kt^2, \quad \boxed{x = x_0 e^{kt^2/2}}$$

2.6 Coordinate Systems

Plane Polar Coordinates (2D)

For a point P in the plane, the polar coordinates (r, θ) are related to the Cartesian coordinates (x, y) by:

$$r = \sqrt{x^2 + y^2}, \quad \theta = \arctan\left(\frac{y}{x}\right)$$

or inversely:

$$x = r\cos\theta, \quad y = r\sin\theta$$

Cylindrical Polar Coordinates

For axial symmetry:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$$

Spherical Polar Coordinates

For spherical symmetry, the coordinates are defined as:

$$0 \leq r, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi < 2\pi$$

where r is the radius, θ is the polar angle, and ϕ is the azimuthal angle.

2.7 Multivariate Integration

Univariate integration is one dimensional:

$$\int f'(x)\, dx = f(x) + C$$

Areas and volumes require multivariate integration.

$$\text{Area} = \iint_A dA = \iint_A dx\, dy = \int_c^d \left(\int_a^b dx \right) dy = (b-a)(d-c)$$

Polar Coordinates

For an infinitesimal area element dA in polar coordinates:

$$dA = r\, dr\, d\theta$$

Volumes

The volume of a sphere:

$$\text{Volume} = \iiint_V dV = \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta\, dr\, d\theta\, d\phi = \int_0^R r^2\, dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$= 2\pi \cdot \frac{R^3}{3} \cdot [-\cos\theta]_0^\pi = \frac{4}{3}\pi R^3$$

Flux Integrals

One can also integrate an arbitrary scalar or vector function over some area or volume.

Vector field integrations are performed over areas to find fluxes, and are only concerned with the normal (perpendicular) component of the field:

$$I = \iint \vec{f} \cdot \hat{n}\, dA$$

Part II

Classical Physics

3 Mechanics

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4 Thermodynamics

> ℹ **Status**
>
> Content to be transcribed from IPhO notes.

5 Waves & Oscillations

> **ℹ Status**
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> Content to be transcribed from IPhO notes.

Part III

Electromagnetism & Optics

6 Electrostatics

> **ℹ Status**
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7 Circuits & Magnetism

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8 Optics

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Part IV

Modern Physics

9 Modern Physics & Relativity

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References