

# MULTI-ROBOT WAYPOINT INSPECTION

## PLAN MIXED INTEGER LINEAR

## PROGRAMMING PROJECT

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ME 6033 Linear and Mixed Integer Optimization

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# INTRODUCTION

- Based on previous work done in an outdoor concrete inspection multirobot framework
- Precast concrete elements require efficient inspection methods after being transported to a site
- Multi-robot approach:
  - Aerial robots locate targets
  - Ground robots perform detailed inspections
- For this project wanted to see if we could extend this idea to planning of multiple robots across an inspection site



## PROBLEM DESCRIPTION

- Two robot types: aerial and ground mobile robots
- Inspection targets as waypoints in a 2D plane
- Depot location for each robot type
- Sequential operation:
  - Aerial robots verify waypoint location first
  - Ground robots perform detailed inspection second
- Robot constraints:
  - Fixed speeds (meters/minute)
  - Limited operation time (battery life time after leaving depot)
  - Required inspection time at waypoints
- Objective: Maximize waypoint visits in a single inspection loop.
- Becomes a MILP problem. Linear objective function and linear constraints but use binary and continuous variables.

## LITERATURE REVIEW

- Prior works demonstrate multi-robot planning applications
- Problem resembles multiple traveling salesman problem (mTSP)
- Traditional MILP for mTSP:
  - Routing variables
  - Subtour elimination constraints
- Initially tried this approach but solving time was too slow and larger problems became infeasible (likely implementation error) so used a simplification
- Route approximation: roundtrip distances from depot to waypoints estimate travel times

# MODEL

$$\text{Maximize } \sum_{i \in N} \sum_{l \in L} w_i^{p,l} \quad (1)$$

subject to:

$$\text{Assignment: } \sum_{k \in K} w_i^{a,k} \leq 1 \quad \forall i \in N \quad (2)$$

$$\sum_{l \in L} w_i^{p,l} \leq 1 \quad \forall i \in N \quad (3)$$

$$\text{Robot Usage: } \sum_{i \in N} w_i^{a,k} \geq \text{use}_k^a \quad \forall k \in K \quad (4)$$

$$\sum_{i \in N} w_i^{a,k} \leq n \cdot \text{use}_k^a \quad \forall k \in K \quad (5)$$

$$\sum_{i \in N} w_i^{p,l} \geq \text{use}_l^p \quad \forall l \in L \quad (6)$$

$$\sum_{i \in N} w_i^{p,l} \leq n \cdot \text{use}_l^p \quad \forall l \in L \quad (7)$$

$$\text{Precedence: } w_i^{p,l} \leq \sum_{k \in K} w_i^{a,k} \quad \forall i \in N, \forall l \in L \quad (8)$$

$$g_i^l \geq \alpha_i^k - M_G \cdot (1 - z_i^{k,l}) - M_G \cdot (2 - \text{use}_k^a - \text{use}_l^p) \quad \forall i \in N, k \in K, l \in L \quad (9)$$

$$\text{Linking } z_i^{k,l}: z_i^{k,l} \leq w_i^{p,l} \quad \forall i \in N, k \in K, l \in L \quad (10)$$

$$z_i^{k,l} \leq w_i^{a,k} \quad \forall i \in N, k \in K, l \in L \quad (11)$$

$$z_i^{k,l} \leq \text{use}_k^a \quad \forall i \in N, k \in K, l \in L \quad (12)$$

$$z_i^{k,l} \leq \text{use}_l^p \quad \forall i \in N, k \in K, l \in L \quad (13)$$

$$z_i^{k,l} \geq w_i^{p,l} + w_i^{a,k} + \text{use}_k^a + \text{use}_l^p - 3 \quad \forall i \in N, k \in K, l \in L \quad (14)$$

$$\text{Time: } \alpha_i^k \geq t_A^{\text{resp}} \cdot w_i^{a,k} \quad \forall i \in N, k \in K \quad (15)$$

$$g_i^l \geq t_G^{\text{resp}} \cdot w_i^{p,l} \quad \forall i \in N, l \in L \quad (16)$$

$$\alpha_i^k + t_{i,d_A}^A \cdot w_i^{a,k} \leq T_A^{\text{max}} + M_A \cdot (1 - w_i^{a,k}) \quad \forall i \in N, k \in K \quad (17)$$

$$g_i^l + t_{i,G}^G \cdot w_i^{p,l} \leq T_G^{\text{max}} + M_G \cdot (1 - w_i^{p,l}) \quad \forall i \in N, l \in L \quad (18)$$

$$\text{Route Length: } \sum_{i \in N} w_i^{a,k} \cdot (2 \cdot t_{d_A,i}^A) + \sum_{i \in N} w_i^{a,k} \cdot t_A^{\text{resp}} \leq T_A^{\text{max}} + M_A \cdot (1 - \text{use}_k^a) \quad \forall k \in K \quad (19)$$

