Social Networks and Price Formation in an Artificial Market

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***Abstract*—This paper investigates the effects of social networks on price formation in an artificial market. Analysing price dynamics where participants are connected using Regular, Scale-Free and Small-World networks, in which agents communicate and change their strategies depending on the prior success of their neighbours and themselves. The market is built off the characteristics of cryptocurrency markets, and validate using Bitcoin price data. The Scale-Free network is found to best emulate real market data, and all network types have a significant impact on volatility when compared to no communication at all.**

I. Introduction

Traditional financial market theory and the Efficient Market Hypothesis suggest that returns should be normally distributed. Empirical statistical regularities, observed in market data which contradict this hypothesis, occur so frequently they are accepted as *stylised facts*. These properties remain unexplained analytically and research has turned to agent-based models of artificial markets for explanation, using behavioural and dynamic properties emergent in real markets [1]. Predominant stylised facts, and those considered in this proposal are: excess kurtosis, autocorrelation of returns, and volatility clustering. Cryptocurrencies, as tradable financial instruments, appear to be continuously rising in popularity. These markets exhibit similar *stylised facts* as those seen in traditional markets, and in many cases to a greater degree [2]. Agent-based research on market dynamics in traditional financial markets is widespread, however models on cryptocurrencies specifically are sparse. Furthermore, the dissemination and spread of information in markets has also been commonly modeled, though no notable applications exist in cryptocurrency markets, which exhibit significant susceptibility to peer-influence and momentum effects [3].

This paper thus implements an agent-based, artificial market, based on prevalent literature and common characteristics of cryptocurrency markets. The effects of social networks as a mechanism for price formation are investigated, and the stylised facts seen in real market data are attempted to be replicated. The goal of this paper is to explore how simple herding behaviour can have complex effects on price formation.

II. Related Literature

Generally, artificial financial market components can be segmented into the following: agent distinction, price determination, strategy representation, learning and social interaction. Each relatively distinct component, and notable corresponding literature (including their relation to real market observations) is discussed below.

Agent Distinction

The type of agents introduced into models depends on the nature of the market simulated. In traditional financial market models, agents are often segmented into either Chartists, fundamentalists or Random Traders [1]. Chartists, or noise-traders, generate expectations through historical prices (with shorter time horizons), while fundamentalists do so using fixed or dynamic fundamental market values (with longer time horizons). In their cryptocurrency market models, due to limited fundamental indicators, Cocco et al. [5] rather segment agents into chartists and random traders. Where random traders issue orders at random with equivalent buy/sell probabilities, providing market and price stability [4]. These traders are aimed to represent persons who enter the market for various reasons other than that of speculation, which is represented by chartists. Hessary & Hadzikadic [6], studying herding phenomena in traditional markets, segment chartists further into Optimists and Pessimists. These behavioural distinctions are interestingly also made in Aspembitova et al. [3] in assessing structures of cryptocurrency market users.

Pricing Mechanism and Definition

Determining how prices are defined and adjusted is central the development of an artificial market, and in many cases the fundamental market mechanism. Historically, pricing mechanisms have been implemented in three ways: a linear function of excess supply or demand, computationally clearing the market at an analytically determined price, or through the construction of a limit order book. A linear function is perhaps the simplest mechanism, and prevalent in literature, though results in the market never reaching equilibrium; while limit order books are the most realistic, simulating true order books seen in real markets, and resulting in the most realistic mechanisms of market microstructures [1]. However, effective order book implementation requires requires additional details to be included in the market architecture, and agent strategies.

Price definition depends on the mechanism(s) used. When a linear function of excess supply and demand is implemented, the output is thus the price in the next time-step. Using order books, the price in the next time-step is often the mid-point between last executed bid and offer prices, common in real markets [4]. In investigating herding behaviour, Yu & Chen found similar qualitative volatility behaviour using limit order books, when comparing results to papers using linear functions [7].

Strategy Representation

Strategies are determined by agent distinction and are most often implemented as a set of rules, either fixed or dynamically re-parameterised over time (through learning or evolutionary algorithms). Chartist rules usually consist of common technical indicators parameterised by historical prices, and are either standalone or exist in a set where the agent has choice over an indicator. Common technical trading indicators are the Simple and Exponential Moving Average (SMA/EMA), the Moving Average Convergence Divergence (MACD) and the Relative Strength Index (RSI). Random Traders employ strategies dictated by pseudo-random number generation, scaled by certain parameters. Other trading strategies involve Neural Networks [8] and Deep Learning Methods [9].

