

Theory of Computation

Problem Set 3

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Please start solving these problems immediately, don't procrastinate, and work in study groups.

Please do not simply copy answers that you do not fully understand;

Advice: Please try to solve the easier problems first (where the meta-problem here is to figure out which are the easier ones). Don't spend too long on any single problem without also attempting (in parallel) to solve other problems as well. This way, solutions ~~to the easier problems~~ (at least easier for you) will reveal themselves much sooner (think about this as a “hedging strategy” or “dovetailing strategy”).

1. Solve the following problems from the [Sipser, Second Edition] textbook:

Pages 83-93: 1.4, 1.5, 1.6, 1.7, 1.11, 1.12, 1.13, 1.16, 1.17, 1.18, 1.19, 1.20, 1.21, 1.23, 1.29, 1.30
2. Prove or disprove: a countable set of parabolas (arbitrarily oriented and placed) can completely cover (every point inside) the unit square in the plane (i.e., the interior and boundary of a square of side 1)
3. Prove or disprove: an uncountable set of pairwise-disjoint line segments can completely cover (every point in) the unit disk in the plane (i.e., the interior and boundary of a circle of diameter 1). What if the segments could intersect each other, but must all have unique slopes?
4. What is the cardinality of the set of all finite-sized matrices with rational entries?
5. What is the cardinality of the set of all infinite matrices (i.e., matrices with a countably-infinite number of rows and columns) with Boolean entries?
6. Is every subset of a regular language necessarily regular?
Is every superset of a regular language necessarily non-regular?
7. Does every regular language have a proper regular subset?
Does every regular language have a proper regular superset?
8. Are the regular languages closed under infinite union? Infinite intersection?
9. Is a countable union of regular languages necessarily regular?
10. Is a countable intersection of regular languages necessarily regular?
11. Solve problems 1.6(b), 1.6(h), 1.6(i) on page 84 of [Sipser]. Use [JFLAP](#) to implement and test each of these deterministic finite automata on various representative input strings.
12. Solve problem 1.17 on page 86 of [Sipser]. Use [JFLAP](#) to implement the NFA and DFA of this question, and test both of them on various representative input strings.

13. Prove or disprove: the set of all languages (i.e. 2^{Σ^*}) is countable.
14. Prove or disprove: a given regular language is a countable set.
15. Prove or disprove: the set of all regular languages is a countable set.
16. Determine as precisely as possible, for a language L , when is the following true: $L^+ = L^* - \{\epsilon\}$
17. What is the infinite union of all of the regular languages?
What is the infinite intersection of all of the regular languages?
18. Let $\text{YESNO}(L) = \{xy \mid x \in L \text{ and } y \notin L, x, y \in \Sigma^*\}$. Does YESNO preserve regularity?
19. Let $\text{PALI}(L) = \{w \mid w \in L \text{ and } w^R \in L\}$. Does PALI preserve regularity?
20. Describe an algorithm that determines for a given pair of regular expressions whether they denote the same language. What is the time complexity of your algorithm?
21. True or false: for any given regular language, there exists a linear-time algorithm for testing whether an arbitrary input string is a member of that language.
22. What is the smallest language, closed under concatenation, containing the languages L_1 and L_2 ?
23. Give a sufficient condition (but as general as possible) for $L_1^* + L_2^* = (L_1 + L_2)^*$ to hold.
24. Given two arbitrary languages S and T , find a new language R (in term of S and T) so that the equation $R = SR + T$ holds.
25. Describe exactly what happens if we apply the “powerset construction” to a finite automaton that is already deterministic?
26. Show that the intersection of two sets of languages can be empty, finite (of arbitrarily large cardinality), countably infinite, or uncountably infinite.