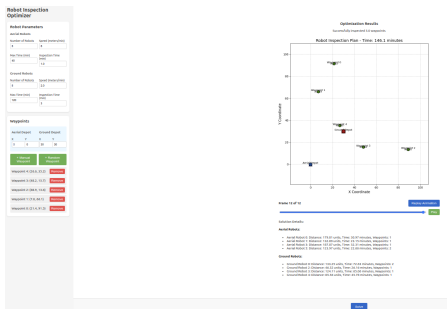
$$\sum_{i \in N} w_i^{p,l} \cdot (2 \cdot t_{i,G}^G) + \sum_{i \in N} w_i^{p,l} \cdot t_G^{\text{resp}} \leq T_G^{\text{max}} + M_G \cdot (1 - \text{use}_l^p) \quad \forall l \in L \quad (20)$$

$$\text{Domain: } w_i^{a,k}, w_i^{p,l}, \text{use}_k^a, \text{use}_l^p, z_i^{k,l} \in \{0, 1\} \quad \forall i \in N, k \in K, l \in L \quad (21)$$

$$\alpha_i^k, g_i^l \geq 0 \quad \forall i \in N, k \in K, l \in L \quad (22)$$

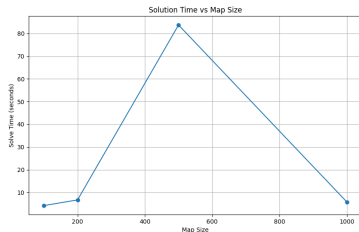
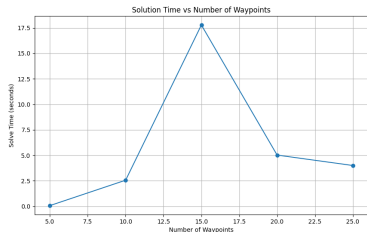
# SOLUTION METHOD

- Implemented in Python using PuLP library
- CBC solver from PuLP used for optimization
- Interactive browser-based GUI developed:
  - Parameter input for robot specifications
  - Waypoint location setting
  - Real-time solution visualization
- Code available at GitHub repository



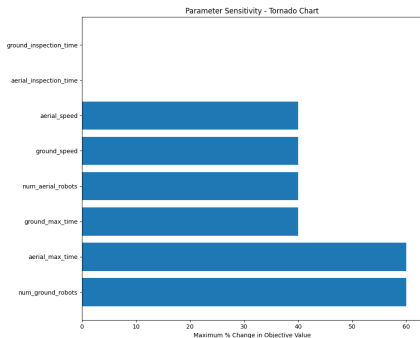
# NUMERICAL RESULTS - COMPUTATIONAL PERFORMANCE

- Testing environment:
  - Intel i7, Python 3.12 Docker container
- Tested waypoint scaling (5 to 25 waypoints) and map size scaling (100 to 1000 m)
- Non-monotonic scaling behavior:
  - Solution time peaks at 15 waypoints then decreases
  - Computation time peaks at 500m map size



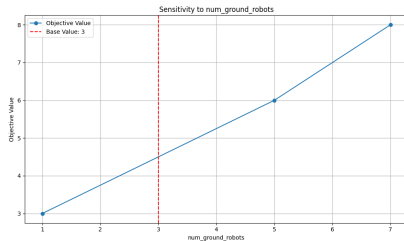
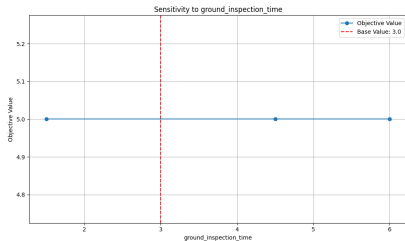
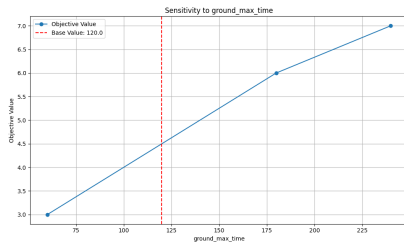
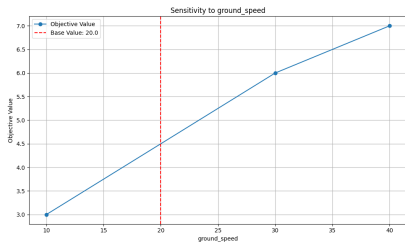
# NUMERICAL RESULTS - SENSITIVITY ANALYSIS