Learning and Social Interaction

Agents learn and adapt their respective strategies as time develops. This learning is either system-wide, or agent specific. System-wide learning is most common, and predominantly implemented through a Genetic Algorithm (GA); whereas agent-specific learning is only possible through either social interaction (discussed below) or implementing strategies which themselves have learning components (as in [8], [9]). The high computational complexity of agent specific learning methods in a multi-agent system, without social networking, results in models with very few heterogeneous agents, and is a limiting factor for providing learning-based strategies to individual agents [9].

Interaction and herding between agents can be represented in a variety of ways, from basic interaction with neighbours [6] to complex social networks. Zhang et al. [8] explore a variety of networks and their effects on trading strategy convergence, observing substantial differences in stable strategy-states between random, regular and scale-free networks respectively. While Hoffman [10] compare torus and scale-free networks in social artificial markets, finding significant results using both networks. Noting that the slower rates of absorption, and higher capacity of diffusion of information in the market through torus networks better represent reality. Social learning can result in strategy correlations, and from such larger realised price volatility [8].

III. Model Design

The proposed market will follow the general architecture of Hessary & Hadzikadic [6] (adapted from [11]), segmenting cryptocurrency-specific agent types identified by Cocco et al. [4], [5] and incorporating realistic behavioural aspects from Aspembitova et al. [3].

Assumptions

1. There are only two types of traders, Random Traders and Chartists, in which the Chartist type is further segmented into Optimists and Pessimists.

2. Short selling is unlimited, and all positions held are closed at the end of each time-step.

3. Only one asset is traded, and each order is one-unit in size.

4. Agents have no wealth, and are only incentivised by their prior returns.

While these assumptions are significant, it is beneficial to estimate cause and effect through a more controlled environment [1], and the focus of this paper thus does not warrant consideration outside of its scope, the limitations of such assumptions are discussed in detail in the respective section. The market architecture and network types are discussed in detail below:

Market Architecture

Following [5], agents are categorized into two *fixed* types: Chartists and Random Traders. Random traders issue actions at random, subject to a uniform distribution. Chartists are further segmented into Optimists or Pessimists, with these distinctions defining how they generate expectations. At each time-step, an agent can either: *buy*, *sell* or *do nothing*. They can only purchase one unit at each given time-step. At initialisation, agents are segmented between Random Traders and Chartists using a fixed parameter, and Optimists and Pessimists are divided equally.

Pricing Mechanism

Following [6], [10], [11] the determination of the Market Price at time $t+1$, for agent i is:

$$P_{t+1} = P_t + \alpha(D_t - S_t) + \epsilon_1$$

Where D_t and S_t are the total market demand (bids) and supply (asks) at time t, respectively. Alpha(α) is a positive, constant parameter dictating price sensitivity. A random component ϵ drawn from a Normal Distribution (with mean 0, and σ) is included to represent the random variation, and dark pools seen in the orderbook process in real markets [1].

Agent Expectations

Chartists generate expectations based on historical price, subject to a heterogeneous memory parameter and their distinction (Optimists or Pessimists). This is implemented using an Exponential Moving Average (MA^i_k) with memory parameter, k. The memory parameter is drawn from a uniform distribution: $k_i \sim U(1, \gamma)$. This expectation of agent i for the price at time $t+1$ can be described as follows:

$$E^i_t[P_{t+1}] = P_t + \beta^i(P_t - MA^i_k) + \epsilon_2$$

Where:

$$\beta_i = \begin{cases} 0 < \beta < 1 & \text{if Agent } i \text{ is an Optimist} \\ -1 < \beta < 0 & \text{if Agent } i \text{ is a Pessimist} \end{cases}$$

A chartist's unique expectation at time t is a function of current price, a moving average (with memory k) and coefficient β_i, determined by their distinction. At each time-step, agents will execute a bid (ask) order if they are an Optimist (Pessimist) and their expectation is higher (lower) than the current price, otherwise they will do nothing. Random Traders generate expectations drawn uniformly between these three possible actions.

