# Manual Calculation Part 3

1. **SHERYL ATIENO OTIENO**

Initial $m = -1$ Initial $b = 1$ $\alpha = 0.1$ $(1,3)$ $(3,6)$

$y = mx + b$

$\hat{y}_1 = (-1)(1) + 1 = 0$

$\hat{y}_2 = (-1)(3) + 1 = -2$

MSE Calculation

$$MSE = \frac{1}{n}\sum(y - \hat{y})^2$$

$$= \frac{1}{2}\left[(3-0)^2 + (6+2)^2\right]$$

$$= \frac{1}{2}(9+64) \quad = 36.5$$

Gradient calculation

$$\frac{\partial MSE}{\partial m} = \frac{2}{n}\sum x(\hat{y} - y)$$

$$= \frac{2}{2}\left[1(0-3) + 3(-2-6)\right] = -27$$

$$\frac{\partial MSE}{\partial b} = \frac{2}{n}\sum(\hat{y} - y) = \frac{2}{2}(0-3) + (-2-6)$$

$$= -11$$

$$m_1 = m_0 - \alpha \frac{\partial MSE}{\partial M}$$

$$= -1 - 0.1(-27) \quad = 1.7$$

$$b_1 = b_0 - \alpha \frac{\partial MSE}{\partial b}$$

$$= 1 - 0.1(-11) \quad = 2.1$$

$\hat{y}_1 = (1.7)(1) + 2.1 = 3.8$

$\hat{y}_2 = 1.7(2) + 2.1 = 7.2$

New $MSE = \frac{1}{2}\left[(-0.8)^2 + (-1.2)^2\right] = 1.04$

## 2. JUSTINE UMUHOZA

Name: JUSTINE UMUHOZA

Iteration 2

Current Values:

| x | y |
|---|---|
| 1 | 3 |
| 3 | 6 |

$m_1 = 1.7 \quad b_1 = 2.1$

Step 1: Compute predictions:

$\hat{y}_1 = mx + b = (1.7)(1) + 2.1 = 3.8$

$\hat{y}_2 = mx + b = (1.7)(3) + 2.1 = 7.2$

Step 2: Compute Errors

$e_1 = \hat{y}_1 - y_0 = 3.8 - 3 = 0.8$

$e_2 = \hat{y}_2 - y_0 = 7.2 - 6 = 1.2$

Step 3: Compute Gradients

$\frac{dJ}{dm} = \frac{2}{n} e_i x_i = \frac{2}{n} \Sigma (mx_i + b - y_i) x_i$

and

$\frac{dJ}{db} = \frac{2}{n} e_i = \frac{2}{n} \Sigma (mx_i + b - y_i)$

So $\frac{dJ}{dm} = \frac{2}{n} \left[ (e_i \cdot x_1) + (e_2 \cdot x_2) \right]$

$= \frac{2}{2} \left[ (0.8)(1) + (1.2)(3) \right] = (0.8 + 3.6) = 4.4$

and

$\frac{dJ}{db} = \frac{2}{n} \left[ e_i + e_2 \right] = \frac{2}{2} \left[ (0.8 + 1.2) \right] = 2.0$

So new values:

$m_2 = 1.26$

$b_2 = 1.9$

Step 4: Updated Values:

$m_{new} = m_{old} - \alpha \cdot \frac{dJ}{dm} = 1.7 - 0.1(4.4) = 1.7 - 0.44 = 1.26$

$b_{new} = b_{old} - \alpha \cdot \frac{dJ}{db} = 2.1 - 0.1(2.0) = 2.1 - 0.2 = 1.9$

3. **JOSUE BYIRINGIRO**

Iteration 3

Current values

$m_2 = 1.26$

$b_2 = 1.9$

| x | y |
|---|---|
| 1 | 3 |
| 3 | 6 |

Predictions

$\hat{y}_1 = mx + b = (1.26)(1) + 1.9$
$= \underline{3.16}$

$\hat{y}_2 = mx + b = (1.26)(3) + 1.9$
$= \underline{5.68}$

Errors

$e_1 = \hat{y}_1 - \hat{y}_i = 3.16 - 3 = 0.16$

$e_2 = \hat{y}_2 - y_i = 5.68 - 6 = -0.32$

Gradients

$\frac{\partial J}{\partial m} = \frac{2}{n} e_i x_i = \frac{2}{n} \sum (mx_i + b - y_i)$

$\frac{\partial J}{\partial b} = \frac{2}{n} e_i = \frac{2}{n} \sum (mx_i + b - y_i)$

$\frac{\partial J}{\partial m} = \frac{2}{n} [(e_1 \cdot x_1) + (e_2 \cdot x_2)] = \frac{2}{2} [(0.16)(1) + (-0.32)(3)]$
$= 0.16 - 0.96$
$= \underline{-0.8}$

$\frac{\partial J}{\partial b} = \frac{2}{n}(e_1 + e_2) = \frac{2}{2}[(0.16) + (-0.32)]$
$= \underline{-0.16}$

New parameters

$m_3 = m_2 - \alpha \frac{\partial J}{\partial m}$
$= 1.26 - 0.1(-0.8)$
$= \underline{1.34}$

$b_3 = b_2 - \alpha \frac{\partial J}{\partial b}$
$= 1.9 - 0.1(-0.16)$
$= \underline{1.916}$

New values

$m_3 = 1.34$

$b_3 = 1.916$

## 4. ULRICH RUKAZAMBUGA

Ulrich RUKAZAMBUGA

### Iteration 4

Current values

$m_3 = 1.34$

$b_3 = 1.916$

$n = 2$

$\alpha = 0.1$

Predictions

$\hat{y}_1 = mx + b$

$= (1.34)(1) + 1.916$

$= 3.256$

$\hat{y}_2 = mx + b$

$= (1.34)(3) + 1.916$

$= 5.936$

Errors

$e_1 = \hat{y} - y_i$

$= 3.256 - 3 = 0.256$

$e_2 = \hat{y}_2 - y_i = 5.936 - 6$

$= -0.064$

Gradient

$\frac{\partial J}{\partial m} = \frac{2}{n} e_i x_i = \frac{2}{n} \sum (m x_i + b - y_i) x_i$

and $\frac{\partial J}{\partial b} = \frac{2}{n} e_i = \frac{2}{n} \sum (m x_i + b - y_i)$

$\frac{\partial J}{\partial m} = \frac{2}{n} [(e_1 x_1) + (e_2 x_2)]$

$= \frac{2}{2} [(0.256)(1) + (0.064)(3)]$

$= 0.256 + (-0.192) = 0.064$

$\frac{\partial J}{\partial b} = \frac{2}{n} (e_1 + e_2)$

$= \frac{2}{2} [0.256 + (-0.064)]$

$= 0.256 - 0.064$

$= 0.192$

Updated values

$m_{new} = m_4 - \alpha \frac{\partial J}{\partial m}$

$= 1.34 - (0.1)(0.064)$

$= 1.34 - 0.0064$

$= 1.3336$

$b_{new} = b_1 - \alpha \frac{\partial J}{\partial b}$

$= 1.916 - (0.1)(0.192)$

$= 1.916 - 0.0192$

$= 1.8968$

New value

$m_4 = 1.334$

$b_4 = 1.897$

**Trend**

The values of m and b gradually move closer to their optimal values with each iteration. The MSE keeps decreasing from high value at the start to a much smaller value after four iterations. This shows that the gradient descent is successfully reducing the error. The changes in m and b become smaller with each step, meaning the model is slowly converging toward the best fit line. Overall, the parameters are moving in the right direction to minimize the error.