- Using fixed waypoints and map size we solve multiple times varying 8 params: robot speeds, operation times, inspection times, and fleet sizes.
- Most influential parameters:
  - Number of ground robots
  - Aerial robot maximum operation time
- Minimal impact: Inspection times

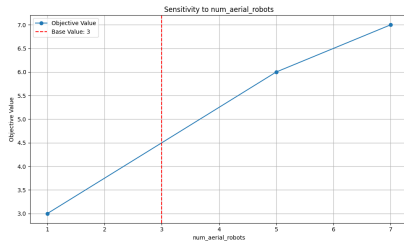
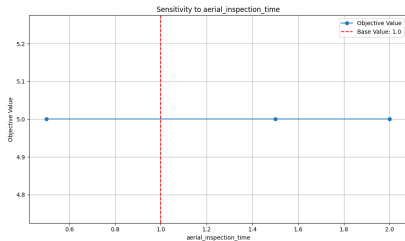
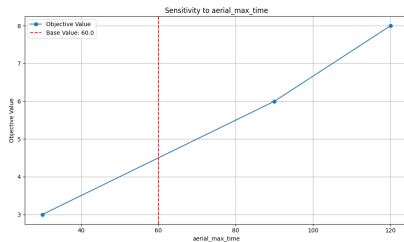
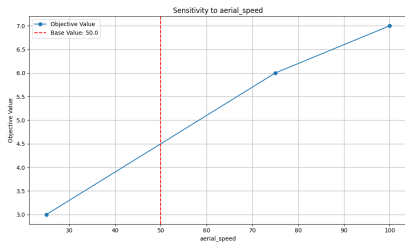




# NUMERICAL RESULTS - GROUND ROBOT SENSITIVITY ANALYSIS



# NUMERICAL RESULTS - AERIAL ROBOT SENSITIVITY ANALYSIS



# DEMO VIDEO

### Robot Inspection Optimizer

#### Robot Parameters

##### Aerial Robots

Number of Robots: 
Speed (meters/sec):

Max Time (min): 
Inspection Time (min):

##### Ground Robots

Number of Robots: 
Speed (meters/sec):

Max Time (min): 
Inspection Time (min):

#### Waypoints

Aerial Depot		Ground Depot	
X	Y	X	Y
0	0	20	20

+ Add New Waypoint

+ Remove Waypoint

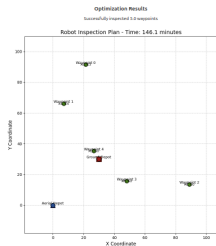
Waypoint 4: (20.6, 35.2) [Remove](#)

Waypoint 3: (48.2, 15.7) [Remove](#)

Waypoint 2: (38.5, 13.4) [Remove](#)

Waypoint 1: (7.6, 46.1) [Remove](#)

Waypoint 0: (21.4, 91.3) [Remove](#)



Frame 12 of 12

[Robot Animation](#)  
[Final](#)

Solution details:

**Aerial Robots:**

- Aerial Robot 0: Distance: 179.81 units, Time: 30.97 minutes, Waypoints: 1
- Aerial Robot 1: Distance: 133.89 units, Time: 23.15 minutes, Waypoints: 4
- Aerial Robot 3: Distance: 187.87 units, Time: 32.51 minutes, Waypoints: 1
- Aerial Robot 5: Distance: 123.87 units, Time: 22.08 minutes, Waypoints: 2

**Ground Robots:**

- Ground Robot 0: Distance: 133.21 units, Time: 72.83 minutes, Waypoints: 2
- Ground Robot 2: Distance: 48.52 units, Time: 26.12 minutes, Waypoints: 1
- Ground Robot 3: Distance: 52.51 units, Time: 30.00 minutes, Waypoints: 1
- Ground Robot 4: Distance: 55.56 units, Time: 43.78 minutes, Waypoints: 1

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Click to watch the demo video