Strategy Fitness

Somewhat similarly to [8] and [6], agents generate a *strategy fitness* parameter at each time-step (S^i_t), signifying their prior success under their current type (Optimist or Pessimist). By observing their historical returns R^i_t, at time t, under their *current* strategy, θ, with a memory parameter, $m \sim U(1,5)$, their strategy fitness as follows:

$$S^i_t = \bar{R}^{\theta}_{t-m}$$

This is only conducted by Chartists. Random Traders cannot switch distinction.

Communication and Decision Making

Once generated, the agents share their strategy fitness among the neighbouring nodes in their network, aggregating the strategies among Pessimists and Optimists alike. Resulting in two sets of $\bar{S^O} = \{S^i_t, S^1_t, S^2_t \cdots S^k_t\}$ and $\bar{S^P} = \{S^1_t, S^2_t \cdots S^j_t\}$, for $1 \ldots k$ optimist and $1 \ldots j$ Pessimist neighbours, note that these sets can also contain the agents own fitness (in this case Agent i is an Optimist). This can be represented as a multi-armed bandit decision process, and a SoftMax strategy is applied to make decisions. This strategy, follows a greedy action, where the best performing type (Optimist or Pessimist) of agents within a certain agent's neighbourhood is given the highest selection probability. More specifically, the probability of choosing a particular type follows a Boltzmann distribution [12] as follows:

$$P(\text{Agent}^i = \text{Optimist}) = \frac{e^{\bar{S^O}/\tau}}{e^{\bar{S^O}/\tau} + e^{\bar{S^P}/\tau}}$$

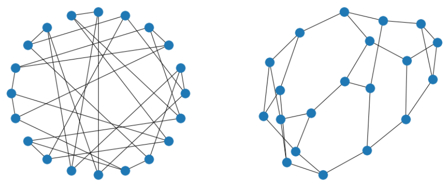
And:

$$P(\text{Agent}^i = \text{Pessimist}) + P(\text{Agent}^i = \text{Optimist}) = 1$$

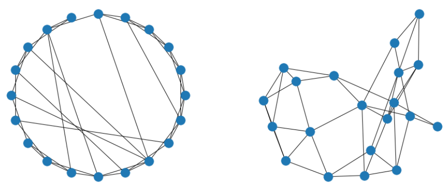
At each time-step, the agents then decide to which strategy to choose based on the above probabilities generated. This is implemented though uniform number generation, where agents then move if the probability is larger than the number generated (and stay vice-versa)

Network Types

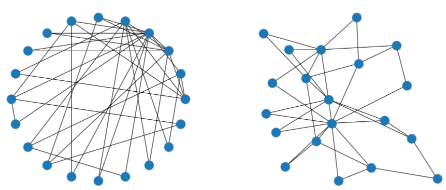
The types of networks assessed, which the agents communicate in are as follows:



1. Regular Network



2. Small-World Network



3. Scale-free Network

1. *Regular Network:* Each node has exactly the same degree, and neighbouring nodes are chosen randomly.

2. *Small-World Network:* This network has two significant properties, short average-path lengths and clustering. It is commonly used to represent social networks (in the sense that very few nodes are connected, through neighbours of neighbours quickly reaches all nodes in the network). The particular network used is the Watts-Strogatz model [13], and the degree of all nodes are relatively homogeneous.

3. *Scale-Free Network:* The networks degree distribution follows that of the power law, and is thus quite distinct from the other two studied. The particular network used is generated using the Barabasi-Albert model [14].

These networks were chosen as they are most prevalent in literature, where [7], [8] found the largest significance between the Scale-Free and Random networks, where [10] found Scale-Free networks have been shown to exhibit agents' actions in financial markets.

IV. Methodology

Due to the stochastic and non-determinate nature of Artificial Markets. The methodology for testing the effects of network types on price dynamics are segmented as follows:

1. For each network, provided a fixed set of parameters, prominent patterns in price evolution, distribution and autocorrelation are identified and discussed.

2. Each model (per-network type) is run multiple times, and aggregate statistics are assessed, with an assessment of volatility and non-normality.

3. The most significant model assessed through observation and aggregation is validated with respect to real market prices, and non-stationarity.

Below is the fixed parameter set used for the analysis and results:

Parameter	Value
Agents (N)	100
Non-Random Agents	70
Timesteps	200
Initial Price (P_0)	50
Price Sensitivity (α)	0.0005
Random Price Component (ϵ_1)	$\alpha/2$
Memory Parameter (γ)	20
Optimist Sensitivity (β^O)	N(0.5,0.1)
Pessimist Sensitivity (β^P)	N(-0.5,0.1)
Random Expectation Component (ϵ_2)	N(0,0.001)
Temperature Parameter (τ)	10

TABLE I. Parameter List

For reference, no network is also included, the network parameters are as follows:

Network Type	Parameter
Regular Network	n = 5
Small-World Network	n = 3, k = 0.9
Scale-Free Network	n = 3, k = 0.9

TABLE II. Network Parameter List

Returns are, often used calculated as:

$$R_t = ln(P_t) - ln(P_{t-1})$$

V. Results and Discussion

Emergent and Prominent Patterns

With the parameters and networks discussed above, the models were run locally and examples of typical results are assessed below. All network types (in this example) exhibit non-normal returns, which is validated in the next section. Areas under consideration for emergent patterns are *clustered volatility*, *autocorrelation* and *price movement*. Graphs of the model output with respect to certain networks are shown below:

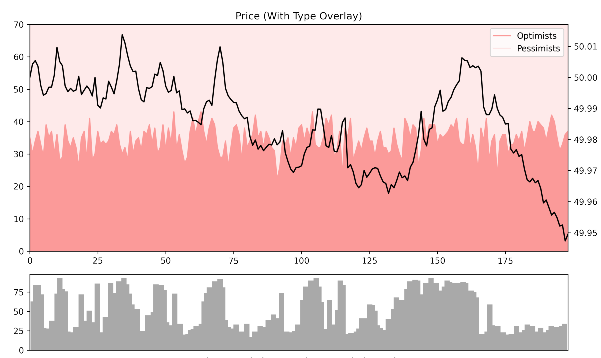


Fig. 2.1(a) Regular Model - Price

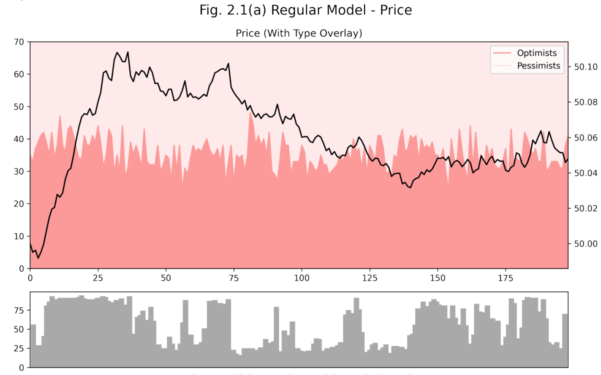


Fig. 2.1(b) Small-World Model - Price

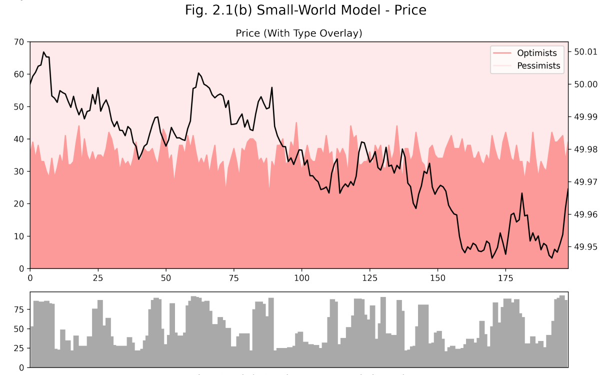


Fig. 2.1(c) Scale-Free Model - Price

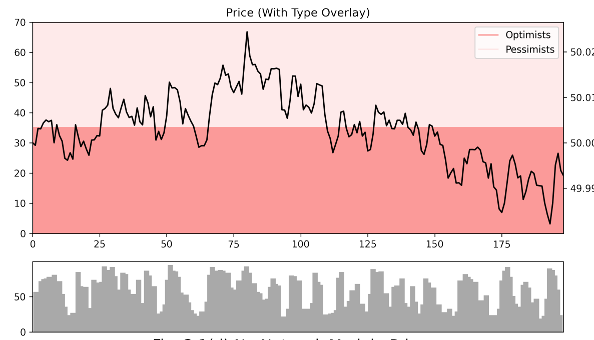


Fig. 2.1(d) No-Network Model - Price

Graphed above are results from the four network types discussed, with the price in black, and a red overlay signifying the proportion of Optimists and Pessimists (excluding Random Traders). In terms of price formation (excluding the No-Network model), differences can be observed between the Small-World model, and the other two models - indicating that the smaller degrees of information dissemination in the market appears to lower the volatility of the price. The Scale-Free model shows signs of more intermittent periods of volume, which last a shorter amount of time, and the movement (herding) between types also appears to last longer (signified by the less jagged overlay). In all cases, examples of bubbles and crashes are evident, as the agents execute trades based on prior price movement (more specifically, the EMA), and this has a compounding effect on return and fitness, resulting in periods of high volume traded (indicated by the grey bars below the graph). The No-Network model shows a far less clustered volatility, with agents executing orders rather continuously throughout the simulation. There appears to be a short term trend, and a long term trend. These periods appear longer in the Regular and Small-World models, resulting in more subtle price movement, outside of large bubbles and crashes. The autocorrelation and distributions are considered based of the graphs (from the same models) below:

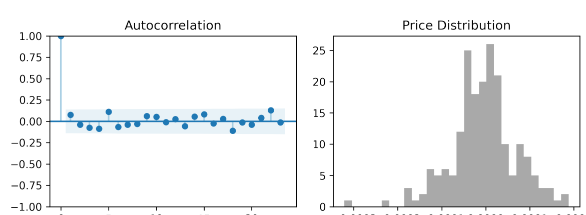


Fig. 2.2(a) Regular Network Model - Returns & Autocorrelation

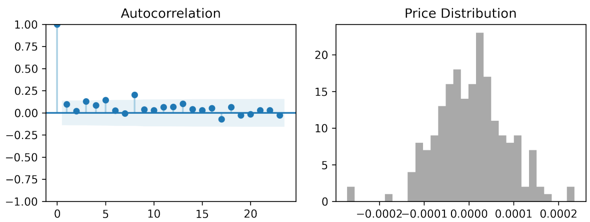


Fig. 2.2(b) Small-World Model - Returns & Autocorrelation

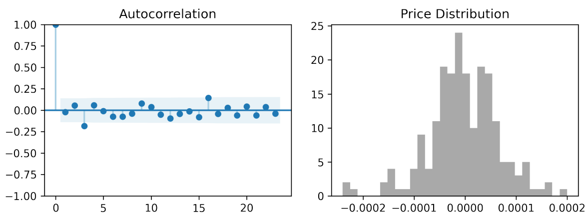


Fig. 2.2(c) Scale-Free Model - Returns & Autocorrelation

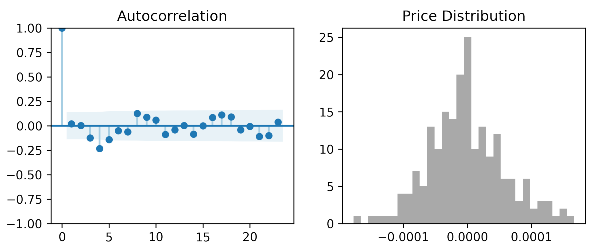


Fig. 2.2(d) No-Network Model - Returns & Autocorrelation

Notably, significant autocorrelation is more often seen with the Scale-Free model, opposed to the Regular and Small-World models. This can potentially be explained as certain agents in this (Scale-Free) network have access to a significantly larger amount of information, better representing that seen in real markets. A similar finding is noted in [10]. All models, excluding the No-Network model, exhibit fat tails, and are significant on the Shapiro-Wilk test for Normality ($p < 10$). While the only model with normally distributed distribution of returns is the No-Network model ($p = 21.2$). This suggests that interaction and learning within markets may cause non-normality of returns.

In terms of autocorrelation, the No-Network model shows a structured pattern, and can be explained by the underlying distribution of the agents memory parameters, in the absence of interaction, while the patterns in the Scale-Free and Small-World model appear to be similar. More robust results on normality and volatility, through aggregate statistics are discussed below.

Aggregate Statistics

Each model was run 20 times, with statistics involving price distribution and normality observed.

Network Type	Variance (e−9)	Kurtosis
Regular Network	5.095	1.136
Small-World Network	4.869	0.934
Scale-Free Network	5.075	1.242
No Network	3.152	0.589

TABLE III. Aggregate Statistics over 20 runs

The above table displays the aggregate variance and kurtosis of Return (specifically, R_t). Excluding the No-Network model, the aggregate variances are all somewhat similar, and are scaled for representation. Indicating that the network types have a minimal effect on the variance of returns, though the implementation of a network significantly increases variance, and can is a substantial factor in explaining the abnormality of returns in a real-world context. These results confirm the suggestions by [1], and are also noted in [10], and [6]. The table below confirms this hypothesis, with the most consistently significant network being the Scale-Free network. With a mean p-value of 0.093 over the runs, indicating that the information disseminates through the Scale-Free model at a much higher rate, as discussed prior. All other models are insignificant when aggregated.

Network Type	Mean	Max
Regular Network	0.149	0.683
Small-World Network	0.122	0.534
Scale-Free Network	0.093	0.311
No Network	0.209	0.642

TABLE IV. Aggregate Shapiro-Wilk Test Statistics

Validation and Real-World Comparison

he closest real world representation of this artificial market, as discussed, is that of a Cryptocurrency market. The data used is the daily prior 6-month closing price of the Bitcoin Cryptocurrency (BTC), sourced from *Investing.com*. The return and price distribution is shown below:

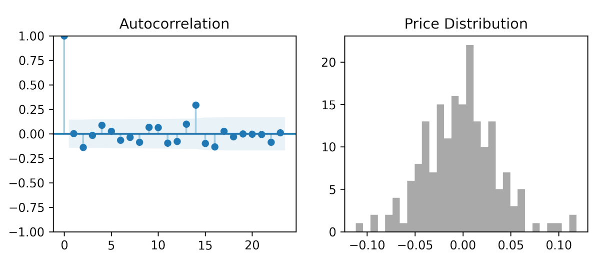


Fig. 3.1 Bitcoin Returns & Autocorrelation

The p-value obtained from the Shapiro-Wilk test statistic for BTC, is 0.225. Which does not reject the null hypothesis of Normality. Given that this data is from daily returns, normality is not often violated [1]. Observing the autocorrelation graph, lags 2 and 14 are significant, and (at an observational level) show a strikingly similar nature to that of the Scale-Free model. As in [3] and [6], Augmented Dicky-Fuller tests for stationarity are conducted on both BTC, and the Scale-Free model. The results are summarised in the table below:

Market	Test-Statistic	p-value
Scale-Free Model	-3215.19	1.91e-25
BTC	-622.81	1.34e-19

TABLE V. Dicky-Fuller Test Results

Indicating that both sets of return data exhibit non-stationarity, indicating that the returns generated by the Scale-Free model is in agreement with realistic market time-series data.

VI. Limitations, Recommendations & Conclusion

Limitations & Recommendations

Due to time constraints, and evident from the assumptions stated, this model is limited in scope a few regards. The assumption of no short selling, and disregard for individual wealth is perhaps the largest limiting factor, though the price formation still produces valid results. The agents' one-dimensional strategy set (built off the EMA) has substantial flaws in terms of convergence with larger parameter values, as all agents can converge to the same type, resulting in a self-fulfilling market and producing either a constant upward, or downward trend. Recommendations are to widen this strategy set, as seen in [4], [7], though as more assumptions are relaxed, determining cause and effect can become more vague. [9] used Deep Learning as a strategy generation mechanism, and (if computationally viable) integration with the interaction and herding processes developed in this report could result in significantly 'smarter' agents. The decision making process, though used widely in literature, is primitive in nature and further recommendations are made to perhaps investigate alternate means of decision making. Furthermore, strategy convergence, as defined in [7] could be assessed with respect to network structure in future.

Conclusion

An artificial financial market was created off a primitive set of rules, generating complex price formation and market dynamics. Agents communicate with one another and make decisions based on the prior success of their neighbours and themselves. Multiple network structures were assessed, and these all had a significant impact in price volatility when compared to no network. Furthermore, it was identified that the presence of a social network, and herding behaviour, produced less normally distributed returns, and more realistic price formation. Of the networks considered, the Scale-Free network best represents the stylised facts seen in real financial markets, and when compared to the Bitcoin market, shows significantly similar levels of stationarity and non-normality. This model is available on Github here, follow the `README.md` for usage instructions.

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